The spectral scale of wave breaking

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Motivation

Momentum input from wind to waves → Wave breaking → turbulent kinetic energy

Turbulence → **vertical mixing** — Heat, gas, momentum, nutrients, pollutants (oil spill)

Wave evolution → **wave energy dissipation**, **wind energy input** — Wave forecast, marine safety

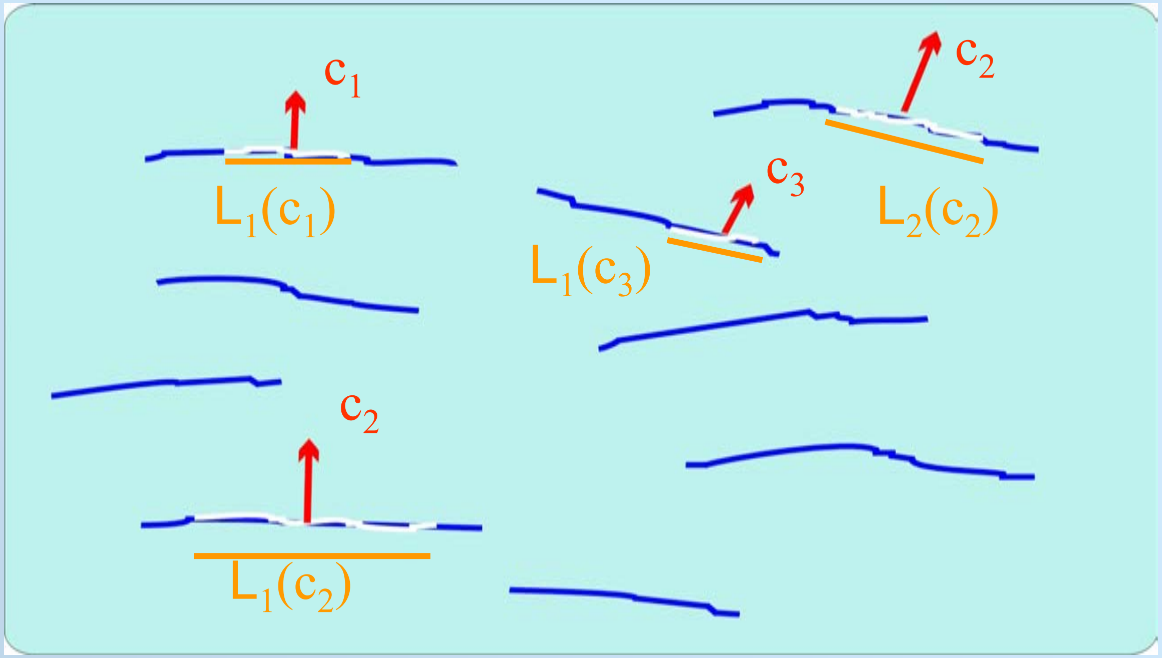
$$\frac{dN(k)}{dt} = S_{in}(k) + S_{nl}(k) + S_{dis}(k)$$

Bubbles → **gas flux**, **sea spray** — Climate, marine weather forecast

Multi-scale breaking rate Λ(c)

Phillips, 1985 JFM

Λ(c): spectral density of *breaking wave crest length* per unit area with velocities in the range (c, c+dc)



analyze wave breaking in terms of:

- breaking crest length L
- propagation speed *c*

$$\Lambda(c_j) = \frac{1}{A}\sum_i L_i(c_j)$$

Kinematic and dynamic relevance of $\Lambda(c)$

$$R = \int c\Lambda(c)dc$$

rate of surface disruption
(equivalent to *breaking rate at a fixed point location*)

$$M(c) = b\frac{\rho}{g}c^4\Lambda(c)$$

momentum flux from waves of scale c to currents

$$\varepsilon(c) = b\frac{\rho}{g}c^5\Lambda(c)$$

wave **energy dissipation** at scale c

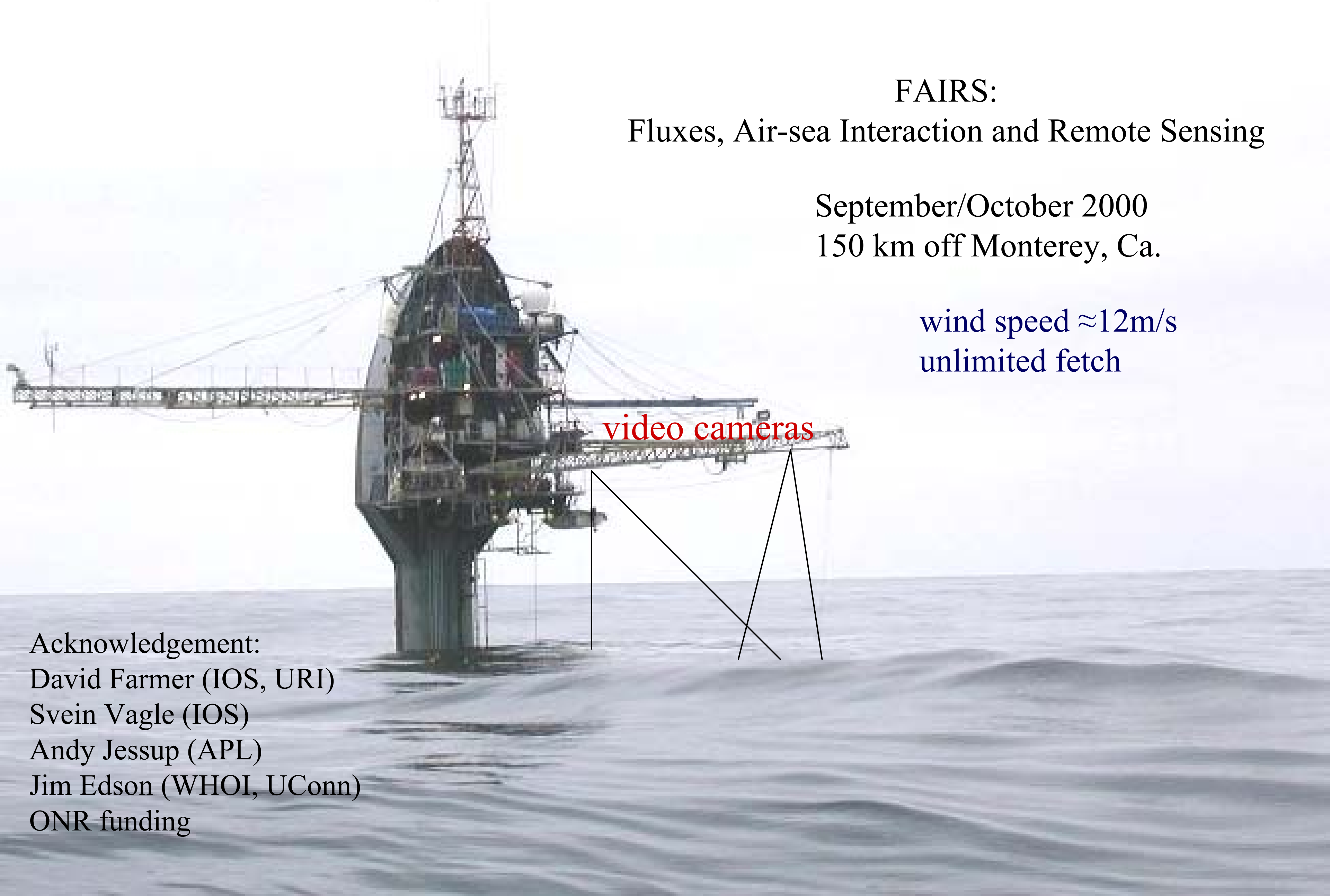
- wave evolution (dissipation)
- mixing (scale, magnitude)

M(c), ε(c) based on similarity scaling, verified in hydrofoil experiment (Duncan, 1981) and scaling of wave tank experiments

unknown factor b

$b \approx 10^{-3}$ (Phillips et al. 2001) $b \approx 10^{-2}$ (Melville & Matusov, 2002) $b = f(c/c_p, \ldots)$?

Multi-scale breaking rate Λ(c) - Data



FAIRS:
Fluxes, Air-sea Interaction and Remote Sensing

September/October 2000
150 km off Monterey, Ca.

wind speed ≈12m/s
unlimited fetch

Acknowledgement:
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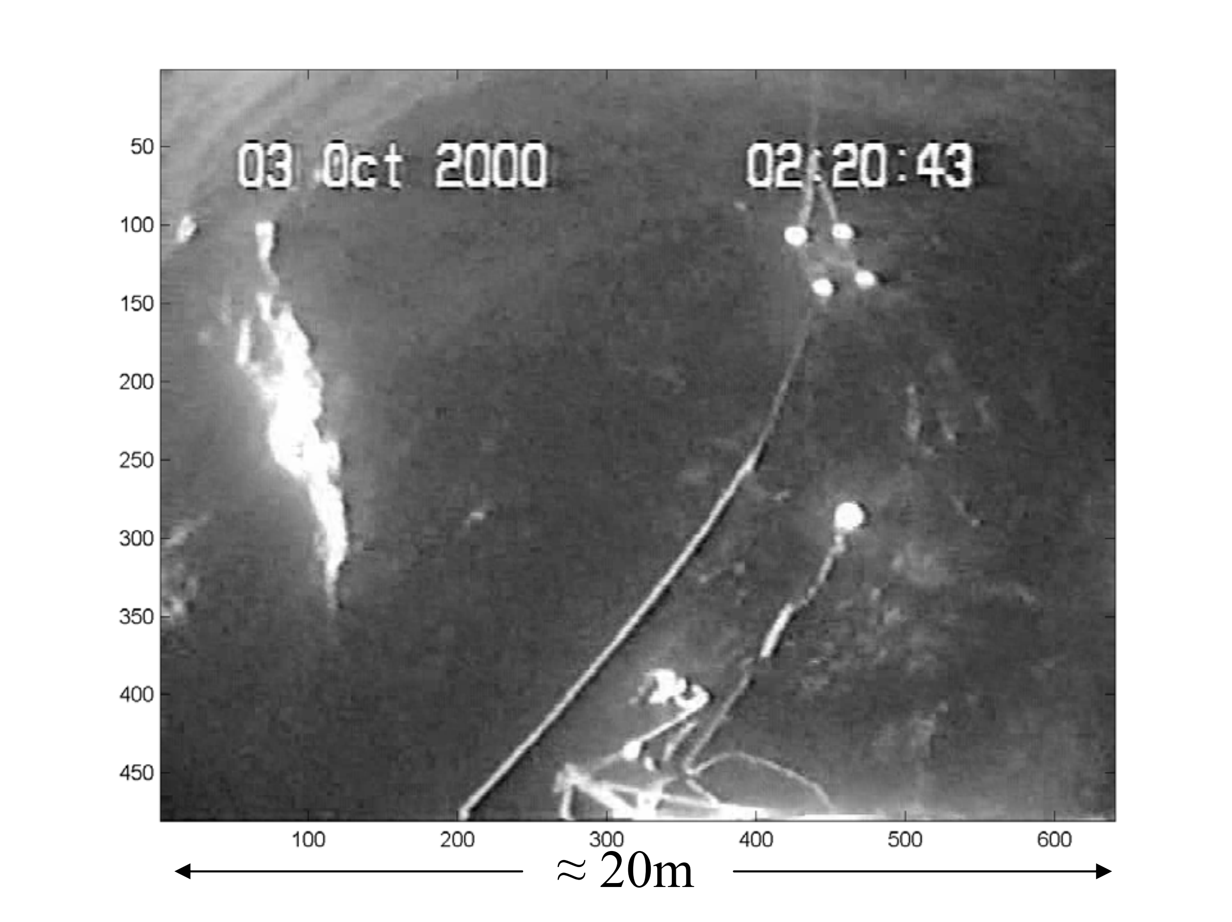
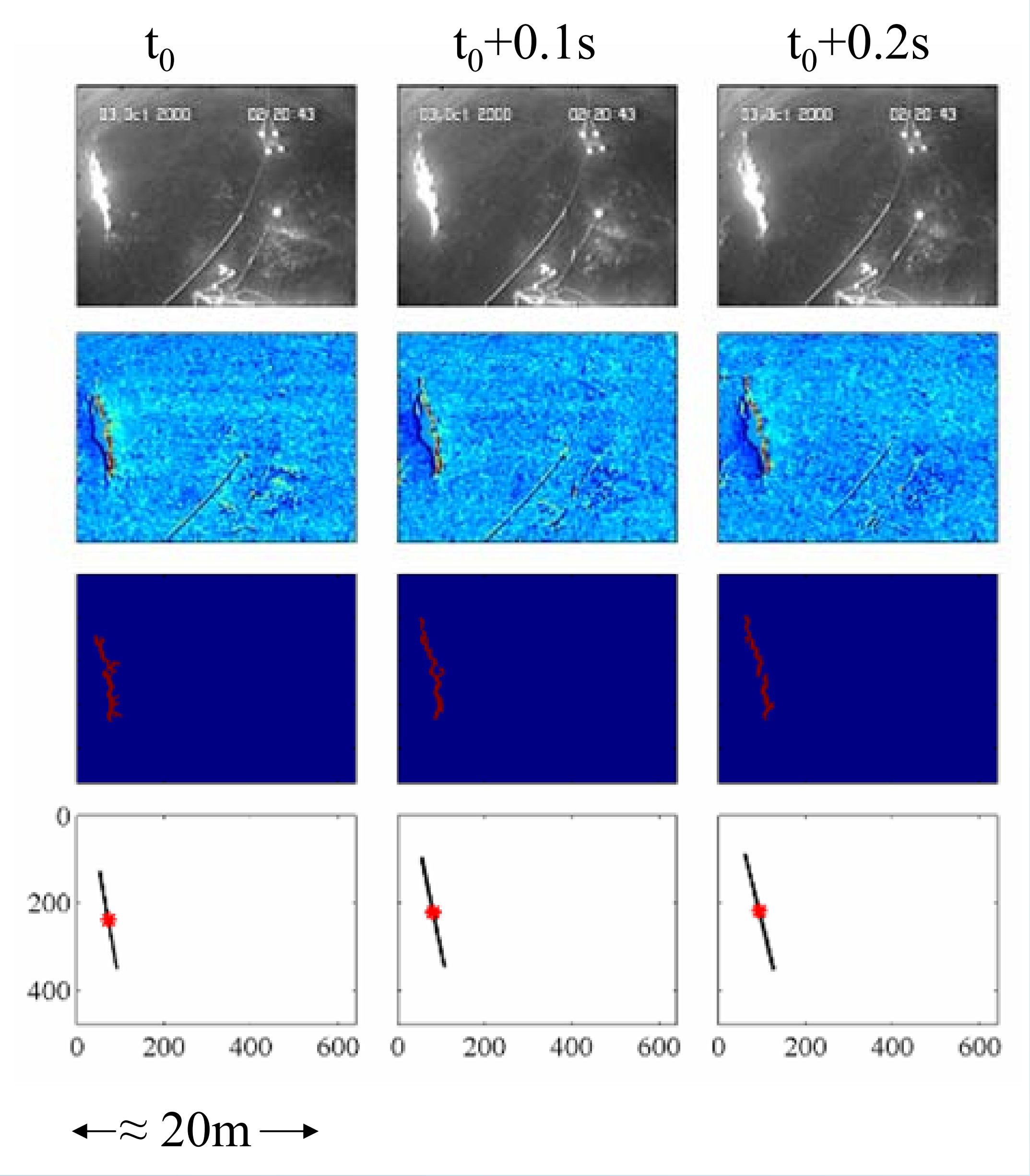
Jim Edson (WHOI, UConn)
ONR funding

Breaking wave scale extraction

digitized VHS video, 10 Hz frame rate



Breaking crest length and speed extraction



digitized VHS video

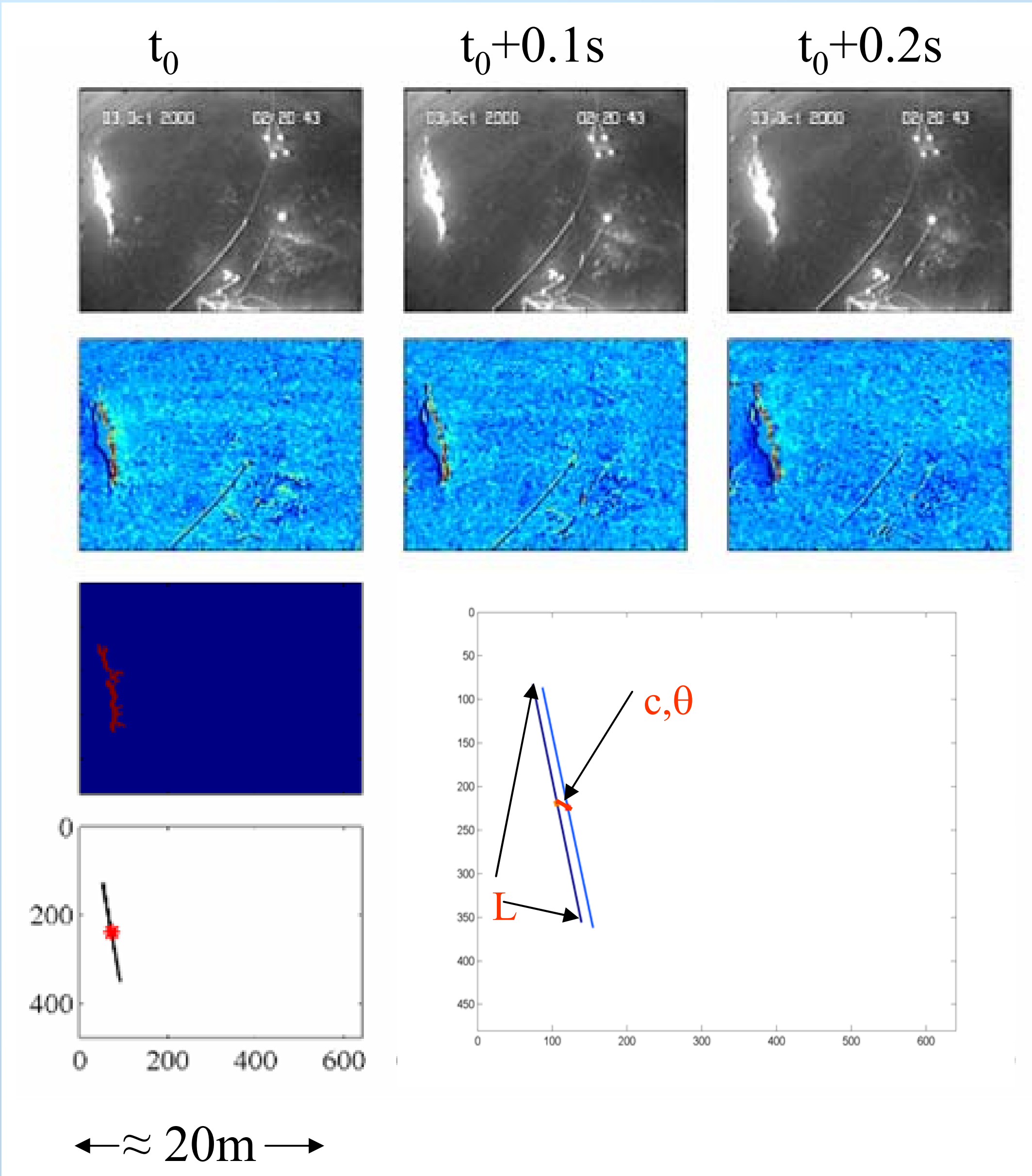
differential image

crest

threshold differential image

crest length L
centroid

Breaking crest length and speed extraction



digitized VHS video

differential image

threshold differential image

crest length L
centroid displacement - c

Doppler correction

$$\Lambda(c_j) = \frac{1}{A}\sum_i L_i(c_j)$$

$\Lambda(c)$

Breaking scale

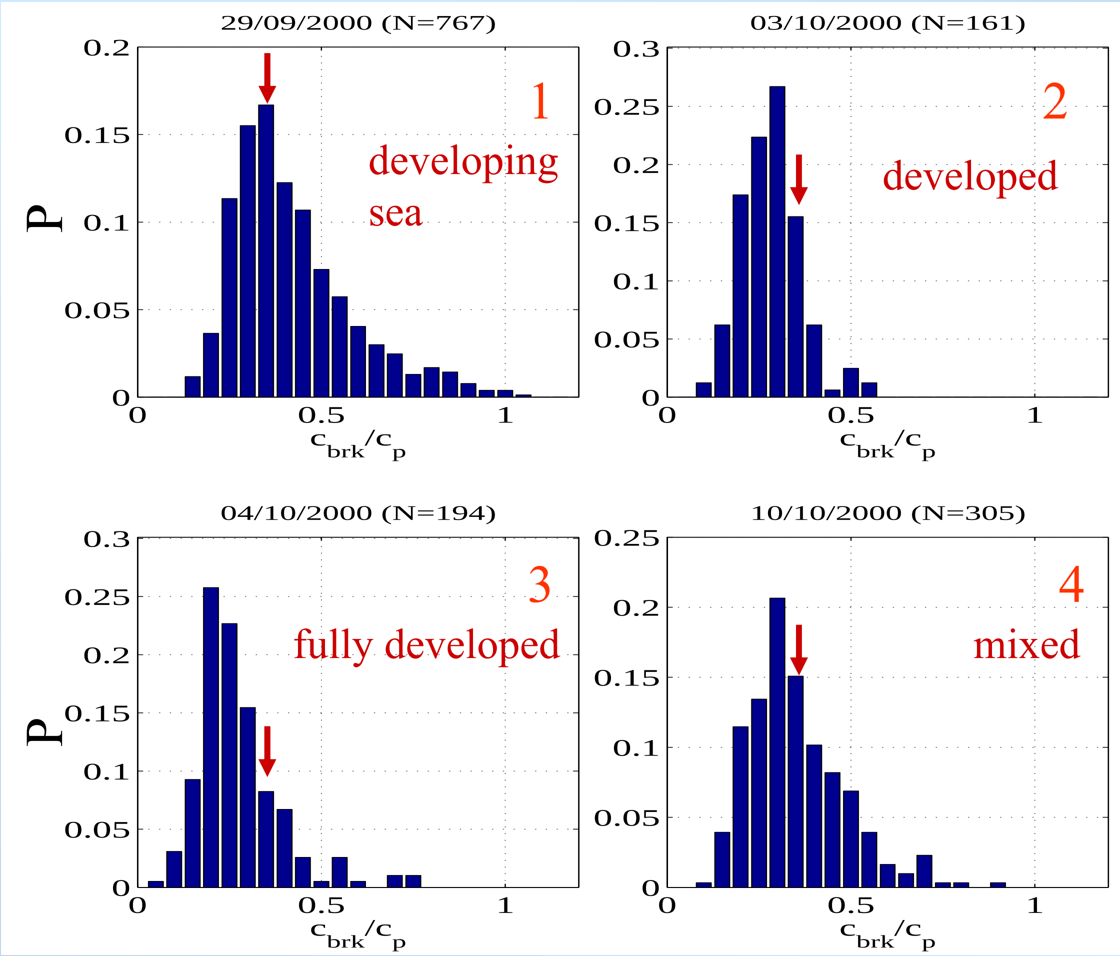
casual observations: wave breaking occurs on a wide range of scales

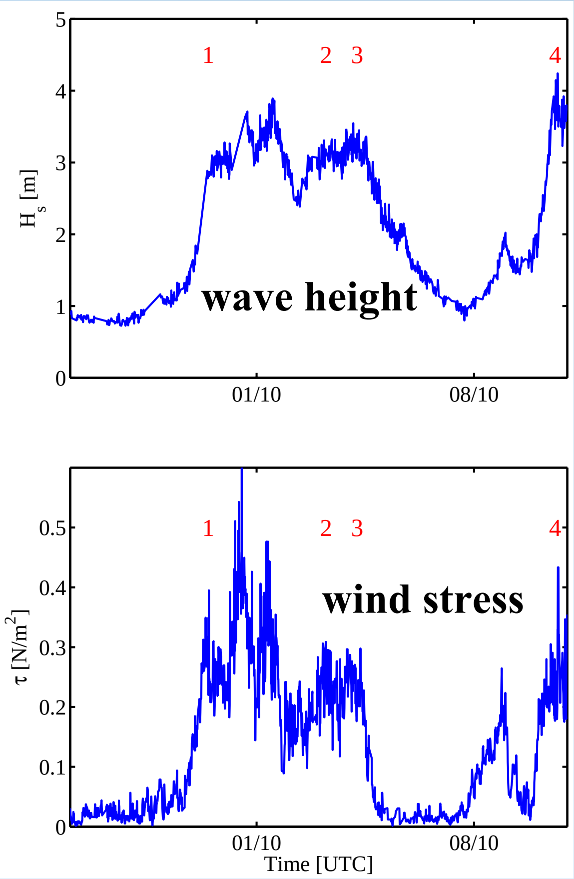
quantify scale of breaking wave: $$\boxed{\frac{c_{brk}}{c_p}}$$

c_p: phase speed of *dominant* wave
c_{brk}: whitecap propagation speed

Note: linear phase speed of breaking wave $c \approx 1.2\, c_{brk}$

Breaking scale





c_p: phase speed of *dominant* wave
c_{brk}: whitecap propagation speed

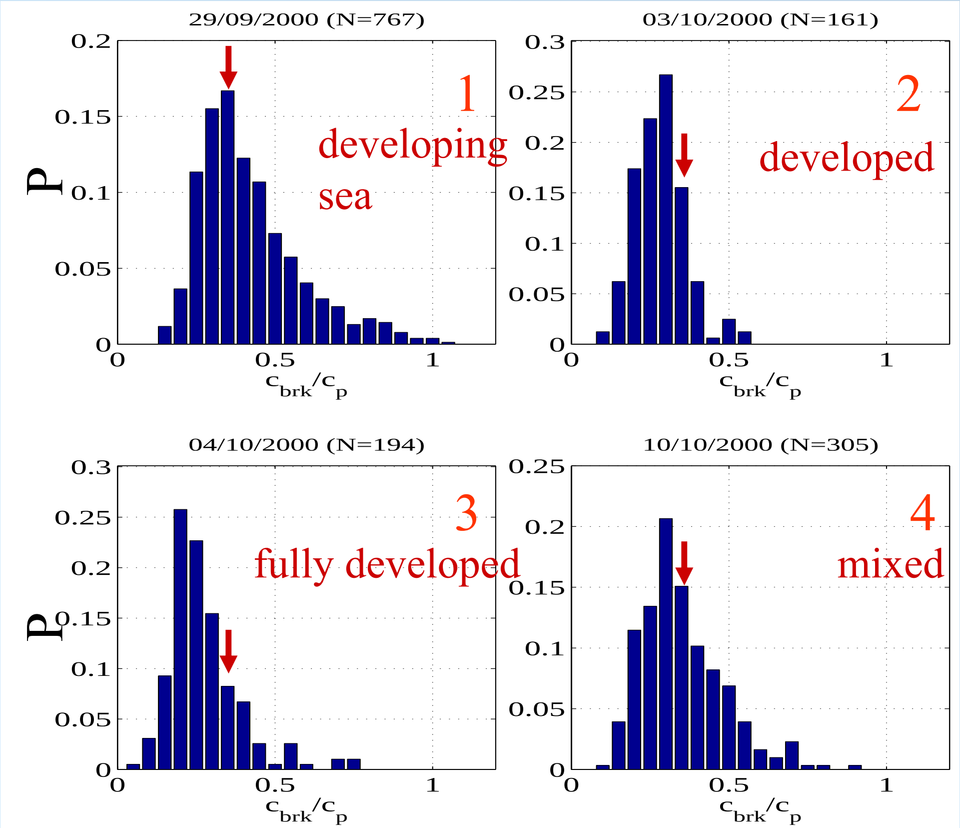
- peak wave breaking scale: 0.2 – 0.4 c_p
- increasing wave age ⇒
  - narrowing of distribution
  - downward peak shift

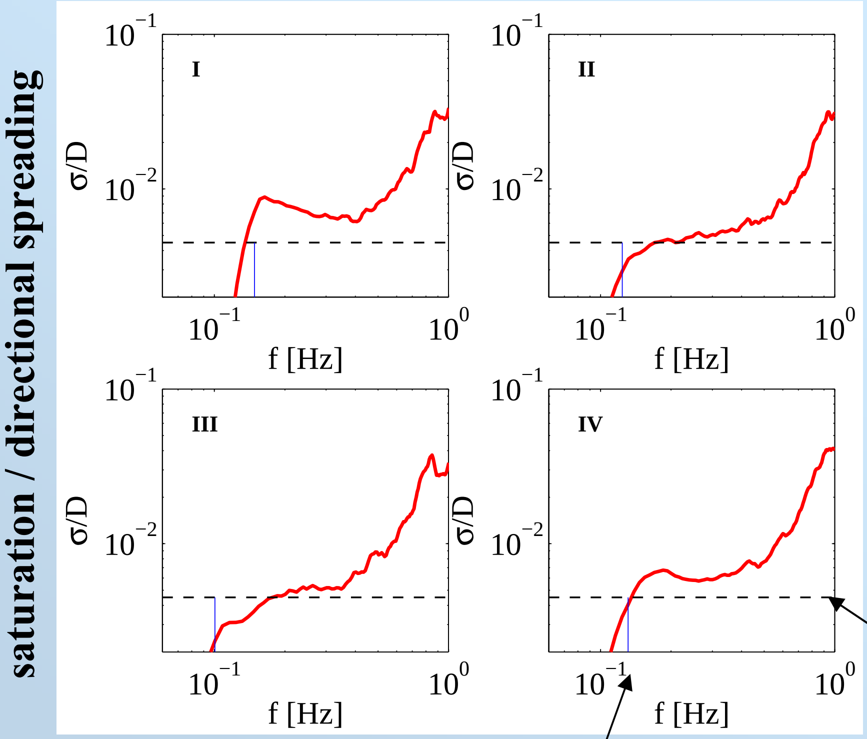
Breaking scale – wave saturation

wave saturation

$$\sigma = 2g^{-2}\omega^5 S(\omega)$$

S(ω): surface elevation spectrum
D: spectral spreading function

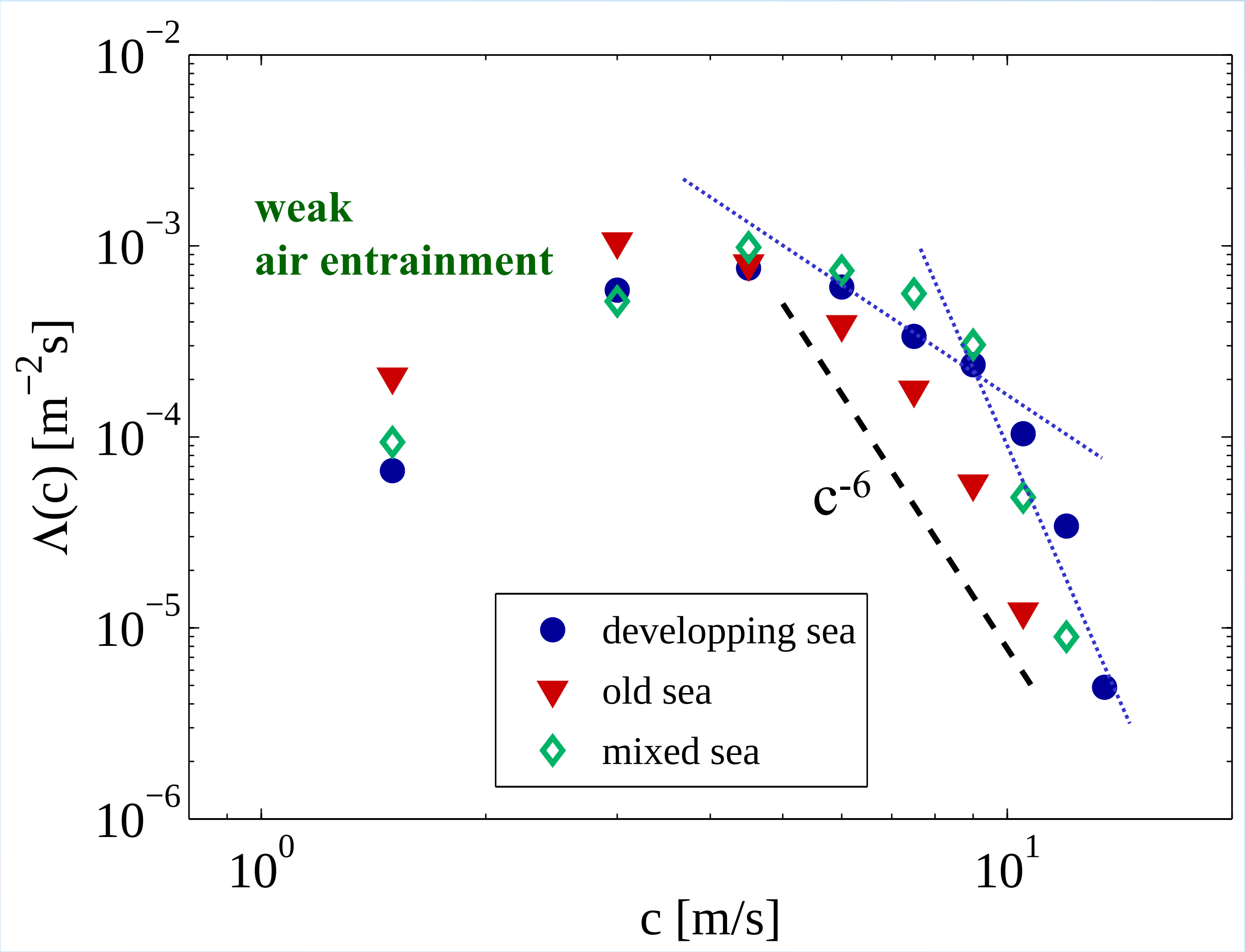




threshold for onset of breaking
(Banner, Gemmrich & Farmer, JPO 2002)

f_p

Multi-scale breaking crest length distribution, $\Lambda(c)$



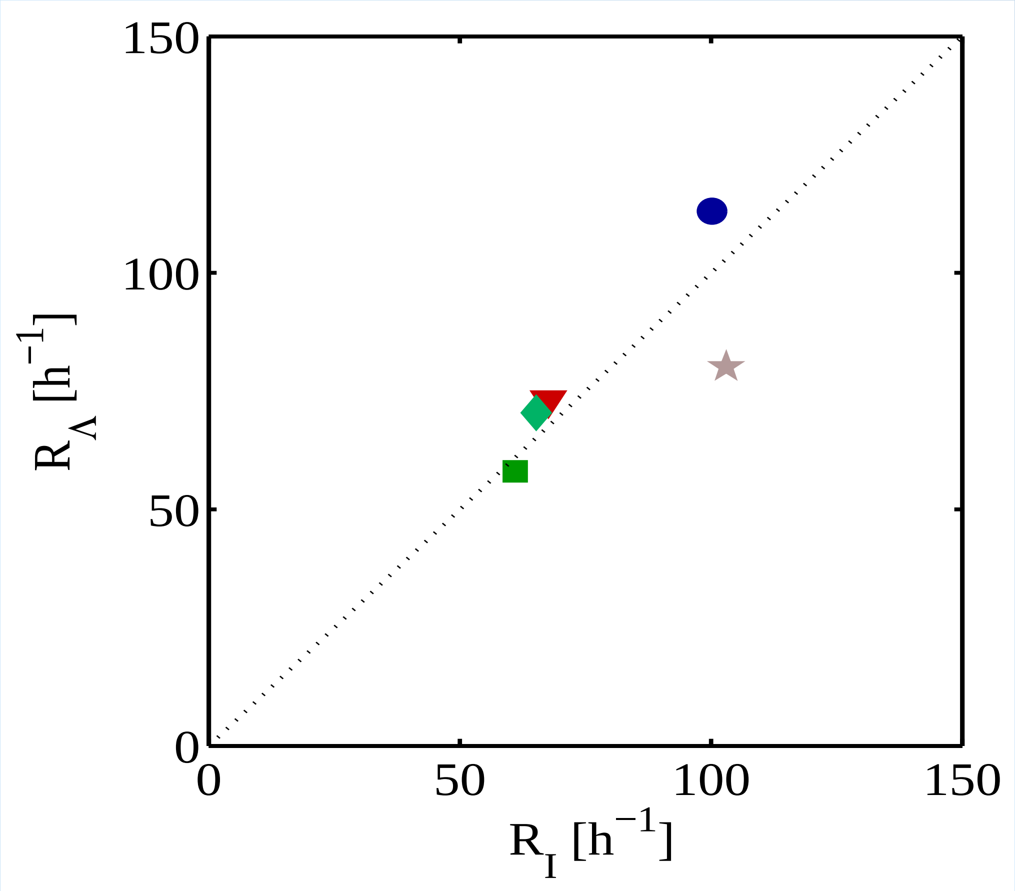
peak at
small – intermediate
wave scales

$\Lambda(c) \sim c^{-6}$: implied by local equilibrium $S_{in}+S_{dis}+S_{nl} \approx 0$ (Phillips, JFM 1985)

Fixed point breaking rate - consistency check

$$R_\Lambda = \int c\Lambda(c)dc$$

R_I: based on fixed point video brightness

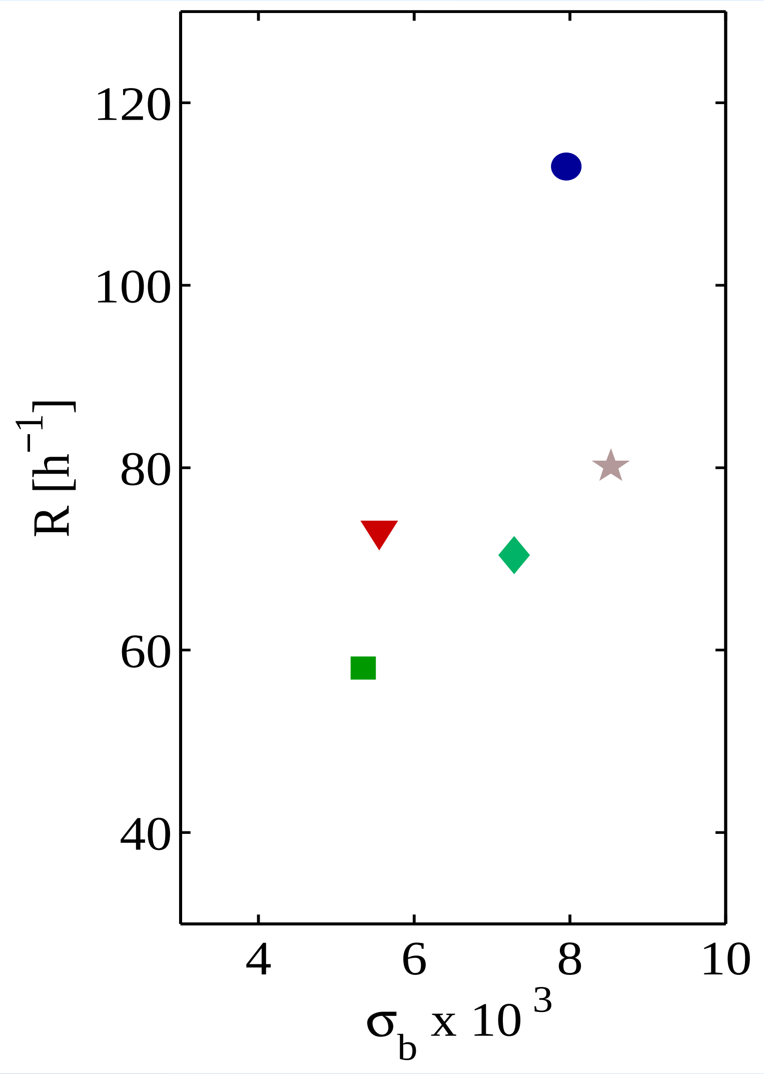


R_Λ

- consistent with independent estimate R_I
- comparable to R = 60 – 120 /h

(Gemmrich & Farmer, JPO 1999)

Fixed point breaking rate - wave saturation



$$R = \int c\Lambda(c)dc$$

$\sigma_{b:}$ mean saturation in band $1 \leq \omega/\omega_p \leq 5$

wave saturation $\sigma = 2g^{-2}\omega^5 S(\omega)$

$S(\omega)$: surface elevation spectrum

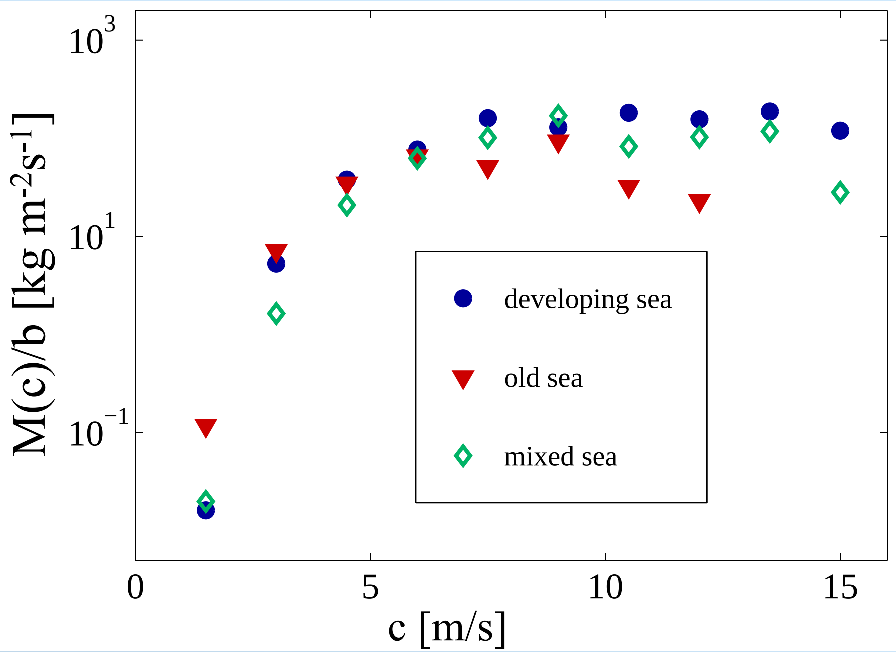
increasing non-linearity of waves
→ increasing breaking rate

Energy dissipation, momentum transfer

based on Λ(c)

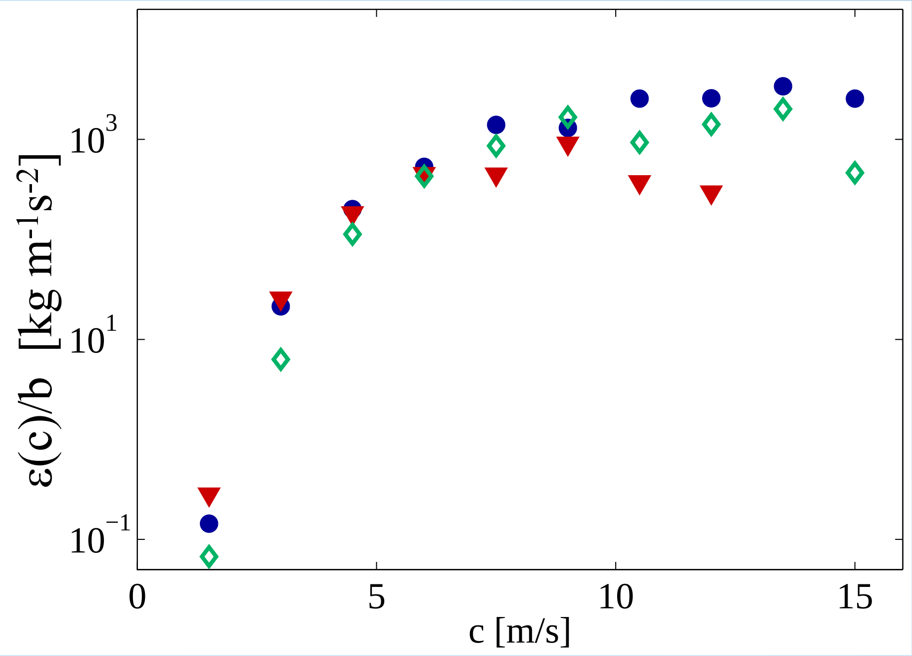
momentum transfer (scaled)

$$M(c) = b\rho c^4 \Lambda(c)/g$$



dissipation (scaled)

$$\varepsilon(c) = b\rho c^5 \Lambda(c)/g$$



short waves

dominant waves

old sea: intermediate wave scales most relevant (c ≈ 7 – 9 m/s)

developing sea: intermediate - large wave scales most relevant (c ≈ 10 – 12 m/s)

Linking direct and remote observations: Estimate of proportionality factor b

$$\varepsilon(c) = b\frac{\rho}{g}c^5\Lambda(c)$$

wave energy dissipation = integrated energy dissipation in water column

$$\int \varepsilon(c)\,dc = \int \varepsilon_{obs}(t,z)\,dtdz \longrightarrow b = \frac{\int \varepsilon_{obs}(t,z)\,dtdz}{\int \rho g^{-1} c^5 \Lambda(c)\,dc}$$

Dissipation measurement

Surface-following float

single beam acoustic Doppler velocity profilers
(Dopbeams, Sontek)

high resolution velocity profiles

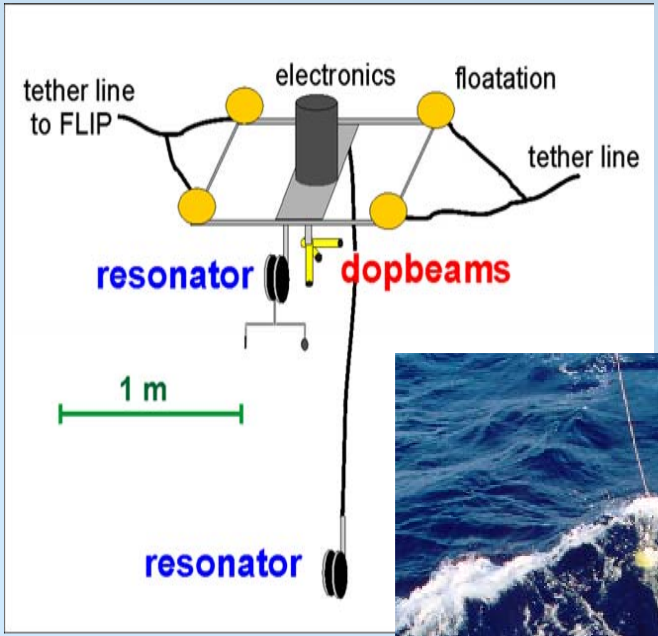
w(z): 1m - 1.7m, 6 mm bin size, 20 Hz

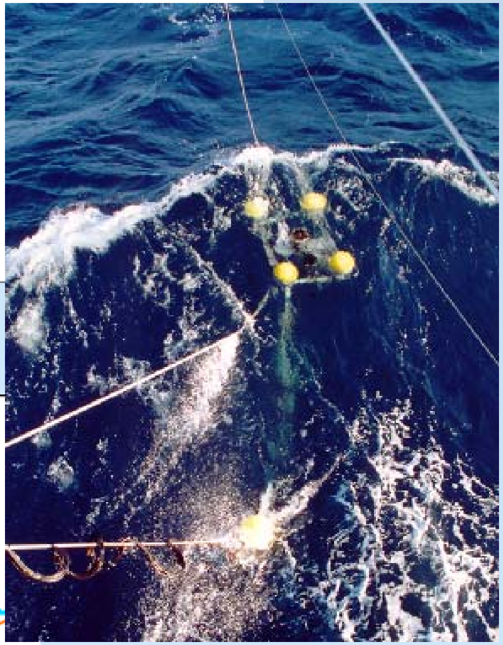
↓

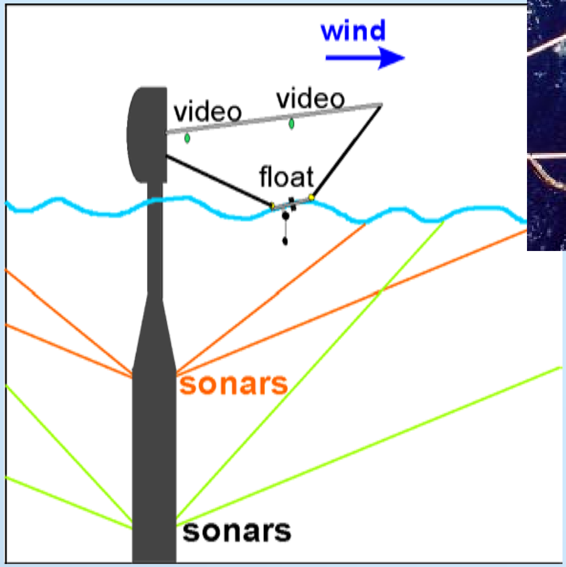
wavenumber velocity spectra

↓ inertial subrange

dissipation time series





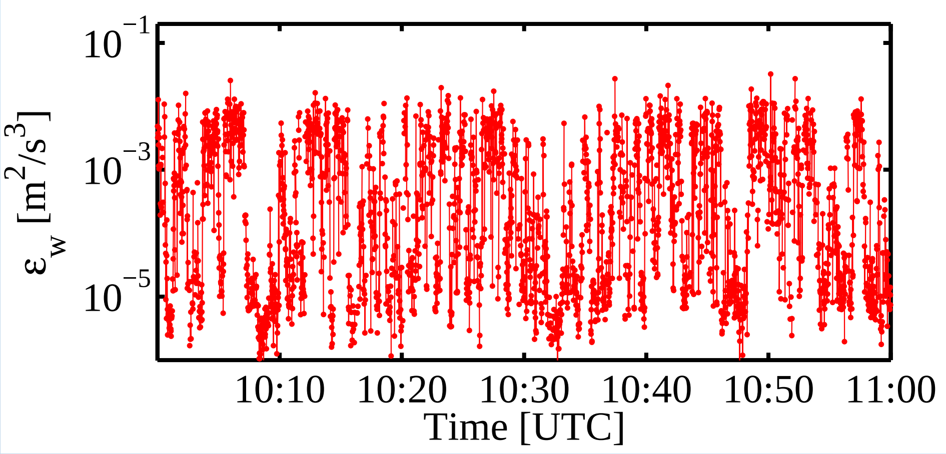


Linking direct and remote observations: Estimate of proportionality factor b

wave energy dissipation = integrated energy dissipation in water column

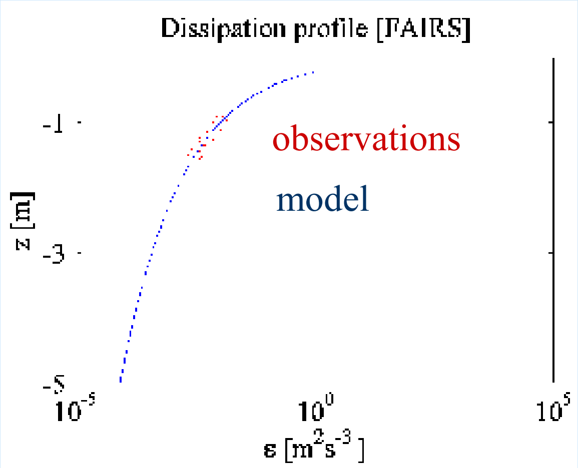
$$\int \varepsilon(c)\,dc = \int \varepsilon_{obs}(t,z)\,dt\,dz \quad \longrightarrow \quad b = \frac{\int \varepsilon_{obs}(t,z)\,dt\,dz}{\int \rho g^{-1} c^5 \Lambda(c)\,dc}$$

Dissipation time series observed at 1 m



Dissipation profile

$\varepsilon \propto z^{-3}$ based on observations



$$b = O(10^{-5})$$

Summary

breaking wave classified by
- speed of advancement
- length of breaking crest

$\Lambda(c)$

- peak wave breaking scale: $0.2 – 0.4\ c_p$

increasing wave age ⇒
- narrowing of distribution
- downward peak shift

onset of wave breaking at saturation threshold

- fixed point total breaking rate R

correlates with *wave nonlinearity*

- spectrally resolved momentum transfer $M(c)$ and energy dissipation $\varepsilon(c)$

dominated by
intermediate wave scales (fully developed seas)
intermediate - large wave scales (developing seas)

- estimate of proportionality factor *b*

$b=O(10^{-5})$

open issues:
- $\Lambda(c)$ of microscale breakers
- functional dependence of b

?