Towards an optimal computation of non-linear four-wave interactions in operational wave models

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Motivation

- Non-linear four-wave interactions S_{nl4} play an important role in wave evolution
- Exact computation of these interactions (Xnl) too time consuming for application in operational models
- Discrete Interaction Approximation (DIA) is present alternative: fast but inaccurate

- When do we need accurate S_{nl4}?
- How to speed-up Xnl or extend DIA?

The problem

- Extending DIA with more configurations difficult;
  no optimal strategy for choosing additional configurations

- Reduced Exact Integration methods start from full solution;
  keep same mathematical structure as full solution

- Various speed-up options suggested by Van Vledder (2006)
- Options tested for academic spectra
- No guarantee they work in ‘real’ cases

Methodology

- Test various speed-up options for deep water fetch-limited wave growth, U_{10}=20 m/s, fetch 25 km
- SWAN 40.51 used as host model freq: 0.08-3.0 Hz, $\Delta\theta$=10°
- Speed-up options tested in DIA and Xnl
- Xnl based on WRT implementation in SWAN

The WRT method

Boltzmann integral (kinetic equation)

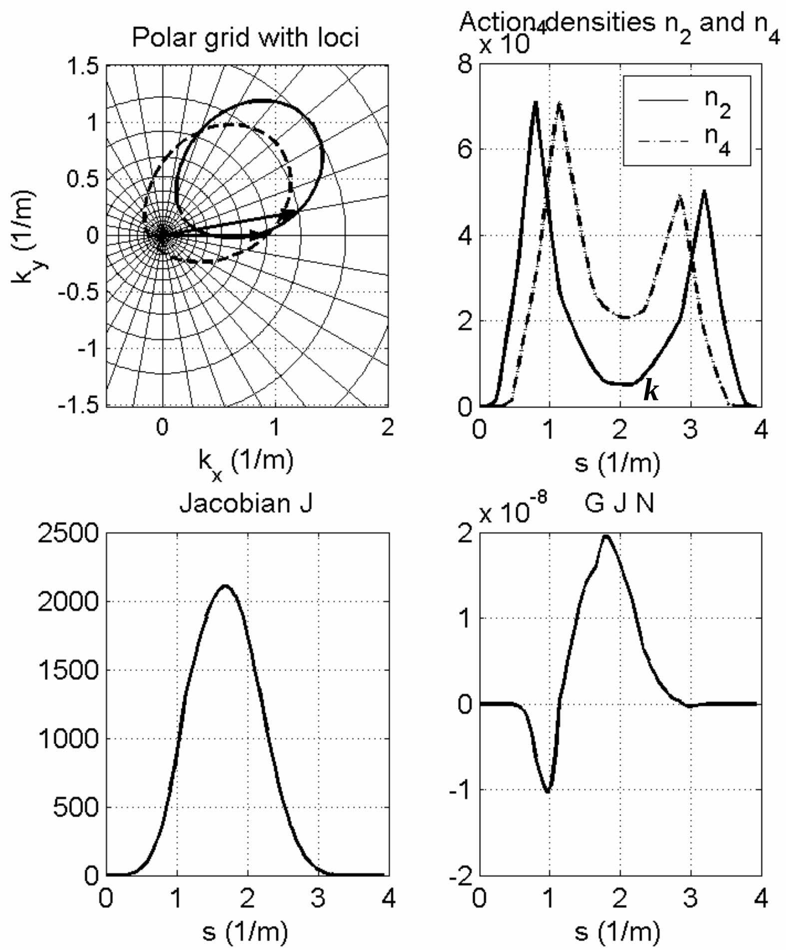
$$\frac{\partial n_1}{\partial t} = \iiint G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \times \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4) \times \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$$
$$\times \left[n_1 n_3 (n_4 - n_2) + n_1 n_4 (n_3 - n_1) \right] d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4$$

Webb (1978)

$$\frac{\partial n_1}{\partial t} = \int d\boldsymbol{k}_3 T(\boldsymbol{k}_1, \boldsymbol{k}_3)$$

$$T(\boldsymbol{k}_1, \boldsymbol{k}_3) = \int_S ds \times G \times J \times N_{1,2,3,4}$$

Integration along locus for $\boldsymbol{k}_2$



$$T(\boldsymbol{k}_1, \boldsymbol{k}_3) = \int_s G(s)\, J(s) N(s)\, ds$$

Discretisation of locus integration

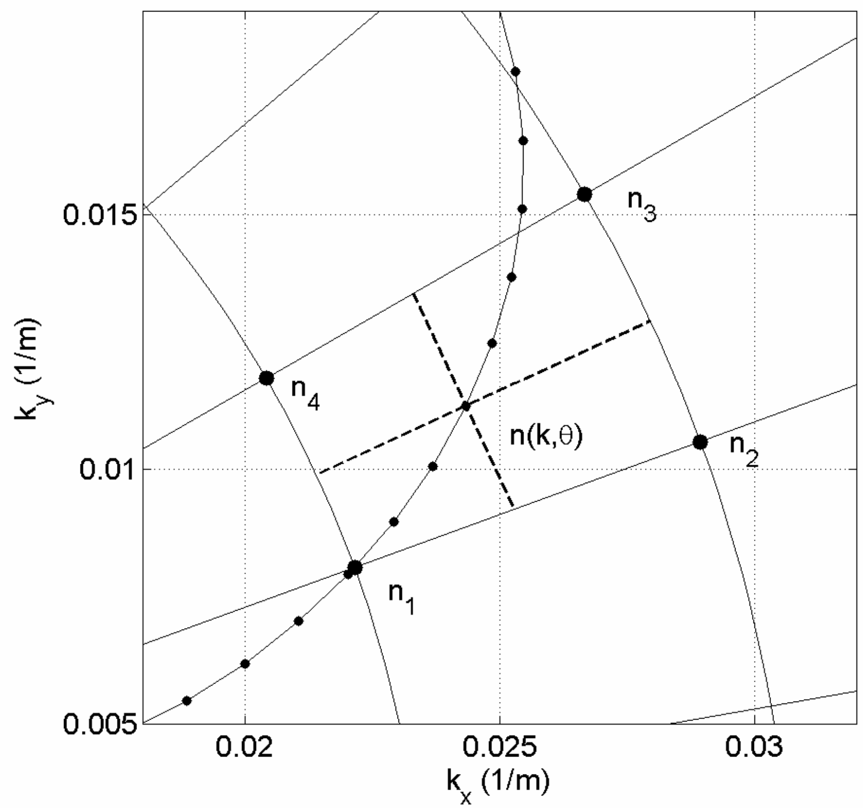
$$T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) = \oint_s G\left(s\right) J\left(s\right) N\left(s\right) ds$$

$$\approx \sum_{i=1}^{N_s} G\left(s_i\right) J\left(s_i\right) N\left(s_i\right) \Delta s_i$$

Divide locus in pieces and obtain values for coupling coefficient *G*, Jacobian *J* and action density product *N* at points s_i

Action densities n_2 and n_4 obtained by bi-linear interpolation in wave number space

Bi-linear interpolation of action density for points on locus

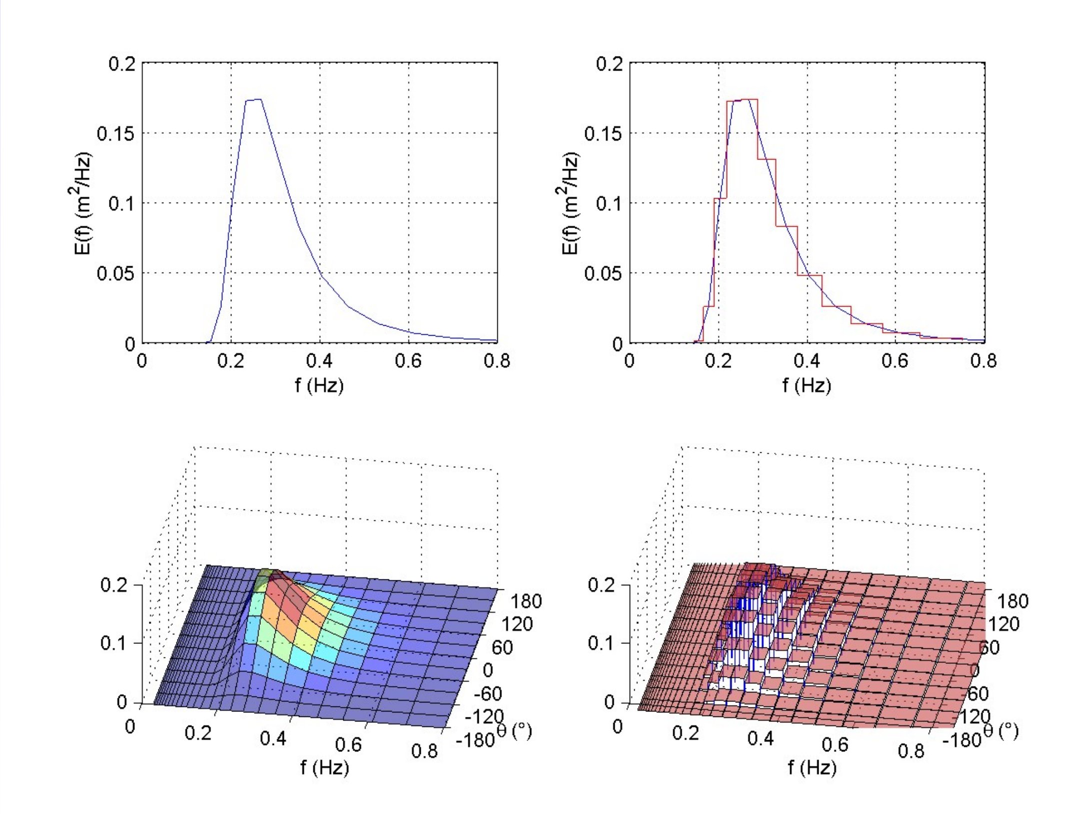


$$n = \sum_{i=1}^{4} w_i n_i$$

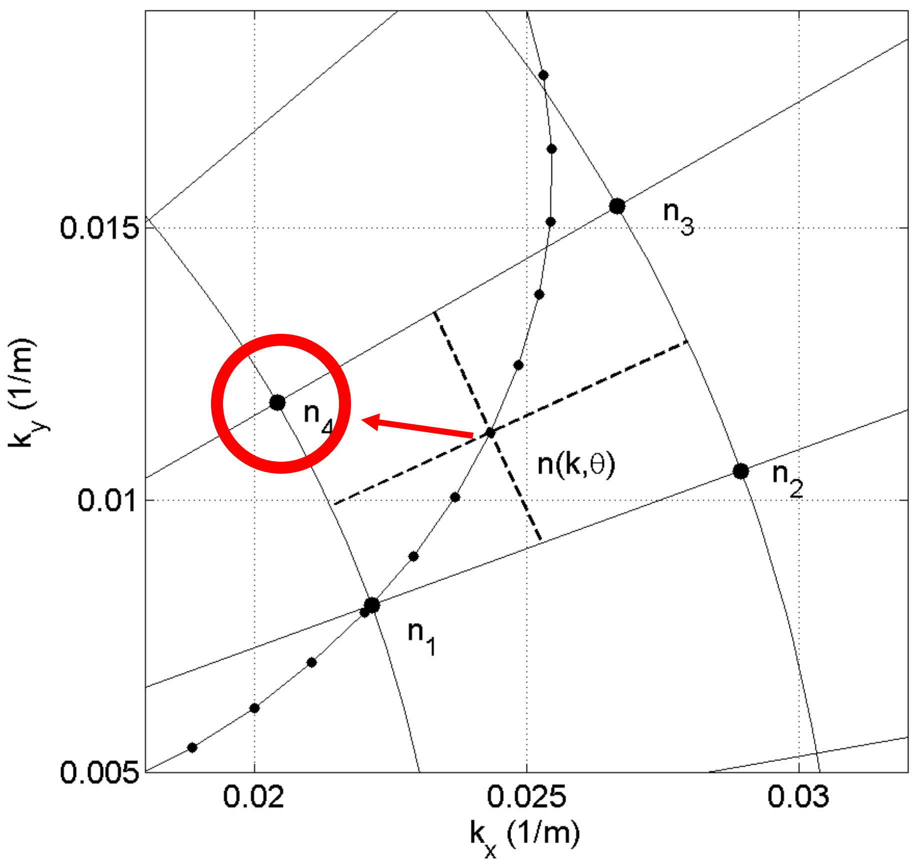
Methods to speed-up Xnl

- replace bi-linear interpolation by nearest bin approach (also tested for DIA)
- reduce the number of points on locus
- replace trapezoid rule by higher order quadrature
- apply filtering (reduce integration space)

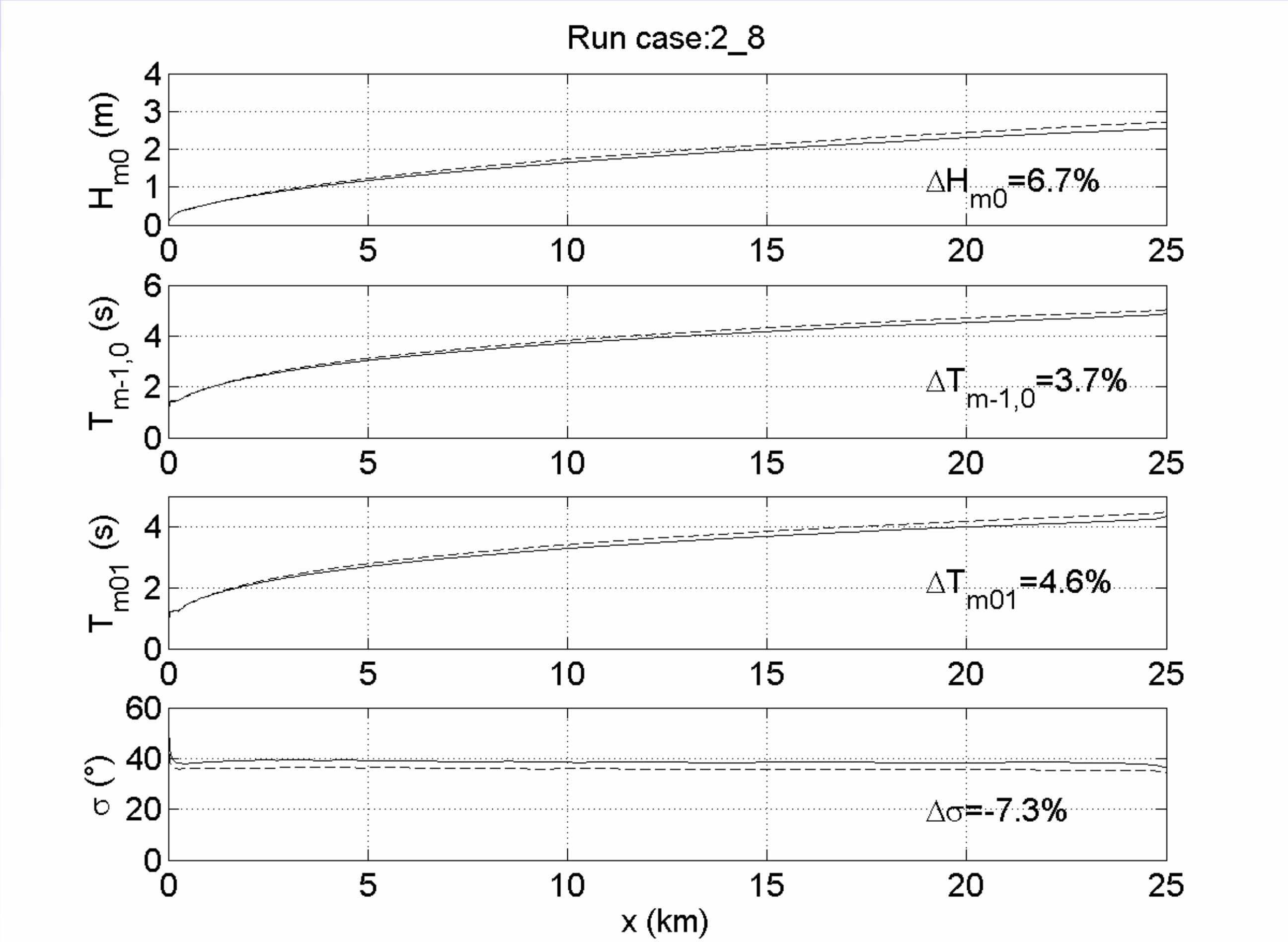
Bi-linear & nearest bin approach: continuous & constant piece wise representation (Snyder et al., 1993)



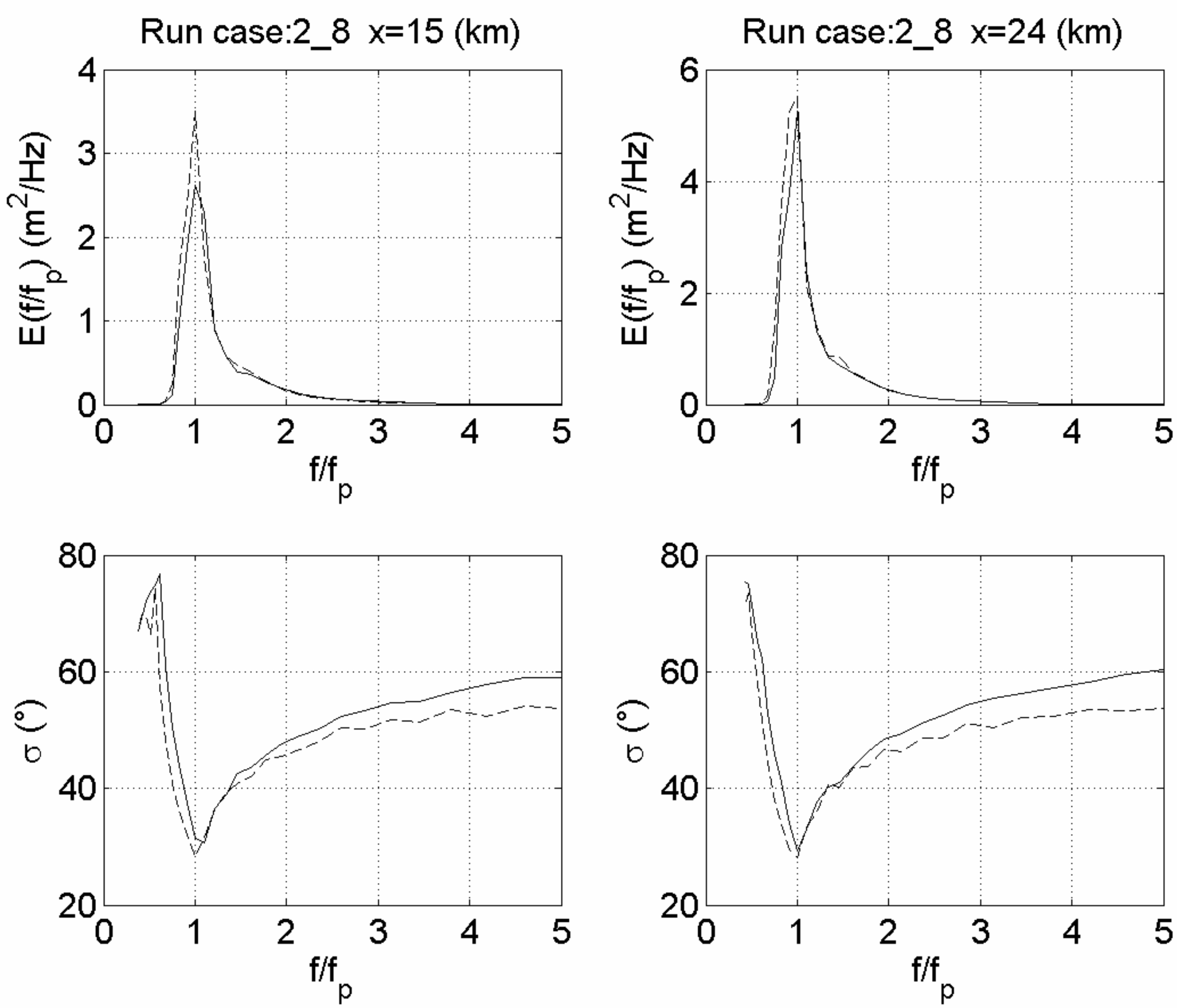
Nearest bin approach to obtain action density for point on locus



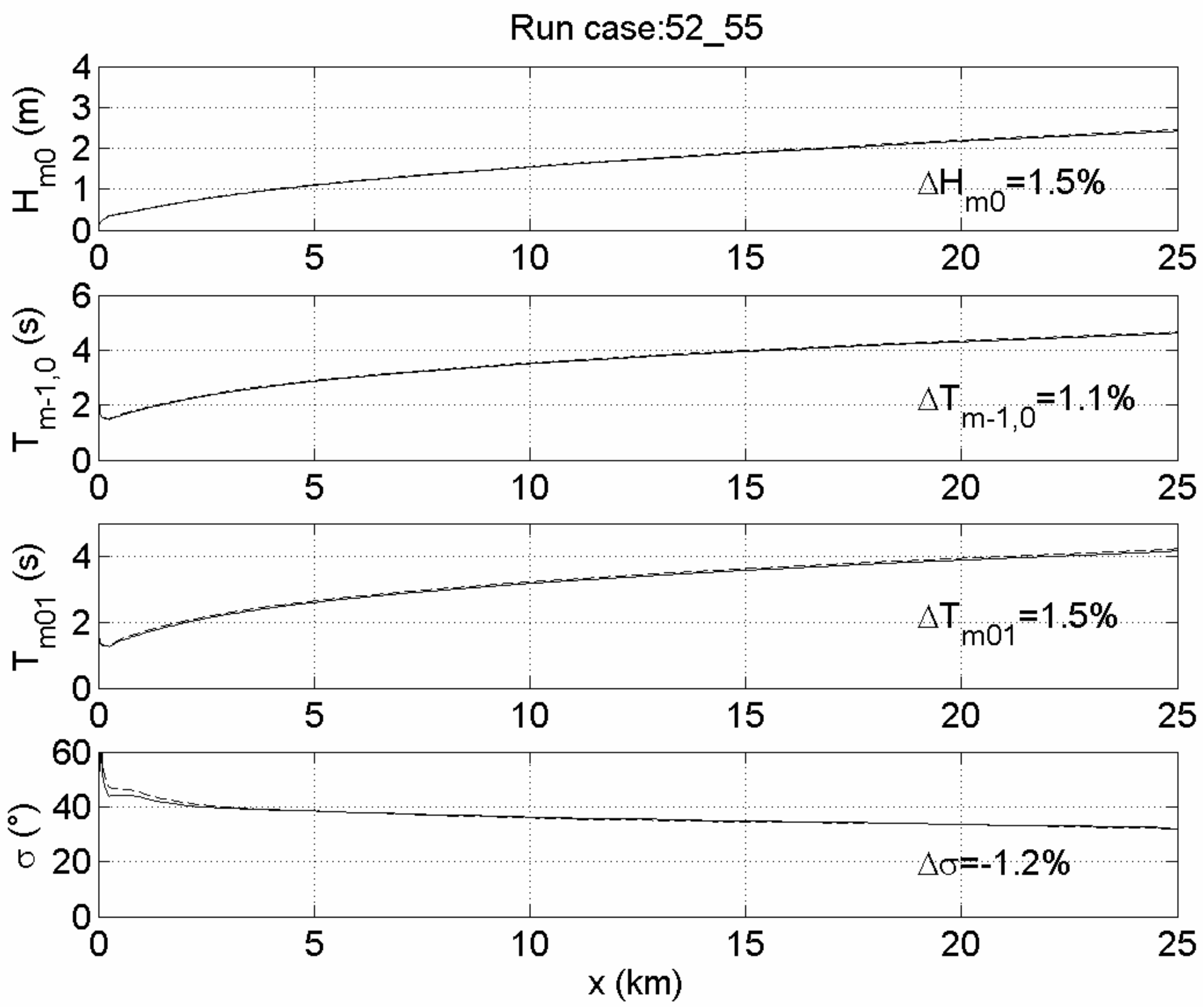
DIA: bi-linear & nearest bin



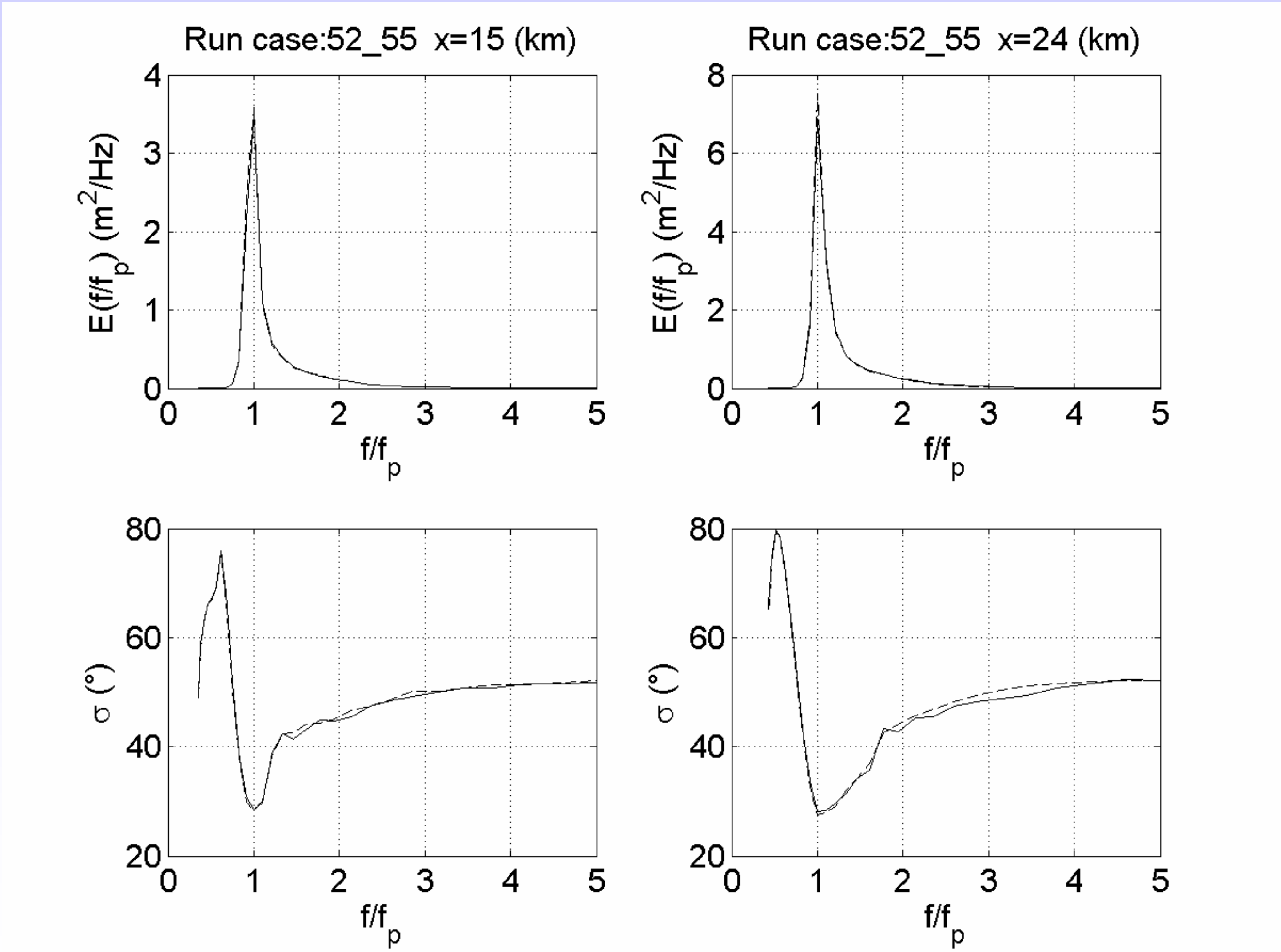
DIA: bi-linear & nearest bin



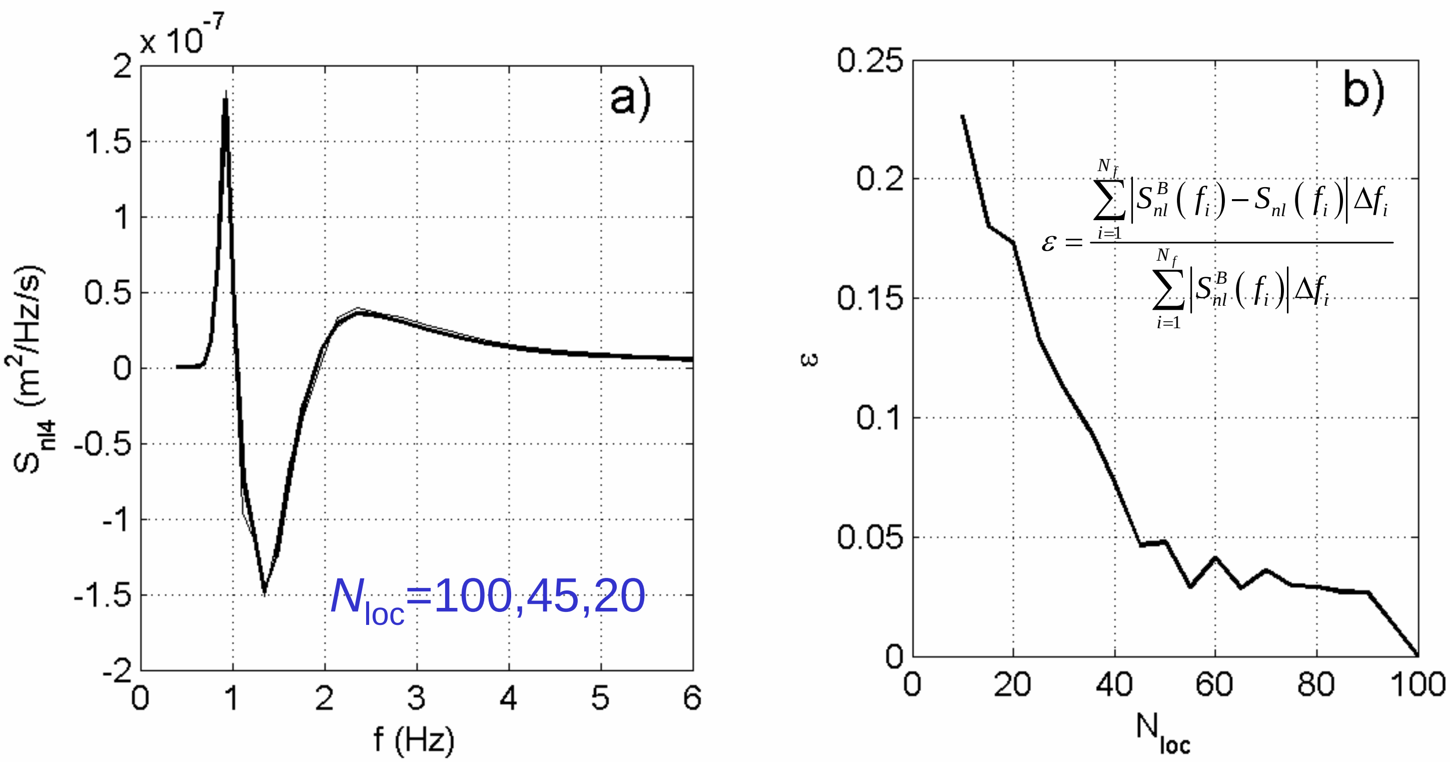
Xnl: bi-linear & nearest bin



Xnl: bi-linear & nearest bin



Number of points on locus for JONSWAP spectrum



Reducing the number of points on the locus N_{loc}

$$T\left(\boldsymbol{k}_1,\boldsymbol{k}_3\right)\approx\sum_{i=1}^{N_{locus}}G\left(s_i\right)J\left(s_i\right)N(s_i)\Delta s_i$$

N_{loc}	ΔH_{mo} (%)	$\Delta T_{m-1,0}$ (%)	ΔT_{m01} (%)	$\Delta\sigma$(%)	T_{nloc}/T_{160}
100	0	0	0	0	1.00
90	-0.5	-0.4	-0.4	0.1	0.81
80	-0.6	-0.4	-0.3	0.4	0.70
70	-0.7	*-0.6*	-0.5	0.5	0.59
60	2.2	1.1	0.7	-0.7	0.38
50	-0.2	-0.1	-0.1	-0.3	0.30
40	2.3	1.1	0.7	-0.5	0.22
30	0.4	0.6	0.7	-0.2	0.16
20	-0.4	-0.8	-1.0	0.6	0.09

Gauss-Legendre quadrature

$$T\left(\boldsymbol{k}_1,\boldsymbol{k}_3\right)\approx\sum_{i=1}^{N_s} w_i G\left(s_i\right) J\left(s_i\right) N\left(s_i\right)$$

20 points with Gauss-Legendre quadrature gives same results as trapezoid rule using 40 points on locus

speed-up with factor 2

Filtering

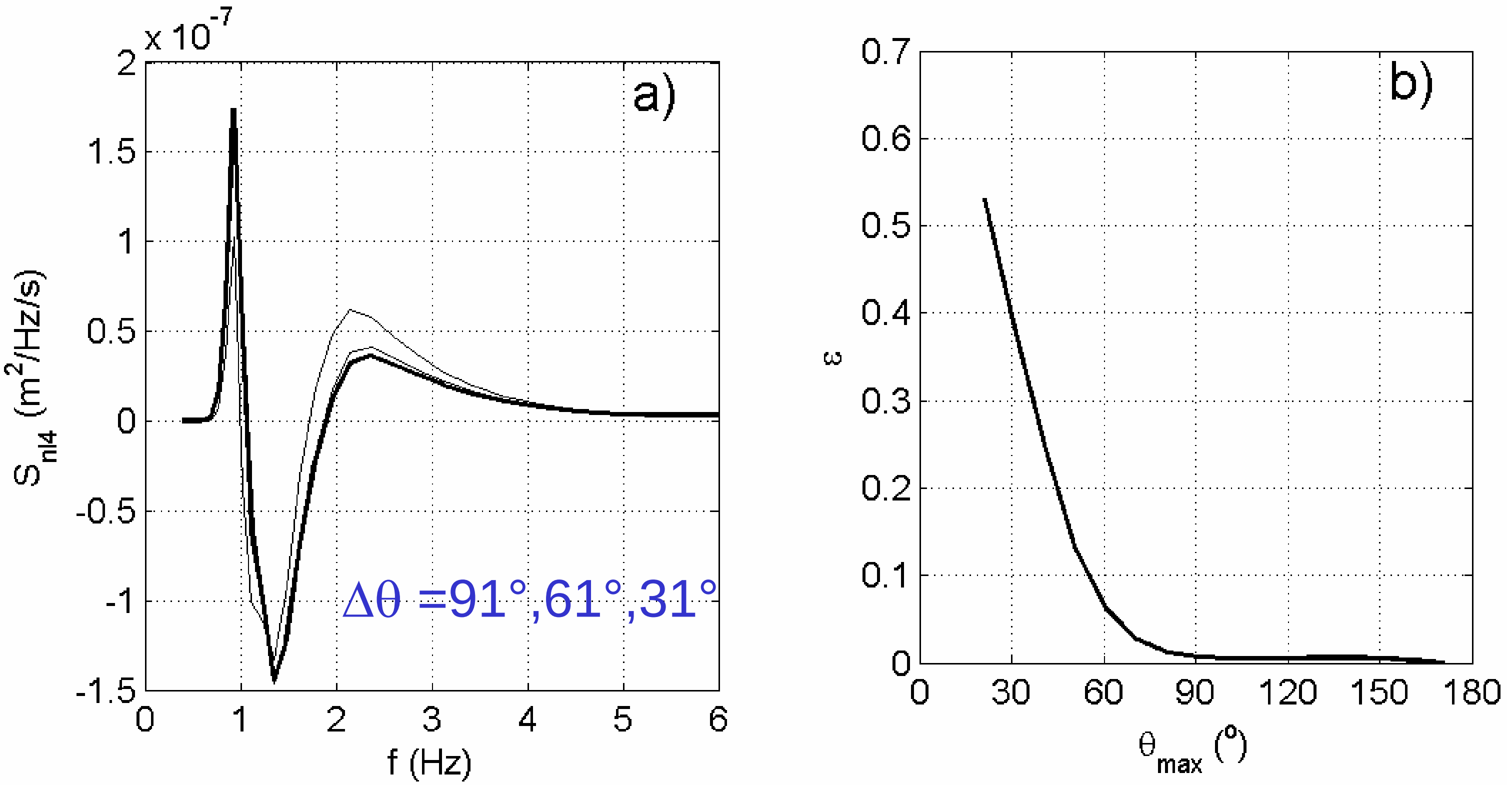
Contribution $T(\boldsymbol{k}_1,\boldsymbol{k}_3)$ becomes smaller when $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$ become more separated in wave number space

$$\frac{\partial n_1}{\partial t} = \int d\boldsymbol{k}_3 T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right)$$

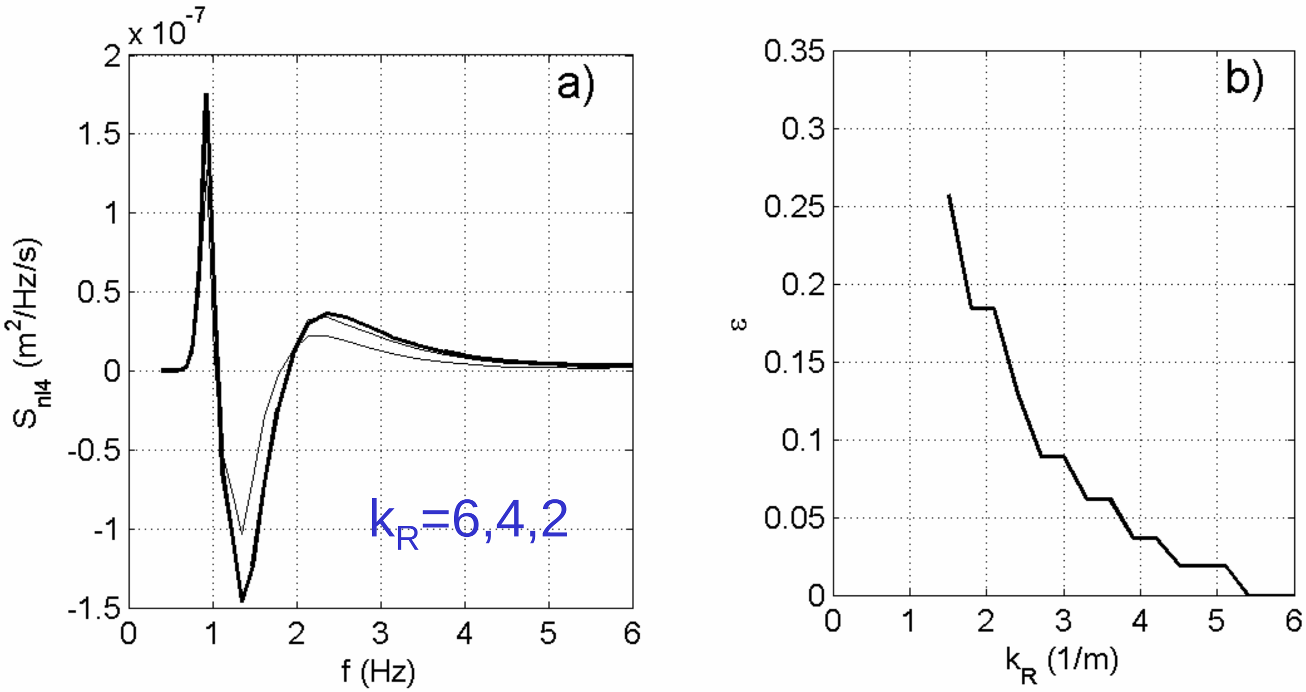
Directional difference: $\Delta\theta=|\theta_1-\theta_3|$

Wave number ratio: $k_R=\min(k_1/k_3,k_3/k_1)$

Effect of directional filtering for JONSWAP spectra



Effect of wave number ratio filtering for JONSWAP spectrum



Effect of directional filtering

F_θ	ΔH_{mo} (%)	$\Delta T_{m-1,0}$ (%)	ΔT_{m01} (%)	$\Delta\sigma$ (%)	$T_{nof}/T_{F\theta}$
91°	-0.3	0.0	-0.1	-0.2	0.25
81°	-3.1	-1.5	-1.1	0.7	0.20
71°	-3.5	-1.8	-1.5	-0.1	0.18
61°	-4.5	-2.4	-2.2	-0.8	0.13
51°	-6.3	-3.6	-3.6	-1.9	0.10
41°	-8.5	-5.2	-5.4	-3.4	0.07

Summary of results of speed-up

- Nearest-bin approach for DIA in SWAN leads differences in H_{m0}, $T_{m-1,0}$ up to 7% with 15% gain in speed
- Nearest bin approach in WRT similar results and 40% gain in speed
- Number of points on locus should be about 40
- Gauss-Legendre quadrature halfs number of points
- Filtering integration space in direction and wave number ratio very effective
- Difference in CPU between Xnl and DIA order of 10^2

Further developments in reduced integration methods

- include combined criterion to filter $T(\boldsymbol{k}_1,\boldsymbol{k}_3)$ contributions

$$\frac{\left|\boldsymbol{k}_1-\boldsymbol{k}_3\right|}{\frac{1}{2}\left|\boldsymbol{k}_1+\boldsymbol{k}_3\right|}<\boldsymbol{k}_f$$

- apply higher-order integration on outer loop in WRT method
- improved distribution of points on locus to better catch discrete points
- parameterisation of transfer rate in spectral tail

When do we need accurate S_{nl4} ?

- For benchmarking and tests of parameterisations
- When we need detail in spectra (peakedness, spreading, bi-modality in tail)
- Complex situations with multi-peaked or directionally sheared spectra
- Turning wind situations (Van Vledder & Holthuijsen (1993)
- Slanting fetch (Pettersson 2004, Ardhuin et al. 2006)
- Short and long time-scale behaviour (Resio & Perrie, 1991, Young & Van Vledder, 1993)

Where to go

- Systematic tests are needed to assess benefits of methods to speed-up Xnl for complex situations like turning winds, slanting fetch situations and other complicated fields cases
- Benefits of higher-order quadrature should be tested for irregular non-smooth spectra
- Extend tests to shallow water
- Verify shallow water scaling of Peter Janssen (2005)
- Objective comparison with other exact methods
- Claims about speed can only be made under controlled situations


questions ?
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