Shell Exploration & Production

Estimating extreme wave design criteria incorporating directionality

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Overview

- Motivation
- Data
- Directional extremal model
- Estimating omni-directional extremes
- Design criteria for directional extremes
- Conclusions

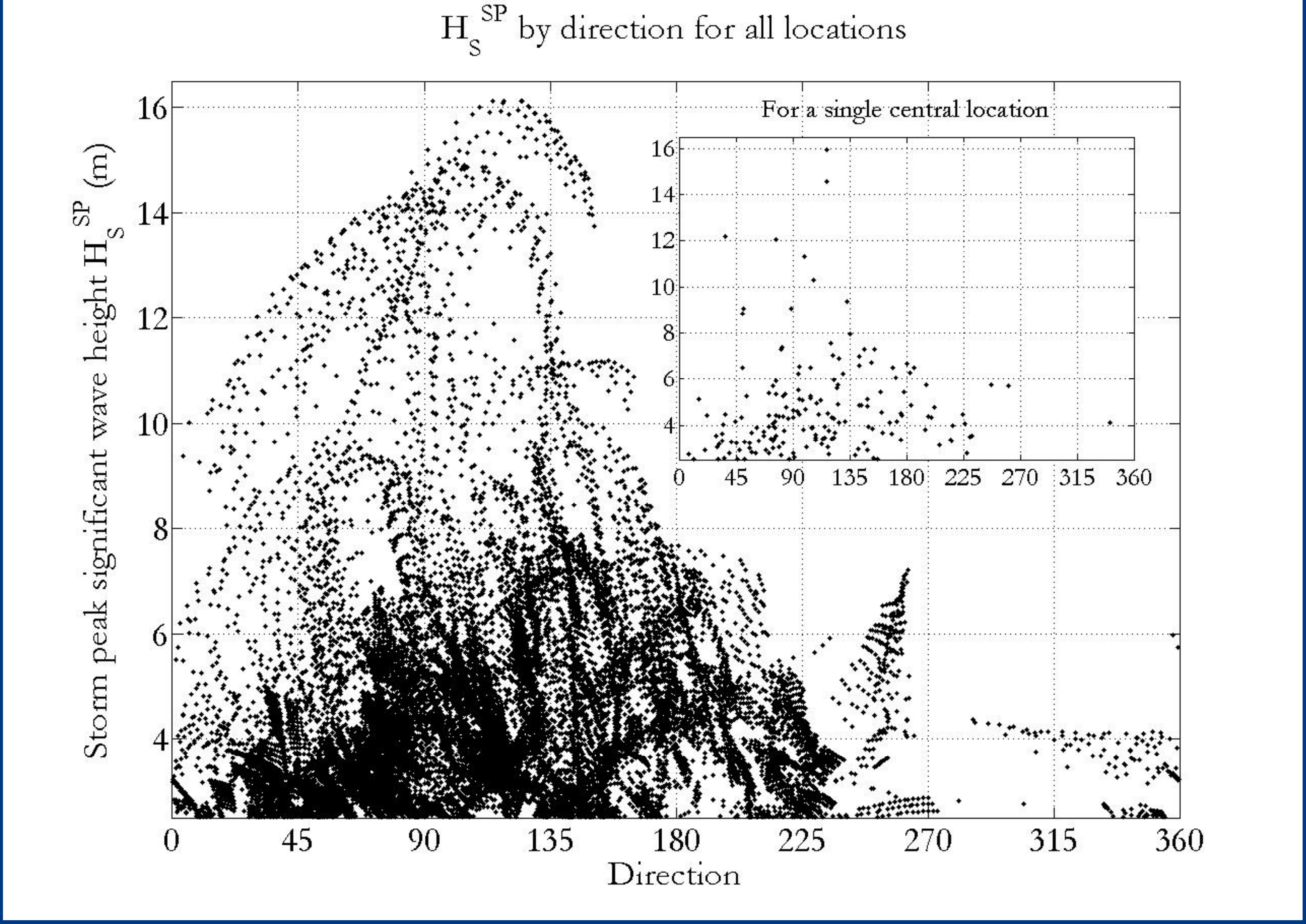
Motivation

- In most regions, but particularly hurricane-dominated regions (e.g. Gulf of Mexico), and in regions where extra-tropical storms prevail (e.g. Northern North Sea), the extremal properties of storms are also highly dependent on storm direction
- Sea state design criteria for offshore facilities are frequently provided by direction to optimise engineering factilities
- Important that these criteria be consistent so that the probability of exceedance of a given wave height from any direction derived from the directional values is the same as for the omni-directional value.
- No consensus on how the criteria should be specified

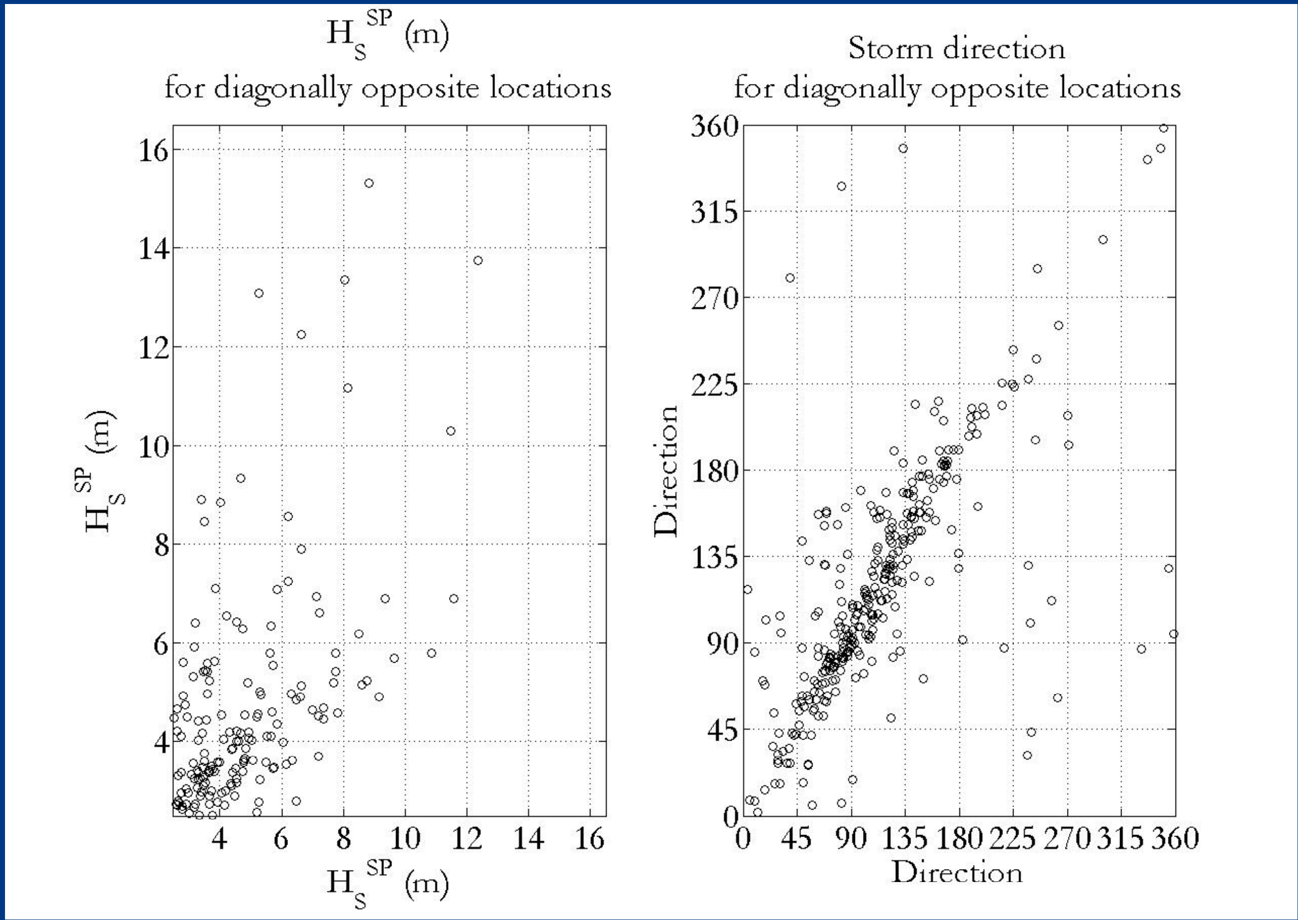
The GOMOS data

- H_s values from GOMOS Gulf of Mexico hindcast study (Oceanweather, 2005)
- September 1900 to September 2005 inclusive
- 30 minute sampling intervals
- 120 grid points on a 15 $\times$ 12 rectangular lattice with spacing 0.125 (14 km)
- For each storm period for each grid point, we isolated a storm peak significant wave height H_s^{sp} and associated direction θ_i
- 315 storms per grid point

The GOMOS data



The GOMOS data



These locations are 250 km apart

The directional extremal model

CDF: $$F_{X_i|\theta_i,u}(x) = P\left(X_i \le x \mid \theta_i, u\right) = 1 - \left(1 + \frac{\gamma(\theta_i)}{\sigma(\theta_i)}(x-u)\right)_+^{-\frac{1}{\gamma(\theta_i)}}$$

$\gamma(\theta_i)$ shape parameter or tail index

$\sigma(\theta_i)$ scale parameter

u threshold (assumed constant with direction)

Parameters characterised by Fourier series expansion (e.g. Robinson & Tawn, 1997)

$$\gamma(\theta) = \sum_{k=0}^{p}\left[A_{11k}\left(\cos(k\theta)\right) + A_{12k}\left(\sin(k\theta)\right)\right]$$

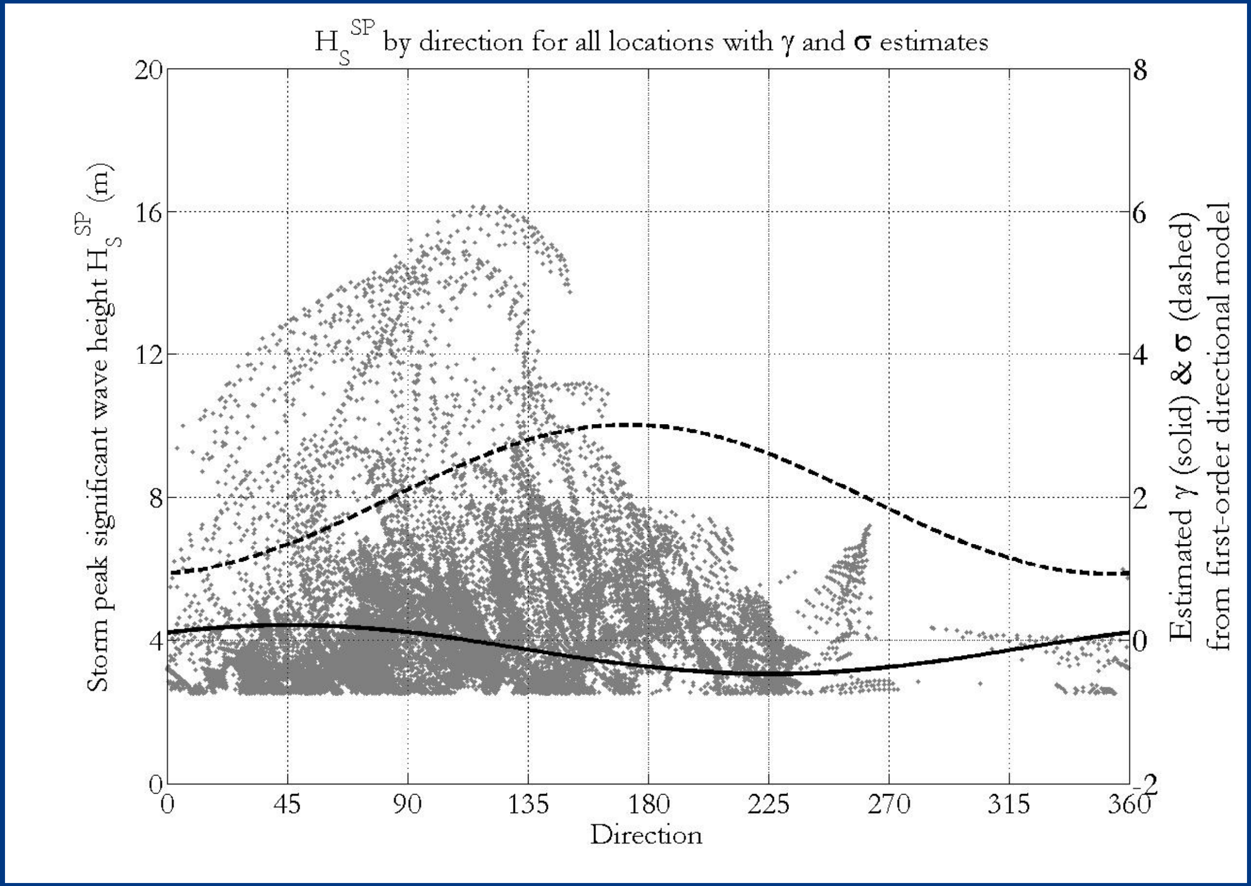
$$\sigma(\theta) = \sum_{k=0}^{p}\left[A_{21k}\left(\cos(k\theta)\right) + A_{22k}\left(\sin(k\theta)\right)\right]$$

Parameters estimated by maximum likelihood

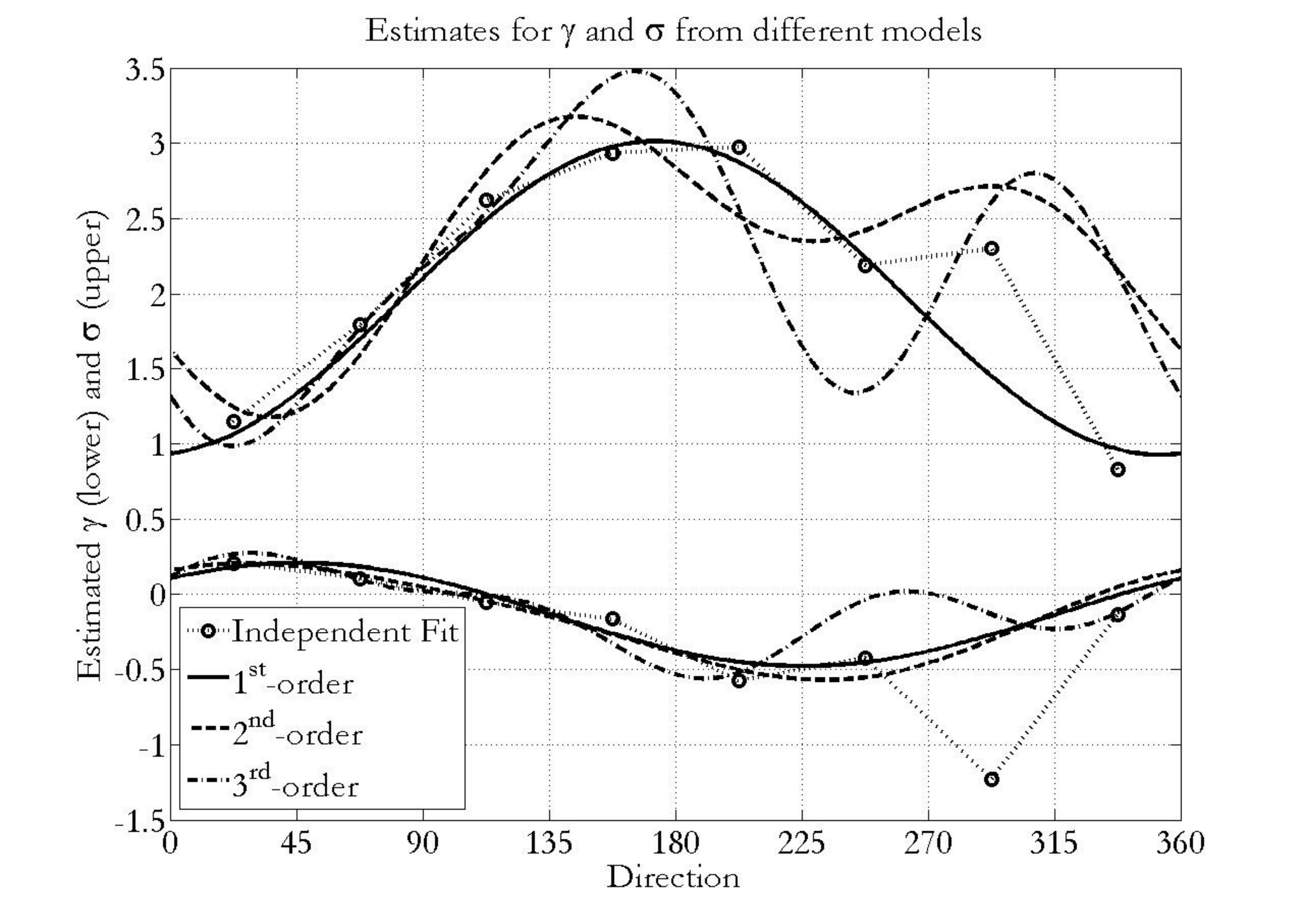
The GOMOS first-order directional model

$$\gamma = -0.13 + 0.24\cos(\theta) + 0.24\sin(\theta)$$

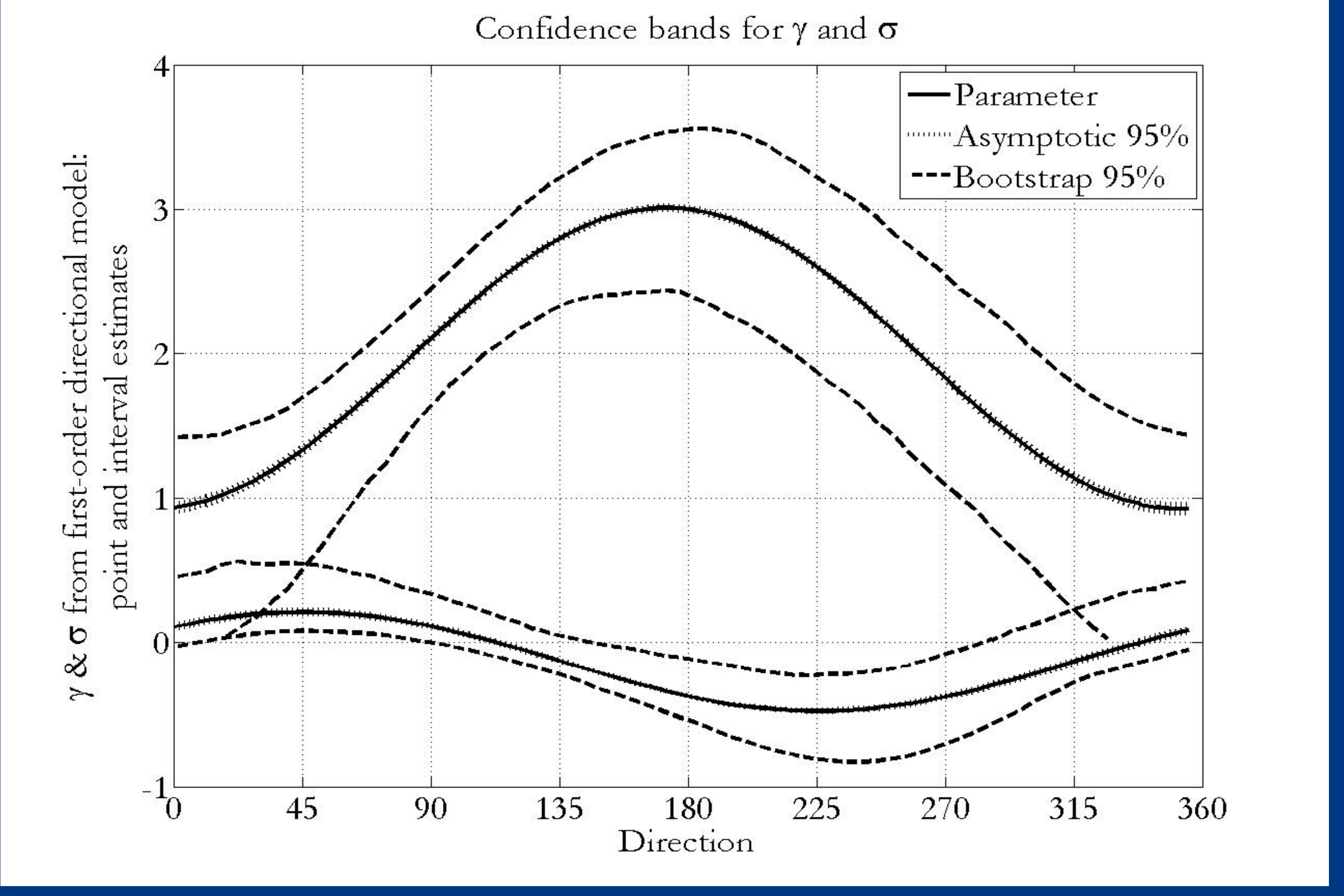
$$\sigma = 1.97 - 1.04\cos(\theta) + 0.14\sin(\theta)$$



Comparison of higher order models



First-order model confidence bands



Estimation of omni-direction Extremes

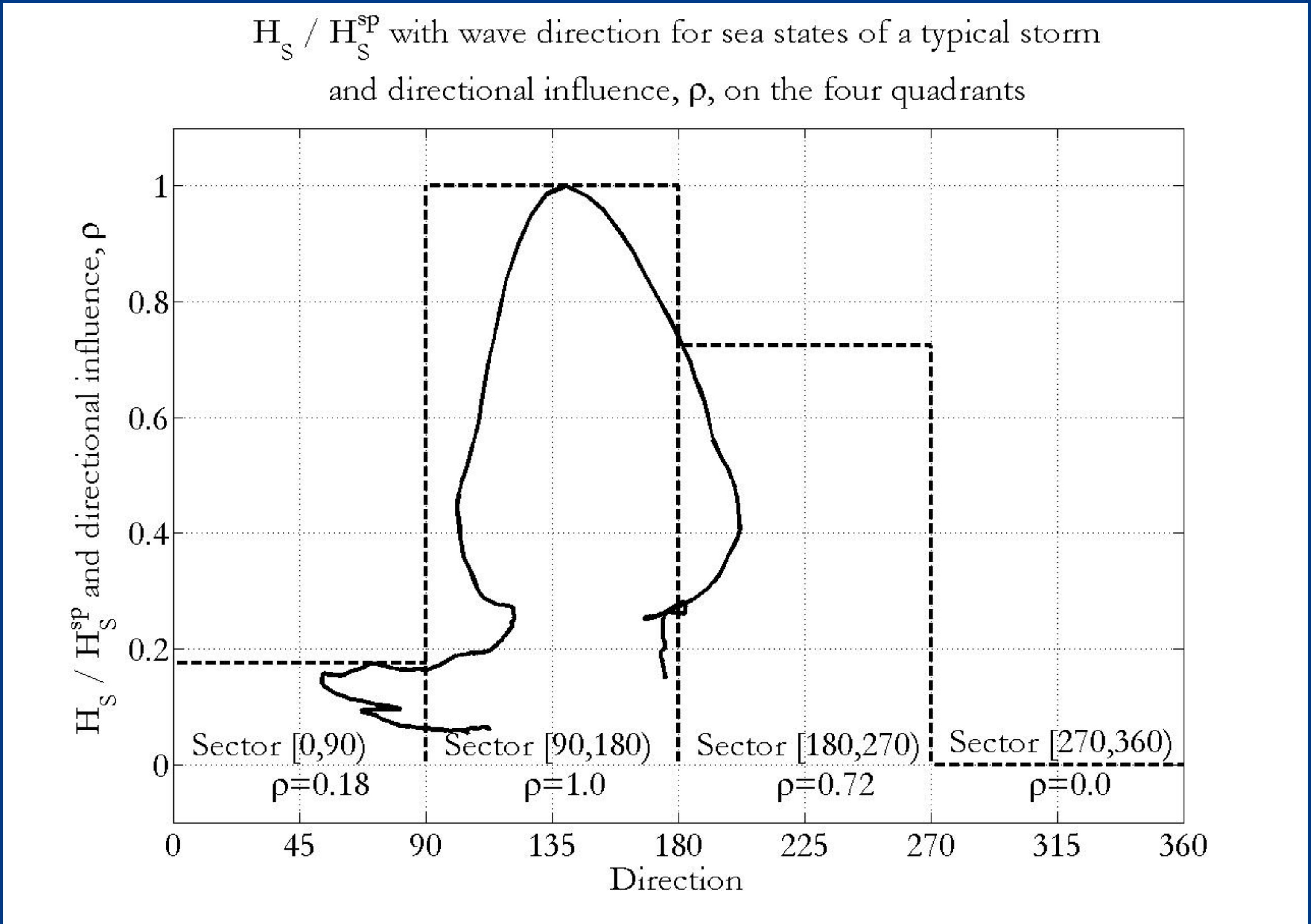
Assume

- Given storm peak direction, θ_i , storm peak H_s above u follows GPD with parameters $\gamma(\theta_i)$ and $\sigma(\theta_i)$

- Storm occurrences are independent Poisson events with expectation $1/P_0$ per annum per storm

- Storm peak directions for any period P are restricted to $\{\theta_i\}_{i=1}^{n}$

Estimation of omni-direction extremes



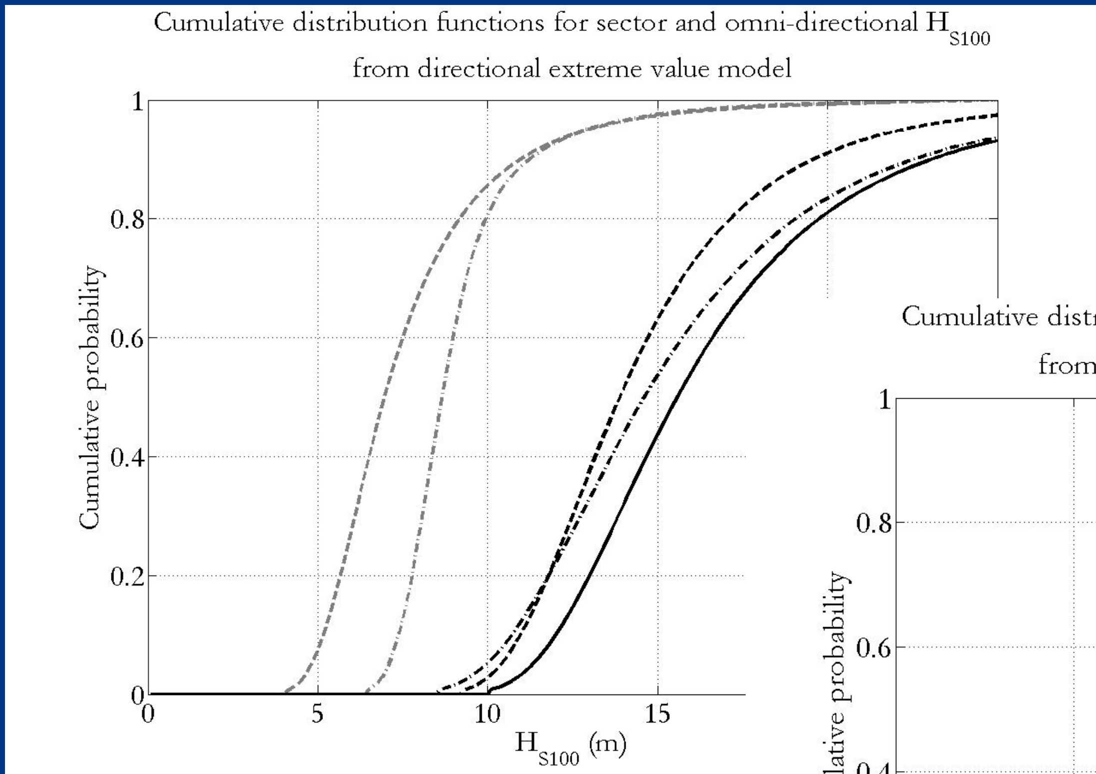
Estimation of omni-direction extremes

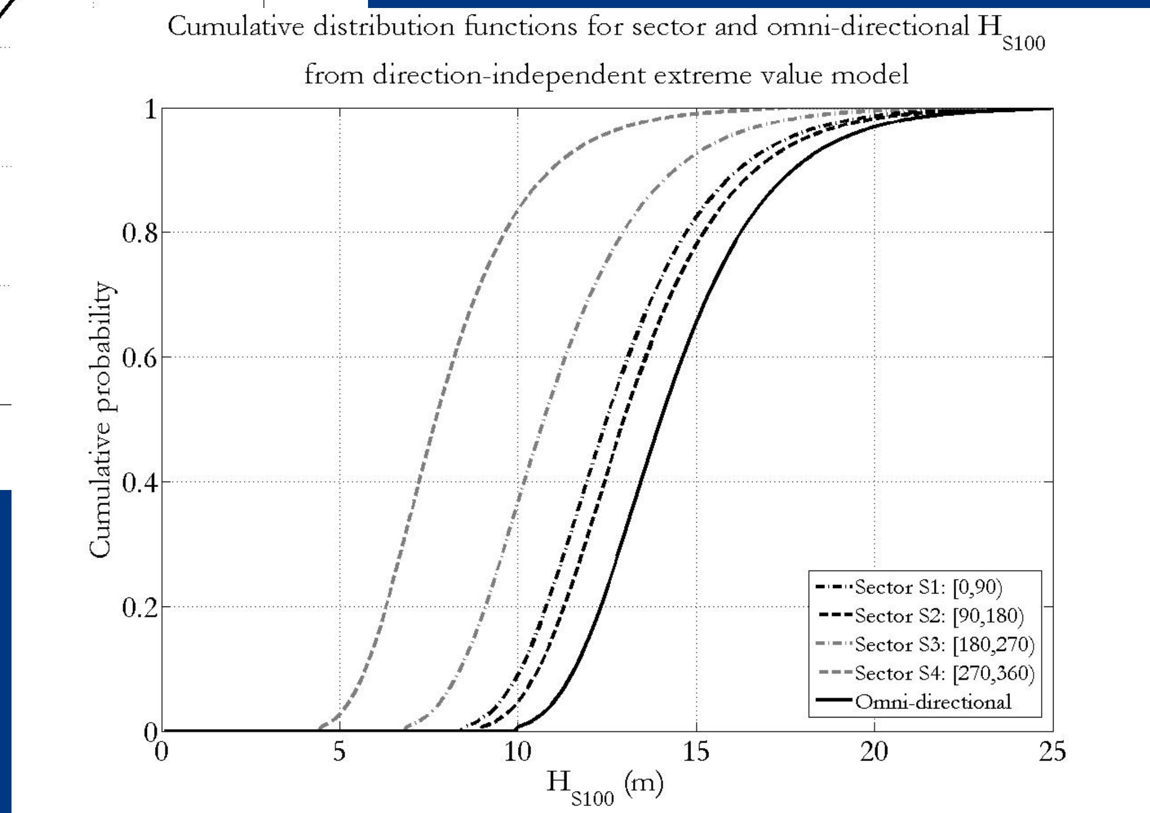
Distribution of the maximum storm H_s within a given sector S in a given period P is given by:

$$
\begin{aligned}
F_{X\max S}(x) &= P\left(X_{\max S} \le x \mid X_i > u \forall i, i \in [1,2,\ldots,n]\right) \\
&= \prod_{i=1}^{n} \left\{ \sum_{k=0}^{\infty} P\left(\rho_i(S) X_i \le x \mid X_i > u, M_i = k\right) P\left(M_i = k\right) \right\} \\
&= \exp\left\{ -m \sum_{i=1}^{n} \left(1 + \frac{\gamma(\theta_i)}{\sigma(\theta_i)} \left(\frac{x}{\rho_i(S)} - u \right) \right)^{-\frac{1}{\gamma(\theta_i)}} \right\}
\end{aligned}
$$

where $m = \dfrac{P}{P_0}$ is the expected value of the number of occurrences of storm which is assumed to follow a Poisson distribution

GOMOS quadrant & omni extremes





Design criteria for directional extremes

Omni-directional 100-yr maximum cumulative probability

$$P\left(X_{\max 100Omni} \le x\right) = \prod_{i=1}^{m} P\left(X_{\max 100S_i} \le x\right)$$

where $P\left(X_{\max 100S_j} \le x\right)$ is cumulative probability for the maximum in the sectors $\{S_i\}_{i=1}^{m}$ in a 100-yr period

The 100-yr design H_s can be calculated for a particular non-exceedance probability $q_{100Omni}$ from

$$q_{100Omni} = P\left(X_{\max 100Omni} \le x_{100Omni}\right)$$

Specification of $q_{100Omni}$ does not allow unique values of the sector 100-yr design H_s

Design criteria for directional extremes

CASE I: All but one sectors have negative shape parameter

Each sector with negative shape parameter has an upper limit of storm peak H_s

Set design values for each of these sectors to the maximum, with $q_{100S_i} = 1$

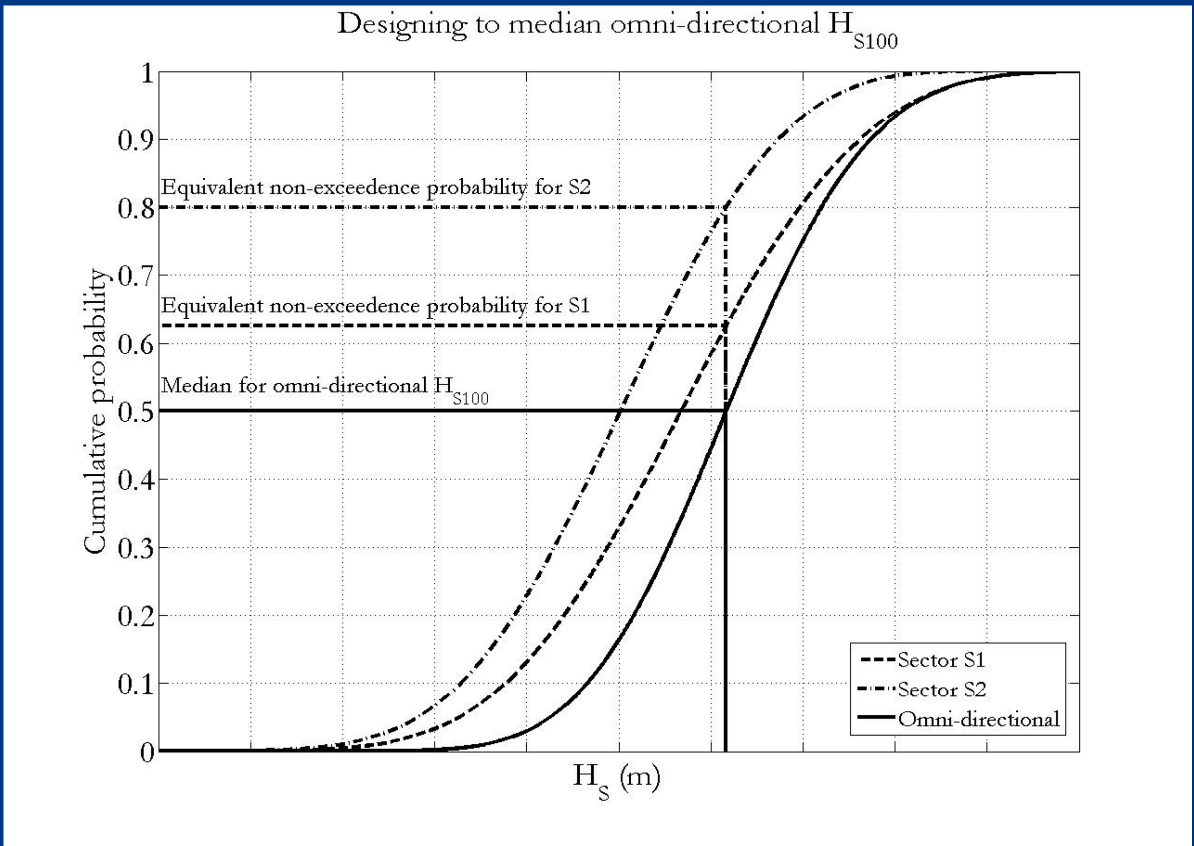
and $$q_{100Omni} = \prod_{i=1}^{m} q_{100S_i} = q_{100S^*}$$

where S^* is the remaining sector with positive shape parameter

Design criteria for directional extremes

CASE II: All sectors set to omni-direction $H_{s100Omni}$

set $$q_{100S_i} = P\left(X_{\max 100S_i} \le x_{100Omni}\right) \quad \forall i$$

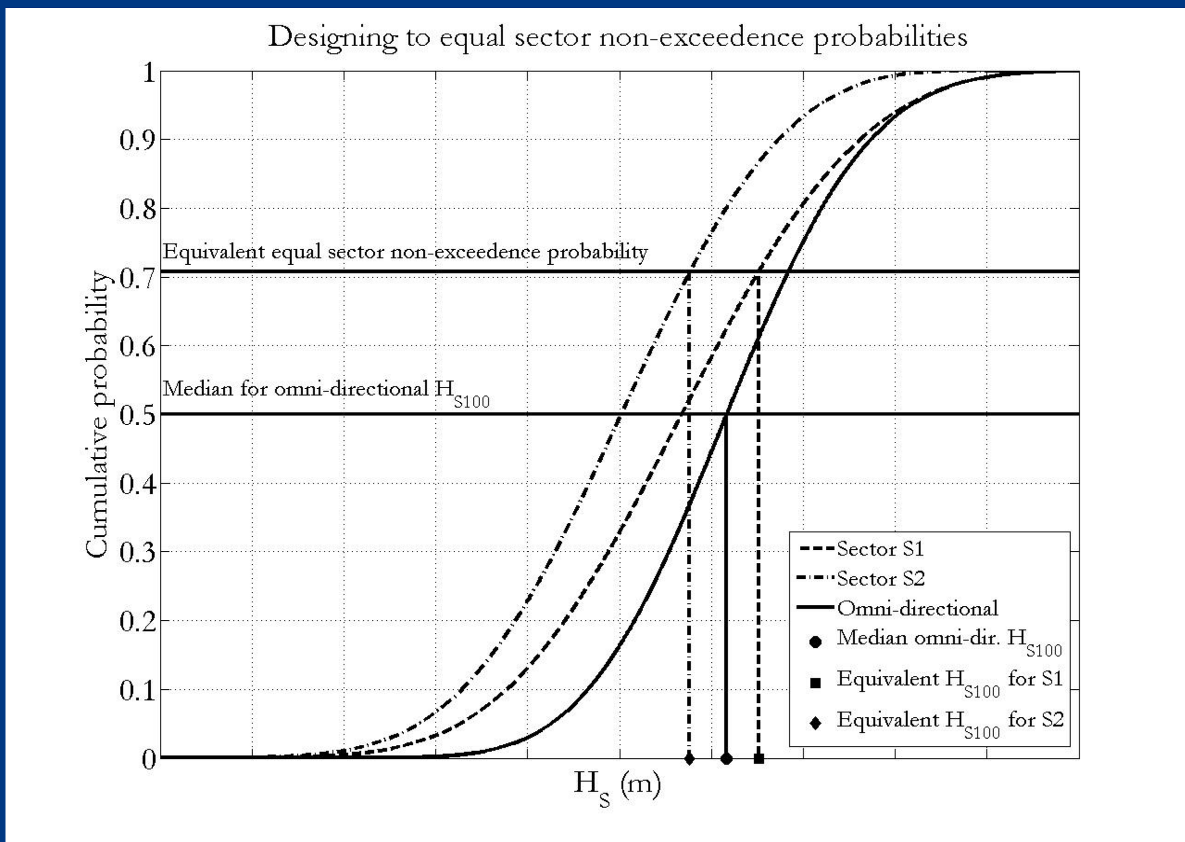


- non-exceedance probabilities different for each sector
- non-exceedance probability for most severe sector less than less severe
- non-exceedance probability for most severe at least as large as omni-direction

Design criteria for directional extremes

CASE III: All sectors have equal non-exceedance probability

set $q_{100Omni} = \prod_{i=1}^{m} q_{100S} = (q_{100S})^m$ where $q_{100S} = (q_{100Omni})^{\frac{1}{m}}$



- sets 100*m* return-period levels for each sector
- most severe sector higher H_{s100S_i} than omni-direction value
- setting the most extreme sector to 100*m*-year level might be "over-conservative"

Design criteria for directional extremes

Risk-cost method

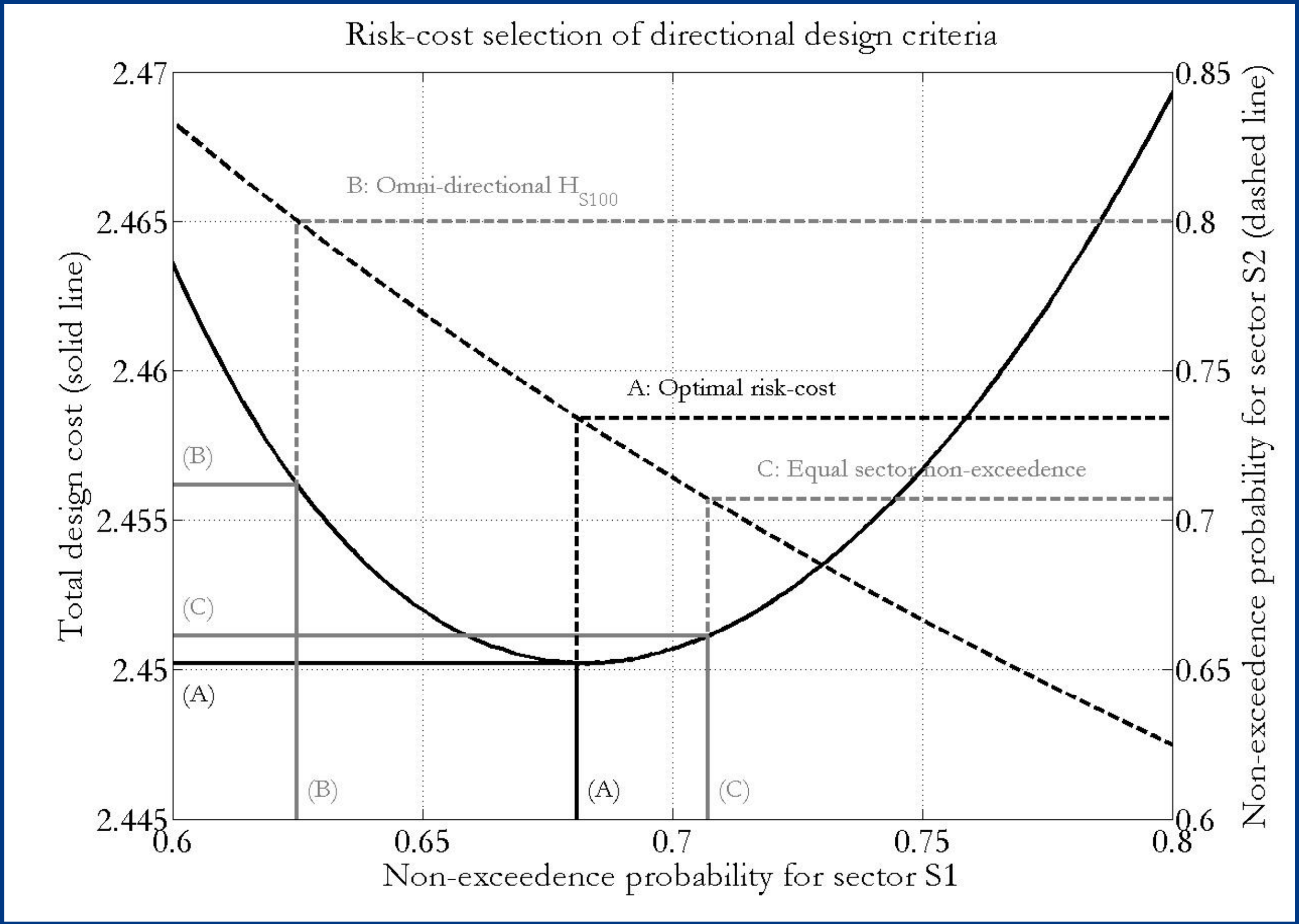
Suppose $c(x)$ is the cost of designing to a storm peak H_s

The overall cost of design is: $$RC = \sum_{i=1}^{m} c\left(x_{100S_i}\right)$$

The optimal design is that which minimises RC subject to $q_{100Omni} = \prod_{i=1}^{m} q_{100S_i}$

Design criteria for directional extremes

Risk-cost method – two sector example $c(x) = 0.001x^2$



- total design cost for $q_{100Omni} = 0.5$
- optimal design cost balance between "omni-directional design" and "equal non-exceedances design"

Design criteria for directional extremes

Design criteria based on median omni-directional $H_{s100Omni}$

		Risk-cost optimal			Omni-direction			Equal sector non-exceedance		
	Sector	RC	x_{100S_i}	q_{100S_i}	RC	x_{100S_i}	q_{100S_i}	RC	x_{100S_i}	q_{100S_i}
Directional Model	[0,90)	8.78	18.03	0.75	9.73	15.60	0.59	9.29	20.20	0.84
	[90,180)	8.78	17.40	0.81	9.73	15.60	0.69	9.29	17.90	0.84
	[180,270)	8.78	11.44	0.91	9.73	15.60	0.98	9.29	10.30	0.84
	[270,360)	8.78	10.90	0.90	9.73	15.60	0.98	9.29	9.70	0.84
Direction-independent model	[0,90)	7.56	15.00	0.82	7.84	14.00	0.72	7.59	15.20	0.84
	[90,180)	7.56	15.40	0.82	7.84	14.00	0.66	7.59	15.70	0.84
	[180,270)	7.56	13.41	0.84	7.84	14.00	0.88	7.59	13.40	0.84
	[270,360)	7.56	10.67	0.89	7.84	14.00	0.98	7.59	10.10	0.84

- Directional model design values > Direction independent values
- RC model avoids large range of q_{100S_i} and x_{100S_i} of other two methods
- RC model design based on directional EV model is preferable

Conclusions

- Strong general case for adopting directional extreme model to storm peak [unless it can be demonstrated statistically that a direction free model is no less appropriate].
- A directional extremes model [e.g. Fourier series expansion] allows directionally consistent extreme values to be derived.
- Important to consider directionality of sea states when developing design H_s criteria [omni-directional extremes from directional data can be significantly different from a direction-independent derivation].
  - Even when the extremal characteristics of storm peak H_s are direction independent rate of occurrence of storms is dependent on storm direction – the distribution of H_{s100} will have directional dependence in general.
- Risk-cost method is an objective approach to optimise directional criteria, while preserving overall reliability [RC method avoids more extreme properties of other design methods].