

TOWARDS AN OPTIMAL COMPUTATION OF NON-LINEAR FOUR-WAVE INTERACTIONS IN OPERATIONAL WAVE MODELS

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1. INTRODUCTION

It is widely accepted that weakly non-linear four-wave interactions play a key role in wind wave evolution (Young and Van Vledder, 1993). These interactions can be computed with great precision on the basis of the Boltzmann integral, originally proposed by Hasselmann (1962). However, this computation is very time consuming, such that it cannot be incorporated in operational wave models. Therefore, various approximations have been developed of which the Discrete Interaction Approximation (DIA), developed by Hasselmann et al. (1985), is the most well known. Despite its important role in the development of third-generation wave models, it suffers from a number of shortcomings (Van Vledder et al., 2000). In practice these shortcomings are commonly compensated by tuning of other source terms (Ardhuin et al., 2006).

In view of this situation it is clear that an accurate and computationally feasible method must be found for computing these interactions in operational models. Such an investigation must address two aspects simultaneously. The first aspect is to have a clear picture in what kind of applications an accurate evaluation of these interactions is necessary. The second aspect concerns the speed-up of exact methods such that they become feasible for application in operational models.

In relatively simple situations, like the open ocean and in fetch limited wave growth the benefits of third-generation wave models over second-generation wave models are not yet evident. In complex situations the performance of the third-generation wave models is poor and deficiencies in model physics become clear. Clear examples of such shortcomings are discussed in Ardhuin et al. (2006), with an emphasis on whitecapping dissipation and non-linear four-wave interactions in slanting fetch-situations. These interactions are currently modelled with the DIA. Differences in model behaviour due to different computational methods for these interactions, either with the DIA or with an exact computation, become apparent in more complex situations. Examples are a turning wind situation as described by Van Vledder and Holthuijsen (1993), but also field cases as described by Van der Westhuysen et al. (2004) and Ardhuin et al. (2006).

Assessing the benefits of a more accurate computation of the non-linear four-wave interactions requires a distinction to the type of application. The answer to this question is relevant for practical applications. Many third-generation models allow the user to choose between various model physics. This is especially tricky for relatively inexperienced users, moreover because there is still no consensus among model developers about the 'best' model physics. Regarding the source term for the non-linear four-wave interactions, the differences between the rather crude but fast DIA and the accurate but time consuming exact methods are still not fully bridged. Multiple DIA's (van Vledder, 2001, Tolman 2004) or reduced accurate methods may help to bridge the gap between both types of methods. Another, issue concerns the type of output. In the case that only information on the significant wave height and peak period is required, one may suffice with relatively cheap second-generation wave models, or with third-generation models using the DIA.

When more detailed information on the wave spectrum is required, differences in model behaviour become more noticeable. It is also expected that in more complex situations, the proper modelling of the various processes becomes more important to obtain realistic spectral shapes. As shown by various authors (Van Vledder and Holthuijsen, 1993 and Van der Westhuysen et al., 2004) spectral shapes are more peaked when an accurate method for computing the non-linear four-wave interactions is used. This seems to hold both for the frequency spectrum and for the directional distribution. An interesting detail of the directional distribution is its assumed bimodality at high frequencies as observed by Hwang et al. (2000) and simulated by Van der Westhuysen et al. (2004). Another example concerns the short- and long-term effect of the non-linear interactions. Numerical experiments by Resio and Perrie (1991) and Young and Van Vledder (1993) indicate that local ‘gaps’ in the spectral shape are quickly filled in by the non-linear interactions, an effect that cannot be reproduced by the DIA. This implies that an optimal method for computing these interactions should include both short- and long time-scale behaviour.

Separate from the question whether or not an accurate computation of the non-linear four-wave interactions is needed, is the question how the accurate computation can be speeded up while retaining sufficient accuracy. Accurate methods are still needed as a benchmark for developing approximate methods. Thus, efficient and accurate methods are also needed for research applications. Various accurate methods were developed than can act as a benchmark. Benoit (2005) provides a summary of various methods of which those presented by Webb (1978), Masuda (1980), and Lavrenov (2001) are among the most commonly used. In this paper the method of Webb (1978), as originally implemented in a numerical model by Tracy and Resio (1982) and Resio and Perrie (1991), is used to illustrate possible ways to speed up an ‘exact’ code. This method is also known as the WRT method for the computation of non-linear four-wave interactions in discrete spectral wave models and its implementation in SWAN is used (Van Vledder, 2006). A brief description of the WRT method is given in the Appendix to this paper.

Van Vledder (2006) describes a number of promising methods to reduce the computational workload for academic spectra. However, the effect of these optimizations in practical applications was not tested. Therefore, in this paper results are presented of a first systematic analysis of different optimizations and its effect on wave growth in a fetch-limited fetch situation. Thereafter, some future developments in computing the non-linear four-wave interactions are discussed.

2. REDUCING THE COMPUTATIONAL WORKLOAD

In this paper two computational methods are used to illustrate the effect of various methods to reduce the computational workload for evaluating the non-linear four-wave interactions. The two methods are the commonly used DIA and the WRT method. Both methods are implemented in the SWAN wave model (Holthuijsen et al., 2004, Van Vledder, 2006).

Van Vledder (2006) shows that the WRT method essentially consists of the repeated computation of one-dimensional integrals along a path in wave number space. At the highest level (the outer loop) interactions between a discrete target wave number k_1 with all other discrete wave numbers k_3 are considered. Then, for each pair of two wave numbers (k_1 and k_3) a line integral in wave number space can be computed yielding the transfer rate between these two wave numbers. This line integral is computed at the lowest level (or inner loop). It consists of a compound function along the (closed) line in wave number space. This compound function consists of the product of

the coupling coefficient, the Jacobian term and the product term of the action densities at four resonating wave numbers.

As with any numerical quadrature procedure, the accuracy of the solution depends on the resolution of the discretisation. Generally, higher resolutions lead to more accurate the solution (Note that this is not true for the DIA, as shown by Van Vledder et al., 2000). Therefore, reducing the resolution of the quadrature method in the WRT method, might be a way to reduce the computational workload, while retaining sufficient accuracy. An attractive feature of this approach is that the (mathematical) structure of the solution method is retained.

In the WRT method various choices can be made regarding the resolution with which the integrals are evaluated. Van Vledder (2006) tested a number of these choices by computing the non-linear transfer rate for smooth academic JONSWAP spectra. The following methods were investigated:

- Reduce the number of points on the locus for the computation of the one-dimensional line integrals;
- Replace the presently used trapezoid rule by a higher order quadrature method to evaluate the line integrals. In this method the points on the locus for the evaluation of the line integral are located at specific points;
- Replace bi-linear interpolation of action densities by the nearest bin approximation. This method assumes a piece-wise representation of the discrete wave spectrum as used by Snyder et al. (1993). Bi-linear interpolation is needed because the discrete points on the locus generally do not coincide with discrete grid points in wave number space. An artist impression of a constant piece-wise representation is given in Figure 1.
- Apply filtering to avoid calculating contributions that marginally contribute to the transfer integral. In the WRT method filtering can be applied to the highest level by omitting the contribution of interactions between the wave numbers k_1 and k_3 that are well separated from each other in wave number space. This is now achieved by specifying a maximum angle $\Delta\theta_{13}=|\theta_1-\theta_3|$ and a maximum ratio of wave number magnitude $\max(k_1/k_3, k_3/k_1)$ between the wave numbers k_1 and k_3 . The basic idea behind this type of filtering is that the coupling coefficients on the corresponding locus become smaller with separation distance in wave number space. This approach is similar to the Reduced Interaction Approximation (RIA) of Lin and Perrie (1998).

The method of replacing bi-linear interpolation by the nearest bin approach has also been implemented in the DIA method in the SWAN model (Sub-option IQUAD=8 in the QUAD command).

Van Vledder (2006) performed numerical experiments to assess the applicability of various methods to speed up the computation of the non-linear transfer integral using JONSWAP spectra. The results indicate that reductions in computational workload can be achieved by allowing a coarser resolution. Especially, filtering, a smaller number of points on the locus, and applying the nearest bin approach were identified as attractive methods. As noted by Tolman (2004), this is no guarantee that such methods work well in a wave model application. Even if they work, small changes in the non-linear transfer rate of an individual spectrum may accumulate, leading to unstable or unrealistic results of wave evolution.

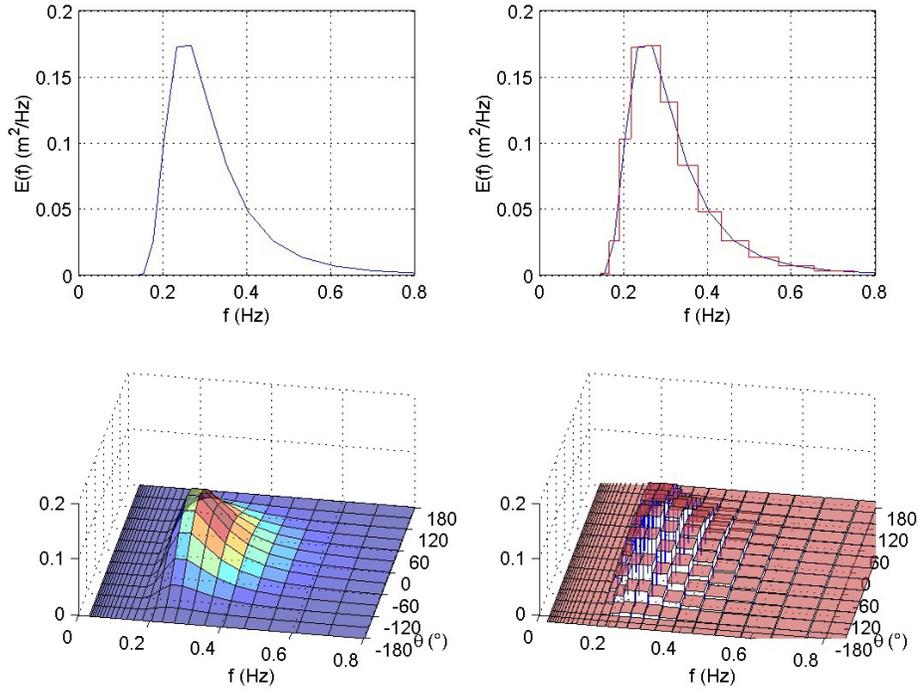


Figure 1: Continuous and piece-wise representation of a discrete wave spectrum.

3. SENSITIVITY COMPUTATIONS

Wave model tests were performed to determine the effect of the above methods on deep-water fetch-limited wave growth. Computations were performed with the SWAN model using a fetch of 25 km and a constant spatial step of 250 m (100 space steps) and a constant wind speed of 20 m/s. The number of directions was 36 (10° resolution) and 39 geometrically distributed frequencies in the interval 0.08 Hz – 3 Hz. In addition, a fixed number of 20 iterations was specified. It is noted that these modelling choices are somewhat arbitrary. They do not necessarily reflect recommended modelling practise, which may lead to a finer spatial and/or spectral resolution or more iterations. However, for the purpose of this sensitivity study they are not of primary concern. It is more important that they are kept constant for all tests.

Sensitivity tests for the effect of bi-linear interpolation versus nearest bin approach were performed for both the DIA and the XNL method (XNL stands for eXact Non-Linear transfer rate, here achieved by using the WRT method). The other tests were only performed for the XNL method.

3.1 Bi-linear interpolation versus nearest bin approach in the DIA and XNL

Figure 2 shows a comparison of the spatial variation of the significant wave height H_{m0} , the spectral wave periods $T_{m-1,0}$ and $T_{m0,1}$, and the directional spreading σ (definition given in Kuik et al., 1988) as computed with bi-linear interpolation (IQUAD=2) and the nearest bin approach (IQUAD=8). The wave number configuration is computed with the shape parameter λ set to the default value of 0.25. The relative difference in the various wave parameters at $x=24$ km is also indicated. The relative difference is defined as $\Delta P = (P_8 - P_2) / P_2 * 100$, where P_2 refers to the wave

parameter computed with SWAN using $IQUAD=2$, and similarly for P_8 . For closer inspection also the frequency spectrum $E(f)$ and the frequency dependent directional spreading $\alpha(f)$ are presented as a function of dimensionless frequency (normalized by the peak frequency of the base case; viz. using bi-linear interpolation in the DIA). Spectra are shown in Figure 3 for the locations $x=15$ km and 24 km.

The results indicate that the nearest bin approach in the DIA leads to non-directional wave parameters that are about 5% higher than using bi-linear interpolation. The directional spreading reduces about 7%. Inspection of the frequency distributions reveals that the nearest bin approach causes a faster growth of energy on the forward face of the spectrum. The differences in directional spreading have always the same sign and the absolute difference become larger for higher frequencies. Only in the area just above the peak frequency nearly equal results are obtained.

A similar exercise was carried out using the XNL method in SWAN. The effects of the nearest bin approach versus bi-linear interpolation on fetch-limited wave growth are shown in the Figures 4 and 5. The differences in wave parameters values are much smaller than with the DIA. Also the computed spectra differ only in some small details.

3.2 The number of points on the locus

The most time consuming part of the WRT method is the repeated integration of 1-dimensional integrals for all loci (a locus is the path in wave number space along which the integration is performed). Presently, a trapezoid rule is applied to evaluate these integrals. using an equi-distant spacing of grid points along the locus. The effect of the number of points on the locus on the computational results at $x=24$ km is shown in Table 1. The results are compared to the results of a reference run using 100 points on the locus.

N_{loc}	ΔH_{mo} (%)	$\Delta T_{m-1.0}$ (%)	ΔT_{m01} (%)	$\Delta \sigma$ (%)	T_{nloc}/T_{160}
100	0	0	0	0	1.00
90	-0.5	-0.4	-0.4	0.1	0.81
80	-0.6	-0.4	-0.3	0.4	0.70
70	-0.7	-0.6	-0.5	0.5	0.59
60	2.2	1.1	0.7	-0.7	0.38
50	-0.2	-0.1	-0.1	-0.3	0.30
40	2.3	1.1	0.7	-0.5	0.22
30	0.4	0.6	0.7	-0.2	0.16
20	-0.4	-0.8	-1.0	0.6	0.09

Table 1: Relative error of computed wave parameters and normalized computational requirement with respect to the case using 100 points on the locus.

The results indicate that choosing 30 to 40 points on the locus produces similar results as for the reference case. A further reduction of the number of points on the locus increases the errors, as expected. The computational requirement is approximately proportional to the number of points on the locus.

3.3 Effect of a higher-order quadrature method

The trapezoid rule for evaluating the 1-dimensional integrals is not the most efficient one available. Therefore, in the WRT implementation of Van Vledder (2006) several more efficient methods were implemented and tested; examples are the Simpson method and Gauss-Legendre quadrature. The latter method requires full control of the discrete points on the locus, since these points are not equidistant. Usually, the benefit of using Gauss-Legendre quadrature above a trapezoid rule is that fewer points are needed to obtain a similarly accurate result. This assumption was tested by comparing results obtained with a trapezoid rule using 40 points on the locus and a Gauss-Legendre method using 20 or 10 points on the locus.

The results (not shown here) indicate that using 20 points the differences in all wave parameters are lower than or equal to 1%. Also the wave spectra show hardly any differences. However, choosing only 10 points in the Gauss-Legendre method caused significant differences. The H_{m0} increased by about 3%, $T_{m-1,0}$ by 7%, T_{m01} by 11% and σ by 3%. These results indicate that comparable results can be obtained between a trapezoid rule and a Gauss-Legendre method when the number of points is reduced by a factor two.

3.4 The effect of filtering

An unrestricted evaluation of all possible wave number combinations k_1 and k_3 in any ‘exact’ method includes many negligible contributions. Omitting these contributions will reduce the computational workload considerably. In the EXACT-NL model (Hasselmann and Hasselmann, 1985) such contributions are filtered out by identifying them using a reference spectrum. The filtered phase-space (containing all relevant contributions) can then be applied to similarly shaped spectra. If the target spectrum differs too much from the reference spectrum an updated filtered phase space must be computed. Therefore, this method is not practical for cases with many different spectral shapes.

Another way of filtering is to assume that the transfer rate in a wave number configuration is small when two of the determining wave numbers k_1 and k_3 of a wave number configuration are well separated in wave number space. Since the present implementation of the WRT method has the wave numbers k_1 and k_3 in the outer loops, such a filtering is achieved by comparing the difference angle $\Delta\theta_{13}=|\theta_1-\theta_3|$ and the ratio of wave number magnitudes $R_k=\max(k_1/k_3, k_3/k_1)$ against some threshold values F_θ and F_k .

The effect of filtering is determined by using a case without filtering as the reference case. After some numerical experiments it was found that the filter settings $F_k=4$ and $F_\theta=91^\circ$ produces nearly identical results as the case without filtering with a gain in speed of a factor 4. Subsequently, the threshold values F_k and F_θ were varied and the result on fetch-limited wave growth determined. The results are summarized in the Tables 2 and 3. Note, that the threshold values F_θ are 1° higher than multiples of the directional resolution.

F_θ	ΔH_{mo} (%)	$\Delta T_{m-1.0}$ (%)	ΔT_{m01} (%)	$\Delta \sigma$ (%)	$T_{nof}/T_{F\theta}$
91°	-0.3	0.0	-0.1	-0.2	0.25
81°	-3.1	-1.5	-1.1	0.7	0.20
71°	-3.5	-1.8	-1.5	-0.1	0.18
61°	-4.5	-2.4	-2.2	-0.8	0.13
51°	-6.3	-3.6	-3.6	-1.9	0.10
41°	-8.5	-5.2	-5.4	-3.4	0.07

Table 2: Relative errors of integral wave parameters for fetch-limited wave growth at $x=24$ km for different values of the threshold angle F_θ .

F_k	ΔH_{mo} (%)	$\Delta T_{m-1.0}$ (%)	ΔT_{m01} (%)	$\Delta \sigma$ (%)	T_{nof}/T_{Fk}
4.0	-0.3	0.0	-0.1	-0.2	0.25
3.5	-3.0	-1.4	-1.0	0.9	0.25
3.0	-3.0	-1.4	-1.0	1.0	0.24
2.5	-2.1	0.0	0.7	0.3	0.24

Table 3: Table 2: Relative errors of integral wave parameters for fetch-limited wave growth at $x=24$ km for different values of the threshold ratio F_k .

The results in the Tables 2 and 3 indicate that filtering with respect to directional differences has more effect than filtering with respect to the wave number ratio. With decreasing threshold value F_θ , the relative error increases steadily and the required computational load decreases. Filtering using the wave number ratio produces errors in the wave height of about 3% irrespective of the threshold value. The results for the other parameters are all smaller than 2%, whereas the computational workload (surprisingly) does not seem to diminish with decreasing threshold value F_k .

5. DISCUSSION

In this study the effect of various methods to speed-up the computational method for the computation of the non-linear four-wave interactions on fetch-limited wave growth was tested. To that end the DIA and XNL implementation in SWAN were used. First, the effect of using the nearest bin versus bi-linear interpolation was tested. This technique is available in both the DIA and XNL. For the DIA differences up to 7% in computed wave parameters were found, whereas for XNL the differences are about 1%. At first sight, the differences for the DIA are quite large and of the same order as other uncertainties in model behaviour. The differences occur for all non-directional wave parameters and tuning other source terms, e.g. the whitecapping dissipation, may compensate them. The gain in speed in the DIA (at least in its implementation in the SWAN model) is only about 10%. Applying the nearest bin approach in XNL produces only slightly different results. In contrast to the DIA, the gain in speed in XNL is about 40%, also the size of the pre-computed database containing indices of interacting wave number configurations, interpolation coefficients, coupling coefficients and Jacobians becomes much smaller (~50%). Thus, it is attractive to apply the nearest bin-approach in XNL, but not in the DIA, at least in fetch-limited wave growth.

A simple method to speed-up the WRT method is to lower the number of points on the locus. This concept was tested by decreasing the number of points from 100 to 20 in steps of 10. The

results indicate that the results hardly change for a number of points as low as 30. A further reduction starts degrading the results too much. It is also evident that the computational workload decreases in proportion to the number of points. In view of the findings of Van Vledder (2006) the slightly higher value of 40 is recommended.

Applying higher-order integration quadrature methods seems to be a simple way to increase the computational speed while retaining sufficient accuracy. However, such methods assume a relatively smooth behaviour of the (compound) function. This is true when academic spectra are tested which by definition produce smooth functions. It appears that this also holds for the spectra and related basis functions obtained in a discrete spectral model applied for fetch-limited wave growth. This finding supports those of Lavrenov (2001) who obtained accurate results using higher-order quadrature rules. Whether this is generally true for modelled spectra, cannot be said on the basis of the present results.

Filtering is also an effective way of reducing the computational workload. It was found that directional filtering has more effect on the results than filtering on the basis of the wave number ratio. The present method of filtering uses separate thresholds for the directional difference and the wave number ratio. Another way of filtering may be based on combining these criteria and computing the relative distance in wave number space according to the measure $|\mathbf{k}_1 - \mathbf{k}_3| / (|\mathbf{k}_1 + \mathbf{k}_3|/2)$. The author is now investigating application of this measure in the WRT method.

The present method of filtering uses fixed criteria in terms of wave number ratios and directional differences. Like the EXACT-NL model, it is also possible to apply adaptive filtering in which the action density in a certain bin is taken into account. Spectral bins with a relatively small amount of action density are assumed to have a small and possibly negligible transfer rate. Applying such a criterion only on the action density n_1 at wave number \mathbf{k}_1 is not recommended, since then, such a bin may never receive any wave action or energy from other spectral bins. It is safer to apply such a criterion on the action densities n_1 and n_3 simultaneously. The author is now testing this kind of filtering.

In the present paper no claims are made regarding the computational efficiency of the WRT method in comparison to other computational methods. Such claims can only be made under controlled conditions. In addition, the computational requirements strongly depend on details of the algorithmic implementation in the host model, the computer hardware, and on the type of wave problem investigated.

6. FURTHER DEVELOPMENTS

In the previous part a follow up was given to the paper of Van Vledder (2006) regarding the applicability of methods to speed up the WRT method for a practical situation. As a first step, a fetch-limited wave growth situation was tested using the SWAN model in 1-d mode. The results indicate that lowering the number of points on the locus, filtering, and using a high order quadrature method can speed up the WRT method while retaining sufficient accuracy. These results confirm the findings of Van Vledder (2006) for academic spectra.

However, the applicability of these methods in more complex situations needs to be determined. It is recommended that such situations should contain directionally skewed and multi-peaked spectra. Such spectra can be found in rapidly turning hurricane winds and slanting fetch situations. The role of non-linear four-wave interactions in rapidly turning winds has been studied by (Van Vledder and Holthuijsen, 1993). In slanting fetch situations it has been studied by Pettersson (2004) and Bottema and Van Vledder (2005). Recently, Ardhuin et al. (2006) present a

detailed analysis of two slanting fetch situations along the USA coast near Duck. Such complex situations are also an interesting test case for models of whitecapping dissipation (e.g., Van der Westhuysen et al., 2006). These above mentioned studies indicate that different parameterisations of the non-linear four-wave interactions lead to different wave model behaviour. Therefore, such situations are recommended as test cases for methods to speed-up the WRT method.

Suggestions for additional filtering methods were presented in the previous section. However, further tests are needed to assess their applicability in more complex wave situations. It must be clear that each test should focus on the balance between the gain in computational speed and the loss of accuracy in integral wave parameters or in spectral shape.

Further improvements in computational speed can be achieved by applying higher-order integration methods in the outer loops in the WRT method. In its present implementation, the loop of all \mathbf{k}_3 wave numbers runs over all discrete wave numbers \mathbf{k}_1 . This is not required and using higher order methods, e.g. Gauss-Legendre quadrature, may improve the computational efficiency. The author is now developing such an option. In this paper, the effect of Gauss-Legendre integration was tested for a relatively simple fetch-limited case, in which all model spectra are rather smooth. Such spectra also produce smooth compound functions along the locus, which might explain the efficiency of such quadrature methods. It is therefore interesting to test such higher order quadrature methods on very irregular or multi-peaked spectra.

The present paper concentrates on future developments in computational aspects of computing the non-linear transfer rate in deep water. In shallow water different results may be obtained. It is therefore recommended to perform sensitivity tests for different water depths to find optimal settings. Also new developments in the description of the non-linear four-wave interactions in shallow water (cf. Janssen, 2006) needs to be included in future developments of computational methods.

APPENDIX

Basic equations of non-linear wave-four interactions

The WRT method for the computation of non-linear four-wave interactions is based on the Boltzmann integral, originally proposed by Hasselmann (1962). The rate of change of action density n_1 at a wave number \mathbf{k}_1 due to all quadruplet configurations involving \mathbf{k}_1 is:

$$\begin{aligned} \frac{\partial n_1}{\partial t} = & \iiint G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \times \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \times \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \\ & \times [n_1 n_3 (n_4 - n_2) + n_1 n_4 (n_3 - n_2)] d\mathbf{k}_2 d\mathbf{k}_3 d\mathbf{k}_4 \end{aligned} \quad (1)$$

where $n_i = n(\mathbf{k}_i)$ is the action density at wave number \mathbf{k}_i and G is the coupling coefficient. The δ -functions ensure that only resonant wave number configurations are taken into account. In the WRT method equation (1) is rewritten as:

$$\frac{\partial n_1}{\partial t} = \int d\mathbf{k}_3 T(\mathbf{k}_1, \mathbf{k}_3) \quad (2)$$

with

$$T(\mathbf{k}_1, \mathbf{k}_3) = \int_S ds \times G \times J \times N_{1,2,3,4}, \quad (3)$$

where J is the Jacobian and $N_{1,2,3,4}$ is the product term of action densities. The subscript s indicates that the integration is only performed around the locus as a function of the coordinate s .

Discretisation

To compute the non-linear transfer rate for a given discrete wave spectrum, it is assumed that the wave spectrum is given in terms of a discrete action density spectrum as a function of the discrete wave numbers k_i (for $i=1, N_k$) and discrete directions θ_j ($j=1, N_\theta$) with a constant spacing $\Delta\theta$. Based on expression (1) the rate of change of action density at a certain discrete wave number $(k_{i_{k1}}, \theta_{j_{k1}})$ is expressed as:

$$\Delta n(k_{i_{k1}}, \theta_{j_{k1}}) = \sum_{i_{k3}=1}^{N_k} \sum_{j_{\theta3}=1}^{N_\theta} k_{i_{k3}} T(k_{i_{k1}}, \theta_{j_{k1}}, k_{i_{k3}}, \theta_{j_{k3}}) \Delta k_{i_{k3}} \Delta \theta \quad (4)$$

for $i_{k1} = 1, N_k$ and $j_{\theta1} = 1, N_\theta$,

in which the factor $k_{i_{k3}}$ is the Jacobian term arising from the transformation from \mathbf{k}_3 to (k_3, θ_3) . The term $T(\mathbf{k}_1, \mathbf{k}_3)$ can be discretised as

$$T(\mathbf{k}_1, \mathbf{k}_3) = \oint_S G(s) J(s) N(s) ds \quad (5)$$

$$\approx \sum_{i=1}^{N_s} G(s_i) J(s_i) N(s_i) \Delta s_i$$

or alternatively using weights

$$T(\mathbf{k}_1, \mathbf{k}_3) \approx \sum_{i=1}^{N_s} w_i G(s_i) J(s_i) N(s_i). \quad (6)$$

REFERENCES

- Ardhuin, F., T.H.C. Herbers, G.Ph. van Vledder, K.P. Watts, R.E. Jensen, and H.C. Graber, 2006: Slanting fetch and swell effects on wind wave growth. In press, *J. Phys. Oceanogr.*
- Benoit, M., 2005: Evaluation of methods to compute the nonlinear quadruplet interactions for deep-water wave spectra. Proc. 5th ASCE Int. Symp. on Ocean Waves, Measurement and Analysis, WAVES'2005, Madrid, Spain.
- Bottema, M. and G.Ph. Van Vledder, 2005: Evaluation of the SWAN wave model in slanting fetch conditions, Proc. 5th ASCE Int. Symp. on Ocean Waves, Measurement and Analysis WAVES'2005, Madrid, paper No. 165, 10 pp.
- Hasselmann, K., 1962: On the non-linear energy transfer in a gravity-wave spectrum. Part 1. General theory, *J. Fluid Mech.*, **12**, 481-500.
- Hasselmann, S., Hasselmann, K., 1985: The wave model EXACT-NL. In 'Ocean Wave Modelling', The SWAMP group, Plenum Press, New York and London, 256 pp.

- Hasselmann, S., K. Hasselmann, J.A. Allender, and T.P. Barnett 1985: Computations and parameterizations of the nonlinear energy transfer in a gravity-wave spectrum. Part 2: parameterizations of the nonlinear transfer for application in wave models. *J. Phys. Oceanogr.*, **15**, 1378-1391.
- Holthuijsen, L.H., N. Booij, A.T.M.M. Kieftenburg, R.C. Ris., A.J. Van der Westhuysen, and M. Zijlema, 2004: SWAN cycle III, Version 40.41, User manual.
- Hwang, P.A., D.W. Wang, E.J. Walsh, W.B. Krabill and R.N. Shift, 2000: Airborne measurements of the wavenumber spectra of ocean surface waves. Part 2. Directional distribution. *J. Phys. Oceanogr.* **30**, 2768- 2787.
- Janssen, P.A.E.M., 2006: Presentation at WISE 2006, Venice, Italy.
- Kuik, A.J., G.Ph. van Vledder, and L.H. Holthuijsen, 1988: A method for the routine analysis of pitch-and-roll buoy wave data. *J. Phys. Oceanogr.*, **18**, 1020-1034.
- Lavrenov, I.V., 2001: Effect of wind wave parameter fluctuation on the nonlinear spectrum evolution. *J. Phys. Oceanogr.*, **31**, 861-873.
- Lin, R.Q., and W. Perrie, 1998: On the mathematics and approximations of the non-linear wave-wave interactions. In Perrie: W. (Ed.), *Nonlinear Ocean Waves, Advances in Fluid Mechanics*, **17**, Comp. Mech. Publ., Southampton (UK), pp. 61-88.
- Masuda, A., 1980: Nonlinear energy transfer between wind waves. *J. Phys. Oceanogr.*, **10**, 2082-2093.
- Pettersson, H., 2004: Wave growth in a narrow bay, PhD Thesis, Contrib. Finnish Inst. of Marine Research 09-2004.
- Resio, D.T., and W. Perrie, 1991: A numerical study of nonlinear energy fluxes due to wave-wave interactions. Part I: Methodology and basic results. *J. Fluid Mech.*, **223**, 609-629.
- Snyder, R.L., W.C. Thacker, K. Hasselmann, S. Hasselmann, and G. Barzel, 1993: Implementation of an efficient scheme for calculating nonlinear transfer from wave-wave interactions. *J. Geophys. Res.*, **98**, C8, 14507-14525.
- Tolman, H.L., 2004: Inverse modeling of Discrete Interaction Approximations for nonlinear interactions in wind waves. *Ocean Modelling*, **6**, 405-422.
- Tracy, B.A., and D.T. Resio, 1982: Theory and calculation of the nonlinear energy transfer between Sea waves in deep water. WIS Technical Report 11, US Army Engineer Waterways Experiments Station, Vicksburg, Mississippi, USA, 47 pp.
- Van Vledder, G.Ph., 2001: Extension of the Discrete Interaction Approximation for computing nonlinear quadruplet wave-wave interactions in operation wave models. Proc. 4th ASCE Int. Symp. on Ocean Waves, Measurements and Analysis, WAVES'2001. San Francisco, California, USA, 540-549.
- Van Vledder, G.Ph., 2006: The WRT method for the computation of non-linear four-wave interactions in discrete spectral wave models. *Coastal Engineering*, **53**, 223-242.
- Van Vledder, G.Ph., and L.H. Holthuijsen, 1993: The directional response of ocean waves to tuning winds. *J. Phys. Oceanogr.*, **23** (2), 177-192.
- Van Vledder, G.Ph., T.H.C. Herbers, R.E. Jensen, D.T. Resio, and B.A. Tracy, 2000: Modelling of non-linear quadruplet wave-wave interactions in operational wave models, Proc. 27th Int. Conf. on Coastal Eng., Sydney, 797-811.
- Van der Westhuysen, A.J., M. Zijlema, and J.A. Battjes, 2004: Improvement of the numerics and deep-water physics in an academic version of SWAN. Proc. 29th Int. Conf. on Coastal Engineering, Lisbon, Portugal, 855-867.
- Van der Westhuysen, A.J., M. Zijlema, and J.A. Battjes, 2006: Nonlinear saturation-based whitecapping dissipation in SWAN for deep and shallow water. In Press in *Coastal Engineering*.
- Webb, D.J., 1978: Nonlinear transfer between sea waves. *Deep-Sea Res.* **25**, 279-298.
- Young, I.R., and G.Ph. van Vledder, 1993: A review of the central role of nonlinear interactions in wind-wave evolution. *Phil. Trans. R. Soc. London, A.*, **342**, 505-524.

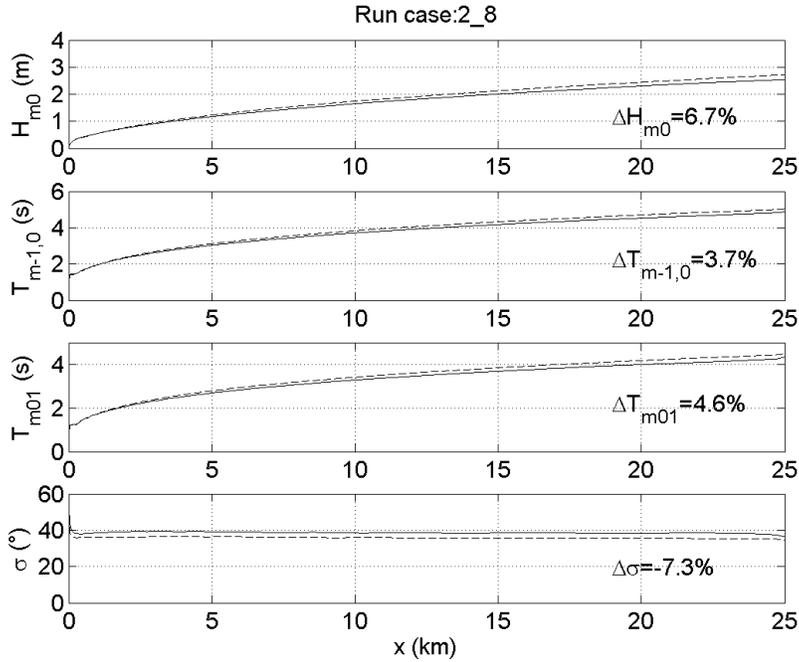


Figure 2: Spatial variation of the significant wave height H_{m0} , the spectral periods $T_{m-1,0}$ and T_{m01} and the directional spreading σ . Using bi-linear interpolation (solid line) and nearest bin (dashed line) approach in the DIA. Computed with SWAN, wind speed 20 m/s.

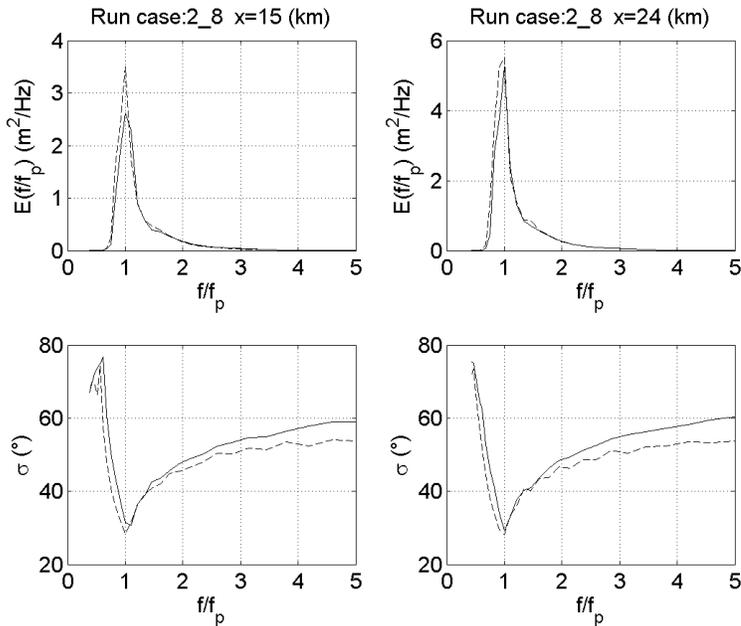


Figure 3: Comparison of the frequency spectrum and frequency dependent directional spreading 15 and 24 km offshore. Using bi-linear interpolation and nearest bin approach in the DIA. Computed with SWAN, wind speed 20 m/s.

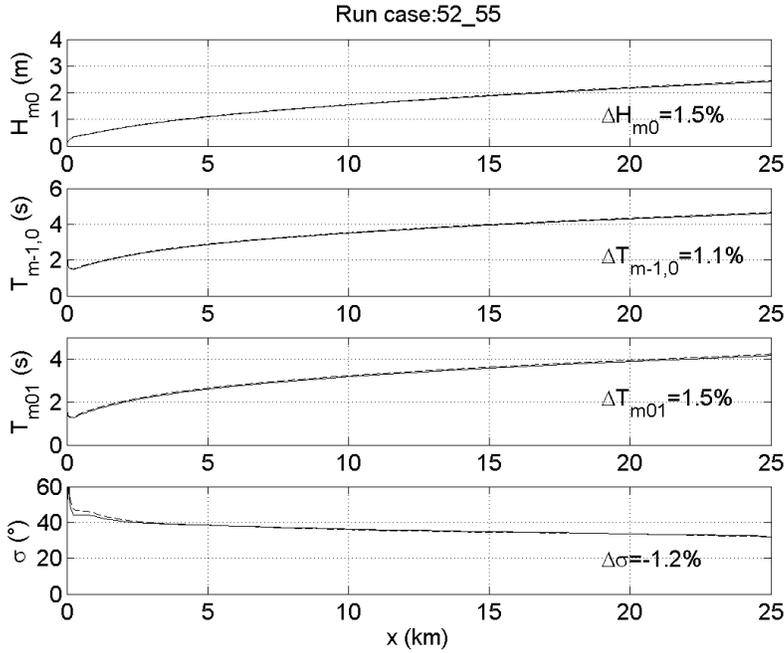


Figure 4: Spatial variation of the significant wave height H_{m0} , the spectral periods $T_{m-1,0}$ and T_{m01} and the directional spreading σ . Using bi-linear interpolation (solid line) and nearest bin (dashed line) approach in the XNL. Computed with SWAN, wind speed 20 m/s.

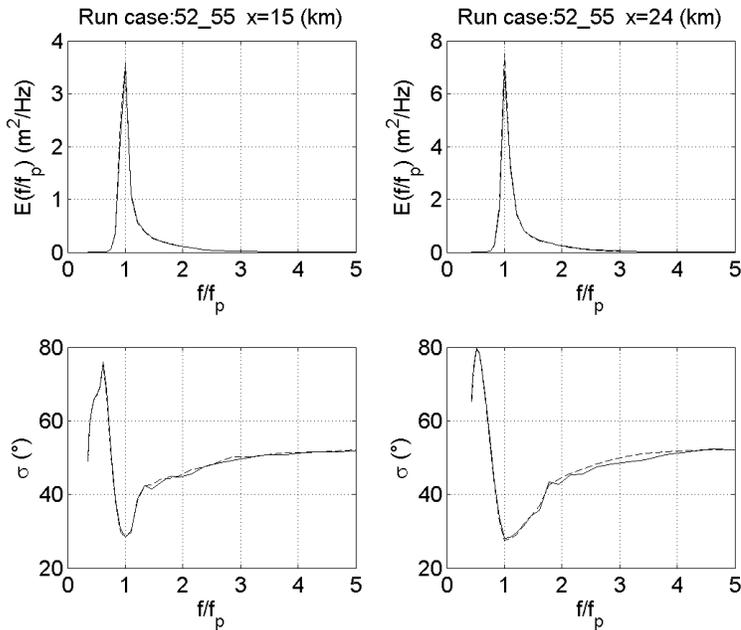


Figure 5: Comparison of the frequency spectrum and frequency dependent directional spreading 15 and 24 km offshore. Using bi-linear interpolation and nearest bin approach in the XNL. Computed with SWAN, wind speed 20 m/s.