EXTREME AND FREAK WAVES:

(The Difference and Similarity)

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1. INTRODUCTION

Presently the main source of wave climate information is based on the results of hydrodynamic simulation (in other words hindcasting). Reanalysis data are the input to hindcasting. Any reanalysis data have to be improved for extreme wave estimation. Lopatoukhin et al (2004) used regression and Kalman filtration for assimilation of additional ship observation and synoptic data. The most known and used wave models are WAM, Wave Watch and SWAN. There exist a lot of other models, though the philosophy of mentioned models remains. In Russia nested models Wave Watch (versions 1.18, 2.22) and SWAN (versions 40.11, 40.31) had been applied for hindcasting waves for Barents, Caspian, Baltic, North, Okhotsk, Black, Azov, Mediterranean, Japan seas and Ladoga Lake. Russian Register of shipping published in 2003 and 2006 two Handbooks of wind wave climate of pointed seas (Wind and wave, 2003, 2006). In both editions extreme and operational wave statistics is published. Firstly in the World practice in the 2006 edition information about climatic wave spectra (i.e. about probability of wave spectra of different shape) is presented. The principles to classification and calculation of climatic wave spectra are published elsewhere (e.g., Lopatoukhin, Boukhanovsky 2005; Lopatoukhin et al, 2005). The most important in applied investigations are extreme statistics (especially joint extremes) and freak (rogue) waves, which have some principal difference from extreme wave.

2. ONE-DIMENSIONAL EXTREMES AT A POINT

There are a lot of approaches to calculations of extreme wave heights at a point (classical unconditional extremes). The main are IDM (Initial Distribution Method), AMS (Annual Maxima Series), POT (Peak Over Threshold), MENU and the BOLIVAR (Review of methods as published by WMO, see Lopatoukhin et al, 2000).

IDM method uses all the waves heights measured or calculated for each synoptic term. The extreme wave height h_{max} of certain return period estimates as quantile h_p of wave height distribution $F(h)$ with probability p. ***AMS approach*** uses only annual maxima and h_{max} defines as the last term (maximum) of the ranked independent series of wave heights h. Hence the possible distribution $F(h_{max})$ is one of three limit distributions. The AMS method has the most solid theoretical background. ***POT approach*** uses k strongest storms with the heights greater than selected threshold. It allows to consider not one, but some the extreme values in separate years and, thus, to increase sample in comparison with method AMS. In method POT distribution $F_{max}(h)$ describes distribution of waves "on the average". Apparently this "averaging" occurs according to the law describing number of storms per year. ***BOLIVAR*** uses random impulses; and takes into account the asymptotic characteristics of AMS and uses a set of stochastic models. As against to method POT, method BOLIVAR uses procedure of definition of extremes without "averaging".

For the description of storm situations basic value (level $Z(t)$) has at least three approaches:

- $Z(t) = const$. This approach is realized in method POT.
- $Z(t)$ – Long-term monthly average value. This approach is realized in a method *MENU* (Athanossoulis, Stephanakos, 1995).
- $Z(t)$ – The current monthly average value, i.e. level corresponds to storm activity of a month. This approach is used in BOLIVAR.

The storm pulses $(h^+, \Im^{(h)}, \Theta^{(h)})$ counted from such level in different years belong to the same general population.

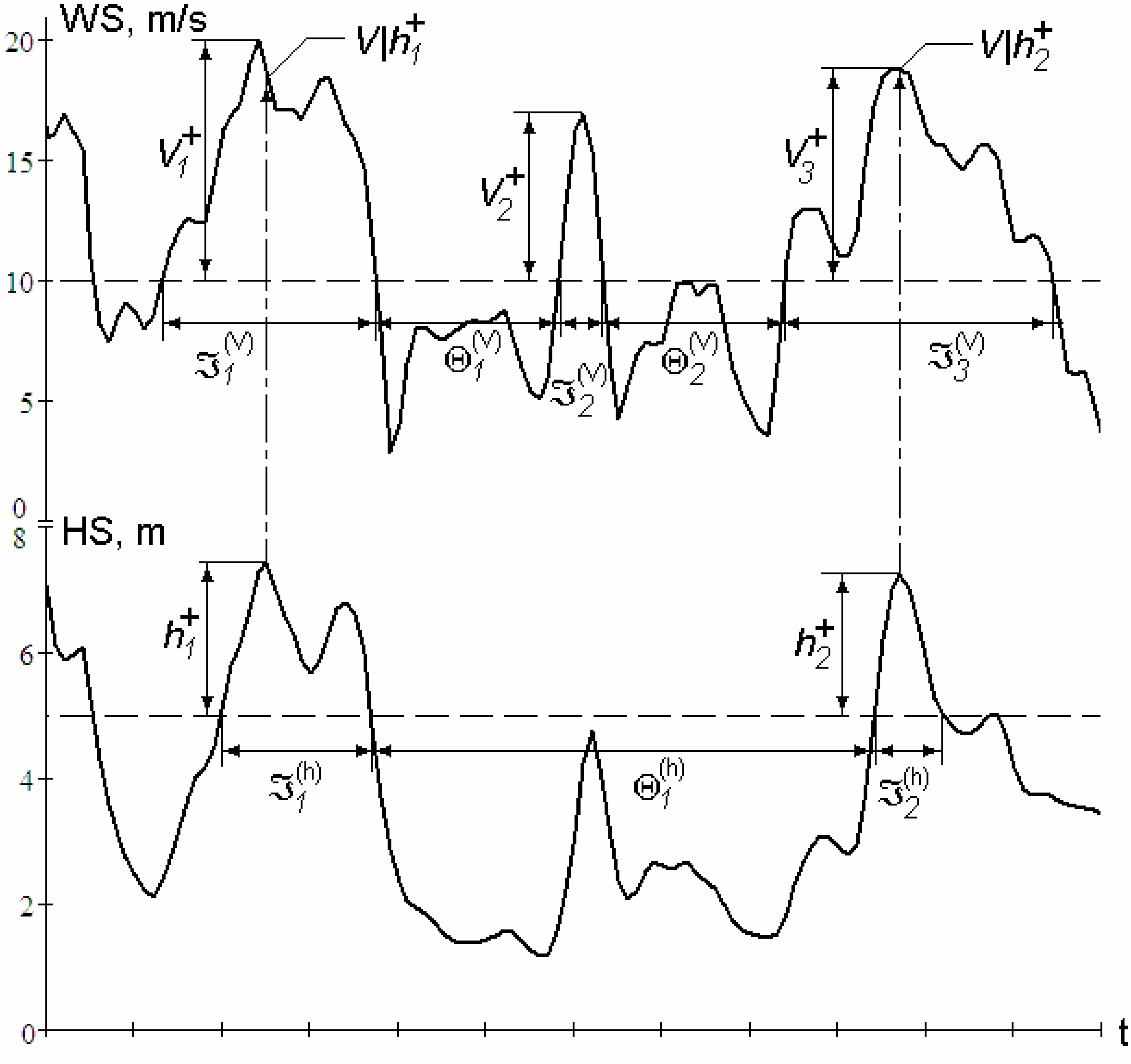

Figure 1. Impulses of time series of significant wave heights HS and wind speed WS. Central part of the North Sea.

Thus, knowing a variable level $Z(t)$ and joint distribution $(h^{+}, \Im^{(h)}, \Theta^{(h)})$, it is possible to define not only intensity of pulses, but also frequency of their occurrence in time as a value $1/(\Im^{(h)} + \Theta^{(h)})$. Intensity of a pulse h^{+} depends on storm duration $\Im^{(h)}$. The background and details are presented in WMO Review (Boukhanovsky et al, 1998). If joint distribution is the product $F_{h^{+}\Im}(x,y) = F_{\Im}(x) F_{h^{+}|\Im}(y\,|\,x)$ for any fixed $\Im$, and h^{+} is a maximum of a section of time series in length $\Im$ (i.e. – *an extreme value* of sample), then conditional Gumbel distribution is acceptable.

$$F(y\,|\,x) = \exp[-\exp[-a(x)(y - b(x))]]. \qquad (1)$$

Parameters $a(x), b(x)$ are scale and shape of conditional distribution. Regressions $f(x) = \xi(1 - \exp(\nu x^{\mu}))$ are shown on the fig. 2. From fig. 2 the seasonal variations of intensity of storm pulses is clearly seen.

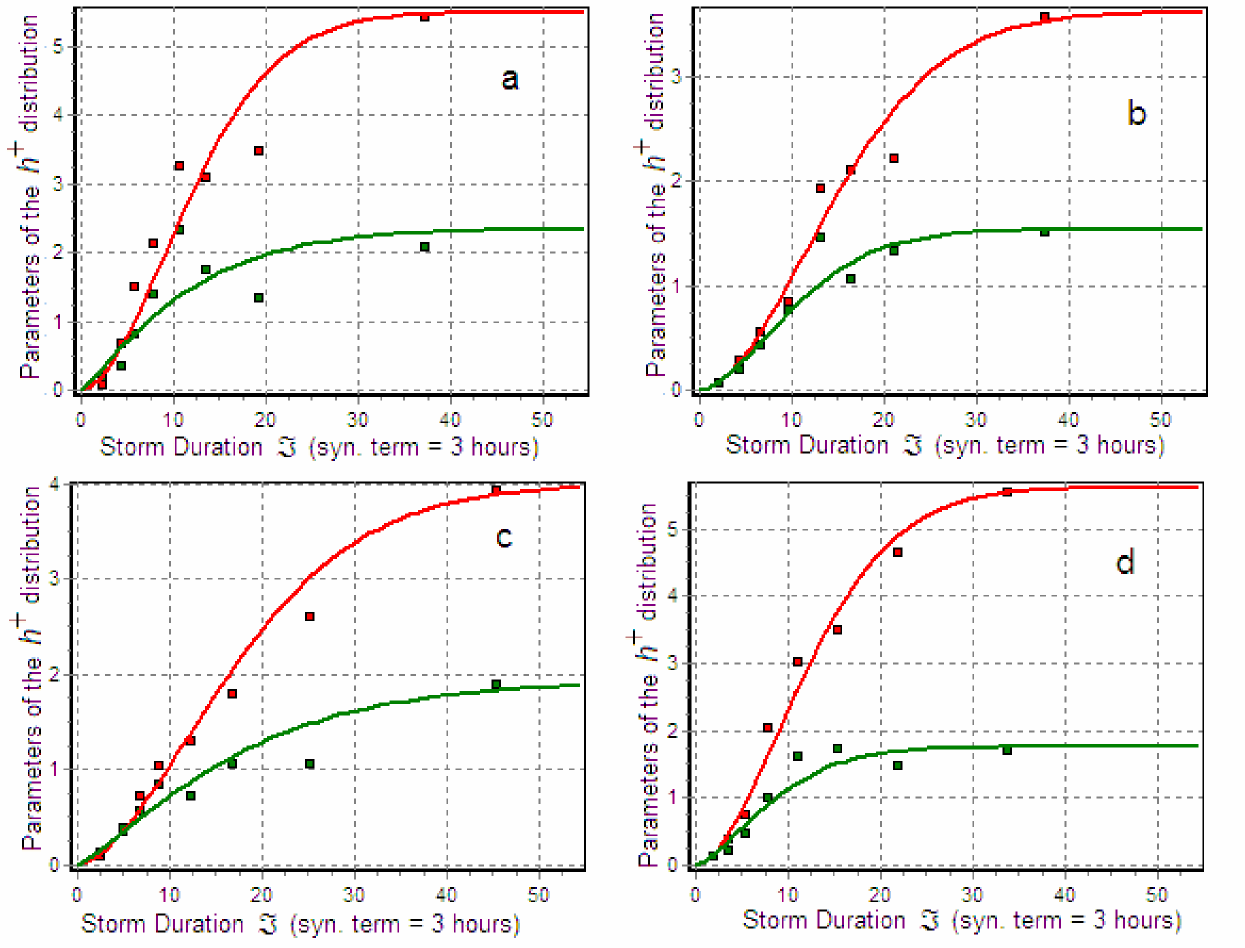

Figure 2. Parameters of conditional distributions (1) plotted against storm durations: a – winter, b –spring, c – summer, d – autumn. Red - $a(\Im)$, Green - $b(\Im)$

3. BACKGROUND OF BOLIVAR APPROACH

Initial data for calculation of extreme heights of waves on method BOLIVAR are continuous (in synoptic terms) realizations of heights of waves – 30 or more years of hindcasting.

3.1. MODEL IDENTIFICATION (DATA ANALYSIS).

1. Calculation of a level $Z(t)$ as current monthly average value of waves heights by means of moving average.

2. Extraction of pulses of variable level $Z(t)$ and deriving of sample of $(h^+, \Im^{(h)}, \Theta^{(h)})_i$ $i = \overline{1, N}$. (i – number of a year, N – total number of years).
3. Estimation of parameters of distributions (1), describing joint variability of a pulse parameters. The data are grouped by seasons.
4. Estimation of seasonal and interannual variability $Z(t)$, e.g. as a model PCSP (coherent or componential).

As a result of steps (1-4) the set of parameters as the input to Monte Carlo method to be obtained. This allows to synthesize ensemble of values of a level $Z(t)$ and not dependent from it pulses $(h^+, \Im^{(h)}, \Theta^{(h)})$.

3.2. SIMULATION (DATA SYNTHESIS)

5. With the help of model PCSP it is modeled N annual realizations of a monthly average level $Z(t)$.
6. The method of conditional distributions simulate sample of independent pulses $(h^+, \Im^{(h)}, \Theta^{(h)})$ so that $\sum_{j=1}^{m_i} \Im_j^{(h)} + \Theta_j^{(h)}$ did not exceed 1 year. (j – number of a pulse within a year, m_i – total number of pulses within a year).
7. For each year highest value (maxima) are extracted: $h^+_{\max_i} = \max_{m_i}\left(h^+_{ij} + Z(t_{ij})\right)i$.

As a result of steps 5-7 simulated sample of N values of waves heights are obtained. Value N can be great enough. In our calculations $N \sim 10^6$ years was used. Method BOLIVAR allows, using stochastic modeling to reproduce synoptic (pulses), annual and interannual variability of sea waves. Modeled ensemble allows to estimate with confidence the characteristics of extremes (as random variables) and also to receive estimations of more rare events – 100 years and more. Besides method BOLIVAR can be used for estimation not only the first, but also secondary maxima.

Each of the considered methods has its advantages and disadvantages and has to be used accordingly. In table 1 methods IDM, AMS, POT and BOLIVAR are compared and all the advantages and disadvantages are seen.

Table 1. Comparing of approaches to estimation of extreme waves.

Criteria	Approaches			
	IDM	AMS	POT	BOLIVAR
Consideration of «tail» of distribution	Heuristic (Ln, W)	Class of limit distributions I, II, III or GEV)	Generalized Pareto (GPD)	Class of limit distributions I, II, III or GEV)
Sample for parameters estimation	365TΔt	T	1÷3T depends from level	40÷70T (depends from location)
Definition of probability for return period T years	By convention, using conditionally independent values in (1)	Exactly (as annual maxima)	In average (as a mean number of storms per year) in (4)	Exactly (as annual maxima)
Consideration of annual and year-to-year variability	Considered in total	Out of consideration	In average (in relation to level)	Considered for each range

4. JOINT EXTREMES

Sea objects and constructions are subjected to complex dynamic loads of different metocean processes (wind, waves, current, etc). The problem estimation and interpretations of one-dimensional extreme of metocean events is well enough developed. At the same time the concept of a multivariate extremes, with return probability T years, till now supposes different treatments, depending on the purposes of research. Necessity of extreme estimation of the natural processes with simultaneously synoptic, seasonal and interannual variability demands use of various approaches.

Let us consider the problems arising at transition from one-dimensional to multivariate extreme on the basis of the most simple IDM approach (omitting known shortcomings). The method (IDM), traditionally developed for one-dimensional extremes, is easily generalized on a multivariate case. It considers sample of values of time series (ζ_t, η_t) in synoptic terms with digitization Δ. The estimation of the extreme phenomenon, possible one time in T years, is determined as quantile of some probability of two-dimensional distribution $F(\zeta, \eta)$.

In some metocean investigations the estimation of a two-dimensional extremes is used as:

$$\Xi_p = (\zeta_p, \eta_p), \text{ where } \zeta_p = F_\zeta^{-1}(p), \eta_p = F_\eta^{-1}(p) \qquad (2)$$

ζ_p, η_p – quantiles of corresponding ***marginal*** distributions. This estimation consider extremes arising simultaneously. This approach can be considered as estimation from above. Relationship between values of processes may be introduced by mean of so called ***"associated"*** estimations of extremes.

$$\Xi_p^{(\zeta)} = (\zeta_p, m_\eta(\zeta_p)); \quad \Xi_p^{(\eta)} = (m_\zeta(\eta_p), \eta_p). \qquad (3)$$

where $m_\eta(\bullet), m_\zeta(\bullet)$ - conditional average (regressions) of sizes η on ζ and ζ on η accordingly. Thus, the associated estimations of extremes give the information about mean probability of the values of one process corresponding to extreme of another process.

The estimation (3) assumes presence of two various estimations of extremes. For the description of combinations of extremes obvious definition can be used

$$F_{\zeta\eta}(x, y) = p \qquad (4)$$

The equation (4) in the implicit form sets a curve in space (ζ, η) with probability p for any point. As against (2,3) definition (4) assume infinite set of possible combinations (ζ, η).

For design decisions, it is necessary to decrease dimension, in particular – introduction of a condition of maximization of function of losses $Q(\zeta, \eta) \rightarrow \max$. Function of losses designates the integrated influence rendered by metocean processes on a construction. For example, for anchorage units functions of losses is $Q = a\zeta^2 + b\eta^2$ which show the total energy influencing to a construction. Coefficients a and b are defined by performance of object.

Definition (4) corresponds maxima of both values ζ, η (i.e. the contour shows probability $P[x > \zeta, y > \eta]$). Such statement corresponds to monotonous function $Q(\zeta, \eta)$. However in some cases in quality $Q(\zeta, \eta)$ no monotonic function is accepted. For example, rolling has strongly pronounced peak at the certain combinations of waves heights and periods. Therefore in this case extremeness can be interpreted not as simultaneous achievement of the greatest value by each of arguments separately, and as ***a rarity of occurrence*** of a combination of arguments, irrespective of their values. The concept of a rarity, or remoteness, demands correlation with the characteristic of centre of the distribution $F_{\zeta\eta}(x, y)$. For example, average value (m_ζ, m_η). The combinations (ζ, η) corresponding to probability of identical "distance" from the center form the closed contour. In some publications they are known as "hat". The problem of construction of "hats" may be solved with the help of methods of transformation of distributions (Winterstein et al, 1993).

Fig. 3 shows estimations of waves and wind extremes in the various interpretations (IDM approach). It is seen, that marginal estimations h_{10}=16.5 m, and V_{10}=38.5 м/c. Lines of regression $m_h(V)$ and $m_V(h)$ diverge for great values of waves and a wind (starting point about h=14.9m, V=30m/s), this leads to the various associated values. For example, $V \mid h_{10}$=35.2 m/s, and $h \mid V_{10}$=16.1 m.

Contour (4) for return period 10 years include marginal values h_{10}, V_{10}, but exclude associated values $h \mid V_{10}, V \mid h_{10}$. It is seen from the fig.3, that while moving along the line of equation (4) (line 4 on the fig.3) equal probability have combinations $(h = 16.5, V = 10)$, $(h = 5, V = 38.5)$, $(h = 15.7, V = 36)$, etc.

The contour of the "hat" corresponding to the same probability (line 1 on the fig 3), differs from (4). So, in this interpretation combinations $(\mathrm{h} = 10, \mathrm{V} = 18.5)$, $(\mathrm{h} = 10, \mathrm{V} = 27.2)$, $(h = 16.9, V = 37.1)$ have the same probability. From fig. it is seen, that the greatest values of waves heights (16.9m) and speeds of a wind (40.2m/s) exceed estimations by (4) (16.5 m and 38.5 m/s, accordingly).

Also on fig. 3 the contour of power function of the losses $Q(\zeta, \eta)$, corresponding to the combinations (ζ, η) resulting in identical loading on a construction is drawn. This contour concerns quantile contour (4) in a point $(\mathrm{h} = 16.1\mathrm{m}, \mathrm{V} = 34.8\mathrm{m/s})$. Thus, this size can be considered as an extreme combination of speeds of a wind and heights of the waves, resulting to the greatest loading on a construction.

Thus, the carried out analysis has shown, that different ways of interpretation of extreme characteristics do not coordinated with each other and lead to different conclusions. One of the most proved is the way of definition of two-dimensional extremes by on (4) as it corresponds to one-dimensional estimations (as the limiting case). Lack of the approach (4) is the opportunity of its application only for objects with monotonous functions of losses $Q(\zeta, \eta)$. Otherwise it is necessary to use "hats".

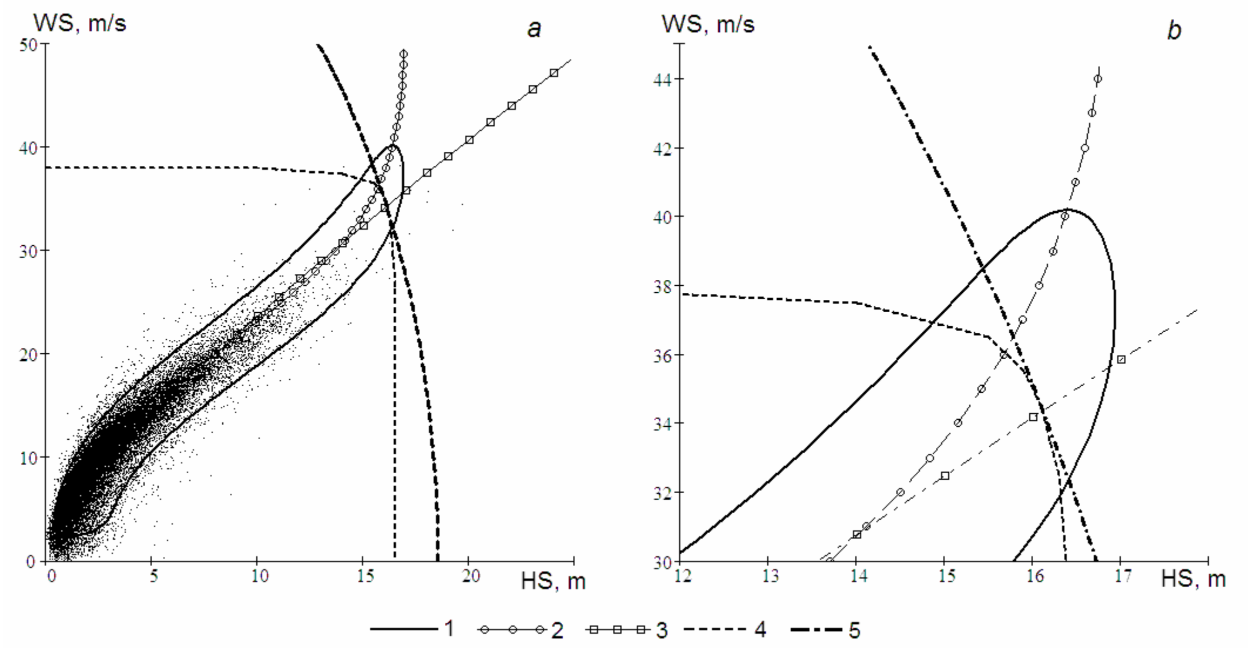

Figure 3. Approaches to joint extremes interpretation. (b– enlarged scale of upper part of a).

1 – line of equal rarity (10 years return period) of wave heights and wind speed, so called «hat», 2 – regression h|V, 3 – regression V|h, 4 – two-dimensional quantile (4), 5 – loss function. Central part of the North Seas.

It is necessary to note, that method IDM, has some specific complexities connected with two-dimensional extremes. In particularly, with definition of the probability corresponding to event of one time in T years. A problem is, that the correlation period to wind is 12 – 36 hours, and to waves 24 –60. Thus, for an estimation of marginal extremes there will be different multipliers for transition to independent observations, which contradicts to (4). It leads to necessity of generalization of method BOLIVAR for calculation of joint extremes.

On fig. 1 joint pulse parameterization of waves heights and wind speeds was shown. It is seen, that the greatest values of h^+ not always correspond to the greatest values of wind V^+. Besides depending on level Z (t), there can be such pulses of a wind and waves, for example, $(h^+, \Im^{(h)}, \Theta^{(h)})_2$ which arise independently from each other; i.e. the wind can be higher $Z_V(t)$, and waves – are lower $Z_h(t)$, and on the contrary. Data shows, that for the level of current monthly average number of such pulses about 20-30 % from all cases.

Thus, joint variability of extreme characteristics of waves and a wind can be described by system of two pulse processes with a set of parameters $(h^+, \Im^{(h)}, \Theta^{(h)}, V \mid h^+)$ and $(V^+, \Im^{(V)}, \Theta^{(V)}, h^+ \mid V)$. The same distributions as to one-dimensional case are valid. As a first approximation for the distribution $V \mid h^+$ three-parametrical Weibull distribution is accepted, and for $h \mid V^+$ log-normal distribution. Introduction of the third parameter is caused by occurrence of the negative associated values (i.e. conformity of a maximum of a storm of one process to a window of weather of another). On fig. 4 examples of diagrams $(h^+, V \mid h^+)$ and $(V^+, h \mid V^+)$ for the different seas are drawn. Characteristics of conditional distributions – a conditional population mean (a curve of regression) and borders of 95 % probability interval are also shown. From fig. 4 it is seen, that for all considered cases regression between the data is similar and well pronounced: for $(h^+, V \mid h^+)$ better, than for $(V^+, h \mid V^+)$. Probability 95 % interval is practically symmetric concerning average. It is connected with are high values (3÷7) of shape parameters for Weibull and log-normal conditional distributions. Use of pulse parameterization BOLIVAR leads to better (with smaller influence of random factors) description of dependence between waves and a wind, than initial distribution. For calculation of extreme characteristics quality of regression is of basic input as it is needed for extrapolation to great values.

In such a manner the joint waves and wind extremes by two-dimensional BOLIVAR calculated. In addition it is simulated: on a step 3 joint distribution $F(h^+, V \mid h^+)$ and $F(V^+, h \mid V^+)$, and on a step 6 the four values $(h^+, \Im^{(h)}, \Theta^{(h)}, V \mid h^+)$ and $(V^+, \Im^{(V)}, \Theta^{(V)}, h^+ \mid V)$. Thus a result of stochastic simulation is a sample of T annual realizations of wave heights and wind speeds. This allow to define annual maxima (as extreme members of sample) and associated values $(h_{\max}, V \mid h_{\max})$ and $(V_{\max}, h \mid V_{\max})$. It follows, that having sample of values of these characteristics, it is possible to estimate their joint distribution $P[h_{\max}, V \mid h_{\max}]$ and $P[V_{\max}, h \mid V_{\max}]$, or their occurrence in years: $T(\bullet,\bullet) = 1/P[\bullet,\bullet]$.

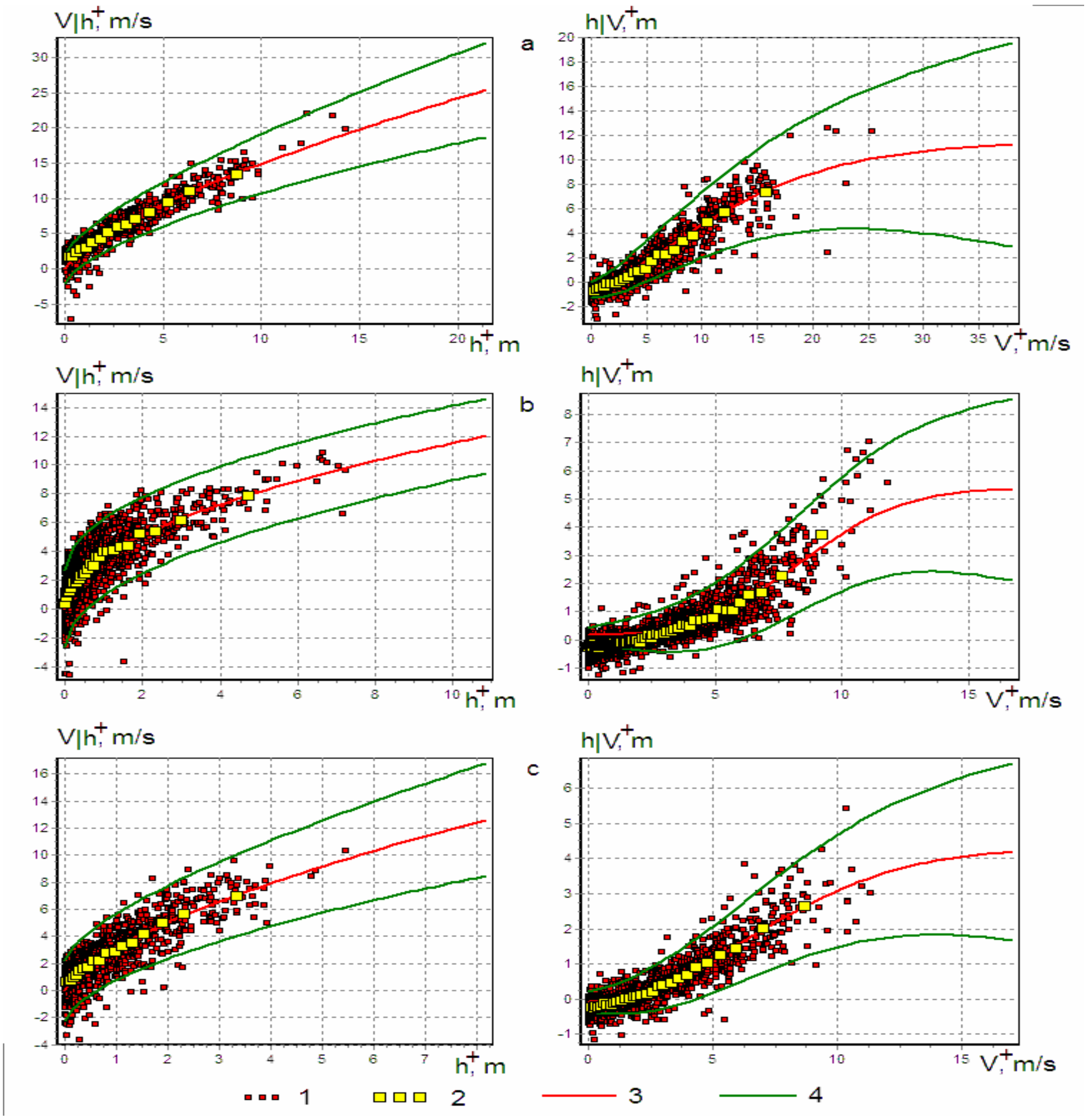

Figure 4. Values of pulses and the associated values of a wind and waves; *a* – North sea; *b* – Mediterranean; *c* – Baltic sea. 1 – The initial data; 2 – estimation of a conditional mean; 3 – line of regression; 4 –95 % probability interval.

Histograms of occurrence (in years) of joint extremes of waves and wind (calculated by BOLIVAR approach) are shown on the fig. 5. From fig.5 it is seen, that occurrence $P[h_{\max}, V \mid h_{\max}]$ differs from $P[V_{\max}, h \mid V_{\max}]$. For example, in the North sea (fig. 5a) height of significant wave with return period 100 years is 23.2 m., and the highest associated value of the wind, laying on the same contour – 37.2 m/s. On the contrary, wind with return period 100 years is 43.7 м/с, and the highest associated value of a wave – only 21.5 m. Similar differences are for other seas. The reason of this difference is that joint extremes of waves and a wind usually do not appear simultaneously (a divergence of curves of regression on the fig.3). Therefore from the point of view of the greatest loading on a object (set by function of losses $Q(\zeta,\eta)$) in some cases the leading part is the wind, and in others – waves. Therefore the estimation of two-dimensional distribution of extremes both waves and wind can be constructed by another rule, e.g.:

$$T[V_{\max}, h_{\max}] = \min\{T[h_{\max}, V \mid h_{\max}], T[V_{\max}, h \mid V_{\max}]\}, \qquad (5)$$

i.e. for the fixed values (*h, V*) the big occurrence is accepted. On fig. 6 it is resulted as histogram for North Sea, calculated by the rule (5).

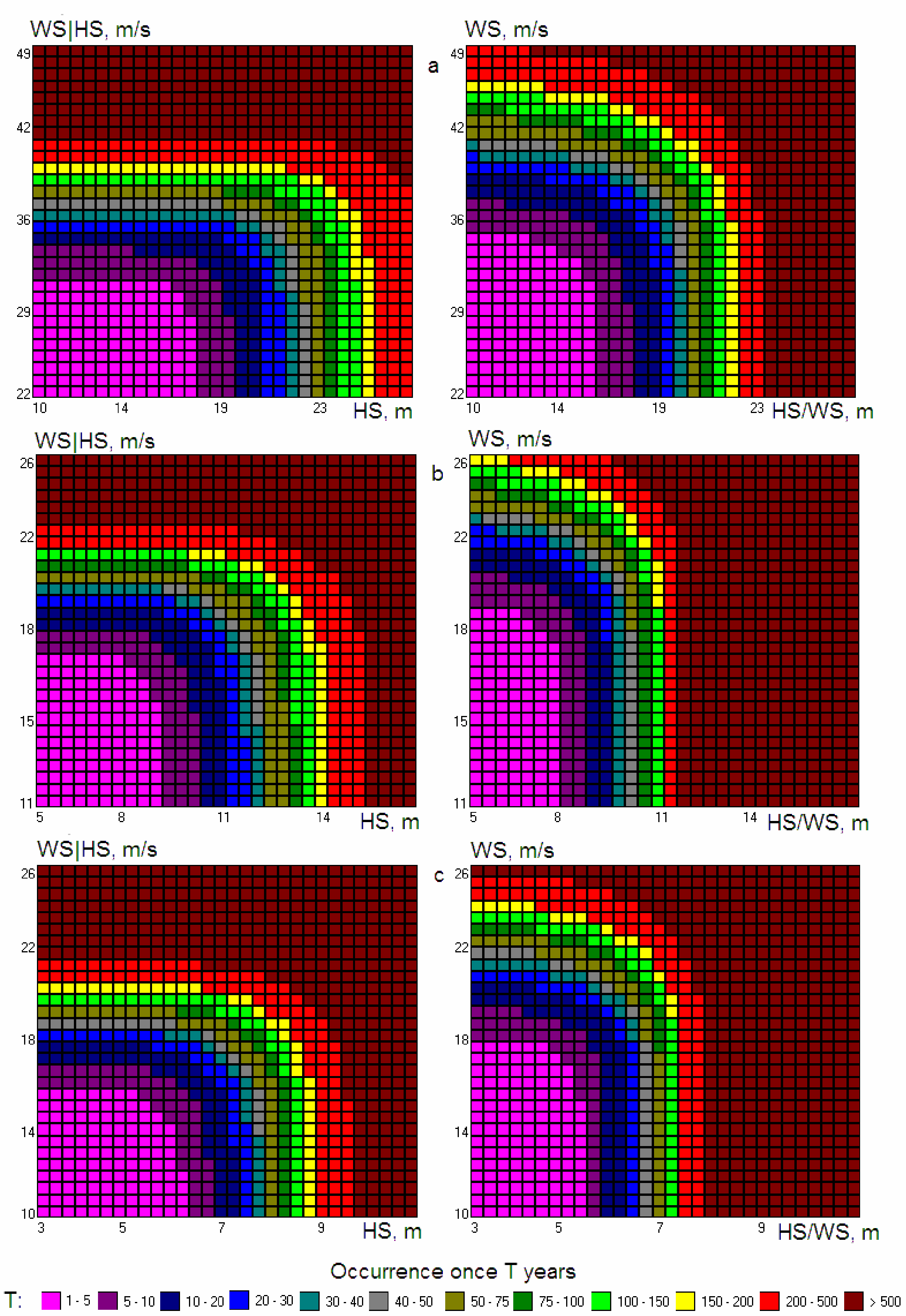

Figure 5. Histograms of occurrence (in years) of wind and waves extremes; *á* - North Sea; *b* – Mediterranean; *c* – Baltic Sea.

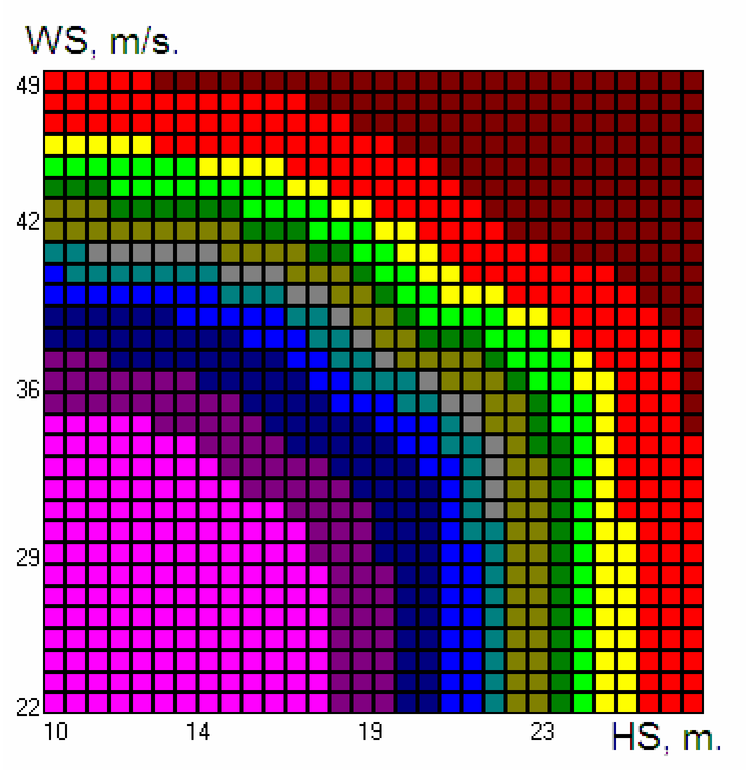

Figure 6. Joint occurrence (in years) of significant wave heights and wind velocity. Central part of North Sea. Notations as on the fig. 6.

5. FREAK WAVES

One of the most interesting extreme phenomena is freak waves – as anomaly steep and high waves. Today a lot of hypotheses try to explain freak wave generation mechanism. All the reasons may be separated on *external* or *internal* (Lopatoukhin, Boukhanovsky, 2003, 2004). The external reasons are metocean, bottom topography and similar, e.g. the opposing wave-current interaction, refraction around shoals or from inclined seabed, wave caustics from diffraction at coastlines, crossing wave systems, etc. The internal reasons are due to specific of wave propagation in wave media and are mainly the frequency and (or) amplitude wave modulation in a random sea, cooperative effect of four- and five-wave interactions, the high-order nonlinearties and nonlinear focusing. Some definitions of freak waves as set of parameters $\Xi = \{h, \tau, c, ...\}$, characterizing the shape of the wave and the steps of it selection from a record are presented in the fig. 7. Really, hypotheses of freak wave generation allow their arising in any place of the Ocean, and not only in the well-known dangerous regions, such as South shore of Africa etc. **Any metocean event described by a system of nonlinear thermo hydrodynamic equations, possesses their own freaks.** Freak wave had been recorded in such "calm" region as the NE part of Black sea (Divinsky et al, 2004). There were three such waves recorded during six years of measurements, i.e. three waves from more than million recorded.

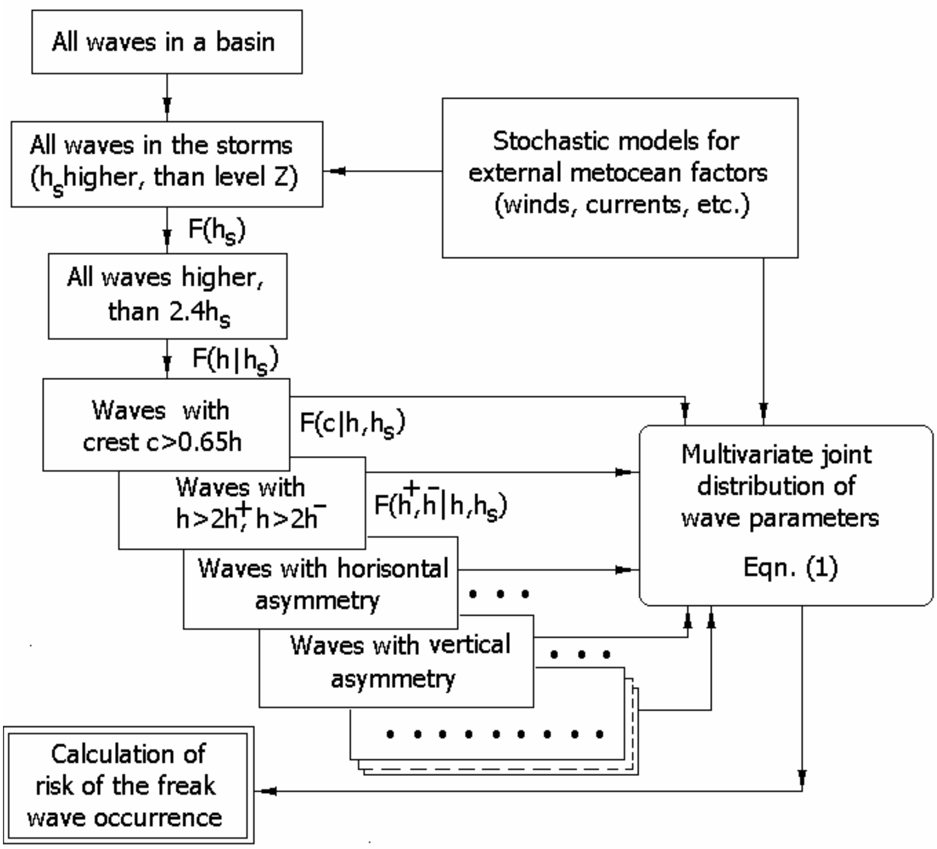

Figure 7. General scheme of freak wave generation scenarios

The example of recent freak wave event is the loss of ship "Aurelia" (Class of Russian Register of shipping, 34000tonn) in February 2005 in the N. Pacific. "Aurelia" sunk during passing of atmospheric front with veering wind, changing wind waves and presence of swell. Fig. 8 shows possible parameter of freak wave during this case.

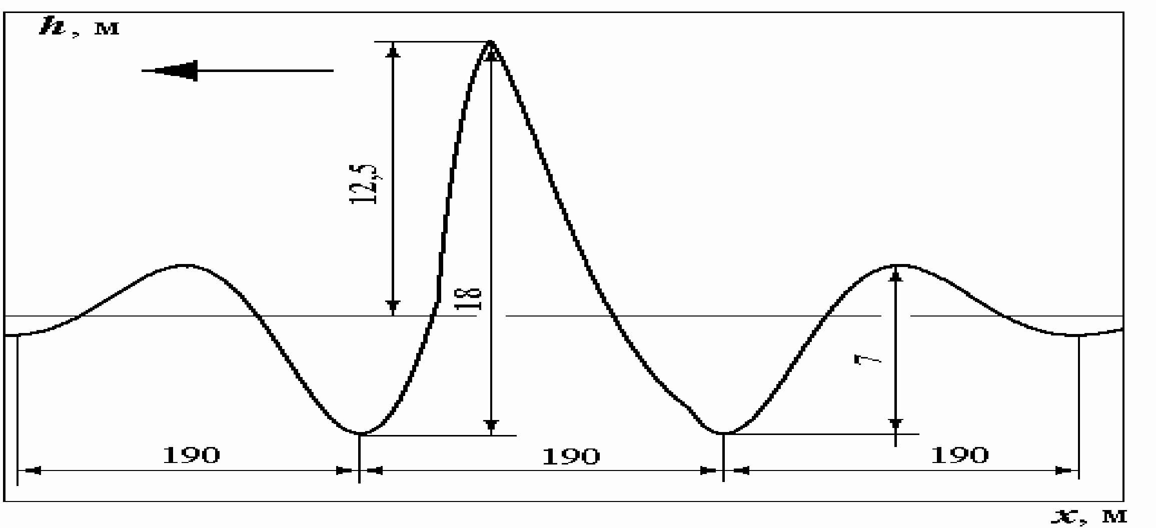

Figure 8. Possible freak wave during loss of ship «Aurelia». February 2, 2005, N. Pacific.

5.1. PROBABILISTIC SCENARIOS FOR FREAK WAVES GENERATION

There are two ways to formulate the conditions of freak waves generation in the Ocean. The first way considers the arising of the different external conditions, leading to possibility of freak wave generation, and computation the joint probability of these conditions (e.g. combinations the severe waves and opposite currents etc.). But the real input of this approach is not obvious, because it is hard to take into account all the driving factors. Another way considers the ensemble of all waves (their heights h, periods τ, crests c etc.) and estimate occurrence of its crucial combinations, leads to freak wave arising. This approach seems more reliable in practice, because it is based on the consideration of freak waves as the elements of the same ensemble, as all the waves. But, it requires the sophisticated statistical techniques for rare events analysis, because the extreme combinations of the waves parameters belong to the tails of its joint probability function.

The problem of freak wave occurrence, and associated scenarios, include the procedures of statistical analysis and synthesis of huge data samples, because freak wave is very rare event. Moreover, due to multiscale and spatio-temporal variability of sea waves, the numerical simulation here is very resource-consuming procedure. It requires the development of special approach for stochastic simulation, that allows investigating the freak waves occurrence efficiently and precisely.

Freak wave is unusual not only by there height, but by their form. This uncommonness specified by means:

- Set of parameters, e.g. h>2.4hs, crest>0.65h, unusual steepness δ of a wave and (or) it front or back slope, deep trough, twice as greater than preceding and subsequent waves, etc. Not all of these parameters are realized simultaneously, but as a rule at least three can be achieved.
- Governs by nonlinear Schrödinger equation.
- Suddenness of arising in some point of a wave field.

One of the main objectives of investigation is a probabilistic treatment of a wave field $\zeta(\vec{r},t)$ as probabilistic contaminated distribution. (Such type of distributions had been introduced by Tukey (1977)).

$$\Phi_{\varsigma}(\vec{x},\vec{r},t) = (1-\varepsilon)F_{\Xi}(\vec{x}) + \varepsilon\hat{F}_{\Xi}(\vec{x}). \qquad (6)$$

Where $F(\vec{x})$ - joint distribution of wave parameters (e.g., height, crest, steepness), $\hat{F}(x)$ - asymptotic distribution of this parameters, $\varepsilon(\vec{r},t)$ - probability of freak wave arising in specific place at a moment t. Ξ -multidimensional system of random values (h,c,δ…).

The first term in (6) describes “background” distribution of Ξ in short-term domain. It is approximated as,

$$F_{\Xi}(X) = F_h(x_1)F_{c|h}(x_2|x_1)F_{\delta|h}(x_3|x_1) \qquad (7)$$

In short-term scale distribution (6) is a set of Weibull distributions with different shape parameters. The second term in (6) incorporate contamination (litters) of a “background” distribution by freak wave. Asymptotic distribution $F(x_1,x_2,x_3)$ may be used.

Rayleigh distribution may be used as marginal $F_h(x_1)$, and Weibull distributions with scale parameter from 2 to 7 as conditional distributions of wave crests c and steepness δ. Joint distributions $F_h(x_1)F_{c|h}(x_2 \mid x_1)$ and $F_h(x_1)F_{\delta|h}(x_3 \mid x_1)$ are presented at fig. 9. This fig. is generalization of about 5000 wave records, but without freaks. The equal probability (p%) curves for values $\{h, c/h\}$ и $\{h, \delta\}$ are drawn. It is seen, that value $\{h/\bar{h} \geq 3.8, c/h \geq 0.65\}$ for any δ, or $\{h/\bar{h} \geq 3.8, \delta \geq 0.5\}$ for any c/h have the probability $5\cdot10^{-6}$. Probability defined from three-dimensional distribution $P\{\delta \geq 0.5 \mid h/\bar{h} \geq 3.8 \cap c/h \geq 0.65\} = 0.12$ This means, that probability of three conditions simultaneously $\{h/\bar{h} \geq 3.8, c/h \geq 0.65, \delta \geq 0.5\}$, will be $5\cdot10^{-6}\cdot0.12 = 6\cdot10^{-7}$. This means, that only one wave from 1.7 million will be with height greater, than $3.8\bar{h}$, crest greater than $0.65h$ and steepness $\delta > 0.5$. This value is the lower limit of probability ε, i.e. probability of freak wave in a specific point not greater than $6\cdot10^{-5}$ %.

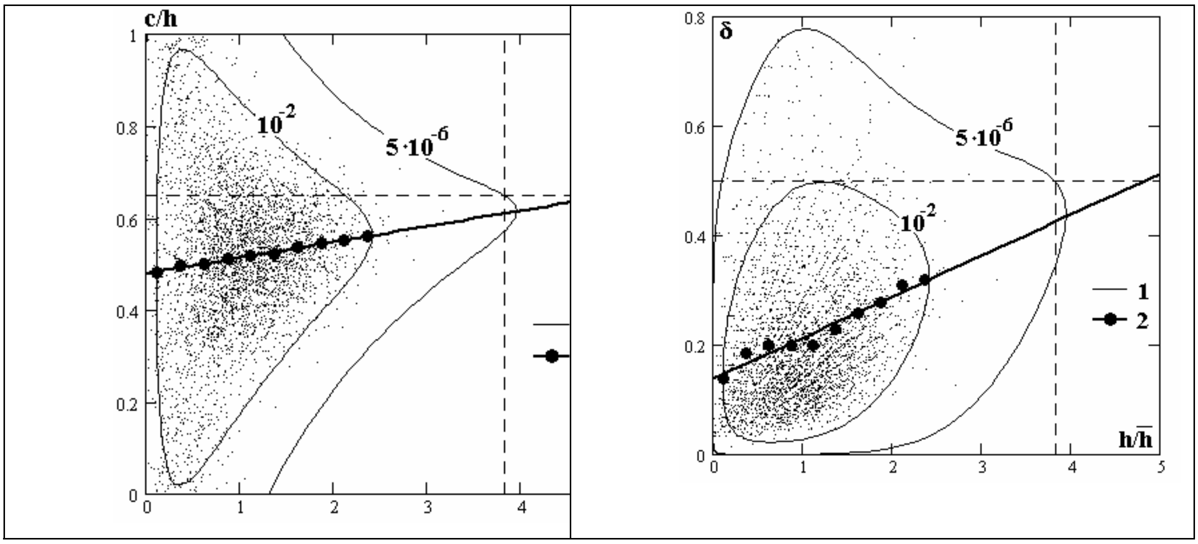

Figure 9. Joint distribution of parameters $\{h, c/h\}$ (*a*) and $\{h, \delta\}$ (*б*). (1) – Lines of equal probability; (2) – Regression.

For short term range with 1000 waves, freak wave may arise in one of 1660 time series. Relation (8) may be adopted as asymptotic distribution in (6).

$$F_{\Xi}(X) = F_h(x_1)F_{c,\delta|h}(x_2, x_3) \qquad (8)$$

E.g., first limit distribution for h and Gumbel-Morgenshtern for (δ,c) may be adopted.

Estimate ε, based on the wave measurements, is about 10^{-8} , i.e. one from 10^{8} waves may be freak. If the short-term interval duration is 1000 waves and wave field consist from 100 points, then freak wave may arise in one of 10^{3} short-term ranges. In the long-term interval wind wave is stochastic process modulated by synoptic, annual and year-to-year variability. This means, that system of random values Ξ with mean ξ has long-term distribution Φξ (x) and the climatic ensemble is approximated by multidimensional combined distribution

$$\Psi_{\Xi}(X) = \int F_{\Xi}(X, \xi) d\Phi_{\xi}(\xi) \qquad (9)$$

Apparently the probability to occur the freak wave anywhere in the sea is higher, than to occur it in the fixed point. This effect is valid for all the scales of wave variability.

Hindcasting of waves do not allow revealing freak wave (Lopatoukhin et al, 2005). Wave measurements shows that in the Black and North seas freak waves arise during transformation of wind wave spectra to waves with swell. In this case both wave spectrum and angular distribution became broader. Prediction of wave conditions by spectral model is routine activity. Therefore prediction of spectral jumps may be one of warning to the possibility of freak wave arising (Lopatoukhin et al, 2005).

6. CONCLUSION

Estimation of one dimensional extremes at a point has no any principal problems. There exist a lot of approach with advantages and disadvantages. Choice of BOLIVAR approach allows improving extreme estimates. Evaluation of joint extremes (in particularly, two-dimensional) meet different treatment. Marginal-conditional two-dimensional distributions proposed for calculations of joint wave heights and wind speed extremes. To obtain unambiguous solution of distribution and loads on the object at a sea the introduction of function of losses is needed. In this case the joint rare effect will be determined.

Occurrences of freak wave have to be regarded as multidimensional random event. This is the main difference between extreme and freak wave. Classical statistical analysis of time series does not allow estimating the probabilities of freak waves and associated weather conditions. Results of hindcasting for any specific time also do not display any suspicion to such a wave. Directional spectrum of wave record does not reveal existence of freak wave. General scheme of freak wave selection from the sample of measured waves is shown on the fig. 7. Special attention has to be paid to investigation of field conditions leading to freak wave generation. These include weather features, current effects and bottom bathymetry.

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