On the Prediction of the Fetch Relations and Dissipation Laws Based on Dual Evolution Equations for Non-linear, Non-conservative Wave Systems.

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Energetic ocean waves are known to be created and to grow due to energy and pumping by the wind, balanced by momentum and energy loses due to breaking, accomplished by small overturning jets at the wave crests; the waves are known to be unstable to near neighbor sideband disturbances and to modulate, causing peaks in the wave amplitude where breaking takes place; the dominant frequency is known to downshift steadily as the wave energy grows, and, correspondingly, the phase and group speeds increase. Nonlinear processes dominate. Field data have provided experimental laws. The provision of coherent predictive theory based on sound mechanical laws remains a challenging goal.

Here we provide a mathematical description of appropriate conservation laws using the variational approach: a modified Hamiltonian principle involving the modulating wave Lagrangian plus a Work Function representing the non- conservative effects of boundary stresses and fluxes due to wind and breaking.

The dual evolution equations which result, represent conservation laws for <energy,momentum> or, correspondingly <energy,celerity>, including the non-conservative effects of wind and breaking. In these equations, the wave state is represented by operations on the wave Lagrangian, which must be approximated. The boundary stresses must also be paramaterized and given semi-empirical representation based on fundamental considerations and field data.

We consider in further detail the case of wave systems symmetric about a steady wind direction, where the wave Lagrangian may be calculated, at least initially, by utilizing the second order solution for a modulating planar wave. Dual evolution equations <energy,group velocity> are produced which are correct to the fourth order in (ak) for energy and to the third order for group velocity.

These dual evolution equations may be expressed in complex form as a single equation which is identical to the usual NLS except for the addition of forcing terms involving the real effects, including an integral over space of the dissipation due to breaking. The proper extension of the NLS to real waves and long fetches is therefore accomplished, and its actual physical nature, in the form of dual evolution equations, is revealed.

Wave evolution in two separate asymptotic regimes is considered: the "modulation", or mid-time; and the "wind driven growth", or long time regimes.

In the case of wind driven growth, where a well known empirical wind pumping law is used, the breaking dissipation law is determined in form and magnitude, by requiring that power law solutions of the dual evolution equations exist and match field observations. It is then shown that the higher order dynamical terms are small in comparison to real effects.

A frequency downshifting solution is then found from the dual evolution equations, which is in excellent agreement with the law determined from field measurements. It is shown that downshifting is ultimately a consequence of the detailed breaking process.

In the case of the modulation regime it is then shown that the real effects are of the same order as the higher order dynamic terms.

The particular case of waves of permanent form is shown readily to arise as a special solution of the dual evolution equations. They are periodic and soliton like waves in the presence of wind.