

THE NATURE OF THE DRAUPNER GIANT WAVE OF 1ST JANUARY 1995 AND THE ASSOCIATED SEA-STATE, AND HOW TO ESTIMATE DIRECTIONAL SPREADING FROM AN EULERIAN SURFACE ELEVATION TIME HISTORY

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1. Introduction

On New Year's Day of 1995 a severe storm occurred in the central North Sea at Statoil's Draupner platform. At just before 15.30, a wave crest of 18.5m was recorded by a downwards pointing laser wave gauge. This is one of the highest wave crests ever recorded in the North Sea and has generated much interest as a possible 'freak' or 'rogue' wave, see for example Haver and Jan Andersen (2000) and Prevosto and Bouffandeau (2002).

Haver (2004) gives a discussion of the meteorology and the storm evolution when the large wave occurred. Unfortunately, only two 20min records are currently available, with start times separated by 1 hour. Henceforth, these will be described as the 15.20 and 16.20 records. The surface elevations for both are shown in Figure 1 below.

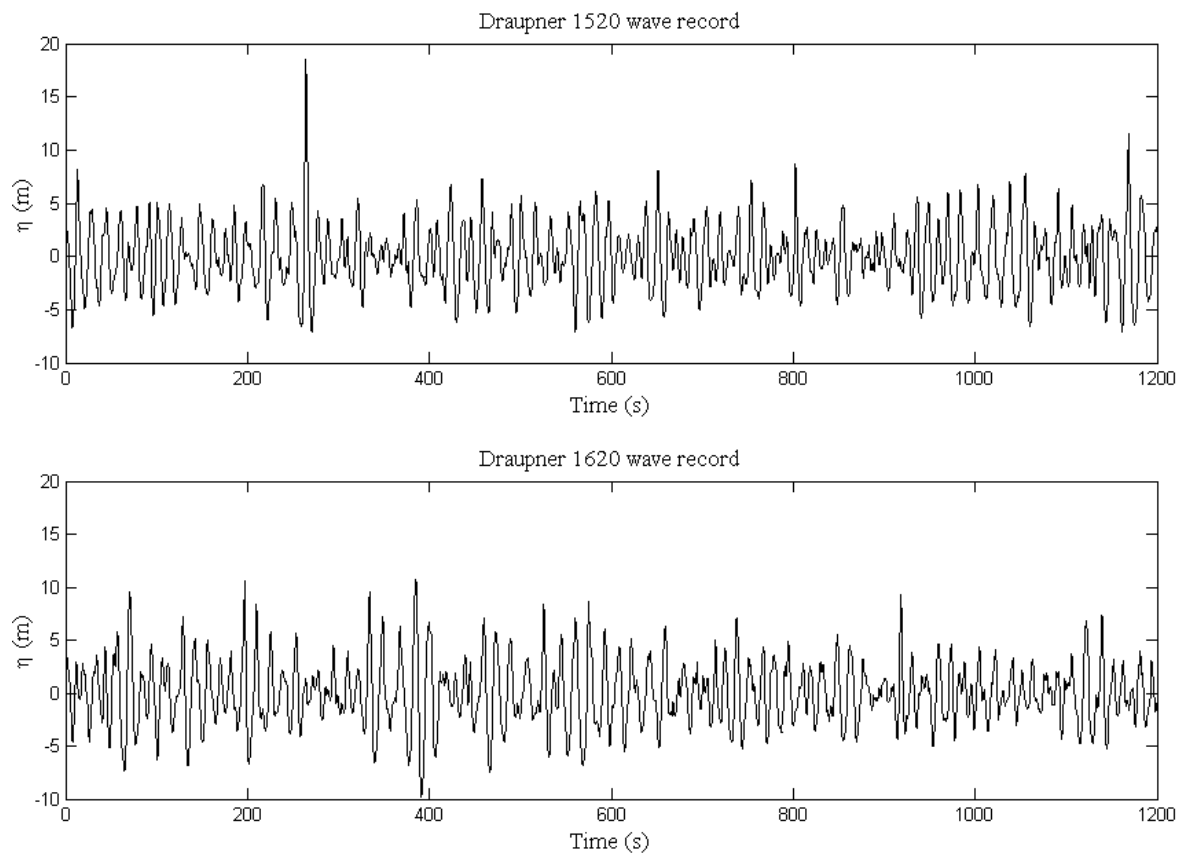


FIG 1. The Draupner wave records

With a zero crossing period of $T_z \sim 12s$, there are approximately 100 waves in each record. The data sampling rate was $2.1Hz$, so the peak of the large wave is relatively well resolved. The Draupner platform consists of two slim jacket structures connected by a bridge. The laser wave gauge was attached to the subsidiary jacket supporting the flare tower, so it is reasonable to assume that the records are uncontaminated by wave-structure interaction. Thus, in the subsequent analysis we assume that the records are accurate representations of the free-field waves. Minor damage was done to equipment below deck level on the platform (Haver, private communication), so we can have confidence that green water reached this level. The significant wave height for each record is $H_s \sim 12m$, corresponding to a severe winter storm in the central North Sea.

In this paper we attempt to analyse various aspects of the waves in both Draupner records, examining both the properties of the New Year wave and the associated wave records. Only the two Eulerian surface elevation records are available. There do not appear to be any other wave measure stations anywhere close to the Draupner platform, so *a priori* there appears to be no information available as to the directional spreading in the New Year storm. This paper aims to address this issue.

2. Overall crest-trough asymmetry: 2nd order sum contributions

It is clear from any Eulerian surface elevation time history that water waves are vertically asymmetric. Crests are spiky and larger, troughs smaller and more rounded. Most of these differences are simple consequences of the harmonic structure of waves – and the dominant contributions are at 2nd order in a Stokes-type perturbation expansion. Initially we demonstrate this vertical asymmetry by simply measuring every maximum elevation between zero up and down crossings (crest) and every minimum between zero down and up crossings (trough). These crest and trough values are then sorted into ascending order and then plotted as pairs: the n -th largest crest with the n -th largest trough. Figure 2 shows the results for the 15.20 and 16.20 records.

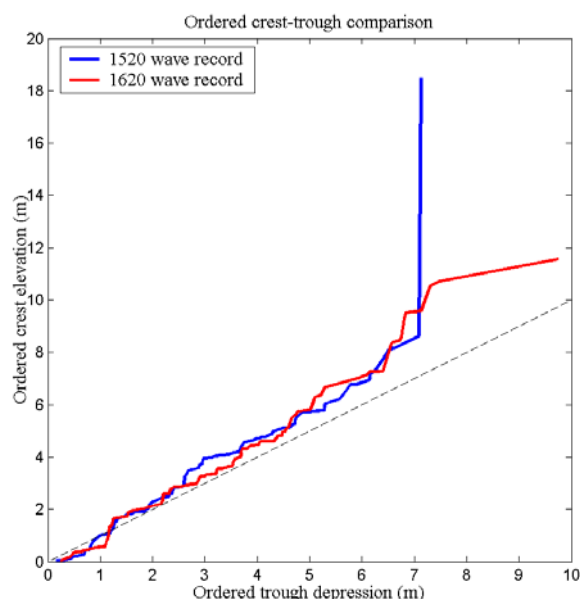


FIG 2.
Ordered crest-trough statistics

The ordered crest-trough distributions for both records are similar, except at the top end where the 15.20 record contains one anomalously large crest, whereas the 16.20 record contains a rather deep trough and no large crest. Other than these single largest extrema, all the records can be approximated by a single line through the origin with a slope slightly steeper than 1:1.

We shall approximate the trough-crest asymmetry using a regular wave model. For details see Walker et al. (2005). The traditional Stokes expansion of the surface elevation time history for a regular wave train on finite depth can be written to 3rd order as (Fenton 1990)

$$\eta(t) = a \cos \phi + a^2 k B_{22} \cos 2\phi + a^3 k^2 B_{33} \cos 3\phi$$

$$B_{22} = \coth(kd) \frac{(1+2S)}{2(1-S)}$$

$$B_{33} = \frac{3(1+3S+3S^2+2S^3)}{8(1-S)^3}$$

$$S = \operatorname{sech}(2kd)$$

This form is awkward to apply to an irregular wave train because of the apparent necessity to estimate the local wave number (k) and the product of wave number and water depth (kd) individually. However, it can be re-written as

$$\eta(t) = a \cos \phi + a^2 \frac{S_{22}}{d} \cos 2\phi + a^3 \frac{S_{33}}{d^2} \cos 3\phi$$

$$S_{22} = kB_{22} \times d$$

$$S_{33} = k^2 B_{33} \times d^2$$

In this form only the local depth (d) and the Stokes coefficients S_{22} etc. are required. At the Draupner location the water depth is known to be $\sim 70m$. Thus, the analysis of 2nd order asymmetry simply reduces to estimation of Stokes coefficient (S_{22}), the local value kd and an approximation $a \cos \phi$ for the linear component for the wave.

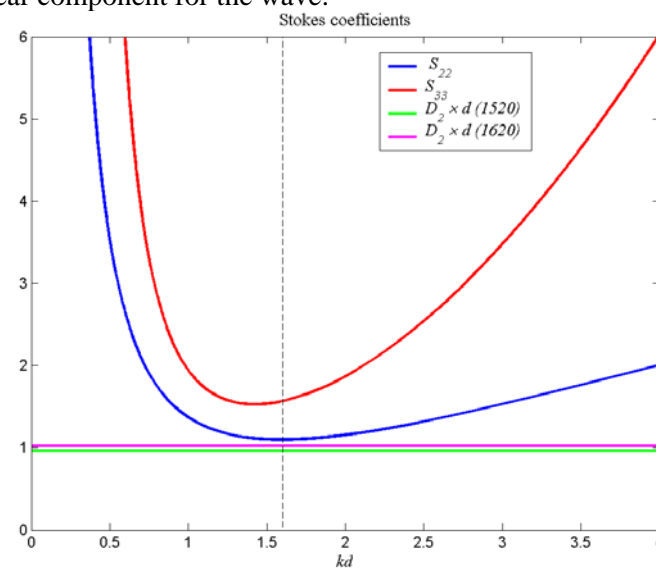


FIG 3.
Variation of the modified
Stokes coefficients S_{22} (and S_{33})
with water depth (kd)

As a helpful coincidence, the combination of the zero-crossing wave period and the water depth results in an estimate of $kd \sim 1.6$ so Figure 3 demonstrates that the Stokes coefficient in the new form should be remarkably insensitive to the precise values of wave number and water depth at the Draupner platform in the storm of 1st January 1995. Thus, the estimation of the expected vertical asymmetry reduces to an estimation of the linear component and the coefficient S_{22} .

If we have a linear signal and its Hilbert transform (easily obtainable via a FFT and then phase-shifting by 90°), then these can be written as

$$\eta_L = a \cos \phi, \quad \eta_{LH} = a \sin \phi$$

A double frequency component can be approximated as

$$a^2 \cos 2\phi = a^2 (\cos^2 \phi - \sin^2 \phi) = \eta_L^2 - \eta_{LH}^2$$

Unfortunately, all we have is the fully nonlinear wave record, not its (assumed) linear decomposition. However, we can estimate the linear part by high-pass filtering to remove the long wave 2nd order difference terms and then forming an approximation

$$\eta_L \approx \eta - \frac{S_{22}}{d} (\eta^2 - \eta_H^2)$$

We then look for the value of the average 2nd order Stokes coefficient S_{22} that completely removes skewness from the resulting η_L record. Histograms for the entire 20min records are shown in Figure 4 below (for the 15.20 record we remove a 45sec period around the giant wave, as this is assumed to be atypical of the rest of the record).

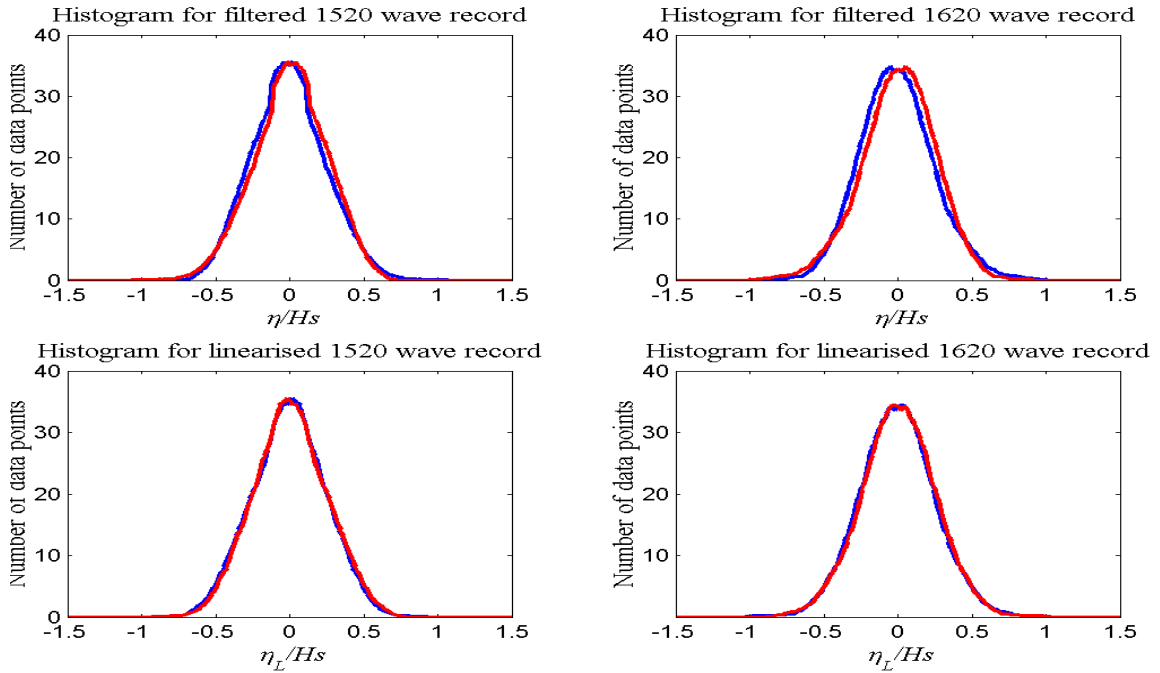


FIG 4. Histograms of the original 15.20 and 16.20 records, and the linearised distributions below. The original histograms are in blue, their mirror images in red.

The estimated mean values of the Stokes coefficient S_{22} from the field data are

15.20 record	0.96
16.20 record	1.02
minimum of theoretical S_{22} curve	1.10

From the data, we obtain estimates ~10% lower than the theoretical minimum of the Stokes regular wave coefficient, as shown on Fig.3. Is this 10% difference significant or just a consequence of statistical variability, the finite duration of the records and the analysis technique?

These values are robust, being consistent with the close to 1:1 linearised crest-trough statistics. Figure 5 shows the paired crest and trough data comparable to Fig.2, but now for the linearised wave data.

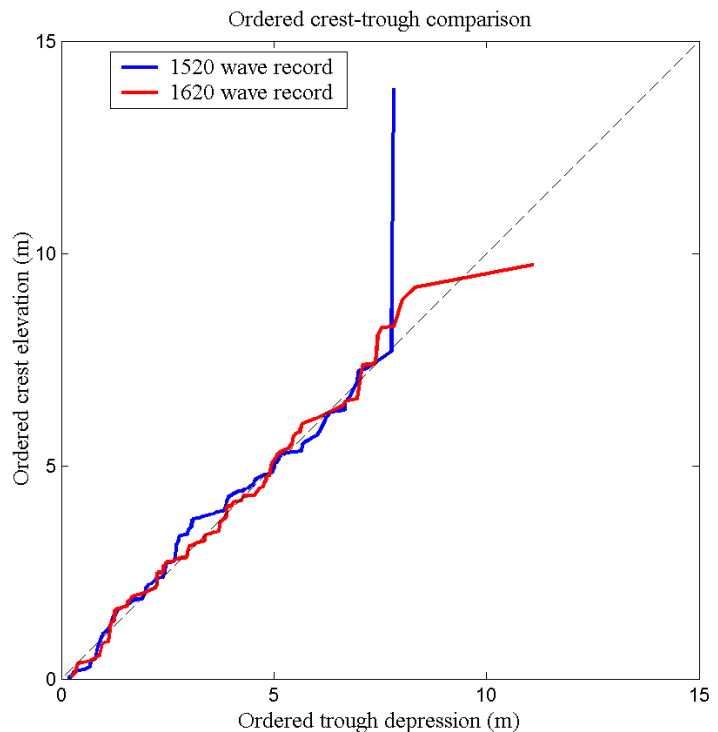


FIG 5.
Ordered crest-trough statistics for linearised wave records

There are three obvious features of waves on the open sea: broadbandness and nonlinearity are key in the vertical asymmetry discussed above. However, the third feature, directional spreading or the short crested nature of waves, has not yet featured in the discussion. It may seem strange to discuss spreading based only on Eulerian time histories but there are consequences even within a time history resulting from directional spreading which we now present – but not of course the mean direction which is completely unknown.

The simple form of the Stokes expansion to 2nd order was first generalised to broadband waves with directional spreading by Dean and Sharma (1981) for finite depth. The equations in the original Dean and Sharma paper contain misprints, so more reliable and consistent sources for the rather complicated interaction kernels are Dalzell (1999) and Forristall (1999).

What does not appear to have been noticed before is the precise shape of the interaction kernel for the 2nd order sum term. In keeping with our previous narrow banded approximation for the Draupner records, we examine the effect of directional spreading on the single cross-interaction term for 2 wave components with the same frequency propagating in different directions separated by a total angle θ . The interaction coefficient is scaled to unity when the two components are parallel. Figure 6 shows the relative change of the kernel with water depth and angular separation.

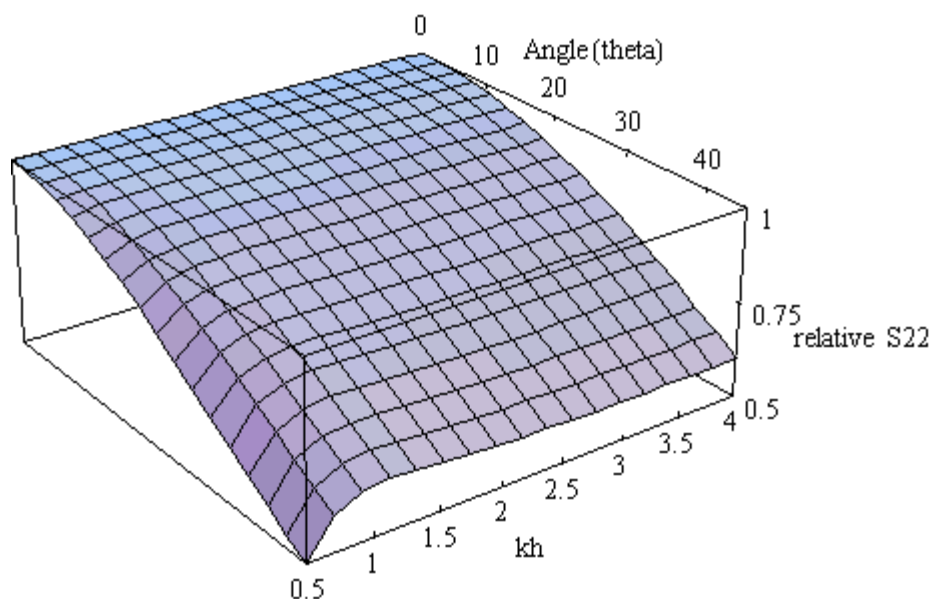


FIG 6. 2nd order sum interaction kernel for waves of the same frequency as a function of the total angle of separation θ and water depth kd .

Unless the water is very shallow and the wave pair is very highly spread, the reduction in the interaction kernel is close to a parabolic variation with angle and virtually independent of water depth. Thus, the reductions in the S_{22} coefficients for the two 20 min Draupner records below the Stokes minimum can be converted directly into albeit rather crude estimates of the mean directional spreading for each record:

	Stokes coefficient S_{22}	rms spreading ($^{\circ}$)
15.20 record	0.96	20
16.20 record	1.02	15

Here we express the spreading angle as the standard deviation of a wrapped normal distribution, see Tucker and Pitt (2001) and Jonathan and Taylor (1997). The only independent estimate of spreading in this storm that we have been able to obtain is from a directional wave buoy in the Auk field where the rms spreading was $\sim 20^{\circ}$ over several hours around the peak of the storm (Ewans, private communication). Auk experienced the same

severe winter storm as Draupner but is located 200km north of Draupner, so the local wave climate could have been significantly different. However, we now turn to the bound wave 2nd order difference terms.

3. Long bound waves : difference terms

Have previously filtered out the long wave terms, we now analyse them to see whether another estimate of spreading can be obtained. Just as for the sum interaction, both Dalzell (199) and Forristall (1999) give the algebraically complicated difference term. We now low-pass filter the measured records to remove the linear and higher components. A cut-off of $\sim 0.04\text{Hz}$ is suitable. Based on the high-pass filtered record we have an approximation for the linear components. Combined with an assumed directional (constant) wrapped normal model for the directional spreading, we compute an estimate of the long bound wave components for each record.

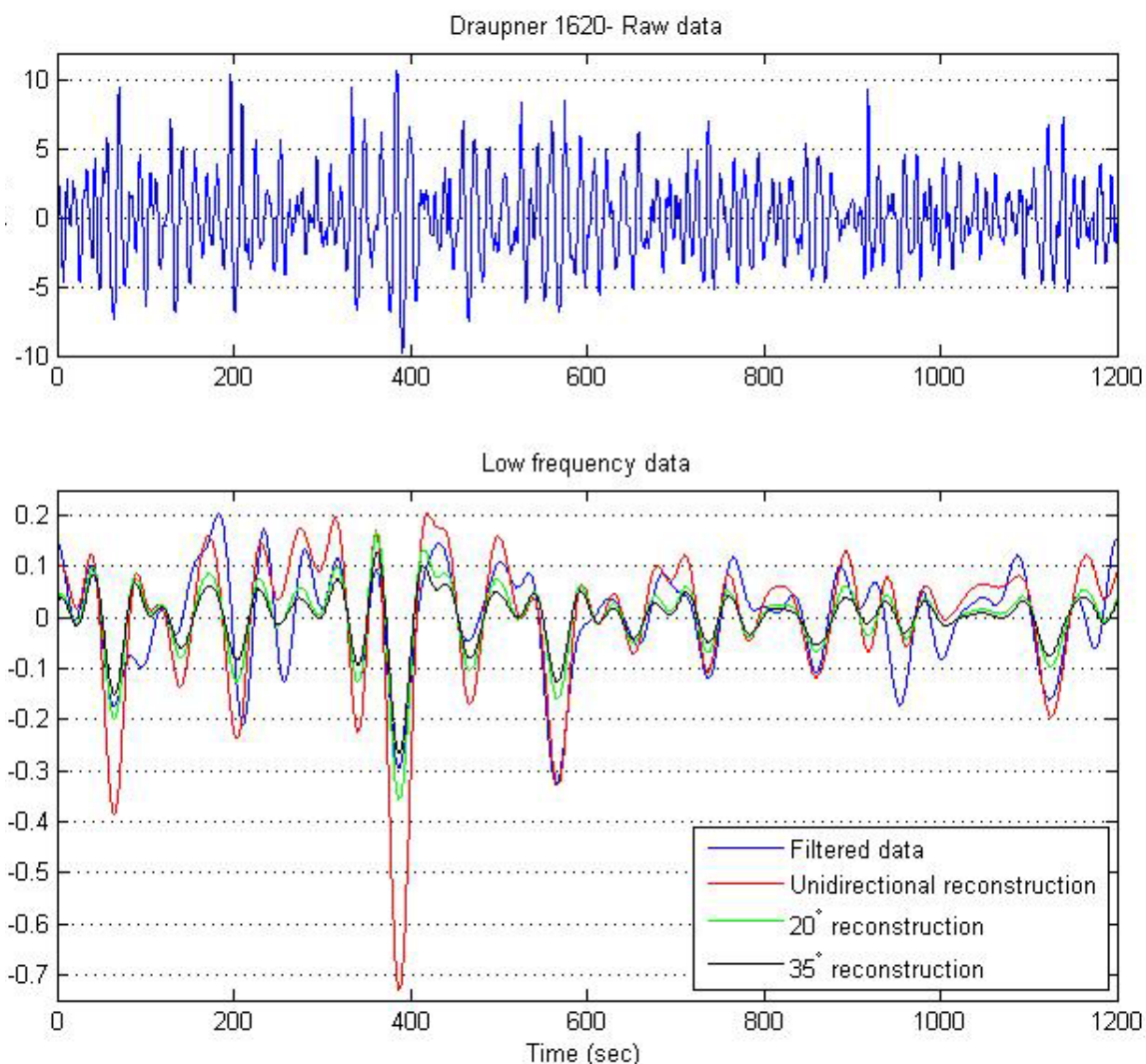


FIG. 7 The 16.20 Draupner record, the measured long wave time history in blue and various approximations based on the linearised wave history and single Gaussian directional distributions with 0, 20 and 35 ° spreading.

As an example, the 16.20 Draupner record is shown in Figure 7. Clearly, the long waves are small $O(10)cm$ components hidden within $O(5)m$ waves. Note also the clear appearance of group structure in the low-pass filtered record. Whenever the wave field is particularly energetic locally, this is associated with long wave set-down. The uni-directional long wave reconstruction is clearly too large, whereas the 35° reconstruction is too small. A spreading $\sim 20^\circ$ perhaps seems reasonable. However a more quantitative measure of the goodness of the fit is required. We use a normalised discrepancy

$$\frac{\sqrt{\sum (\eta_{sim} - \eta_{LowPass})^2}}{\sum \eta_L^2 k_P}$$

where η_{sim} is the simulated 2nd order difference signal, $\eta_{LowPass}$ is the low-pass measured record and η_L is the linearised signal as estimated previously. The wave number corresponding to the peak of the frequency spectrum is used to produce a non-dimensional quantity. Before presenting the results of the long wave estimation procedure, there are 2 other models for spreading that we will also consider.

In a very elegant analysis, Ewans (1998) presented very careful fits to a large amount of quality controlled spreading data as recorded with a directional buoy of the coast of New Zealand in almost perfect JONSWAP-type fetch limited seas. He conclusively showed that spreading is frequency dependent, being smallest at $\sim 15^\circ$ close to the spectral peak and bi-modal at high frequencies. His proposed double Gaussian spreading function is shown in Figure 8.

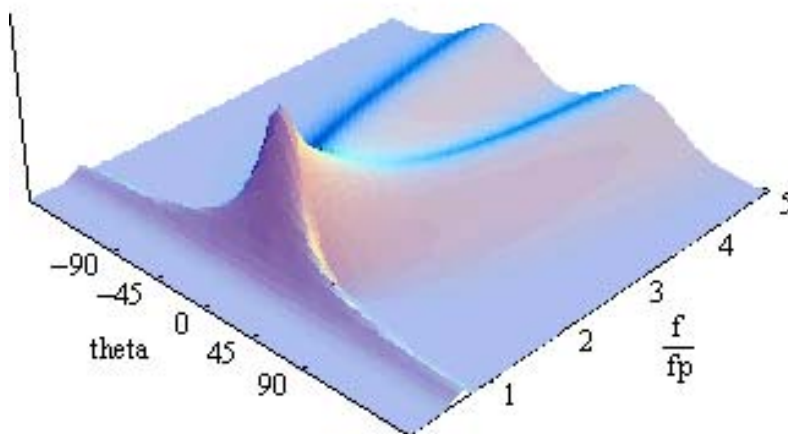


FIG 8. Double Gaussian model of directional spreading for fetch-limited seas, Ewans (1998)

We will fit 3 types of spreading function to the Draupner records:

1. A simple frequency-independent wrapped Normal angular distribution,

2. A modified version of the Ewans model with the bi-modal shape of the frequency dependent distribution retained but the whole angular width simply scaled,
3. A frequency-independent distribution of two overlapping Gaussians, each with an rms width of 15° separated by an arbitrary angle.

For each representation of directional spreading, the spreading is shown as the overall rms value in Figure 8. The 15.20 record now has *2mins* around the large wave removed, longer than previously as the timescales are now associated with group structure not single waves.

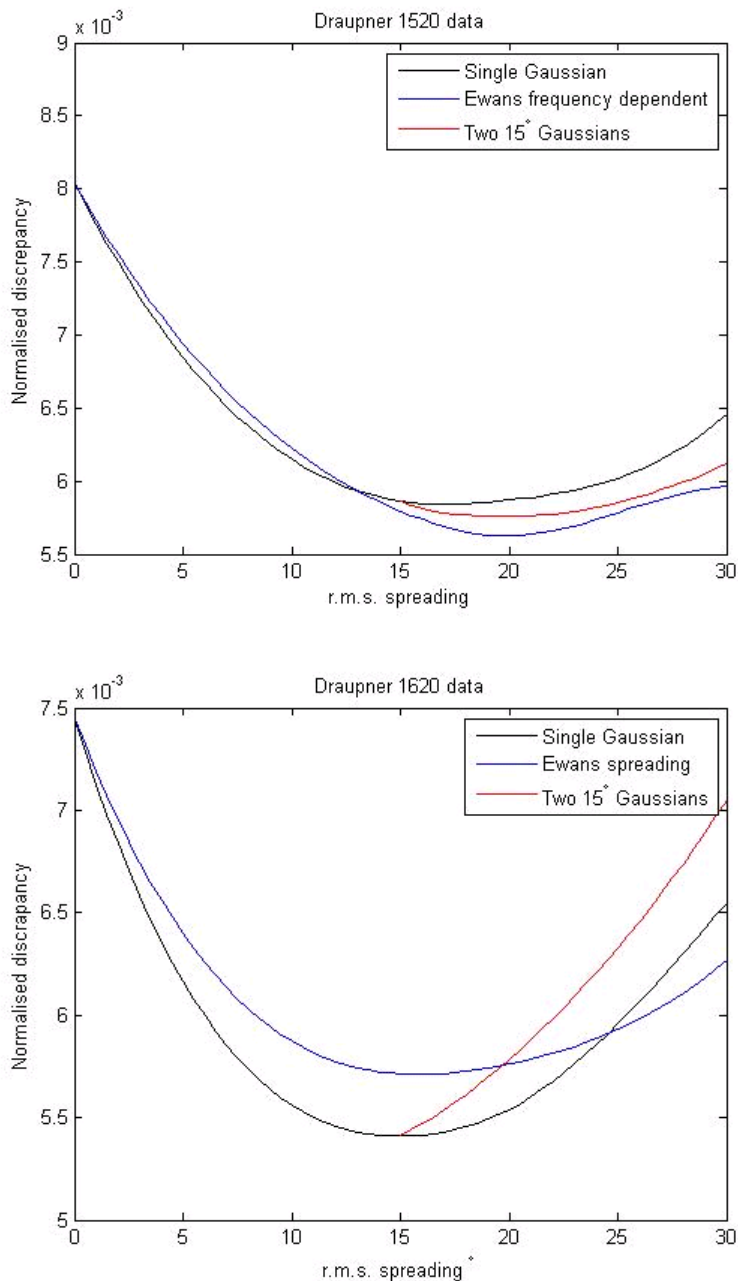


FIG. 8 Normalised discrepancy between the measured long waves in the field and the approximations based on linear waves and 3 models of directional spreading.

Both records show a clear minimum for the discrepancy function. For the 15.20 record the best fitting spreading model has an rms spreading of 20° , whereas the 16.20 record has a best estimate of 15° - both in good agreement with the estimates from the 2nd order sum analysis in the previous section! Further, there is no evidence that a split or bi-modal angular distribution could improve the 16.20 fit. However, the 15.20 fit is improved by splitting the single Gaussian into 2 narrower ones and slightly further improved by making this splitting frequency dependent.

Interestingly, Haver (2004) refers to two meteorological lows generating the waves in the Draupner storm. With different sizes and fetches, there could have been two superimposed wave fields crossing at a relatively small angle initially, which then merged into one an hour later. With different peak periods, this would imply that the initial overall sea-state was uni-modal at low frequency, becoming bi-modal or at least more spread at frequencies above the peak of the second wave field. This would be consistent with the different optimal fits for the 15.20 and 16.20 records.

4. The large wave – NewWave and 1-D simulations

Having discussed at length various features of the two 20min records, we now turn our attention to the Draupner large wave which occurred at approx. 15.30. Initially we discuss a linear model. The average shape of a large extreme in a linear random Gaussian process is simply proportional to the auto-correlation function of the underlying process. This remarkable result, due to Lindgren (1970) and Boccotti (1983), has become known as NewWave in the offshore engineering literature, see Walker et al. (2005).

Thus, we compare the measured shape of the Draupner wave to the suitable scaled auto-correlation function based on the measured spectrum for the 15.20 record. We use the Stokes 5th order expansion as presented by Fenton (1990) using a generalization of the approach in Section 2 above to re-create the associated bound harmonics. The harmonic sum components for the NewWave and a comparison with the measured extreme is shown in Figure 9. Initially this comparison is performed for the long assumed bound wave components filtered out. The overall shapes match relatively well for a linear NewWave with amplitude of 14.7m. The local crest shape is reasonable but the troughs either side of the large crest are slightly too deep. However, we only have one example of a crest of this size and the NewWave shape is a statistical average shape, so we probably should not expect any better matching.

Given a significant wave height $H_s \sim 12m$, a linear crest elevation of 14.7m is an unlikely event. We estimate the return rate as ~ 1 in 200,000 waves. Thus, this is an unlikely occurrence in a single record of ~ 100 waves! However, it should be recalled that there are many other 20min records from Draupner without a wave of this severity.

In deep water, the coalescence of independent linear components results in significant third-order wave-wave interactions. In uni-directional groups this leads to dramatic local

evolution and a much taller group than expected, see Taylor and Haagsma (1994) and Baldock, Swan and Taylor (1996). In a directional spread group, shape changes occur with contraction of the group in the mean wave direction and expansion along the crests but no significant extra elevation, Gibbs and Taylor (2005). One interesting question is whether any comparable processes could occur to a focussed wave group based on the Draupner sea-state. We make use of a recently developed high order Boussinesq model to simulate NewWaves based on the Draupner large wave. These results confirm previous work using a fully nonlinear potential flow solver, Vijfvinkel and Taylor (1998).

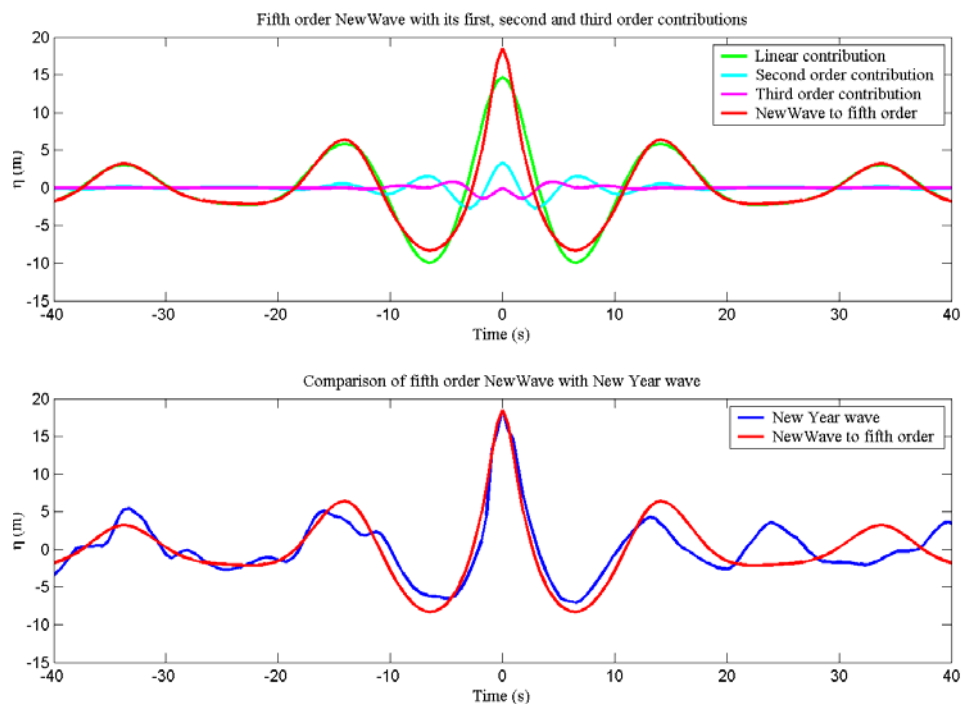


FIG 9. Comparison of 5th order NewWave of linear amplitude 14.7m to the measured Draupner wave

Results from the Boussinesq model for NewWave simulations of the Draupner extreme event for $kd=1.5$ are shown in Figure 10. The initial condition for each is a linear NewWave at 20 periods before focus. Thus, the shorter wave components are ahead of the longer ones. The runs cover 40 periods. Each plot shows the initial and final wave profiles as well as the profile corresponding the maximum crest elevation at any time throughout the simulations. Since the waves are weakly nonlinear, the maximum crest elevations occur at times and positions slightly shifted from those of linear focus. We find very slight nonlinear de-focussing as the amplitude of the wave group is increased rather than the positive focussing expected on deep water. The peak crest for steepness $ka=0.3$ is slightly less than 3x that for $ka=0.1$ and the steeper wave group is slightly wider at focus. Here, ka defines the linear crest amplitude of perfect linear focus.

This slight de-focussing can be associated with the switch of the nonlinear Schrodinger equation from focussing to de-focussing at $kd=1.36$, see Johnson (1997). We conclude that 3rd order resonant wave-wave interactions appear to play little or no role in the local evolution of the large Draupner crest – or at least the various nonlinear processes appear to cancel for these

uni-directional simulations. Thus, the underlying structure of the Draupner large crest appears to have formed by linear dispersion and the return rate value of 1 in 200,000 may be a relatively robust estimate.

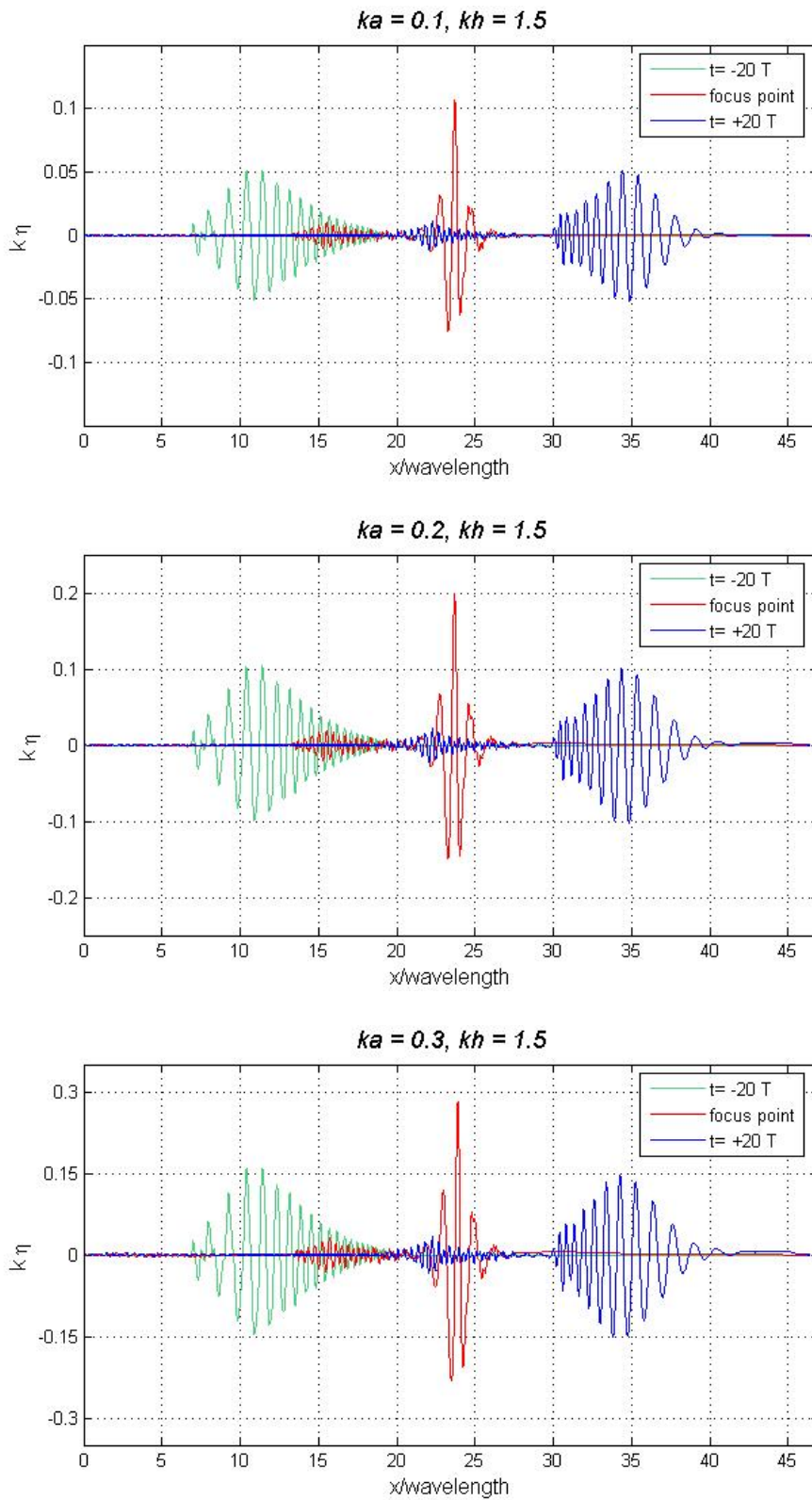


FIG. 10 Evolution of frequency focussed uni-directional NewWave groups based on the 15.20 Draupner wave spectrum

5. Is the large wave a freak ?

The 18.5m crest as recorded at Draupner is undoubtedly a large wave. Is it a freak wave? The fit to the scaled auto-correlation including the contributions from the harmonic sum terms is reasonable but not good. This may be because the wave is locally sufficiently steep that high-order wave processes become significant, see Dyachenko and Zakharov (2001). After all, we do not know whether the wave was breaking or at least about to break

There is one remarkable robust property of the wave, which is not shared by any other large waves in either record. The large wave has a long wave set-up not a set-down. This is demonstrated in Fig 11, showing the largest wave in each record in the upper panels and the same wave histories low-pass filtered in the lower panels.

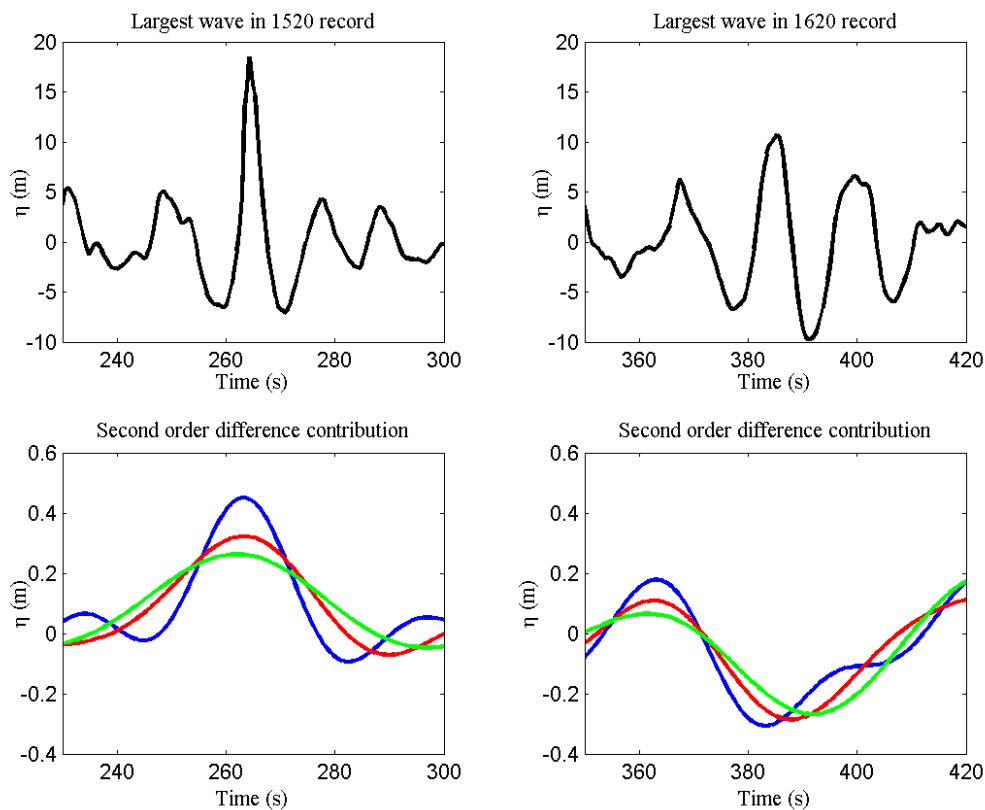


FIG 11. The largest waves in the 15.20 and 16.20 records, low pass filtered at 0.04Hz (blue), 0.03Hz (red) and 0.02Hz (green)

The occurrence of a set-up rather than a set-down for the largest wave is a robust feature. Actually, the magnitude of the long wave set-down at $\sim 0.4m$ is correct for a NewWave of 14.7m in a directional spread sea – just the sign is wrong! We have no explanation for this anomaly – perhaps it is a feature shared by all true freak waves.

To reinforce the significance of this observation, we compare the entire 15.20 record low pass filtered to the predicted 2nd order difference terms based on the linearised wave signal with the best-fit scaled Ewans spreading model in Figure 12. Every other large crest is associated with a clear local set-down. The largest set-downs are associated with the 2nd largest crest in the entire 15.20 record, occurring approximately 25 sec before the end, and the 1st large crest in the entire record.

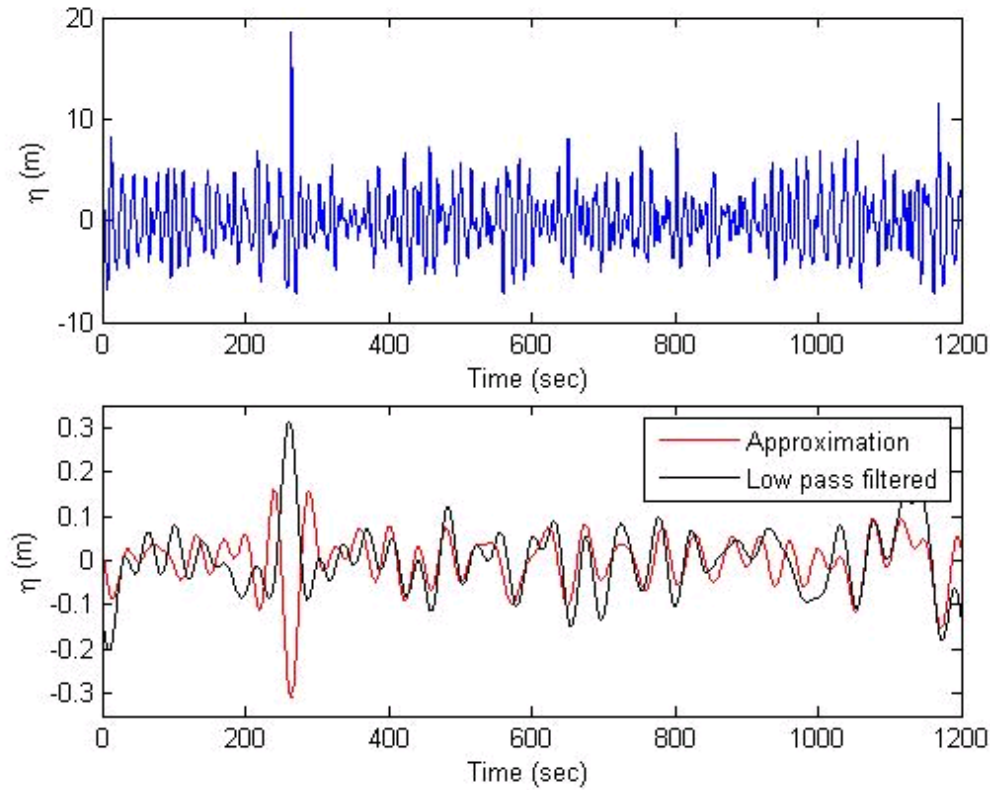


FIG. 12 The alignment of large set-downs with individual large wave groups in the 15.20 record

Actually, the shape of the long wave set-up/down record can equally well be approximated as

$$\eta_{set-down} \sim - (a^2 - \overline{a^2}) \approx - (\eta^2 + \eta_H^2) + \overline{(\eta^2 + \eta_H^2)}$$

However, estimation of the magnitude of the signal requires a more sophisticated 2nd order analysis.

6. Discussion

This paper has summarised detailed analysis of the two Draupner records associated with the violent storm on 1st January 1995. The sea-states are broadbanded, the waves steep and vertically asymmetric. The magnitude of this asymmetry is close to but slightly smaller than that based on the minimum of the Stokes 2nd order coefficient for regular waves when suitably

re-defined. This ~10% reduction is assumed to be due to directional spreading of the wave fields.

Both the 2nd order sum and difference terms are analysed in the records and consistent estimates of overall directional spreading are derived: ~20° for the 15.20 record and ~15° for the 16.20 record. Further, the long wave analysis is most consistent with a uni-modal directional spreading modal for the narrower 16.20 sea, but the broader sea-state for the 15.20 record may in fact have been two superimposed crossing wave fields, as would be consistent with the meteorology described by Haver (2004).

One point that should be stressed is the actual meaning of these estimates for the rms spreading. In commonly used parlance the spreading of a wave field is measured with respect to a defined mean wave direction, as estimated for a whole record. For the Draupner data the mean wave direction is unobtainable since only a single point Eulerian surface elevation time history is available. The spreading estimates obtained should be viewed as measurements with respect to a local mean wave direction inherent for every individual wave, and this local mean direction might well be different for every wave. Thus, it would not be surprising if the local spreading estimates developed in this paper would be smaller than orthodox spreading measures defined with respect to a common mean direction.

The observed form of the Draupner wave crest at ~15.30 is reasonably consistent with a NewWave formulation, allowing for Stokes-type sum harmonics. We estimate the return rate of such a crest as ~ 1 in 200,000. Uni-directional wave simulations of Draupner-type extreme waves show no significant 3rd order wave-wave interactions, hence the return rate of the large wave may be a robust estimate. However, the measured wave is slightly more localised than the NewWave model suggests. This may be a feature of the very high and steep crest form. In some ways, the local detail of the Draupner wave is reminiscent of the 'freak' wave resulting from modulational instability of a Stokes wave train as modelled with a highly accurate computational scheme by Dyachenko and Zakharov (2001).

One dramatic and unique characteristic of the Draupner wave is a relatively small but robust set-up rather than a set-down. We have no explanation for this. All other, admittedly smaller, large waves are associated with a local set-down.

Acknowledgements. The authors would like to thank Dr. Sverre Haver of Statoil for providing the Draupner data studied at length here, Dr. Kevin Ewans of Shell for the directional spreading measurements observed at the directional buoy near the Auk platform and Prof. Rodney Eatock Taylor for helpful discussions. DAGW and TAAA were supported by EPSRC studentships during this work.

REFERENCES

- Baldock TE, Swan C and Taylor PH, 1996: A laboratory study of nonlinear surface waves on water. *Philos. Trans. R. Soc. A*, **354**, 649-76.
- Boccotti P, 1983: Some new results on statistical properties of wind waves. *Appl. Ocean. Res.*, **5**, 134-140.
- Dalzell JF, 1999: A note on finite depth second-order wave-wave interactions. *Appl. Ocean. Res.*, **21**, 105-11.
- Dean RG and Sharma JN, 1981: Simulation of wave systems due to nonlinear directional spectra. *International Symposium on Hydrodynamics in Coastal Engineering*, Trondheim, Norway, 1211-22
- Dyachenko AI and Zakarov VE, 2001: Modulation instability of Stokes wave \rightarrow freak wave. *JETP Lett.*, **81**, 255-9.
- Ewans K, 1998: Observations of the directional spectrum of fetch-limited waves. *Jn. Phys.Oceanogr.*, **28**, 495-512.
- Fenton J, Nonlinear Wave Theories. Chapter 1 in *The Sea*, volume 9 – Ocean Engineering Science. New York: Wiley, 1990, 3-25.
- Forristall GZ, 1999: Wave crest distributions: observations and second-order theory. *Jn. Phys.Oceanogr.*, **30**, 1934-43.
- Gibbs RH and Taylor PH, 2005: Formation of walls of water in ‘fully’ nonlinear simulations *Appl. Ocean. Res.*, **27**, 142-157.
- Haver S, 2004: Freak wave event at Draupner Jacket January 1 1995. Available at <http://www.math.uio.no/~karstent/seminarV05/Haver2004.pdf>
- Haver S and Jan Andersen O, 2000: Freak waves: rare realizations of a typical population or typical realizations of a rare population? *In Proceedings of 10th ISOPE Conference*, Seattle USA, 123-30.
- Johnson RS, *A Modern Introduction to the Mathematical Theory of Water Waves*, Cambridge UK: Cambridge University Press 1997
- Jonathan P and Taylor PH, 1997: On irregular, non-linear waves in a spread sea. *Jn. Offshore Mech. Artic Eng.*, **119**, 37-41.
- Lindgren G, 1970: Some properties of a normal process near a local maximum. *Ann. Math. Stat.*, **41**, 1870-83.
- Prevosto M and Bouffandeau B, 2002: Probability of occurrence of a ‘giant’ wave crest. *In Proceeding of OMAE2002*, paper 28446.
- Taylor PH and Haagsma IJ, 1994: Focussing of steep wave groups on deep water. *In International Symposium: Waves – Physical and Numerical Modelling*, UBC, Vancouver, 862-870.
- Taylor PH and Vijfvinkel E, 1998: Focussed wave groups on deep and shallow water. *In Ocean Wave Kinematics, Dynamics and Loads on Structures ASCE Conference*, Houston, Texas, 420-7.
- Tucker MJ and Pitt EG. *Waves in Ocean Engineering*, Oxford UK: Elsevier 2001
- Walker DAG, Taylor PH and Eatock Taylor R, 2005: The shape of large surface waves on the open sea and the Draupner New Year Wave. *Appl. Ocean Res.* **26**, 73-83.