WAVE FOCUSING ON DEEP WATER - BANDWIDTH, SPREADING AND NON-LINEARITY

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1 INTRODUCTION

In recent years there has been increased interest in the possibility of unexpectedly large ocean waves, termed '*freak*' or '*rogue*' waves, inconsistent with standard statistical distributions. Seafarers have logged sightings of such waves for many years, and there are now field measurements to suggest their existence. On 1st January 1995, during a storm with significant wave height of 12 m, Statoil's 'Draupner' gas platform was struck by a wave of measured crest height 18.5m, causing damage to equipment on the deck, Haver (2000) . Inspection of the time history of the surface elevation at the platform containing the large crest shows that the depth of the preceding trough was only 7.5 m.: there was no obvious indication of the extreme wave about to strike. The Schiehallion FPSO, a BP Amoco vessel moored in the North Atlantic, was hit by a large wave on 9th November 1998. The impact pushed in bow plating at 20m above the mean water level, during a sea state with an estimated significant wave height of 14m. In the autumn of 1995, the ship's master of the Queen Elizabeth 2 recalls seeing "*a wall of water for a couple of minutes*" during a severe storm off Newfoundland. These examples highlight some characteristics commonly used to describe '*freak*' waves.

The largest ocean waves are probably highly transient, arising from a random background and both the wave nonlinearity and directionality significantly affect the local properties of the extreme event, Johannessen and Swan (2001) . The frequency focusing of a wave group, whereby many wave components have maxima that coincide at a particular point in space and time, provides a model that is consistent with the description of a transient event. The unidirectional, fully nonlinear numerical simulations of Taylor and Haagsma (1994) showed that as a group focuses it becomes narrower and much higher than linear theory would predict. In wave flume experiments, Baldock, Swan and Taylor (1996) found that this extra elevation could be up to 30% greater than the corresponding linear solution for narrowbanded groups. Both the works showed that the extra elevation increases with the input amplitude of the group, but decreases with its bandwidth. However, the wave basin experiments of Johannessen and Swan (2001) showed that, for fixed initial input amplitudes, even a small amount of directional spreading leads to a significant reduction in the extra elevation of a focused wave group, which is now almost completely captured by a second order model. They also found local directionality changes requiring a greater initial input amplitude to initiate wave breaking at the nonlinear focus. These results were confirmed by the numerical simulations of Bateman (2000).

This paper considers the significant changes in bandwidth that occur during the focusing of a relatively narrowbanded wave group on a spread sea. Section 2 details how such groups may be simulated numerically and how the input wave group is determined from the observations of real seas. The surface shape and resulting amplitude spectra are shown in section 3 for a particularly steep wave group at different stages of its evolution. The importance of these results is discussed in section 4.

2 NUMERICAL SCHEME

The formation of extreme deep water waves is studied by the frequency and directional focusing of many wave components. It is assumed that such events may be modelled on an incompressible, inviscid, irrotational fluid so that the motion is governed by Laplace's equation for the velocity potential. In addition there are no underlying currents and the effects of surface tension and the interaction of the air with the water surface are neglected. This allows the specification of two free surface boundary conditions, which lead to the explicit expressions for the time derivatives of the surface elevation ($\eta(x, y, t)$) and the velocity potential ($\phi(x, y, z, t)$) below.

$$\eta_t = \phi_z - \phi_x \eta_x - \phi_y \eta_y \tag{1}$$

$$\phi_t = -\frac{1}{2}\left(\phi_x^2 + \phi_y^2 + \phi_z^2\right) - g\eta \tag{2}$$

Given a suitable initial condition, the equations above may be time-stepped to give the evolution of the water surface. However, their evaluation requires computation of the tangential and normal derivatives of ϕ on the surface. For calculation of the normal derivative, knowledge of ϕ in the fluid interior is necessary and becomes the crucial step in any accurate and efficient time evolution scheme.

For unidirectional waves, Craig and Sulem (1993) showed that a Dirichlet-Neumann (G-) operator could be used for the accurate conversion of the velocity potential on the surface into its normal derivative on the surface. The surface elevation and velocity potential are represented by Fourier series, and the G-operator is evaluated by performing multiplications in physical space and differentiations in wavenumber space with the frequent switching between the two done by the FFT. As this is a pseudospectral method, the surface elevation must be a single valued function of the horizontal coordinates and so simulations are restricted to the early stages of wave breaking before the surface becomes vertical. Bateman, Swan and Taylor (2001) extended this scheme to the simulation of directional seas, validating it against the high quality wave basin data of Johannessen and Swan (2001) for groups with a range of bandwidths, directional spreading, and steepness up to the physical breaking limit. This scheme is efficient, robust and suitable, even on a PC, for modelling broadbanded, highly nonlinear directional sea states.

An initial condition is now required that will evolve over a given time to form an extreme wave. The average shape of such an extreme is a useful concept and Lingren (1970) showed that in a linear, random sea state the average shape of a large crest tends to the scaled autocorrelation function of the surface. This idea was introduced to the offshore industry by Tromans et al (1991), where it is now known as NewWave. An important feature of this model is that it links an averaged local extreme event to the global properties of the sea state, since the autocorrelation function is the Fourier transform of the underlying power spectrum. The NewWave model has been validated, by Jonathan and Taylor (1997), against the average shape of large deep water waves measured during severe winter storms in the northern North Sea. The NewWave model was consistent with both single point measurements of the surface elevation in time and the more complex test of a time history at a second location given a wave crest of known size at an adjacent point.

A common method of generating a directional frequency spectrum, $F(\omega, \theta)$, is from the simple product of a unidirectional spectrum, $S(\omega)$, and a spreading function, $D(\theta)$, where ω is the radian frequency and θ the direction of wave propagation.

$$F(\omega, \theta) = S(\omega) D(\theta) \tag{3}$$

The JONSWAP spectrum, derived from field measurements, is considered to be a suitable description of a broadbanded ocean spectrum in a fetch-limited sea. However, much of the energy of this spectrum is concentrated in a narrow range of frequencies around its peak and so we consider the modelling of components close to the peak of the spectrum only as an important first step in understanding the behaviour of directional, broadbanded events. A Gaussian spectrum is used for such a model with a peak period of 12sec. and a bandwidth chosen such that it fits the shape of the JONSWAP peak with $\gamma = 3.3$. Analysis of directional ocean wave spectra by Ewans (1998) , measured off the west coast of New Zealand for a large range of wind and wave conditions, showed that for frequency components below twice the spectral peak the spreading is less than 20°, whereas the higher frequencies are considerably more spread. These findings suggest that the use of a wrapped normal Gaussian distribution with a constant r.m.s. spreading parameter (σ) of 15° will give realistic spreading for our spectral peak model.

This paper presents the analysis of the formation of an extreme wave event, which on a linear basis would focus to a NewWave of amplitude (A) of 10.7m and steepness (Ak_p) of 0.3. The area of ocean in the computational domain is 3.6km in the mean wave direction by 5.8km in the transverse direction, with 256 spatial points in each direction. The simulation is started 20 periods before the linear focus so that a linear model, corrected for the second order bound wave terms of Longuet-Higgins (1963), provides a sufficiently accurate description of the surface when the effects of resonant interactions are negligible. Several checks have been made to establish the integrity of the results. As there is no energy input or dissipation in the scheme the conservation of the total energy in the domain is monitored and results are presented only for cases with a non-physical variation in the total energy of less than 0.5%. The reversibility of the simulations is verified by using the values of the surface elevation and surface velocity potential at the end of one run as the initial conditions of a second run, which is stepped backwards in time. This is a stern test of the numerical scheme and visual inspection shows that the initial conditions of the first run are recovered at the end of the second run. In addition, cases with twice the spatial resolution in each direction and different mean wave directions relative to the grid show that the results are grid independent.

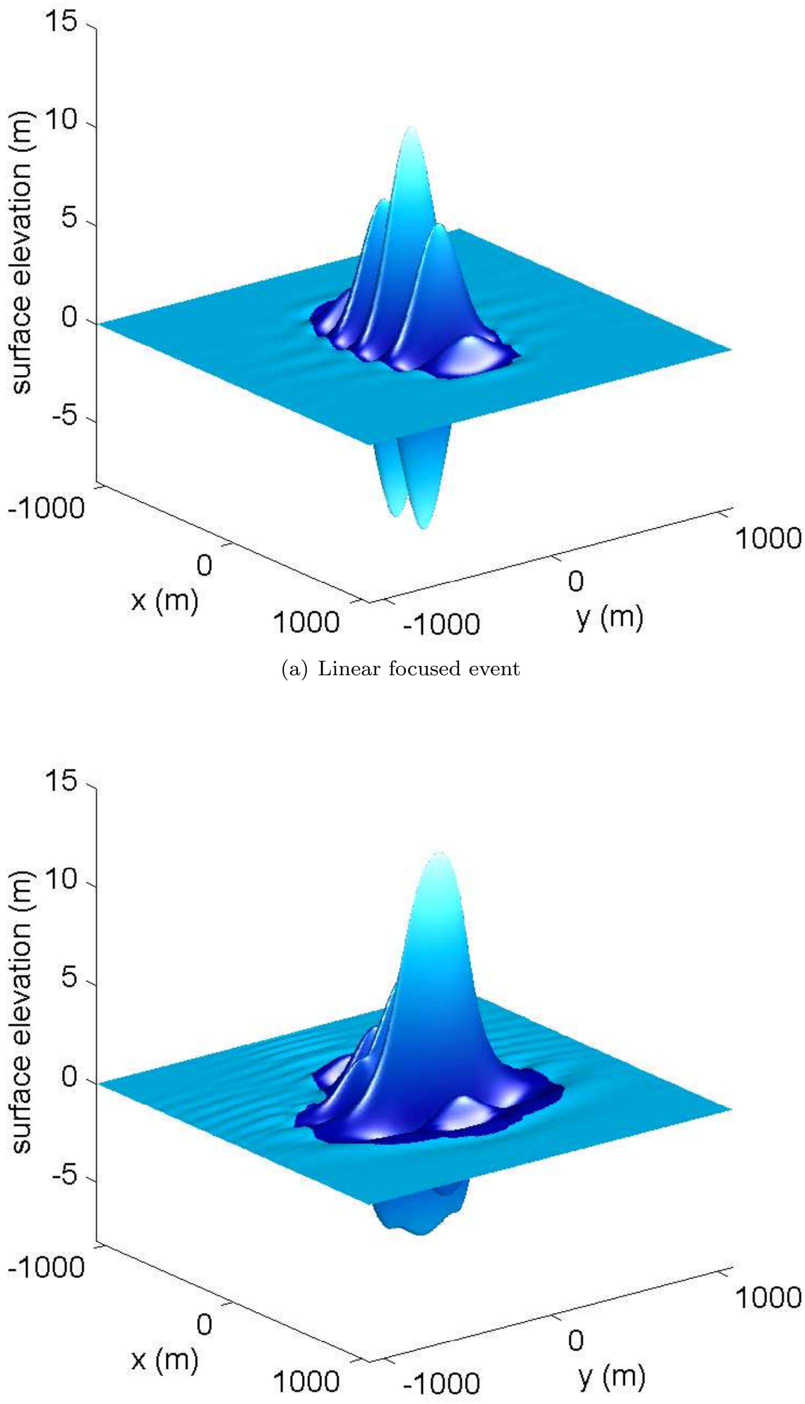
3 WAVE GROUP EVOLUTION

There are striking differences between the behaviour of the linear and nonlinear focused events. We define the focused event as the point in time when the peak surface elevation is a maximum with respect to the entire simulation. Linear evolution results in a wave group that is symmetrical about a transverse direction, and has its highest crest at the centre of the group, fig 1(a). In contrast, the nonlinear focused event, fig 1(b), is highly asymmetrical; the highest crest has moved to the front of the group. The length of this crest, in the transverse direction, is much greater in the nonlinear event than in the linear one, and its amplitude is greatest for the nonlinear group. Another important feature of the nonlinear group is the lack of structure ahead of the main wave, the deep trough seen in fig 1(b) is directly behind the main wave. The height and transverse width of the main wave, with minimal preceding wave structure, in the nonlinear event are suggestive of a '*wall of water*'.

Cross-sections of the nonlinear wave group along the mean wave direction are shown before, at and after the focused event in fig 2(a), fig 2(b), fig 2(c) respectively; the cross section of the linear focused wave group is given in fig 2(d). The figures are presented for an observer moving at the linear group velocity, based on the peak wavenumber of the underlying uni-directional spectrum. The nonlinear simulation exhibits fairly linear behaviour for ten periods from the start, fig 2(a), as the group is only slightly asymmetrical. Comparison of fig 2(b) and fig 2(d) shows the prominence of the largest wave in the nonlinear focused group. The waves ahead give little indication of the extreme event that follows, as the trough ahead of the tall crest is only 4m deep and the crest infront of this trough is 1.1m high. Non-linearity also leads to considerable steepening of the largest crest and its much thinner form, in the mean wave direction, than in the linear focused wave group. The shape of the nonlinear focused wave group suggests that not all of the components are in phase. Although a relatively narrow band of wave components has been specified for the initial condition, it is clear that a more select collection of waves actually contribute to the extreme event. Evolution after the nonlinear extreme event is also striking, since many of the features in fig 2(b) persist ten periods later in fig 2(c), long after it might be expected that linear dispersion would cause the group to fall apart. The main crest is still rather tall and steep, 50% higher than a linear prediction, and the heights of its preceding waves are almost unchanged compared to the focused event. There is also clear evidence, seen from the double-peaked trough in the centre of figure 2(c), of the group beginning to split into two in the mean wave direction. The group of waves not contributing to the focused event is left behind the main group. Similar behaviour has been observed by Lo and Mei (1987) in their numerical simulations of the 2-dimensional nonlinear Schrodinger equation.

A focused trough event (η_{tr}) can be driven by a linear input that is π radians out of phase (ie. inverted) compared to that required to form a focused crest (η_{cr}). A remarkable feature of the nonlinear evolution in forming an extreme trough is that its global behaviour is effectively identical to that for the formation of an extreme crest, even for steep events close to breaking. The troughs in the trough focused simulation are aligned, in space and time, with the crests of the crest focused simulation and vice versa (fig 2) throughout the simulation. This behaviour is obvious for wave groups that evolve linearly, see fig 2(d), but is an important feature of nonlinear evolution given the different local shapes of the two extreme events. Therefore the *formation of an extreme event is dependent on the group properties of the wave field*, such as amplitude and shape, and not the absolute phasing of the waves within the group. Consideration of a Stokes-like perturbation expansion implies that these large-scale changes in group structure can only be consistent with odd order nonlinear interactions. These global changes appear to be examples of the 3rd order resonant interactions of Phillips (1960) and Hasselmann (1962).

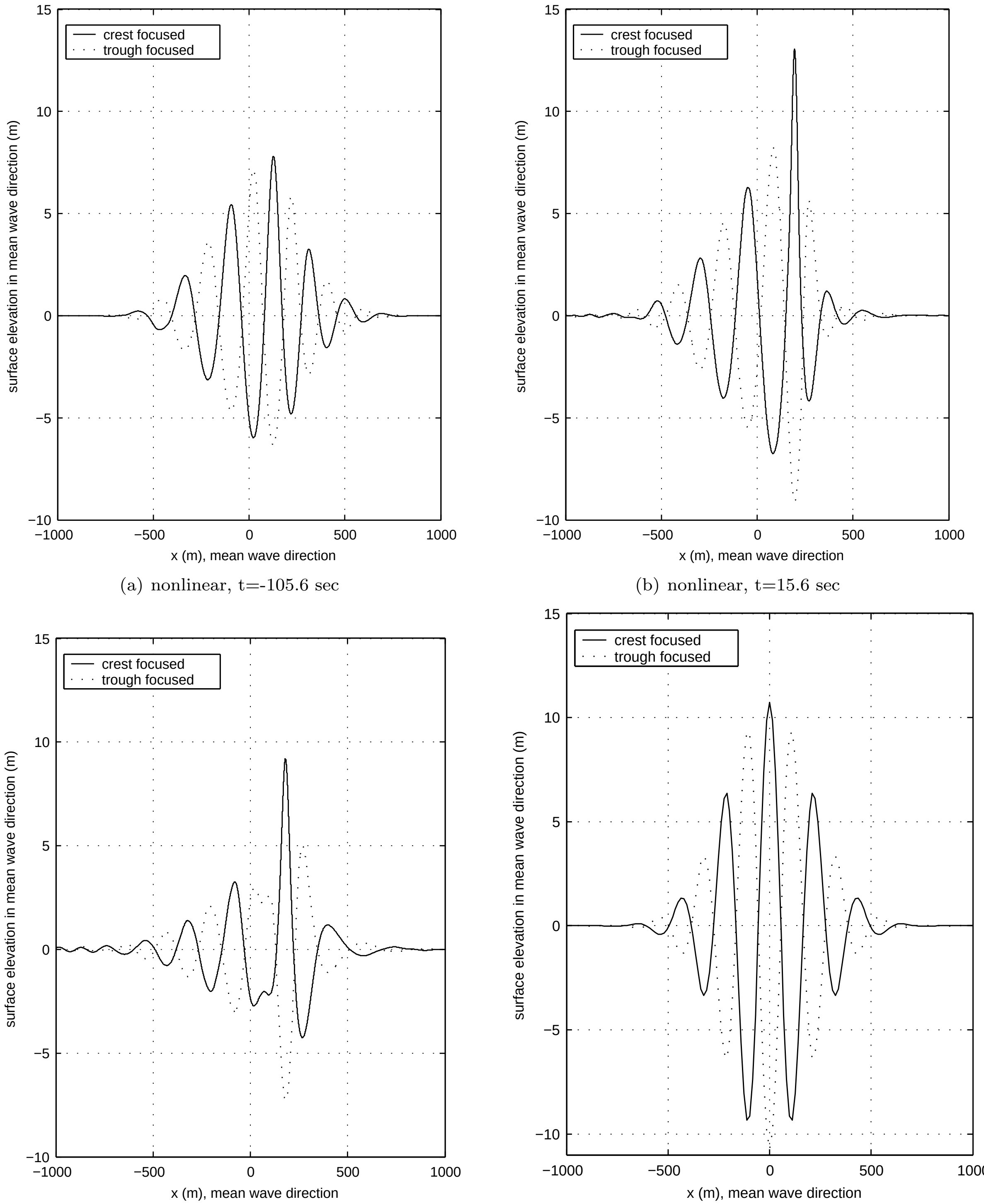
The symmetry property of the crest and trough focused evolutions described above allows a new, mean-



(a) Linear focused event

(b) Nonlinear focused event

Figure 1: Surface elevation



(a) nonlinear, t=-105.6 sec

(b) nonlinear, t=15.6 sec

(c) nonlinear, t=129.6 sec

(d) linear, t=0 sec

Figure 2: Cross-sections of surface elevation along mean wave direction

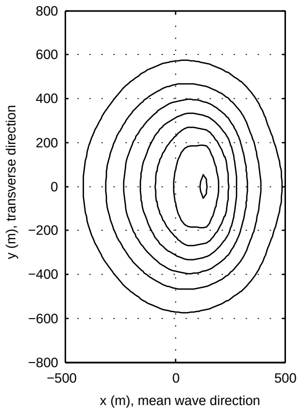
ingful signal to be computed from the difference between the crest and trough focused signals. The linearised surface elevation, $\eta_{odd}\,(x, y, t)$ is defined as

$$\eta_{odd} = (\eta_{cr} - \eta_{tr})\,/2 \tag{4}$$

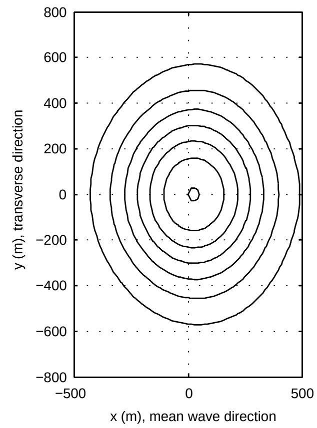
and represents a signal in both space and time which has all the even order bound wave structure removed. Visualisation of the envelope of this locally linearised wave group can be useful in highlighting changes in bandwidth. Figure 3 compares the contours of the amplitude, at 1m intervals, of the linearised envelope from the fully nonlinear runs against those of the linear solution before, at and after the nonlinear focus. At ten periods from nonlinear focus, the departure of the nonlinear evolution from linear is small. However, there are signs of the group contracting in the mean wave direction as the peak of the envelope starts to move towards the front of the group. It appears that there is a delay period, while linear dispersion brings the waves together, before a critical concentration of energy triggers significant nonlinear wave-wave interactions. In the following ten periods, dramatic changes take place to the shape of the group. There is a major contraction at the front of the group, in the mean wave direction, resulting in the near vertical main crest. In a sense this is balanced by the vigorous expansion of the group in the transverse direction, giving a *significantly more long-crested focused event.* Indeed, comparison of the 10m contour reveals that the transverse extent of the tallest peak is 2.5 times wider in the nonlinear focus than in the linear one. This rather thin, but long crest sustains a height of at least 10m across a distance of 230m. Beyond focus the nonlinear group remains much more long-crested.

The amplitude spectrum of the linearised surface elevation from the fully nonlinear runs exhibits significant transfer of energy both before and after the formation of the focused event. The linearisation of the second order accurate initial conditions returns the linear input spectrum, fig 4(a). Leading up to the focused event there is a broadening of the spectral peak in the mean wave direction (wavenumber component k_x) and a contraction in the transverse direction (wavenumber component k_y), fig 4(b), consistent with the changes in the shape of the envelope, fig 3(c). There is a downshift in the position of the spectral peak and clear evidence of energy transfer to higher wavenumbers. Twenty periods after the focused event, the evolution of the wave group is sufficiently linear to infer that the changes seen in the spectrum must be permanent, fig 4(c). A large amount of energy has been moved to higher wavenumbers, much of this transfer has occurred after the nonlinear focused event. The overall, permanent downshift of the spectral peak is about 10%, an effect which has been observed for non-breaking spread seas by Trulsen and Dysthe (1997) in their simulations of the modified NLS equation.

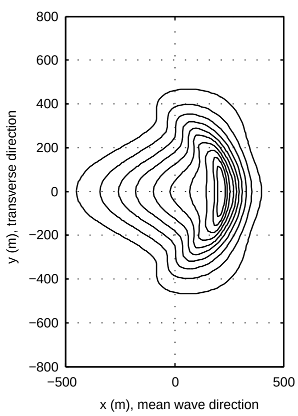
The wave group described in section 2 has been run for a range of steepness (Ak_p based on the linear focused amplitude and the peak wavenumber of the underlying unidirectional spectrum). The maximum surface elevation during each simulation is shown as a function of steepness in fig 5 for the nonlinear focusing of both crest and trough (ie. inverted) events, with the linear values given as a guide to the degree of nonlinearity. For low wave steepness there is little difference between the maximum surface elevation of the nonlinear focused event in comparison to the linear event. However, as the wave steepness increases there is an increase in the maximum elevation above a linear solution, which is about 20% for the steepest group. A comparison of the maximum value of the linearised signal with the linear solution shows that only for the tallest wave groups is there extra elevation much beyond a prediction based on simple second order theory.



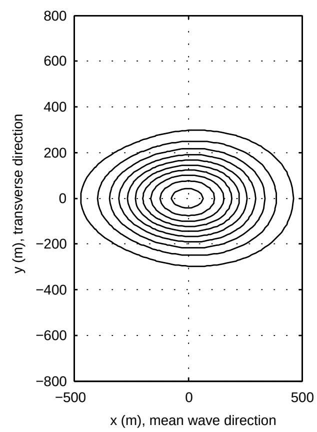
(a) nonlinear, t=-105.6 sec



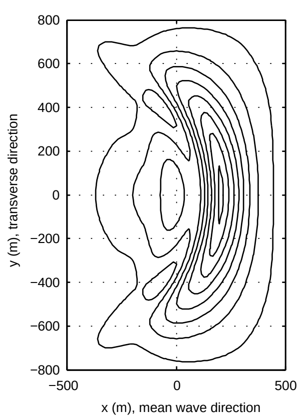
(b) linear, t=-105.6 sec



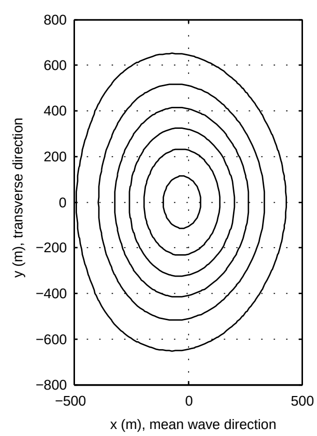
(c) nonlinear, t=15.6 sec



(d) linear, t=15.6 sec

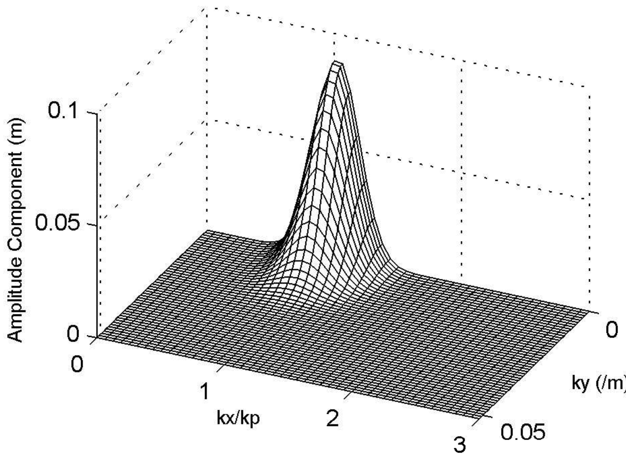


(e) nonlinear, t=129.6 sec

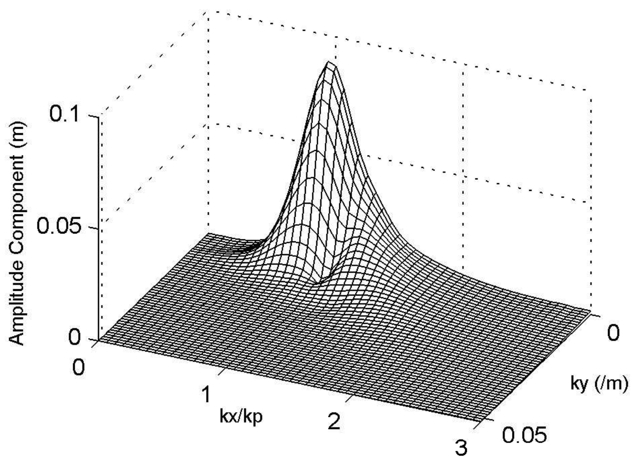


(f) linear, t=129.6 sec

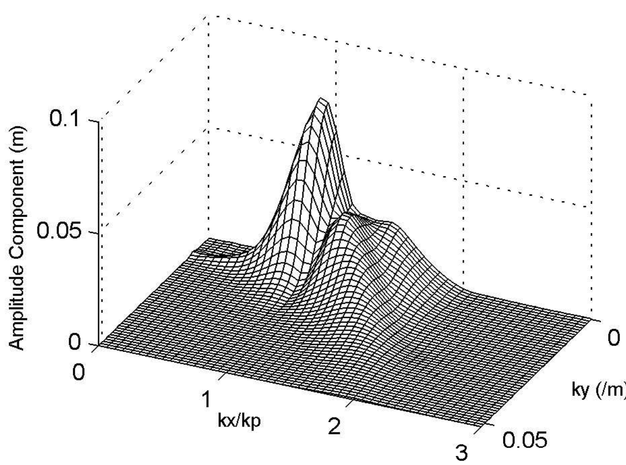
Figure 3: Amplitude contours of linearised envelope



(a) t=-240sec



(b) t=15.6sec, nonlinear focused event



(c) t=240sec

Figure 4: Linearised amplitude spectra

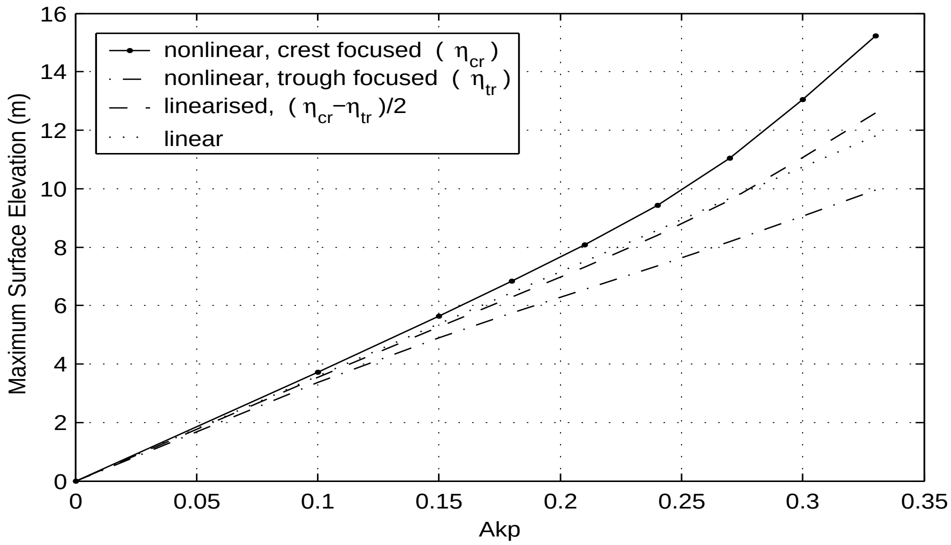


Figure 5: Maximum surface elevation against input steepness

4 DISCUSSION

The nonlinear focusing of relatively narrowbanded wave groups with realistic spreading exhibits many of the qualitative features that are associated with '*freak*' waves. A prominent, tall crest forms at the front of the group with an elevation and transverse width that suggest the formation of a '*wall of water*'. The shape of such a nonlinear focused event shares features with the modulational instabilities of the 2-dimensional nonlinear Schrodinger equation reported by Osborne, Onorato and Serio (2000). Although, unlike in unidirectional focusing, the gain in the peak surface elevation from the nonlinear focusing seems to be captured by a second order model, at least for the combination of frequency and directional spreading used here, there are strong structural changes to the group. The significant expansion of the group in the transverse direction and the contraction of the group in the mean wave direction appear almost to balance at the expense of the possible extra elevation of the envelope. Only for the tallest groups is the peak linearised elevation higher than the input value. The changes in the amplitude spectra are consistent with the observed changes in the group structure, and also reveal a permanent downshift in the position of the spectral peak without wave breaking. These simulations of the peak of a broadbanded spectrum suggest that the formation of a '*freak*' or '*rogue*' event is unlikely, but not impossible. The reluctance of these events to defocus, rather than their height above a linear solution, is likely to have important consequences for the likelihood of seeing such events on the open ocean.

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