Comparison of Hassellmann and Dynamical Equations

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Dynamical equations :

$$\eta_t = \hat{k}\psi - (\nabla(\eta\nabla\psi)) - \hat{k}[\eta\hat{k}\psi] + \hat{k}(\eta\hat{k}[\eta\hat{k}\psi]) + \tfrac{1}{2}\Delta[\eta^2\hat{k}\psi] + \tfrac{1}{2}\hat{k}[\eta^2\Delta\psi] + \hat{\gamma}\eta$$

$$\psi_t = -g\eta - \tfrac{1}{2}[(\nabla\psi)^2 - (\hat{k}\psi)^2] - [\hat{k}\psi]\hat{k}[\eta\hat{k}\psi] - [\eta\hat{k}\psi]\Delta\psi + \hat{\gamma}\eta$$

$$\hat{k}\psi = \frac{1}{2\pi}\int k\psi_{\vec{k}} e^{-i\vec{k}\vec{r}}\, dk_x dk_y$$

Hasselmann (kinetic) equation :

$$\frac{\partial n}{\partial t} = \int |T_{k123}|^2 (n_2 n_3 (n_1 + n_k) - n_1 n_k (n_2 - n_3))\delta(k + k_1 - k_2 - k_3)\delta(\omega_k + \omega_1 - \omega_2 - \omega_3) dk_1 dk_2 dk_3 + \gamma_k n_k$$

Two reasons why the weak turbulent theory could fail:

1. Presence of the coherent events -- solitons, quasi - solitons, wave collapses or wave-breakings

2. Finite size of the system – discrete Fourier space:

Quazi-resonances

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 + \delta$$

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$$

Known macroscopic exhibition of quasi-resonances:

Frozen turbulence of capillary waves

Zakharov, Pushkarev, 1996, 2000

Mezoscopic turbulence of gravity waves

Zakharov, Korotkevich, Pushkarev, 2005

Dynamic equations:

$2\pi \times 2\pi$ domain of 4096x512 point in real space

Hasselmann equation:

domain of 71x36 points in frequency-angle space

Three damping terms:

1. ***Hyper-viscous damping*** $\gamma_k = C(k - 1024)^2$

2. ***WAM cycle 3 white-capping damping***

3. ***WAM cycle 4 white-capping damping***

WAM Dissipation Function:

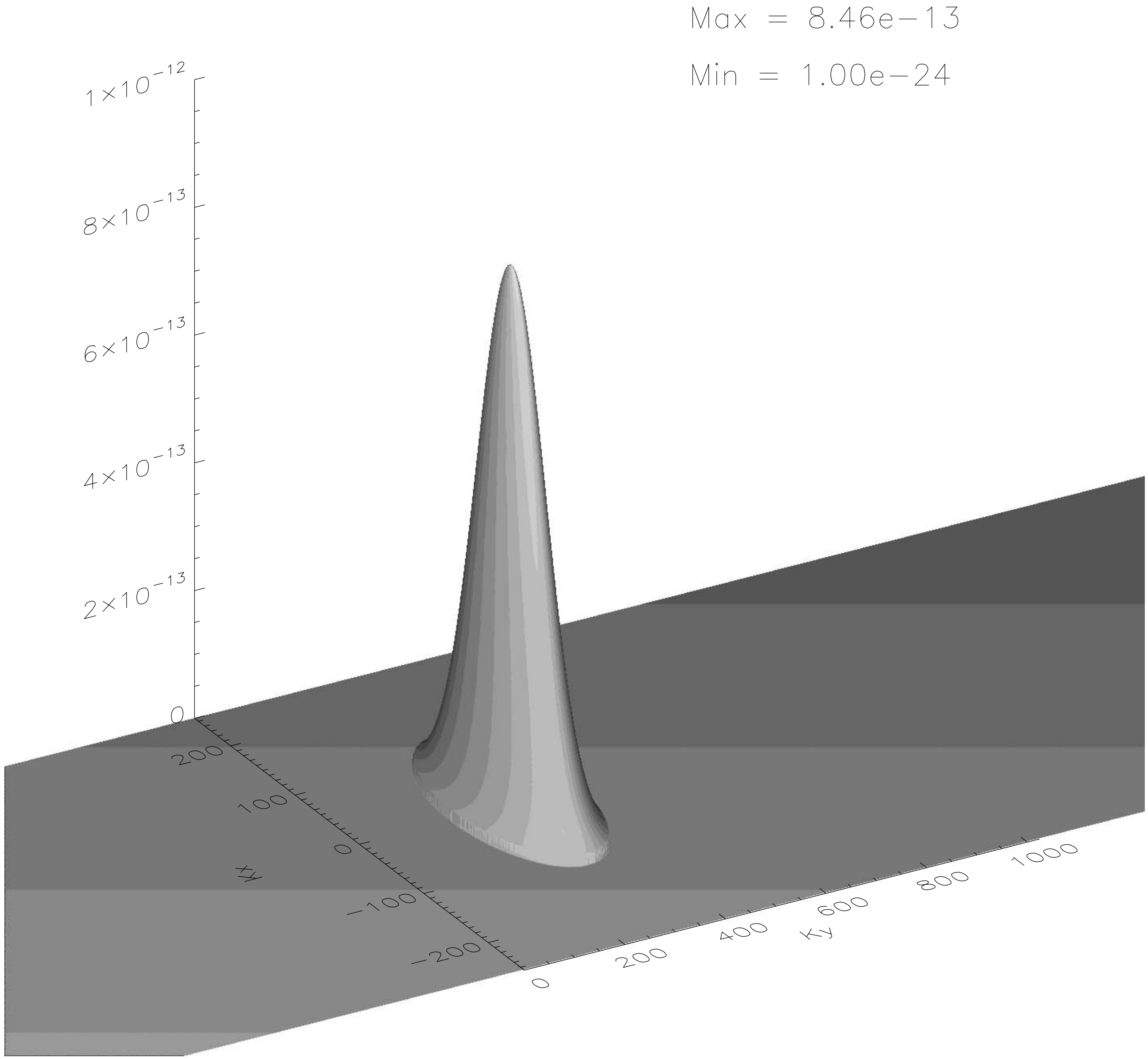
$$S_{ds}(\omega,\theta) = -C_{ds}\left(1-\sigma+\sigma\frac{k}{\tilde{k}}\right)\left(\frac{\tilde{S}}{\tilde{S}_{PM}}\right)^{4}\tilde{\omega}\frac{k}{\tilde{k}}E(\omega,\theta)$$

$$\tilde{S} = \tilde{k}\sqrt{E_{tot}}$$

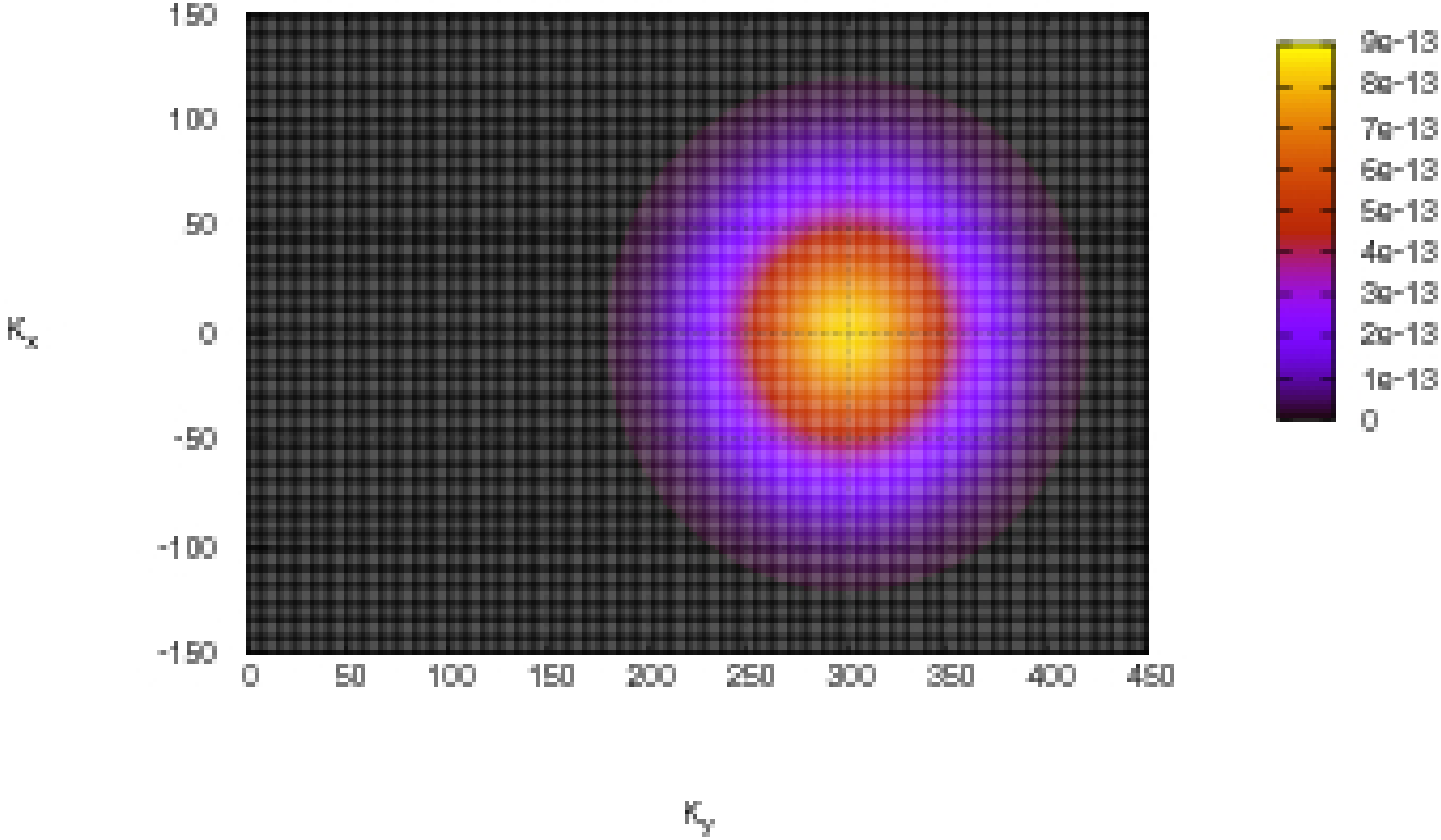
$$\tilde{S}_{PM} = (3.02\cdot 10^{-3})^{1/2}$$

WAM cycle 3: $C_{ds} = 2.36\times 10^{-5}, \delta = 0.5$ *Komen 1984*

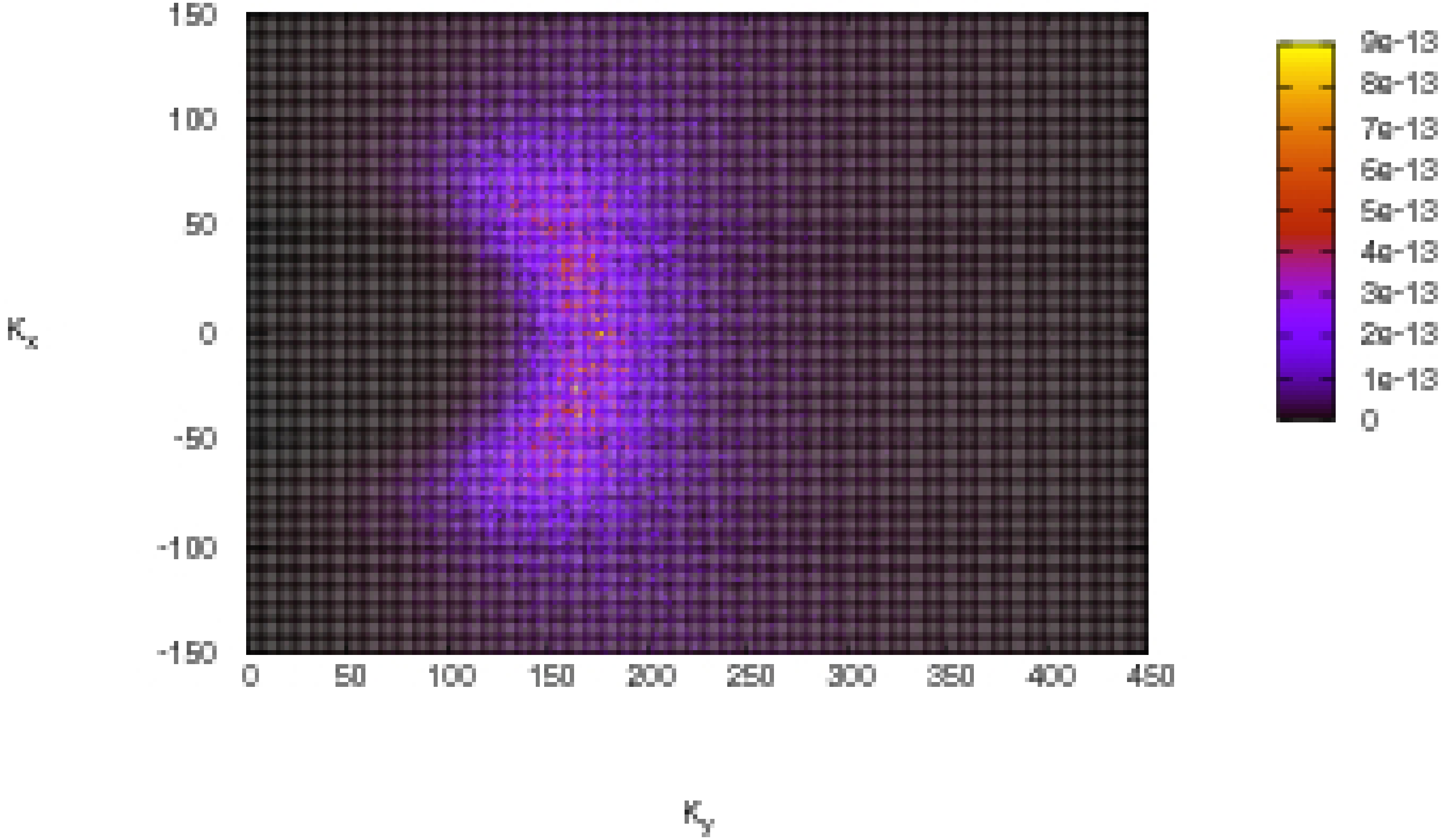
WAM cycle 4: $C_{ds} = 4.10\times 10^{-5}, \quad \delta = 0.5$ *Janssen 1992*
Gunter 1992
Komen 1994



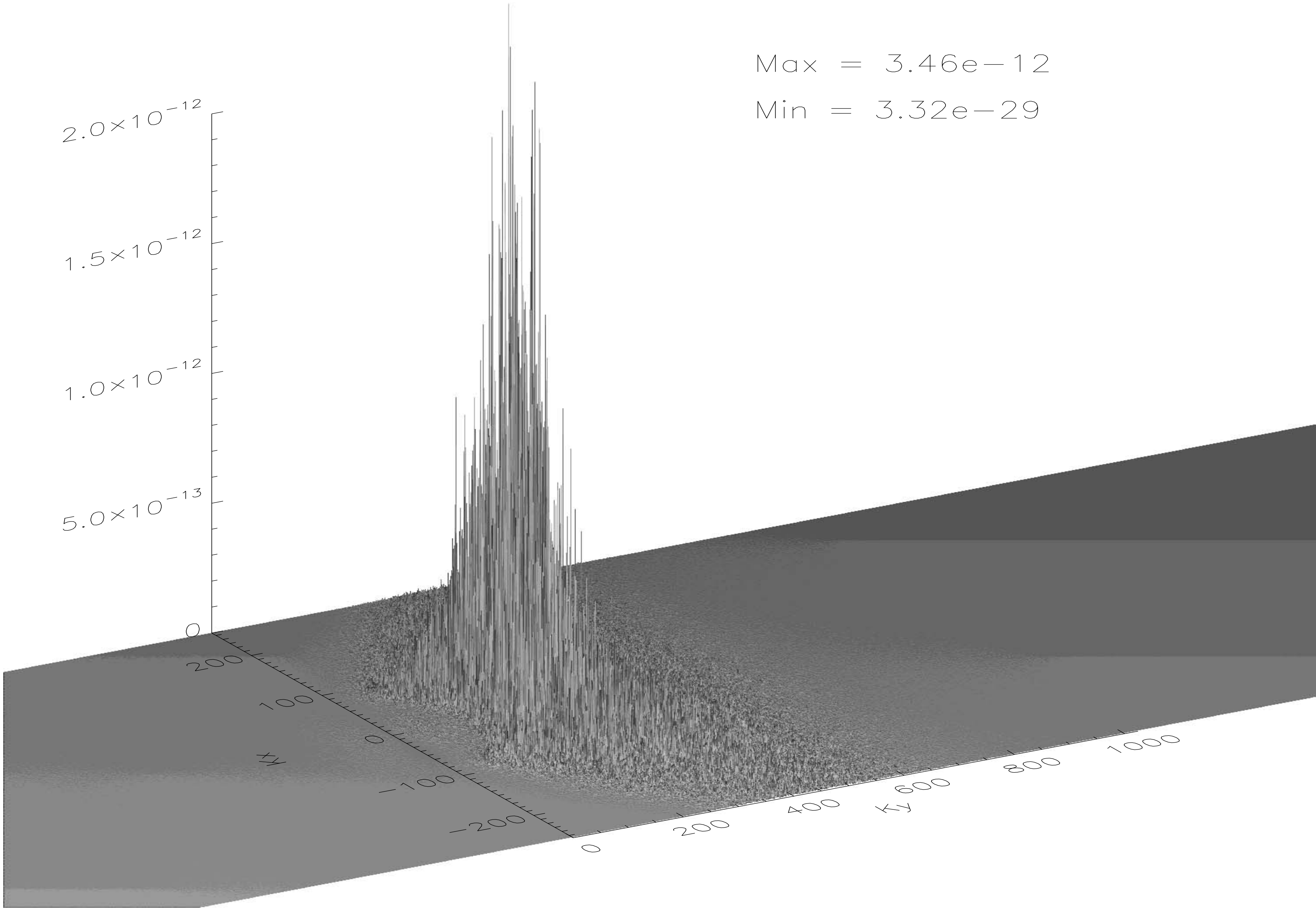
Max = 8.46e−13
Min = 1.00e−24
1×10^−12
8×10^−13
6×10^−13
4×10^−13
2×10^−13
0
200
100
0
−100
−200
Kx
0
200
400
600
800
1000
Ky



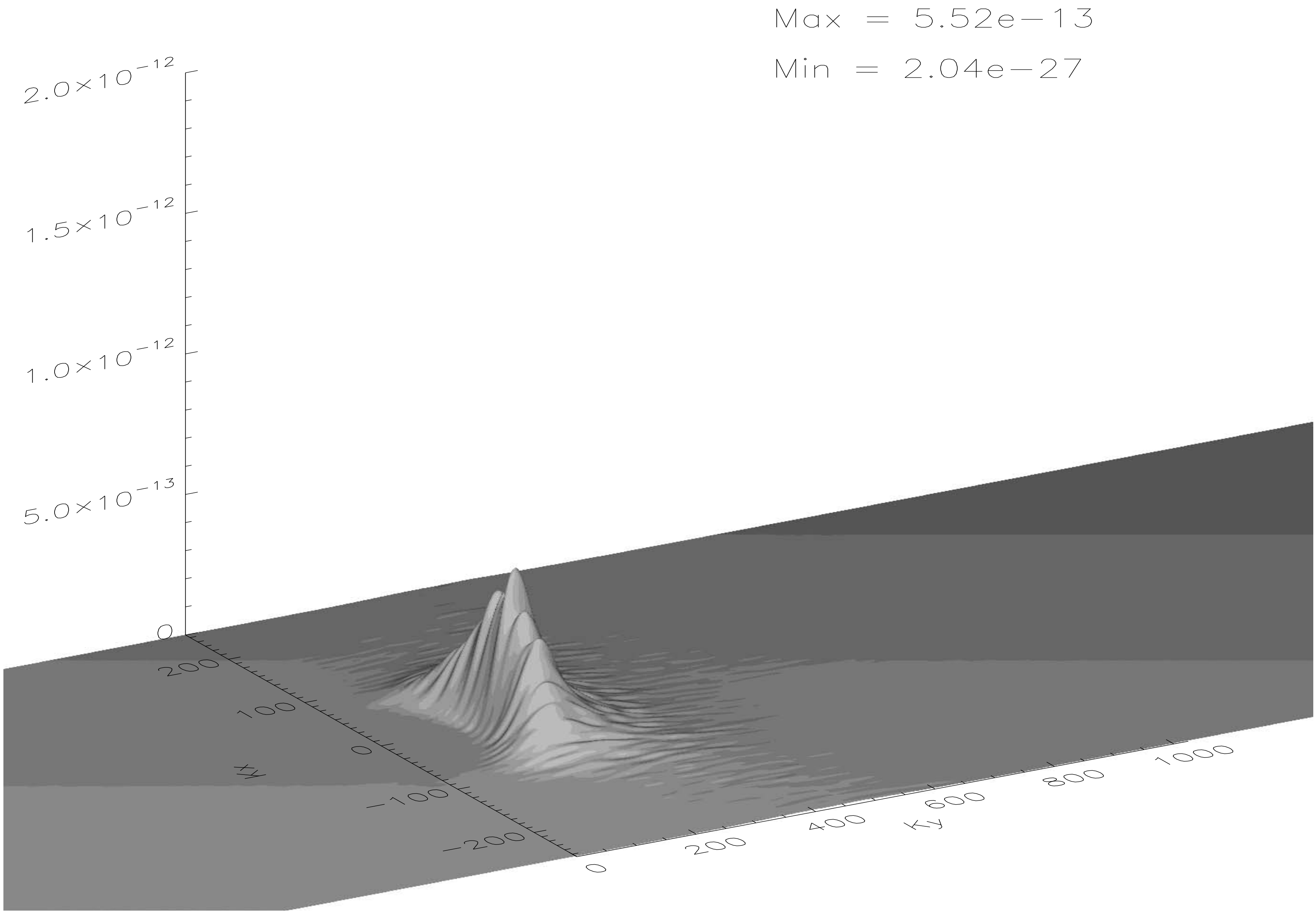
150
100
50
0
-50
-100
-150
K_x
0
50
100
150
200
250
300
350
400
450
K_y
9e-13
8e-13
7e-13
6e-13
5e-13
4e-13
3e-13
2e-13
1e-13
0

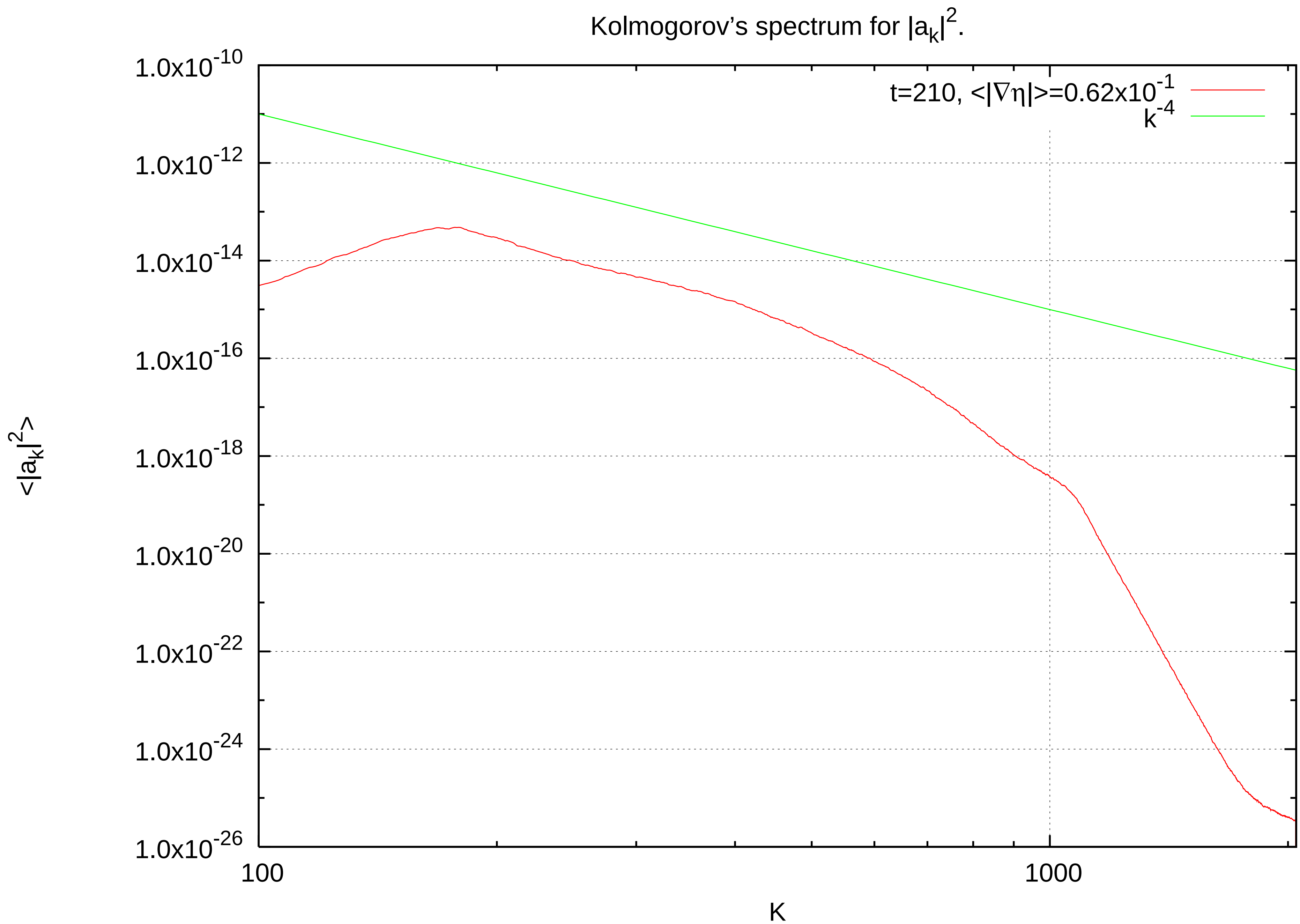


150
100
50
0
-50
-100
-150
K_x
0
50
100
150
200
250
300
350
400
450
K_y
9e-13
8e-13
7e-13
6e-13
5e-13
4e-13
3e-13
2e-13
1e-13
0

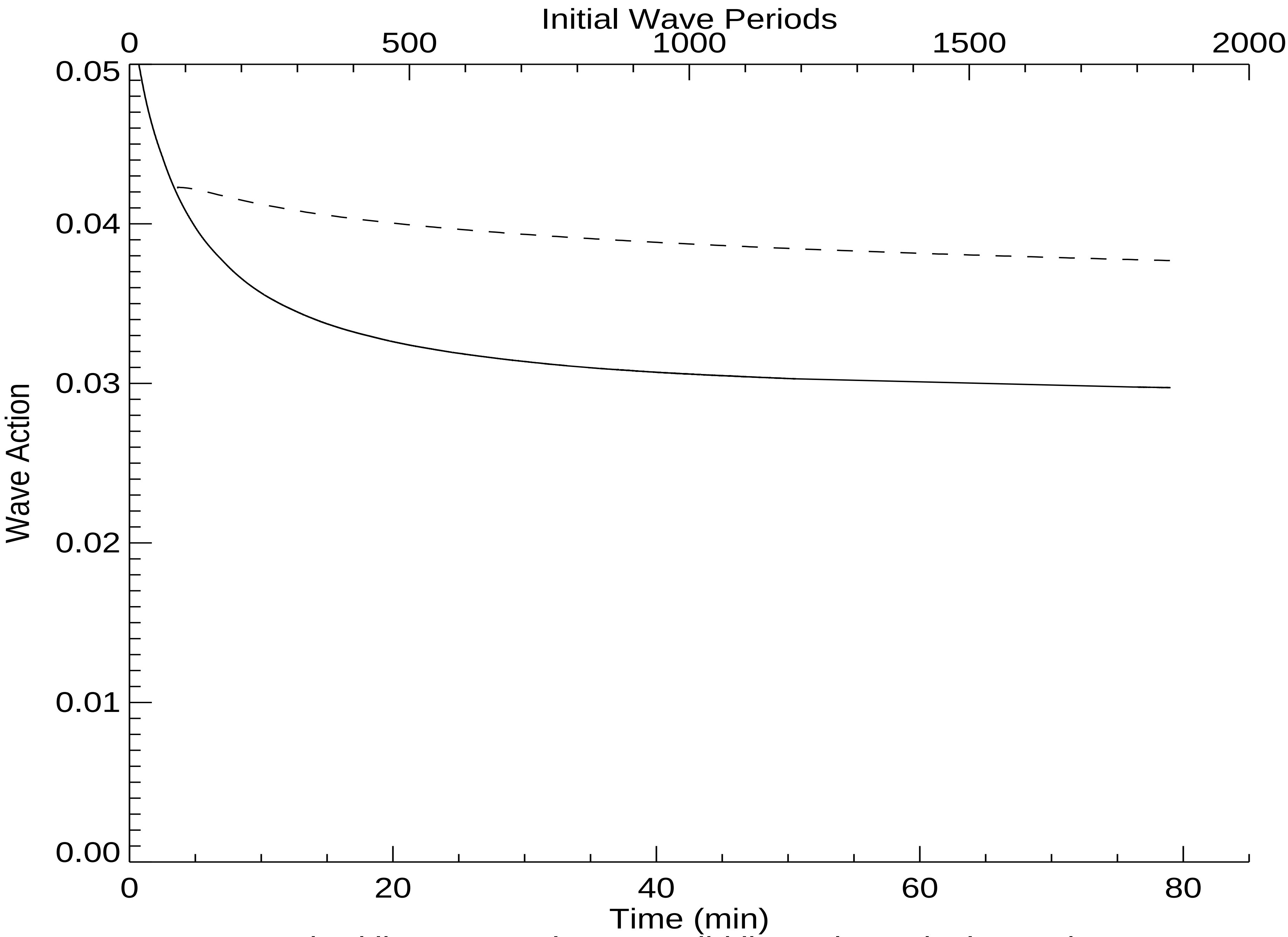


Max = 3.46e−12
Min = 3.32e−29
2.0×10⁻¹²
1.5×10⁻¹²
1.0×10⁻¹²
5.0×10⁻¹³
0
200
100
0
−100
−200
Kx
0
200
400
600
800
1000
Ky

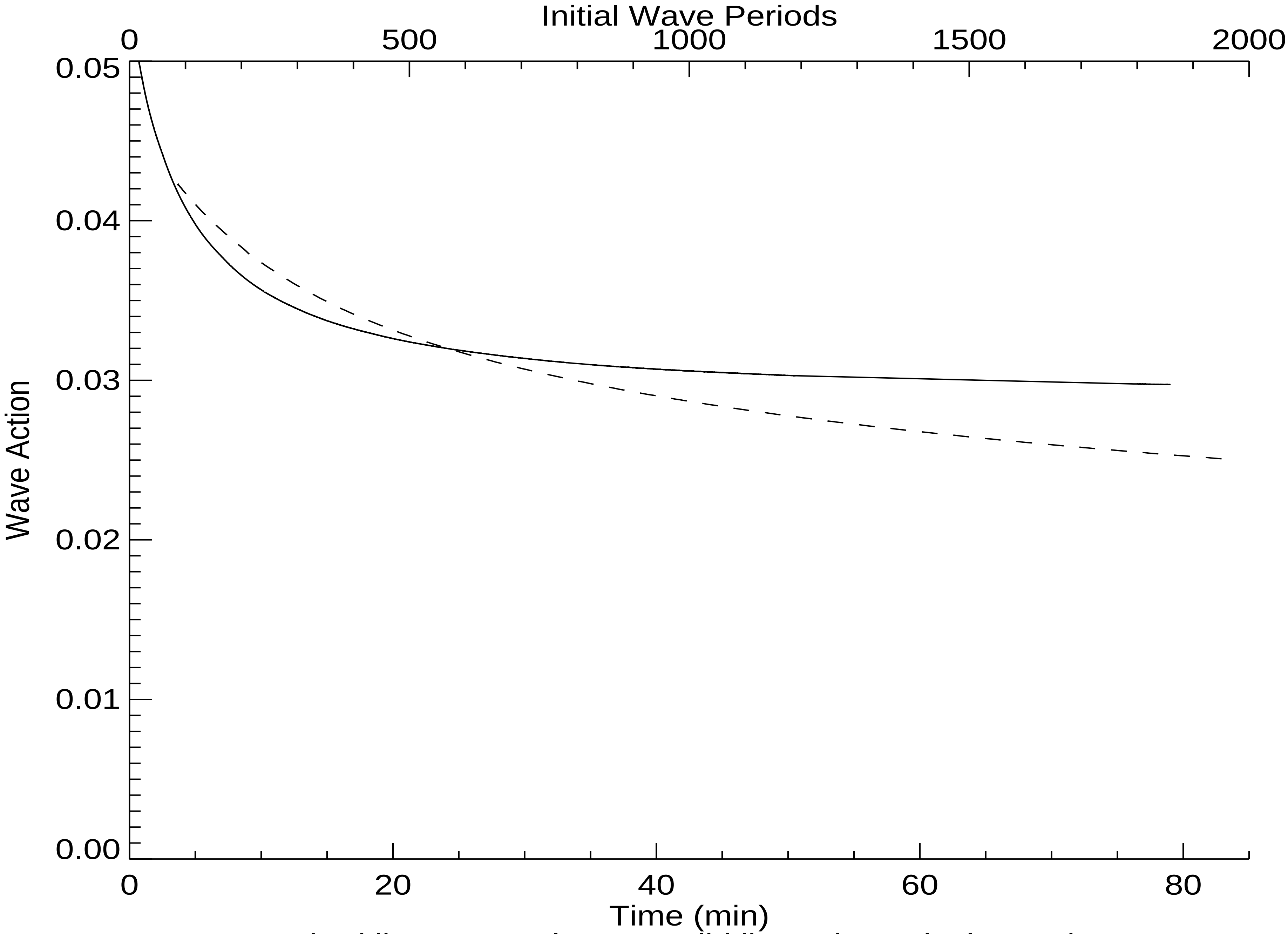

Max = 5.52e−13
Min = 2.04e−27
2.0×10⁻¹²
1.5×10⁻¹²
1.0×10⁻¹²
5.0×10⁻¹³
0
200
100
0
−100
−200
Kx
0
200
400
600
800
1000
Ky



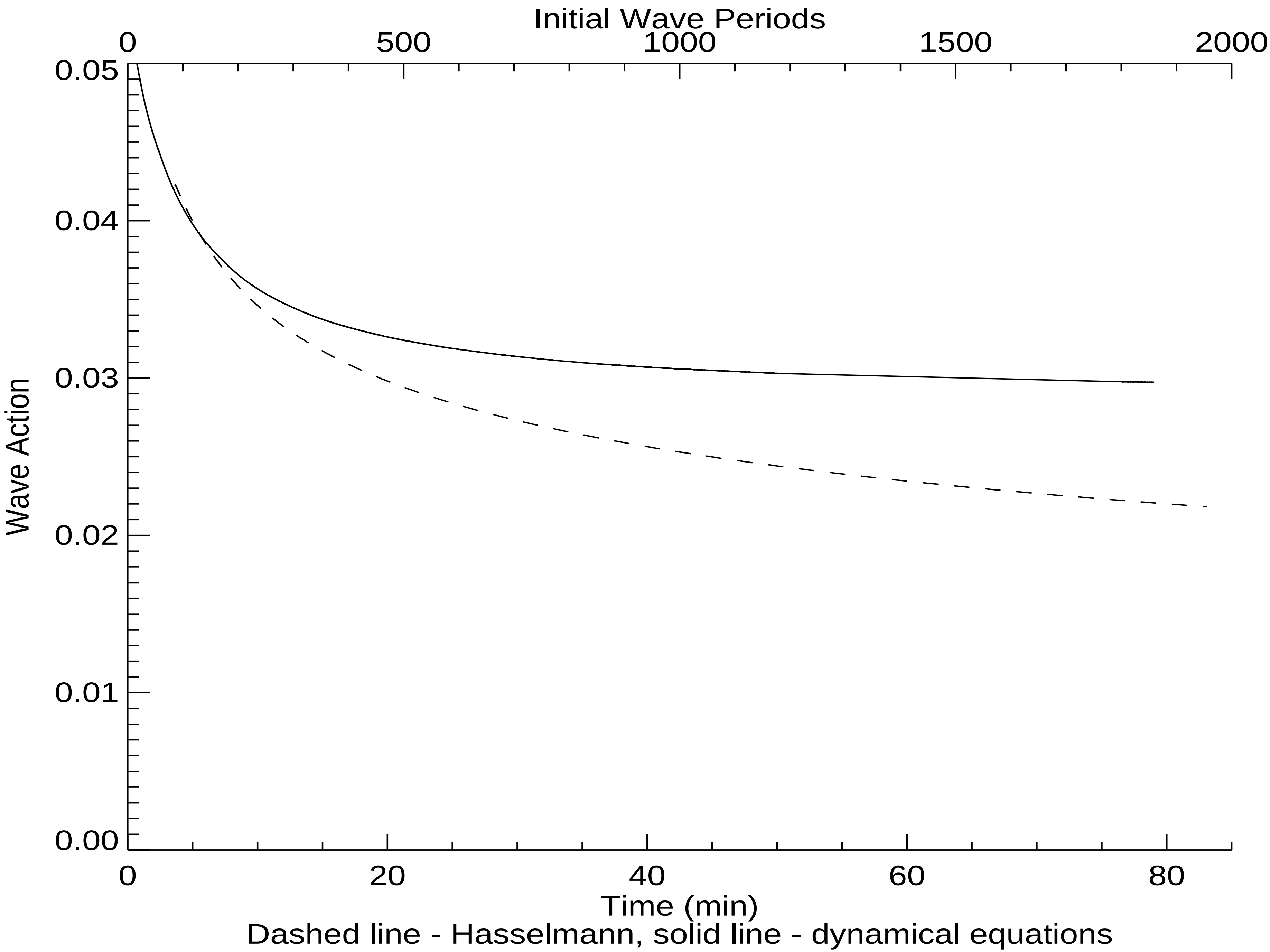
Kolmogorov's spectrum for $|a_k|^2$.
t=210, <|∇η|>=0.62x10^-1
k^-4
1.0x10^-10
1.0x10^-12
1.0x10^-14
1.0x10^-16
1.0x10^-18
1.0x10^-20
1.0x10^-22
1.0x10^-24
1.0x10^-26
100
1000
K
<|a_k|^2>

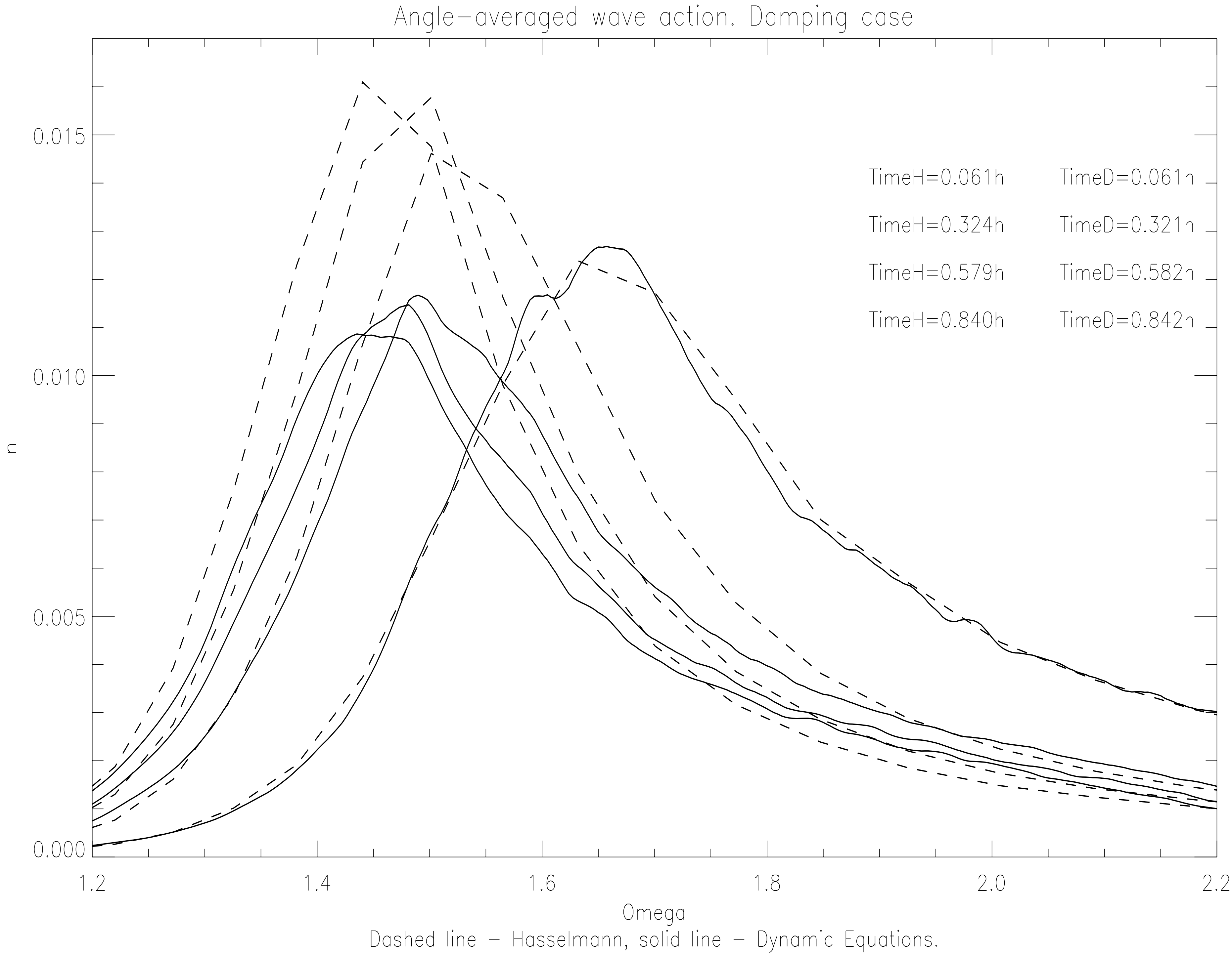


Dashed line - Hasselmann, solid line - dynamical equations

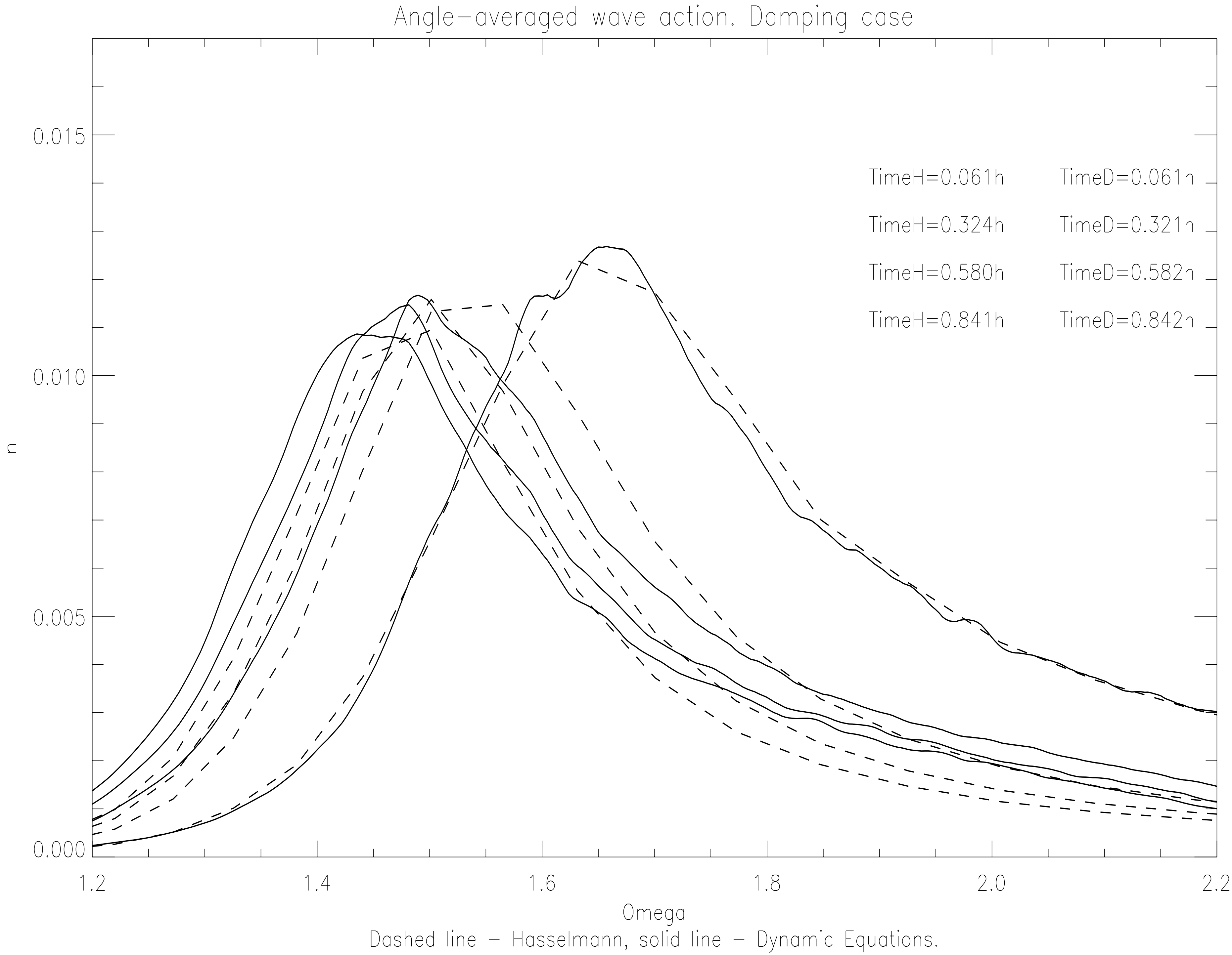


Dashed line - Hasselmann, solid line - dynamical equations

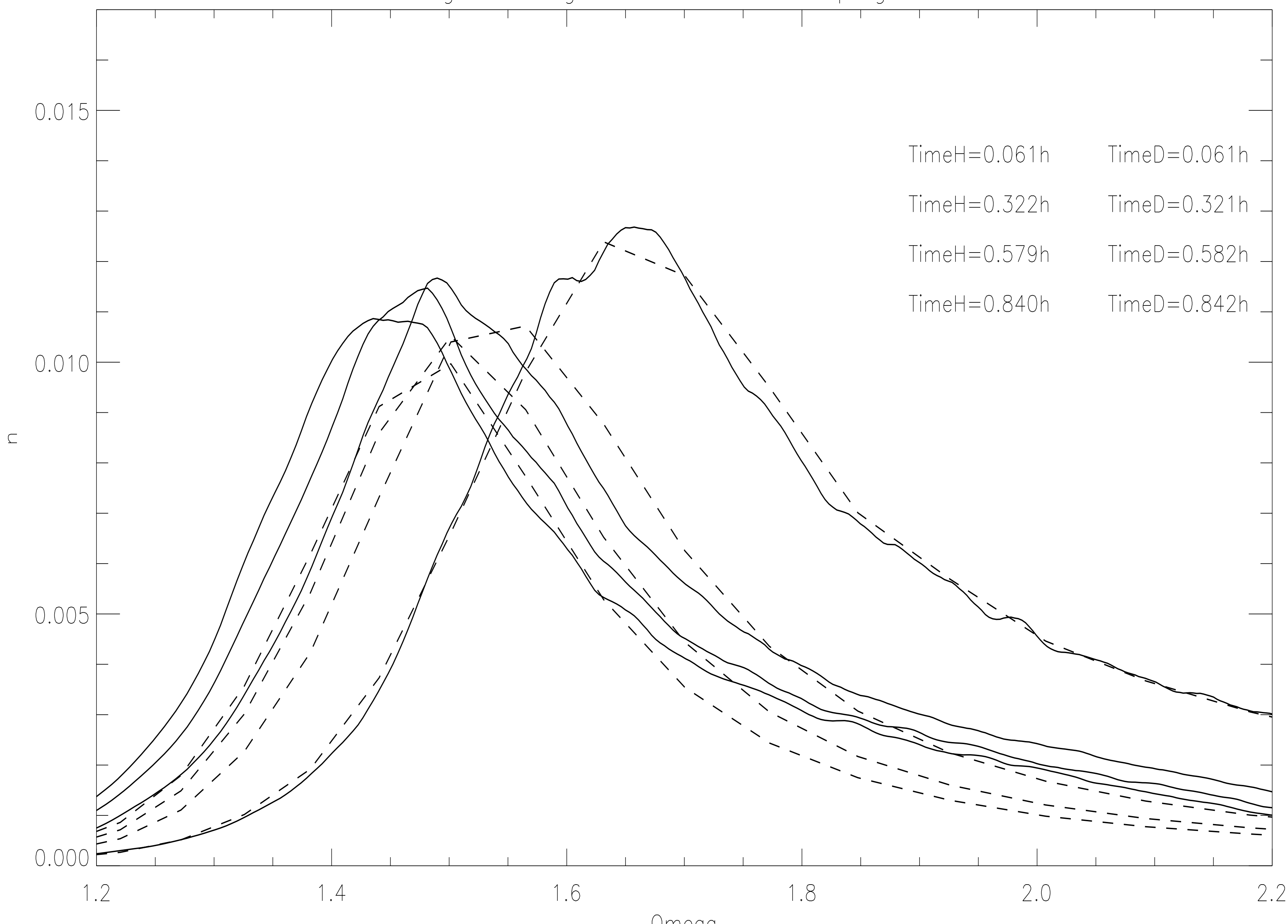

Initial Wave Periods
0
500
1000
1500
2000
Wave Action
0.05
0.04
0.03
0.02
0.01
0.00
0
20
40
60
80
Time (min)
Dashed line - Hasselmann, solid line - dynamical equations



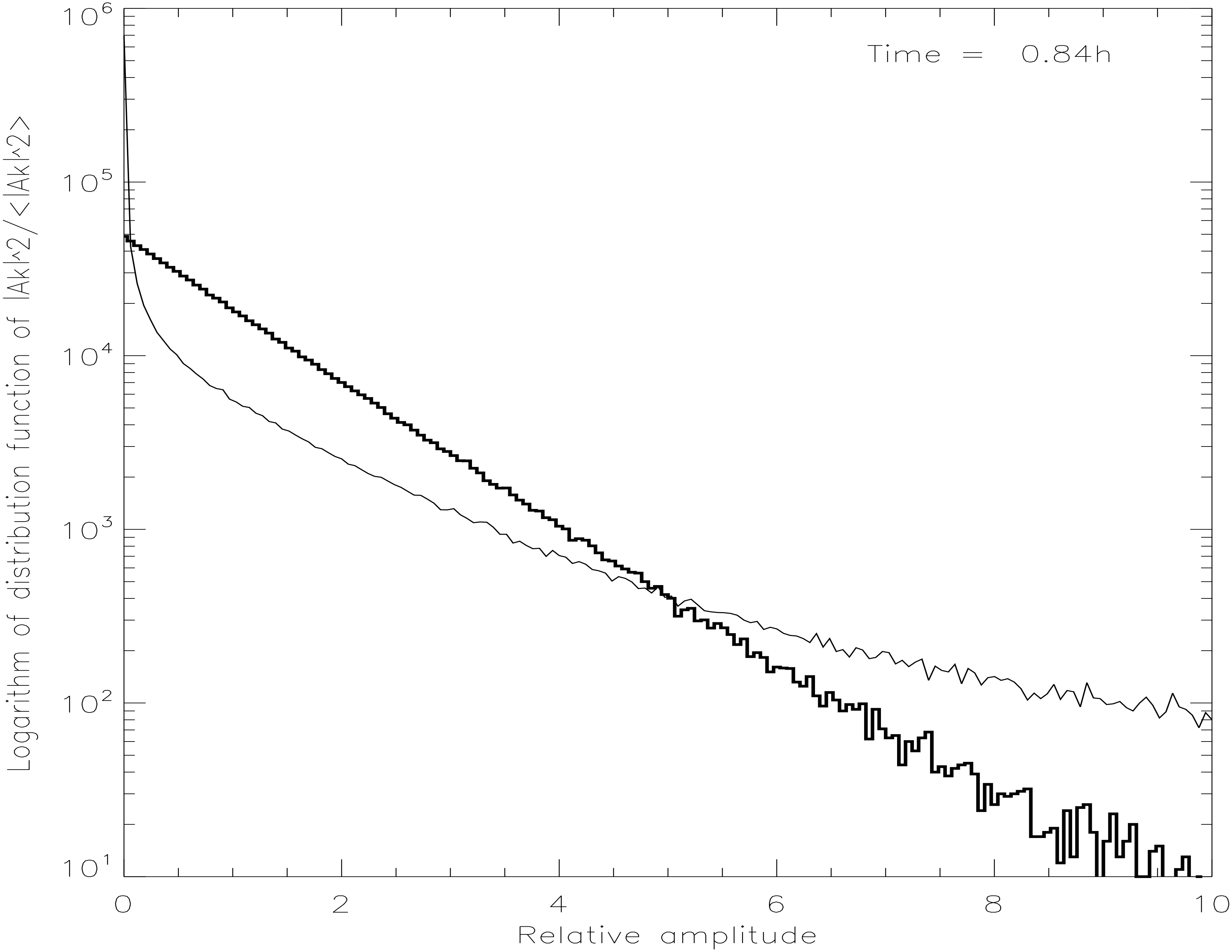
Angle−averaged wave action. Damping case
TimeH=0.061h TimeD=0.061h
TimeH=0.324h TimeD=0.321h
TimeH=0.579h TimeD=0.582h
TimeH=0.840h TimeD=0.842h
0.015
0.010
0.005
0.000
n
1.2
1.4
1.6
1.8
2.0
2.2
Omega
Dashed line − Hasselmann, solid line − Dynamic Equations.

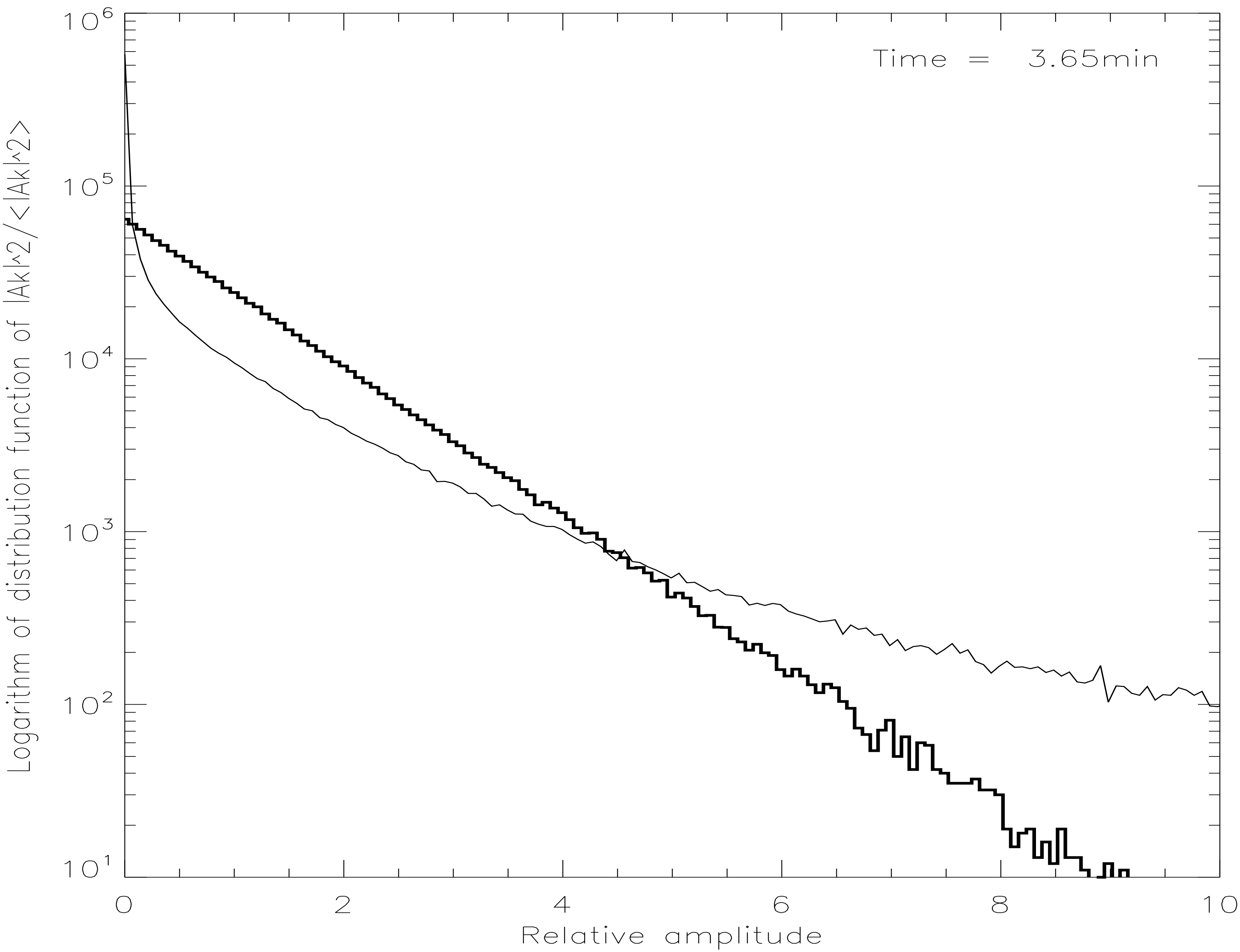

Angle−averaged wave action. Damping case
0.015
0.010
0.005
0.000
n
1.2
1.4
1.6
1.8
2.0
2.2
Omega
Dashed line − Hasselmann, solid line − Dynamic Equations.
TimeH=0.061h
TimeD=0.061h
TimeH=0.324h
TimeD=0.321h
TimeH=0.580h
TimeD=0.582h
TimeH=0.841h
TimeD=0.842h

Angle-averaged wave action. Damping case



Dashed line – Hasselmann, solid line – Dynamic Equations.


Time = 0.84h
Logarithm of distribution function of |Ak|^2/<|Ak|^2>
Relative amplitude
10^6
10^5
10^4
10^3
10^2
10^1
0
2
4
6
8
10



Time = 3.65min
Logarithm of distribution function of |Ak|^2/<|Ak|^2>
Relative amplitude

Conclusions:

1. ***Weak turbulence is confirmed by direct simulation of dynamical equations if the real-space domain is big enough***

2. ***Experimental pools have to be longer that 200m to get the physics equivalent to the open ocean conditions***

Thanks for continuing support

to

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and

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