Evaluation of diffraction behind a semi-infinite breakwater in the SWAN wave model

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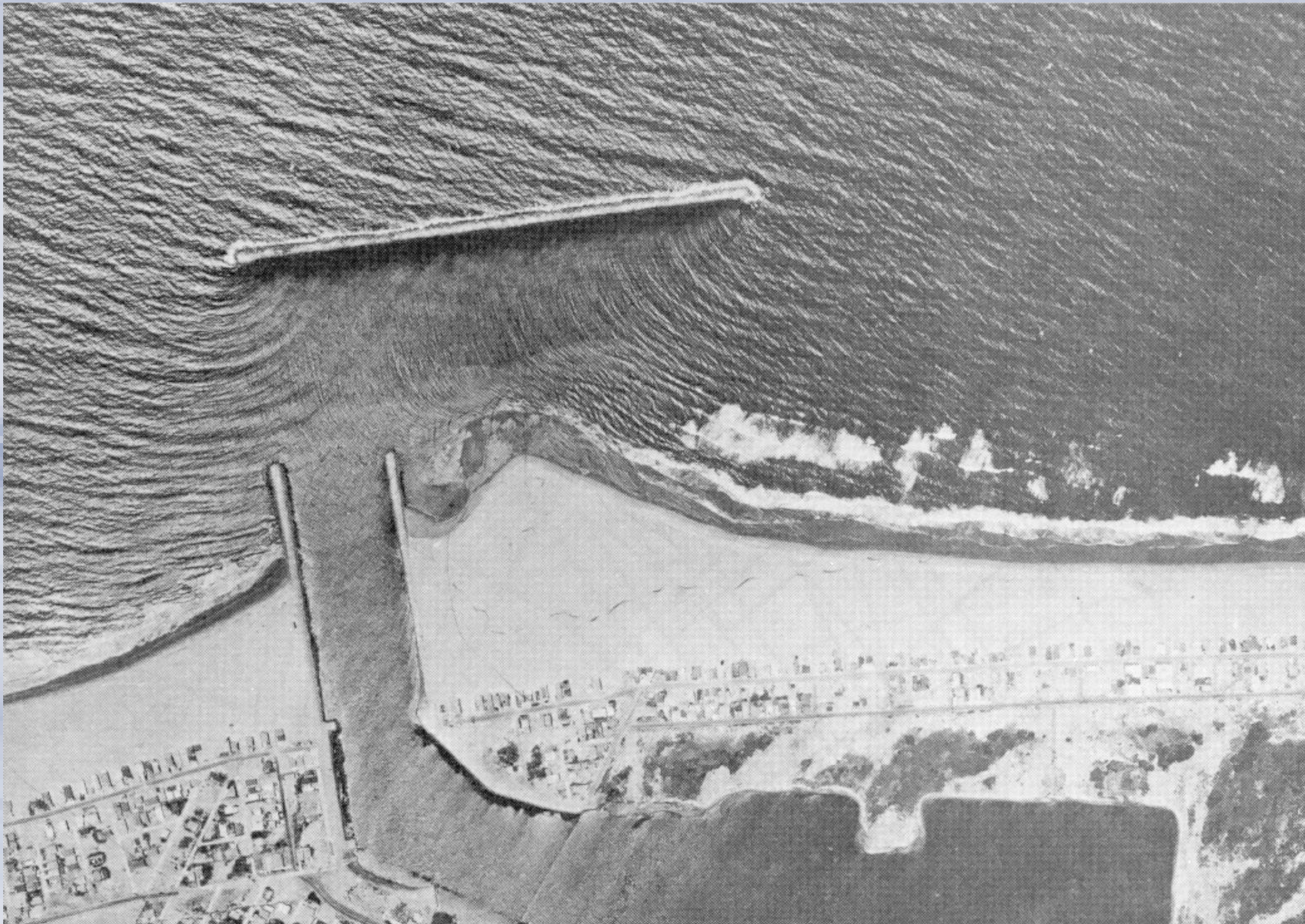
Problem statement

- How effective is the diffraction formulation in SWAN?
- When is the diffraction important?



Diffraction behind Nukuoro atoll

Introduction

- **Wave penetration around breakwater is an important item of harbor design**
- **Diffraction inside harbor usually computed with phase-resolving models (mild-slope, Boussinesq)**
- **However spectral models are more efficient, so attempts have been made to include diffraction in models like SWAN.**

Correction terms to velocities in action balance equation

(Rivero et al. 1997; Holthuijsen et al., 2003)

$$\frac{\partial N}{\partial t} + \frac{\partial (c_x N)}{\partial x} + \frac{\partial (c_y N)}{\partial y} + \frac{\partial (c_\sigma N)}{\partial \sigma} + \frac{\partial (c_\theta N)}{\partial \theta} = \frac{S}{\sigma}$$

$$\vec{C}_g = \vec{c}_g \left(1 + \boldsymbol{\delta_E}\right)^{1/2} + \vec{U}$$

$$c_\theta = -\left[c_g \left(1 + \boldsymbol{\delta_E}\right)^{1/2} \left(\frac{1}{\kappa}\frac{\partial \kappa}{\partial m} + \frac{1}{2\left(1 + \boldsymbol{\delta_E}\right)}\frac{\partial \boldsymbol{\delta_E}}{\partial m} \right) + \frac{\vec{\kappa}}{\kappa}\frac{\partial \vec{U}}{\partial m} \right]$$

$$c_\sigma = \frac{\partial \sigma}{\partial d}\left[\frac{\partial d}{\partial t} + \vec{U} \cdot \nabla d \right] - c_g \vec{\kappa} \cdot \frac{\partial \vec{U}}{\partial s} \left(1 + \boldsymbol{\delta_E}\right)^{1/2}$$

Diffraction parameter δ_E

$$\boldsymbol{\delta_E} = \frac{\nabla \cdot \left(c c_g \nabla \sqrt{E} \right)}{\kappa^2 c c_g \sqrt{E}}$$

Diffraction parameter δ_E estimated from smoothed wave energy distribution in surrounding grid points

$$E_{i,j}^{n} = E_{i,j}^{n-1} - \mathbf{0.2} \times \left[E_{i-i,j} + E_{i,j-1} - 4E_{i,j} + E_{i+1,j} + E_{i,j+1} \right]^{n-1}$$

Smoothing performed *n* times

Methodology

- **Domain selected:**
  - **Simple rectangular domain**
  - **Semi-infinite and impermeable breakwater**
  - **Constant water depth**
- **Base case:**
  - **wave traveling from left to right, H=1m**
  - **Constant depth of 10m**
  - **T ~ 8s**

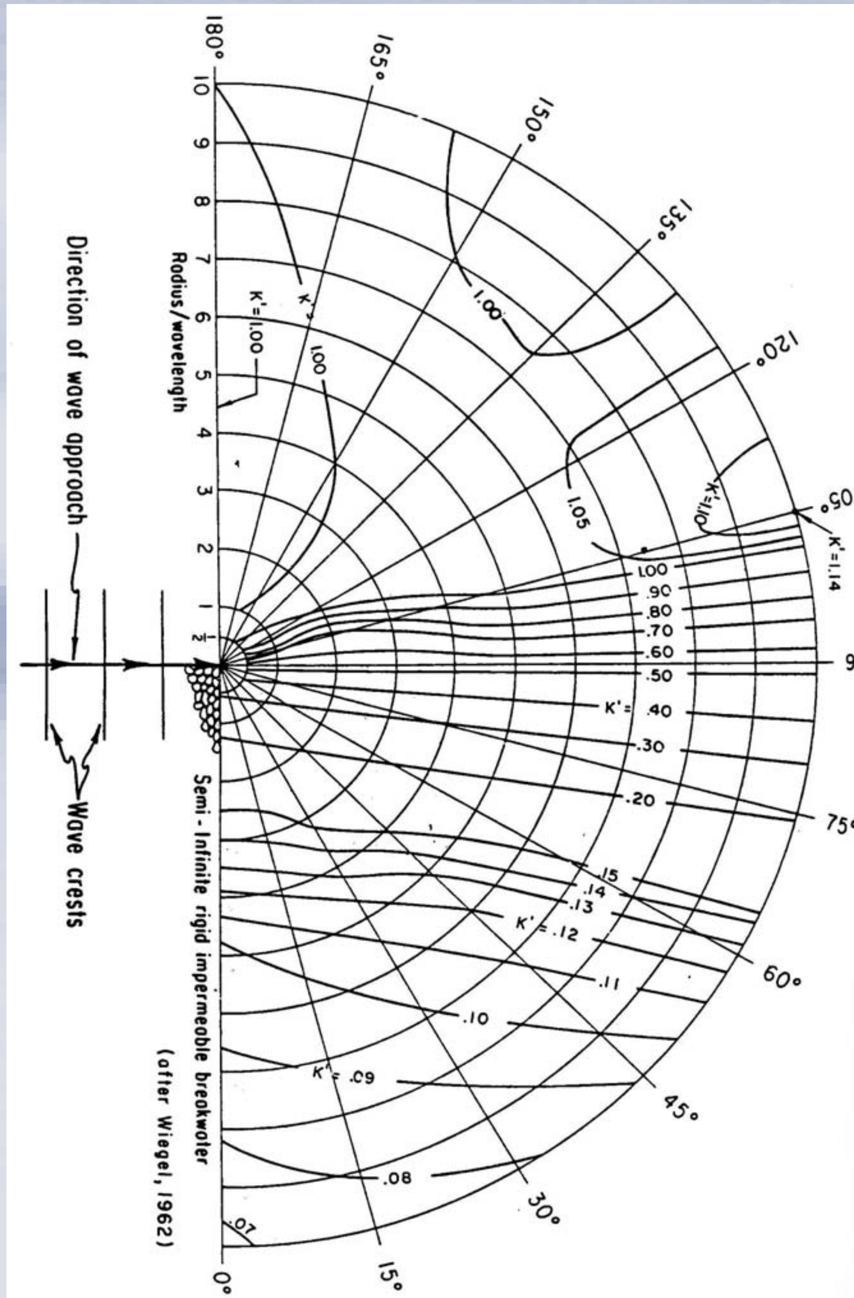
Cases used

- **Directional spreading: three cases**
  - **σ = 1.5° (uni-directional)**
  - **σ = 10° (swell)**
  - **σ = 30° (wind waves)**

(See Kuik et al., 1988 for definition of σ)

Approach

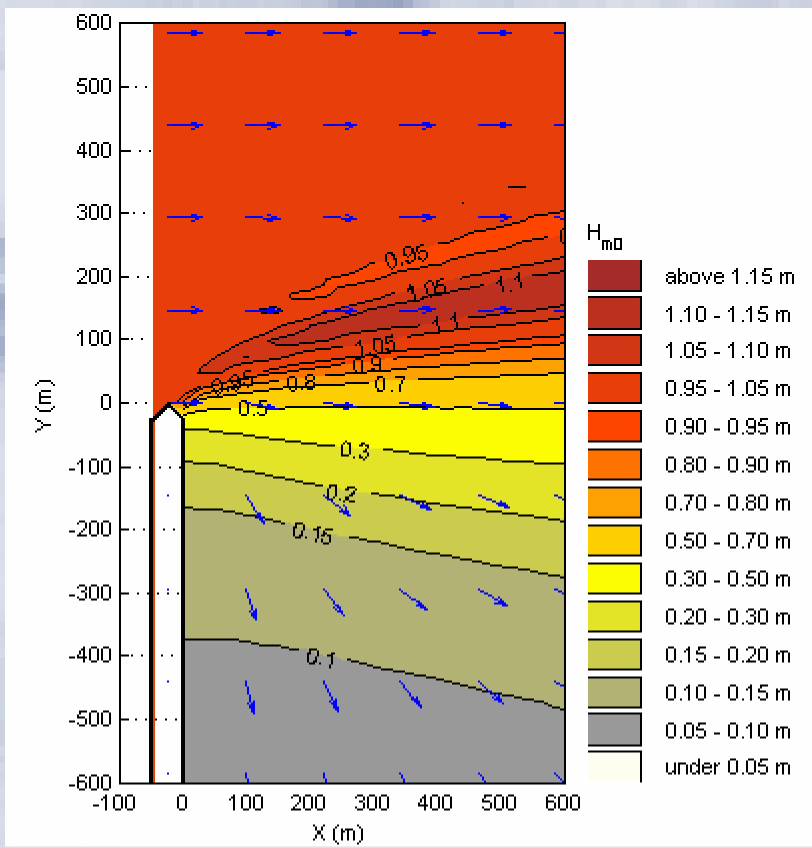
- Phase 1: sensitivity analysis over following parameters:
  - Spatial resolution: Δs
  - Size of the computational domain
  - Smoothing parameters
- Phase 2: validation of the diffraction approximation by:
  - Comparing SWAN results with analytical solutions
  - Determining the areas where diffraction is important
  - Highlighting how much directional spreading is important



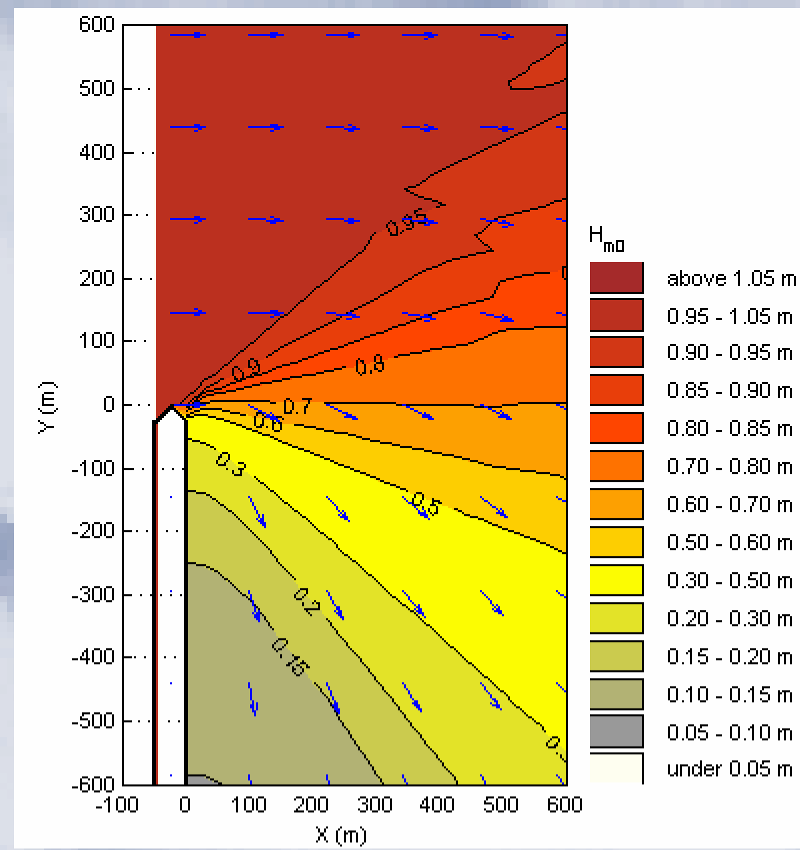
Analytical solution: Wiegel (1962)

- Introduction
- Methodology
- **Results**
- Conclusions

Results: analytical benchmarks



H_{m0} ($H_{incident}$ = 1.0 m, T = 8 s, θ = 0° and σ = 1.5°)

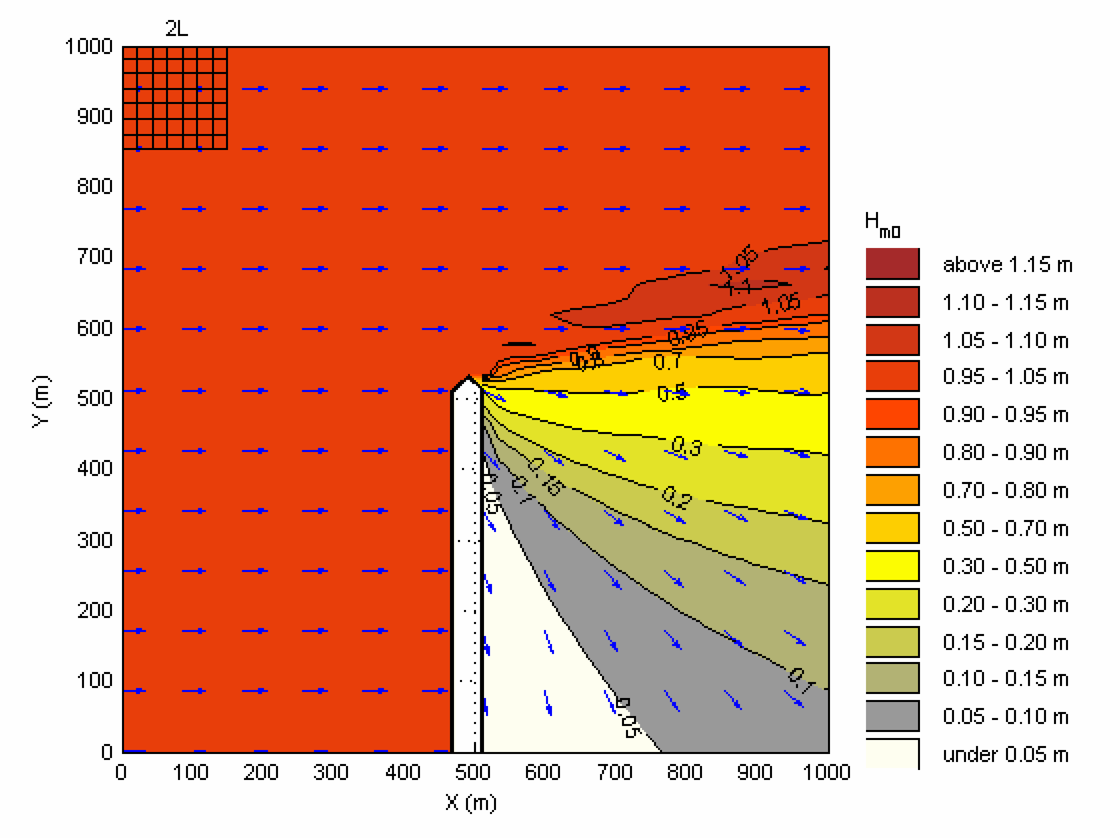


H_{m0} ($H_{incident}$ =1.0 m, T = 8 s, θ= 0° and σ=30°)

Influence of spatial resolution (1)

- **Wide range of wave periods (T) covered**
- **Various water depths**
- **Results become unstable for $L/\Delta s \geq 3.5$**
- **Spatial resolution significant for accuracy**

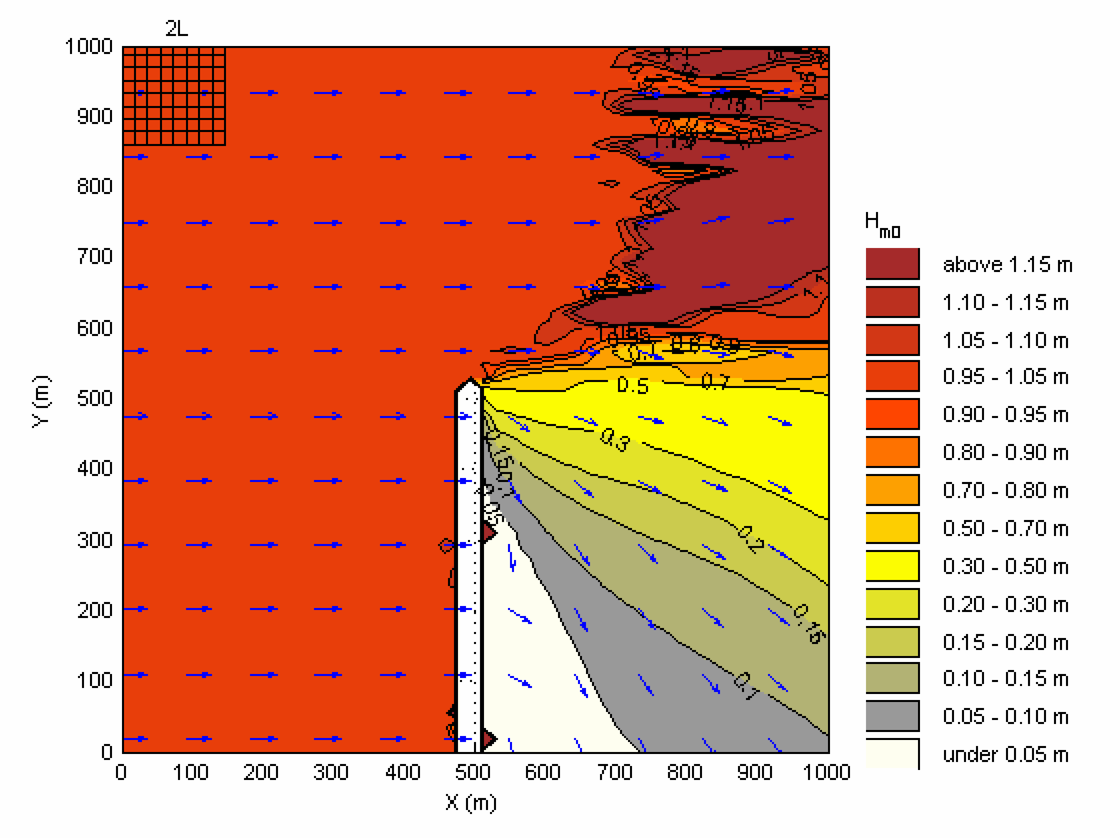
Influence of spatial resolution: stability (2)



H_{m0} , Resolution *L/Δs=3.5*. (depth = 11 m, $H_{incident}$=1.0 m, *T* = 8 s, θ=0° and σ=1.5°)

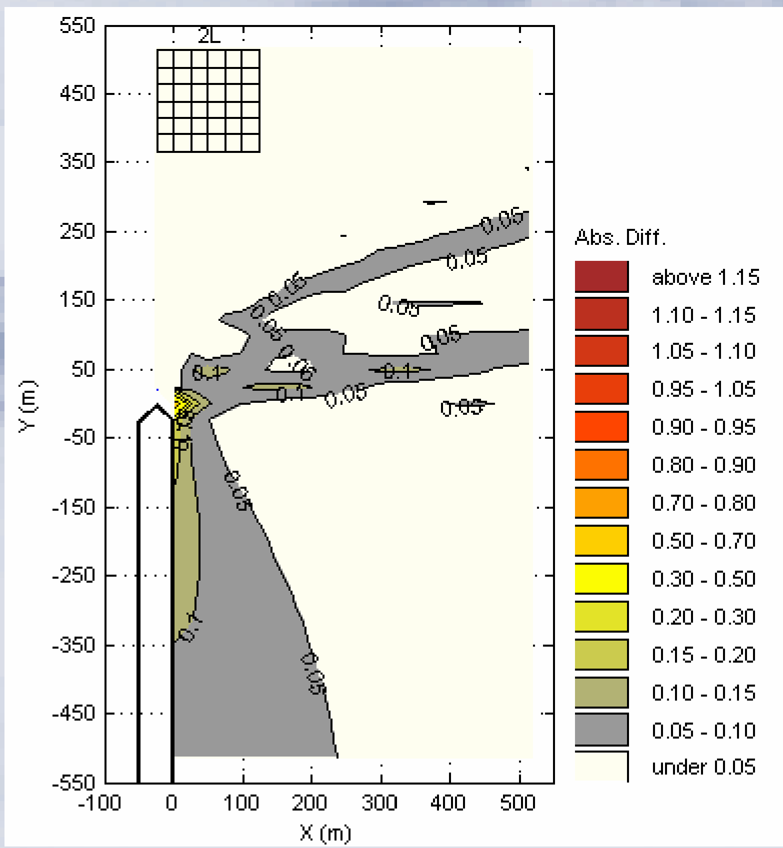
Influence of spatial resolution: stability (3)

- Introduction
- Methodology
- **Results**
- Conclusions

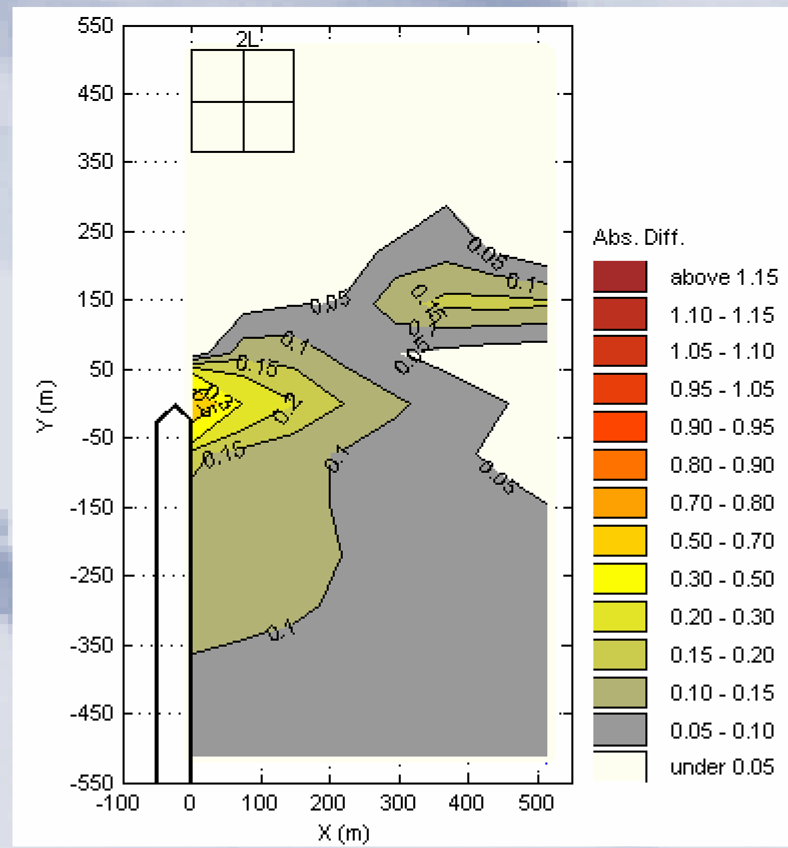


H_{m0} , Resolution $L/\Delta s$=4. (depth = 11 m, $H_{incident}$=1.0 m, T = 8 s, θ=0° and σ=1.5°)

Influence of spatial resolution: accuracy (4)



H_{SWAN}-H_{ANAL}, *L/Δs* =3.0

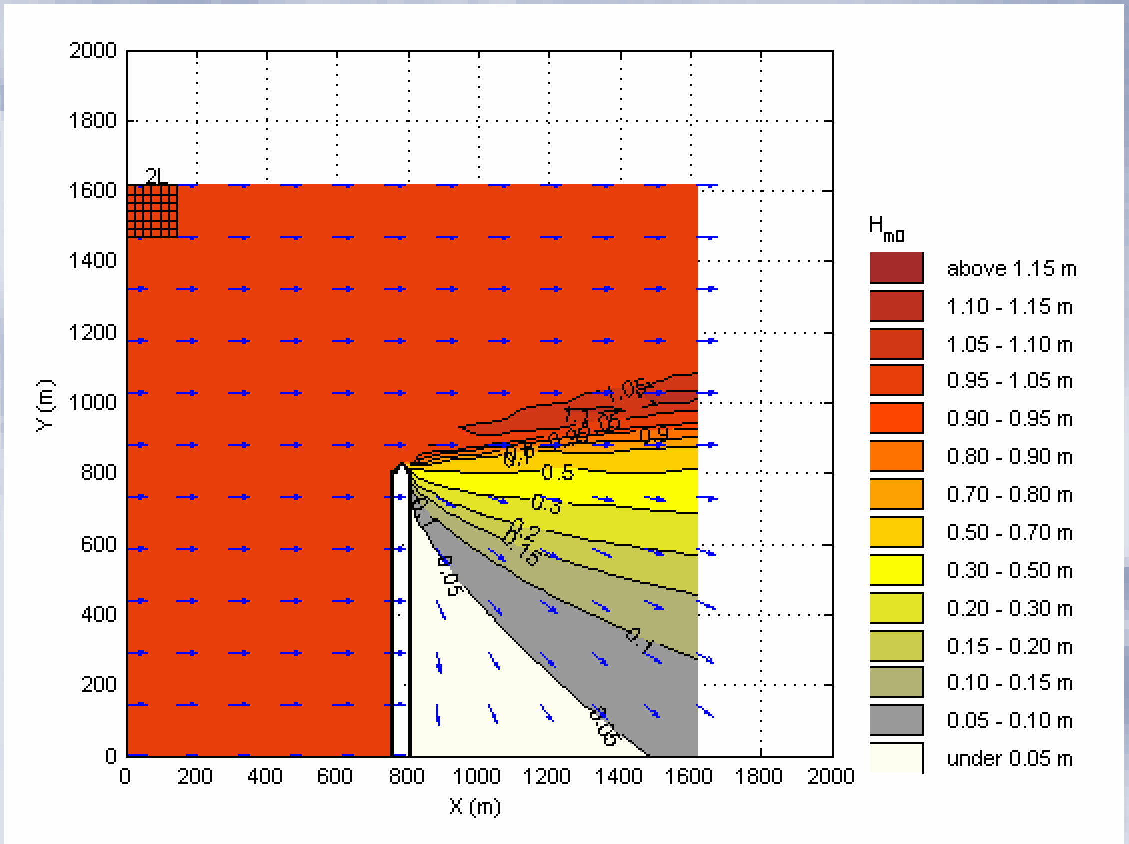


H_{SWAN}-H_{ANAL}, *L/Δs* =1.0

Size of the computational domain (1)

- **For low directional spreading (σ=1.5°) instabilities occur if domain contains too many points (max: 65 pts * 65 pts)**
- **For larger directional spreading (σ= 30°), no limitation**

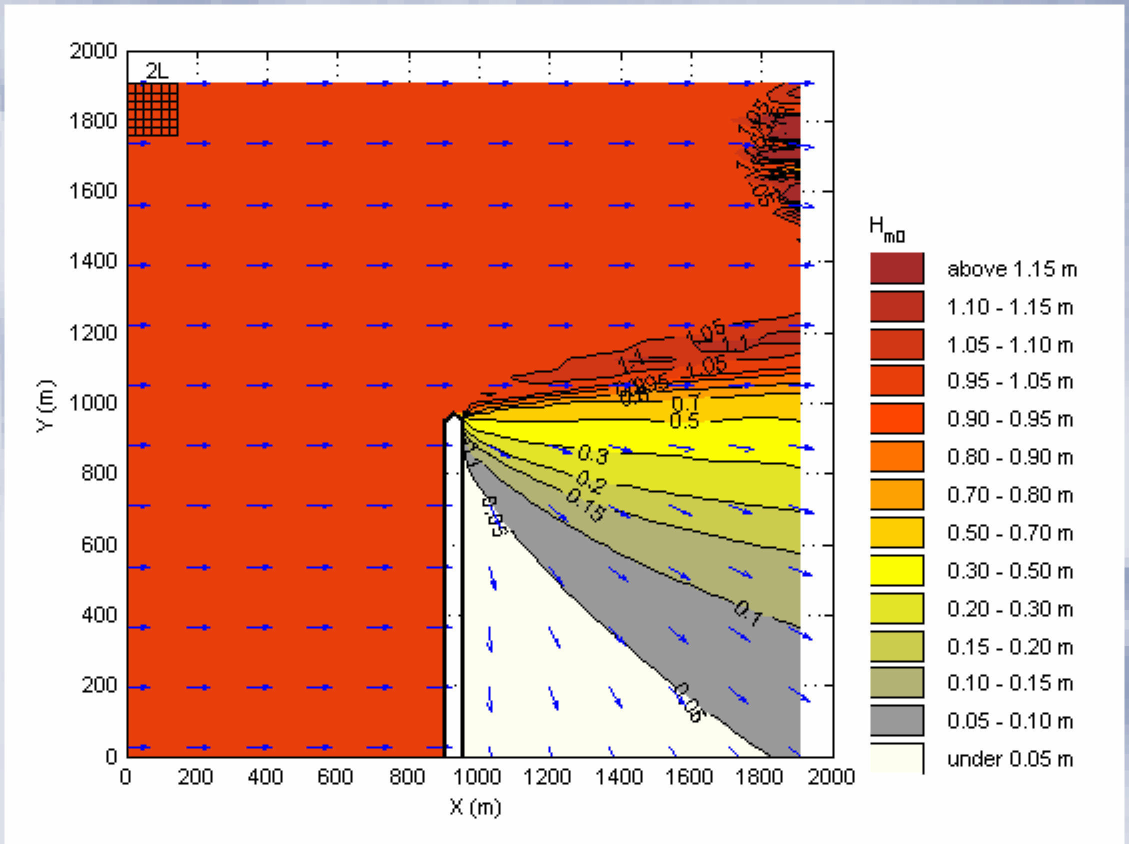
Size of the computational domain (2)



H_{m0} , domain 22 by 22 wavelengths, *L/Δs*=3

Size of the computational domain (3)

- Introduction
- Methodology
- **Results**
- Conclusions

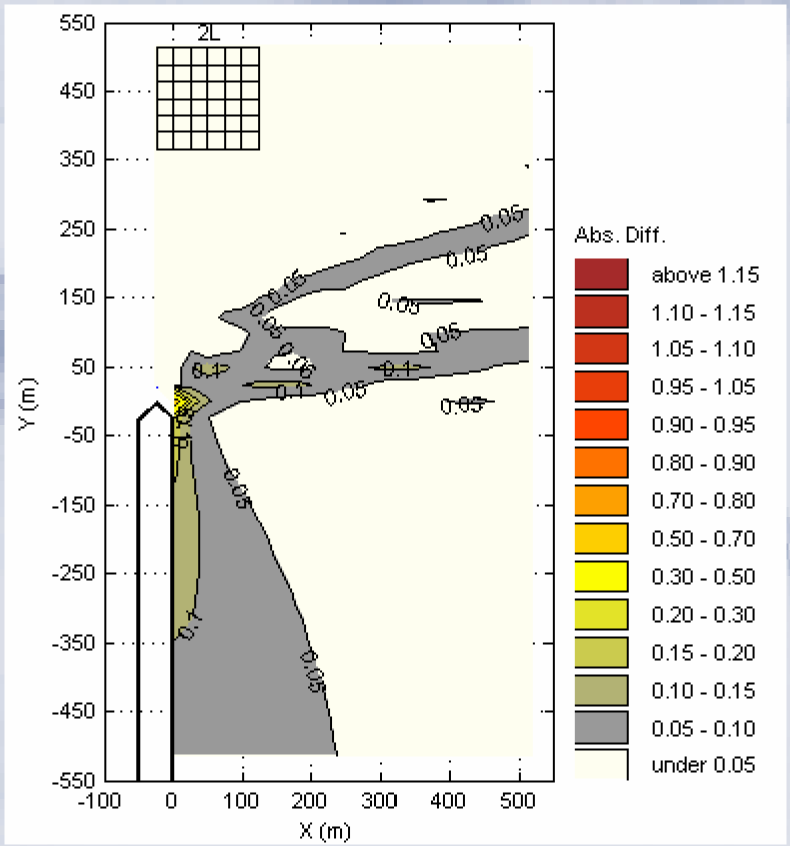


H_{m0} , domain 26 by 26 wavelengths, $L/\Delta s$=3

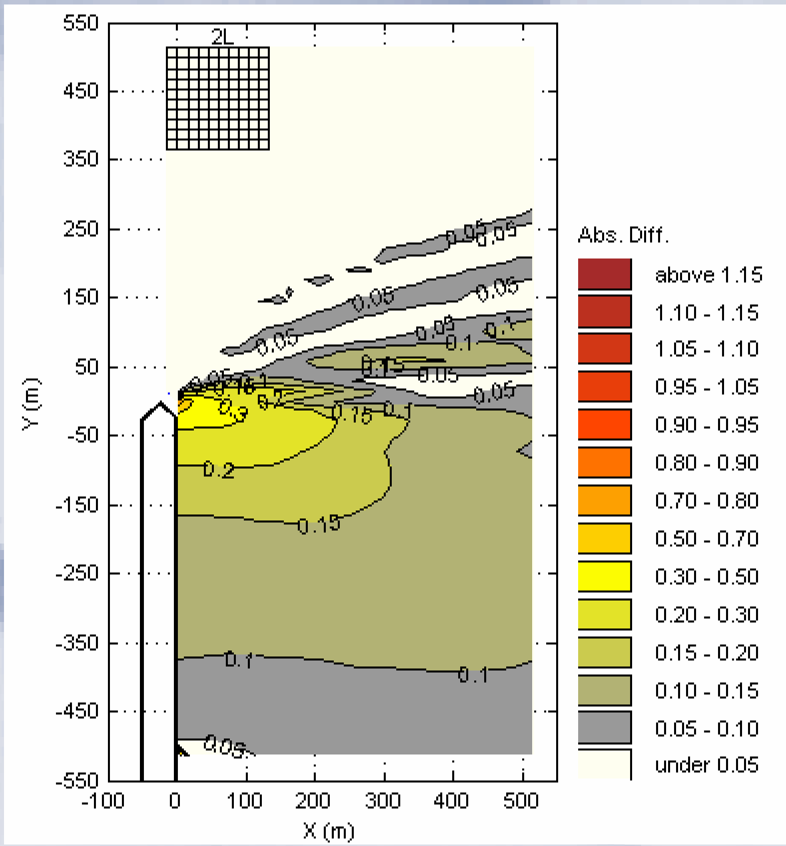
Smoothing (1)

- **Stable results are obtained for higher spatial resolution $L/\Delta s = 5$**
- **But results are worser than case without smoothing when compared with analytical solution**

Smoothing (2)



H_{SWAN}-H_{ANAL}, no smoothing, $L/\Delta s$ =3.0

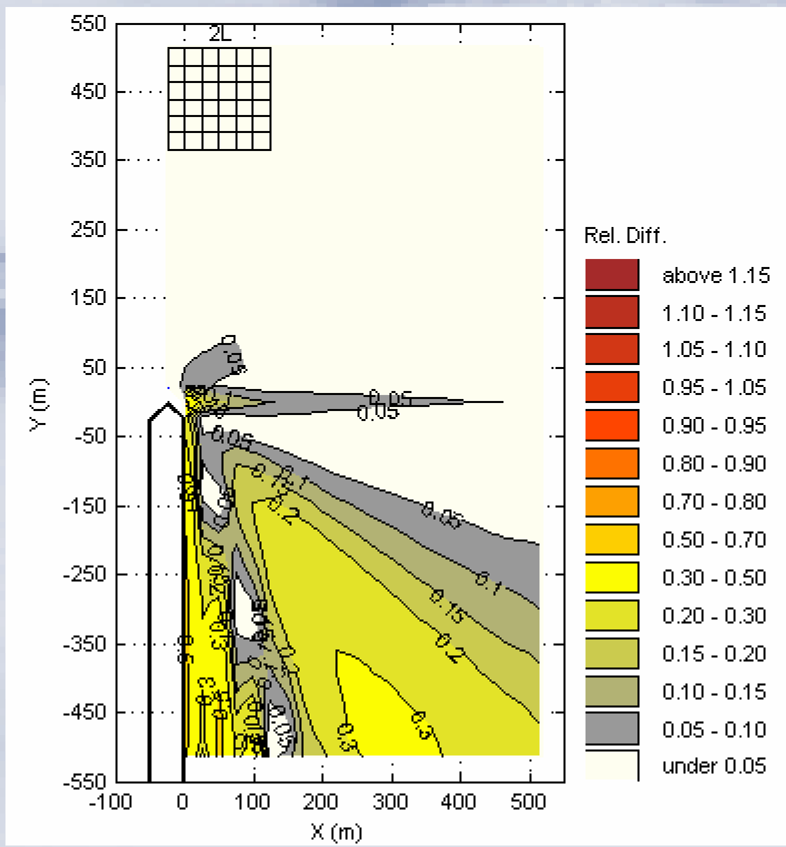


H_{SWAN}-H_{ANAL}, smoothing (SMNUM=6), $L/\Delta s$ =3.0

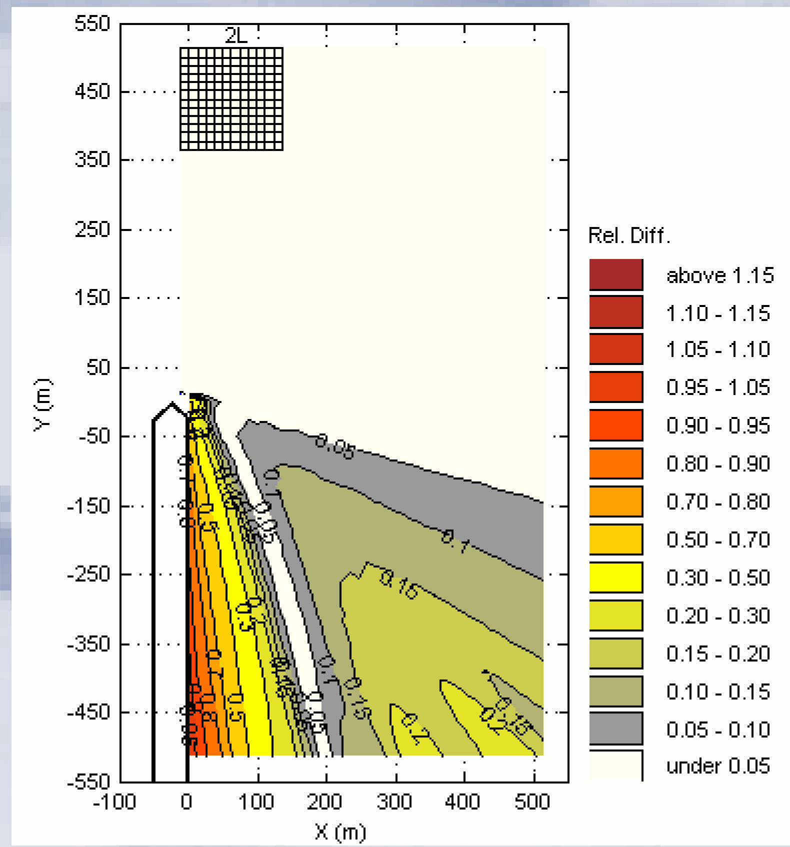
Requirement for diffraction (1)

- **For narrow directional spectra, diffraction should be used**
- **For wide directional spectra, including diffraction leads to worser results**

Requirement for diffraction: wide directional spectrum (2)

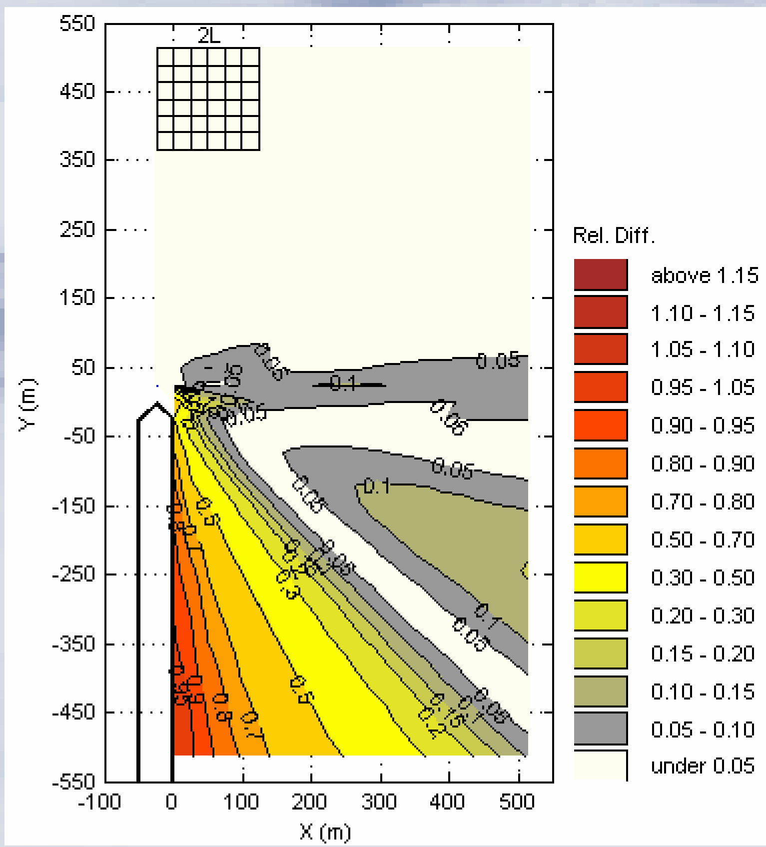


$(H_{SWAN}-H_{ANAL})/H_{ANAL}$, diffraction included, $\sigma = 30°$

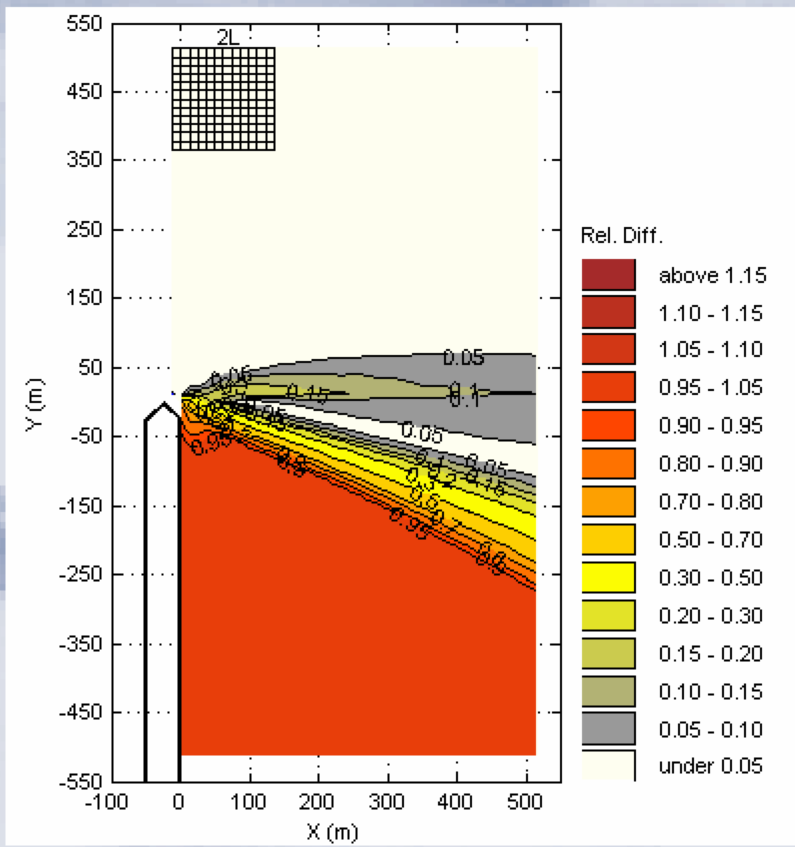


$(H_{SWAN}-H_{ANAL})/H_{ANAL}$, no diffraction, $\sigma = 30°$

Requirement for diffraction: narrow directional spectrum (3)



$(H_{SWAN}-H_{ANAL})/H_{ANAL}$, **diffraction included**, $\sigma = 10°$



$(H_{SWAN}-H_{ANAL})/H_{ANAL}$, **no diffraction**, $\sigma = 10°$

Conclusions

- *L/Δs* <=3.5 for stability
- Computational domain size < 65 by 65 points for narrow directional spectra
- Smoothing gives more stable results, but too much smoothing gives worser results
- Effect of directional spreading decreases the importance of diffraction for simple cases with broad directional spectra, and better results are obtained if diffraction is not included

Suggestions of future work

- **Investigate the role of directional resolution**
- **Investigate the effects of diffraction in a channel by comparing SWAN with a mild-slope (or other type) model**
- **Investigates a real case study: harbor area and compare with measurements**

References

- Wiegel, R.L., 1962: Diffraction of Waves by a Semi-infinite Breakwater. Journal of the Hydraulics Division, 88, No. HY1, Jan. 1962, pp 27-44.

- Holthuijsen, L.H., A. Herman, and N. Booij, 2003: Phase-decoupled refraction-diffraction for spectral wave models. Coastal Engineering, 49, 291-305.

- Rivero, F.J., A.S. Arcilla, and E. Carci, 1997: Analysis of diffraction in spectral wave models. Proc. 3rd Int. Symp. Ocean Wave Measurement and Analysis, WAVES '97, ASCE, New York, 431-445.

- Kuik, A.J., G.Ph. van Vledder, and L.H. Holthuijsen, 1988: A method for the routine analysis of pitch-and-roll buoy wave data. J. Phys. Oceanogr., 18, 1020-1034.


Questions ?