A COMPARISON OF THE ENERGY FLUX COMPUTATION OF SHOALING WAVES USING HILBERT AND WAVELET SPECTRAL ANALYSIS TECHNIQUES

Paul A. Hwang, James M. Kaihatu and David W. Wang
Oceanography Division, Naval Research Laboratory, Stennis Space Center, MS 39529
Email: phwang@nrlssc.navy.mil, Kaihatu@nrlssc.navy.mil, dwang@nrlssc.navy.mil
Phone: (228)-688-4708 (PH), -5710 (JK), -4735 (DW); Fax: (228)688-5379

1. INTRODUCTION

Fourier-based spectral analysis methods have been widely used for studying random waves. One major weakness of the Fourier-based spectral analysis methods is the assumption of *linear* superposition of wave components. As a result, the energy of a nonlinear wave is spread into many harmonics, which are phase-coupled via the nonlinear dynamics inherent in nearshore wave propagation. In addition to the nonlinearity issue, strictly speaking Fourier spectral analysis should be used for *periodic* and *stationary* processes only. Wave propagation in inhomogeneous near shore regions is certainly neither stationary nor periodic.

Recently, Norden Huang and his colleagues developed a new analysis technique, the Hilbert-Huang-Transformation (HHT). Through analytical examples, they demonstrated the superior frequency and temporal resolutions of HHT for analyzing nonstationary and nonlinear signals (e.g. Huang et al. 1998, 1999). Furthermore, using the HHT analysis, the physical interpretation of nonlinearity is frequency modulation, which is fundamentally different from the commonly-accepted concept that nonlinearity results in harmonic generation. Huang et al. argued that the harmonic generation is caused by the perturbation method used in solving the nonlinear equation governing the physical processes, thus it is produced by the mathematical tools used for the solution rather than a true physical phenomenon.

Here we examine the cross-shore evolution and energy flux of shoaling waves using HHT and compare the results with those obtained by the wavelet method (a Fourier-based technique). The Fourier-based analysis tends to underestimate the magnitude of the energy flux. This is attributed to the dispersive nature of water waves. Because the phase and group velocities of each free wave component are frequency-dependent, yet the harmonics of nonlinear waves are bounded to the dominant component, the calculated energy flux (the product of group velocity and spectral density) of the harmonics associated with nonlinear waves cannot be distinguished from those of the free waves at the same frequency. Other issues of nonlinearity (e.g., the dispersion relationship) will be addressed also.

2. EVOLUTION OF THE ENERGY FLUX OF SHOALING WAVES

An airborne scanning lidar system is used to measure the 3D surface wave topography of shoaling waves (Hwang et al., 1998, 2000 a-c). Fig. 1a displays a small segment of the data covering a region from the beach to 2.2 km offshore. The swath of the scanning lidar measurement is slightly less than 300 m. The swell system is left over from an extratropical storm passing through the region two days before the airborne measurement. The wave period is about 12 s, and the wave height measured by an offshore buoy is about 2 m. During the measurement period, the wind speed is very light, less than 2 m/s, so the influence of wind on the swell in negligible.

Applying the HHT technique, the spatial evolution of the wave spectrum is shown in Fig. 1b. The energy content is distributed in a narrow wavenumber band. The evolution shows a distinctive modulation of the spectral density synchronized with the surface wave form (especially in the region of x = 400-1000 m). This property is called "intrawave modulation" by Norden Huang, and is an important indicator of nonlinearity. Using the wavelet analysis method, the resulting spatial evolution of the wave spectrum is shown in Fig. 1c. The wavenumber resolution is obviously coarser, manifesting in the smeared spectral density shown in the image. In the more nonlinear region (x = 400-1000 m), harmonic generation is severe. As commented earlier, the most significant difference between HHT and Fourier-based analysis results is in the interpretation of nonlinearity: intrawave frequency modulation in HHT and harmonic generation by Fourier methods.

The energy fluxes computed by the two methods are shown in Fig. 2a. Although both sets of computation show similar qualitative features of the up and down evolution of the energy flux in the wave field, quantitative details are quite different. The magnitude of energy flux computed by HHT is obviously higher than that obtained by the wavelet method. As discussed in the last section, due to the dispersive nature of surface waves, energy flux calculated by Fourier spectrum will always be below the true value. Although it is probably premature to state that HHT produces the true spectrum of the nonlinear waves, the increased energy flux computed from the HHT spectrum represent, a correction in the right direction.

The rate of change of the energy flux represents the sink and source of the wave system. Approximated by finite difference, the results are shown in Figs. 2b and 2c, respectively for the HHT and wavelet spectra. The HHT result is dominated by pulses of "ringing" in the spatial distribution while the wavelet result is much smoother (notice the difference of vertical scales in the two figures). The meaning of the high frequency fluctuations in the HHT result is still under investigation. The fluctuation appears to be associated with nonlinearity and directionality of the wave field. Interestingly, if the spatial resolution of the HHT result is degraded, the smoothed version can be made to approach the wavelet result. For example, Fig. 2d shows the running average of HHT over 60 resolution cells, the result is essentially similar to the wavelet computation. The ability of HHT to offer high resolution analysis of nonlinear processes may prove to be quite valuable for further detailed analysis of the shoaling wave dynamics.

3. DISPERSION OF NONLINEAR WAVES

Numerical simulation is used to obtain idealized wave records for more detailed investigation of the nonlinear analysis techniques. Fig. 3 and 4 show two sets of the simulations; the first one is without nonlinear effects, while the second one accounts for nonlinearity in refraction and diffraction of shoaling waves (Kaihatu and Kirby 1995; Kirby and Kaihatu 1996). Both simulations are initialized with an offshore monochromatic wave train of 12 s period and 2 m wave height. For the linear case (Fig.3) the evolution of wave propagation is dictated by refraction during shoaling. The spectra computed by HHT and wavelet are basically similar although the spatial and wavenumber resolutions of HHT are obviously higher as reflected in the sharper spectral distribution shown. The representative wavenumber of the system follows the linear dispersion curve, as expected (Fig. 3d).

When nonlinearity is introduced, the HHT spectrum develops significant intrawave modulation (Fig. 4b) and the wavelet spectrum develops higher harmonics (Fig. 4c), as expected. It is worth commenting that wavelet spectrum also displays intrawave modulation in the spectral distribution although the effect is not as dramatic as that of the HHT result. The representative wavenumber calculated from the nonlinear wave spectrum shows considerable fluctuations as a result of the intrawave modulation (Fig. 4d). Of special interest is that the wavenumber from HHT spectrum generally oscillates along the dispersion curve. For a better illustration, the results are averaged to reduce intrawave oscillations (Fig. 5). The HHT result seems to indicate that the mean dispersion relation does not deviate from the linear dispersion until very high nonlinearity where the waveform has severe asymmetry. This is an intriguing conclusion that merits further investigation.

4. SUMMARY

Analyzing nonlinear and nonstationary signals remains a very challenging task. Presently, most methods developed to deal with nonstationarity are based on the concept of Fourier decomposition; therefore all the shortcomings associated with Fourier transformation are inherent in those methods also. The recent introduction of empirical mode decomposition by Huang et al. (1998, 1999) represents a drastic shift in signal decomposition. The associated spectral analysis (HHT) seems to render much superior spatial (temporal) and wavenumber (frequency) resolution for handling the nonstationary signals. The HHT spectrum also yields better interpretation of nonlinearity as illustrated by Huang et al. (1998, 1999) using analytical examples. Applying the technique to water wave problems, the superior resolution offers considerably finer details in the computed spectral properties such as the spectral energy flux and the resulting source and sinks functions. We believe that such ability to reveal the details will be quite useful for the investigation of nonlinear wave properties. Several interesting results presented in this paper include the intrawave modulation, the quantitative results of energy flux and the dispersion of nonlinear waves.

ACKNOWLEDGMENTS

This work is supported by the Office of Naval Research (Naval Research Laboratory Program Elements N61153 and N62435 NRL Contribution PP/7330--02-0060).

REFERENCES

Huang, N. E., Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N. C. Yuen, C. C. Tung, and H. H. Liu, The empirical mode decomposition and the Hilbert spectrum for nonlinear and nonstationary time series analysis, *Proc. R. Soc. Lond. A*, **454**, 903-995, 1998.

Huang, N. E., Z. Shen, S. R. Long, A new view of nonlinear water waves: The Hilbert spectrum, *Annu. Rev. Fluid Mech.*, **31**, 417-457, 1999.

Hwang, P. A., E. J. Walsh, W. B. Krabill, R. N. Swift, S. S. Manizade, J. F. Scott and M. D. Earle, 1998: Airborne remote sensing applications to coastal wave research. *J. Geophys. Res.*, **103**, 18791-18800.

Hwang, P. A., W. B. Krabill, W. Wright, E. J. Walsh, and R. N. Swift, Airborne scanning lidar measurements of ocean waves, *Remote Sensing of Environment*, **73**, 236-246, 2000a.

Hwang, P. A., D. W.C. Wang, E. J. Walsh, W. B. Krabill, and R. N. Swift, Airborne measurements of the directional wavenumber spectra of ocean surface waves. Part 1. Spectral slope and dimensionless spectral coefficient, *J. Phys. Oceanogr.*, **30**, 2753-2767, 2000b.

Hwang, P. A., D. W.C. Wang, E. J. Walsh, W. B. Krabill, and R. N. Swift, Airborne measurements of the directional wavenumber spectra of ocean surface waves. Part 2. Directional distribution, *J. Phys. Oceanogr.*, **30**, 2768-2787, 2000c.

Kaihatu, J.M, and J. T. Kirby, Nonlinear transformation of waves in finite water depth, *Phys. Fluids*, **7**¸ 1903-1914, 1995.

Kirby, J. T., and J.M. Kaihatu, Structure of frequency domain models for random wave breaking, *Proc. 25th Intl. Conf. On Coastal Eng.,* Orlando, FL, ASCE, 1144-1155, 1996.

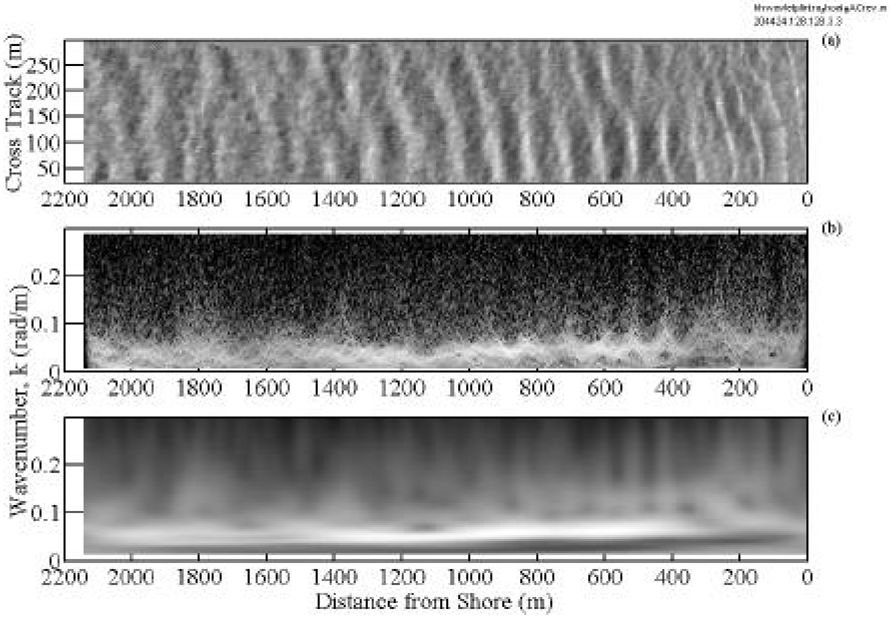

Fig. 1. (a) 3D surface wave topography measured by an airborne scanning lidar system. (b) The HHT spectrum calculated from the wave data. (c) Same as (b) but using the wavelet method

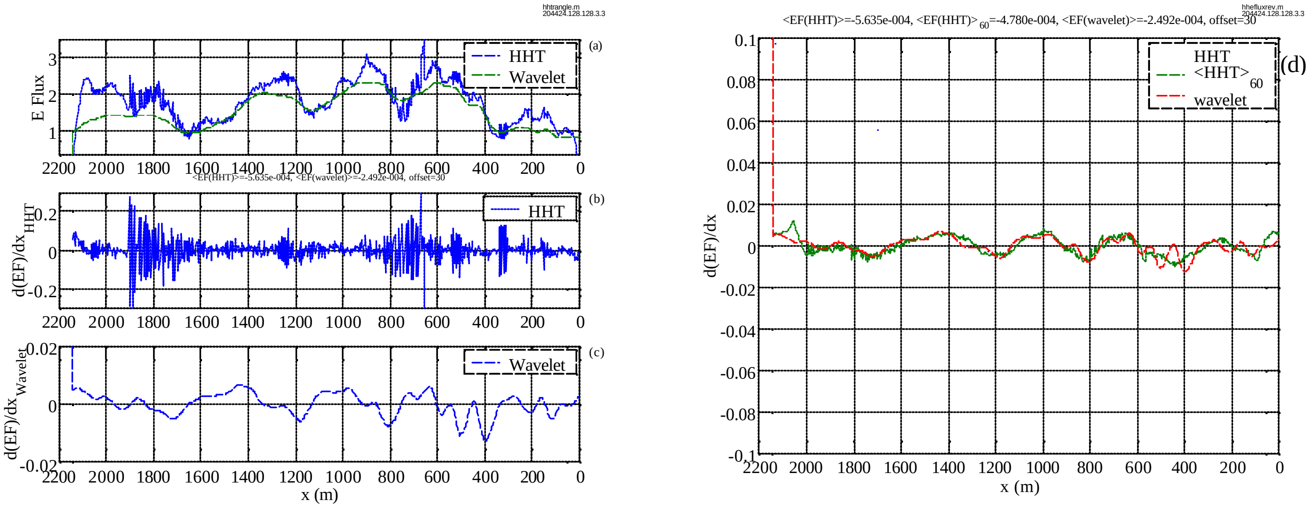

Fig. 2. (a) Spatial distribution of the wave energy flux computer by HHT and wavelet methods. (b) The spatial gradient of energy flux based on the HHT method. (c) Same as (b) but based on the wavelet method. (d) Comparison of wavelet and HHT results with spatial resolution degraded 60 times in the HHT data.

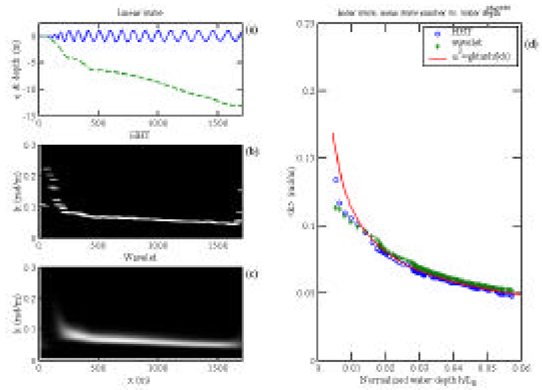

Fig. 3. Analysis of numerically simulated wave data, the linear case. (a) Waveform and bathymetry. (b) HHT spectrum. (c) Wavelet spectrum. (d) Dispersion relation.

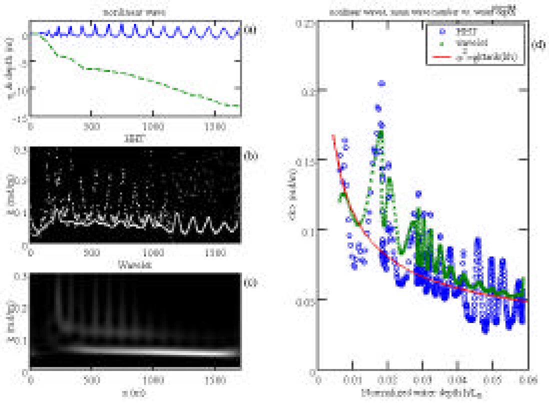

Fig. 4. Same as Fig. 3 but for the nonlinear case.

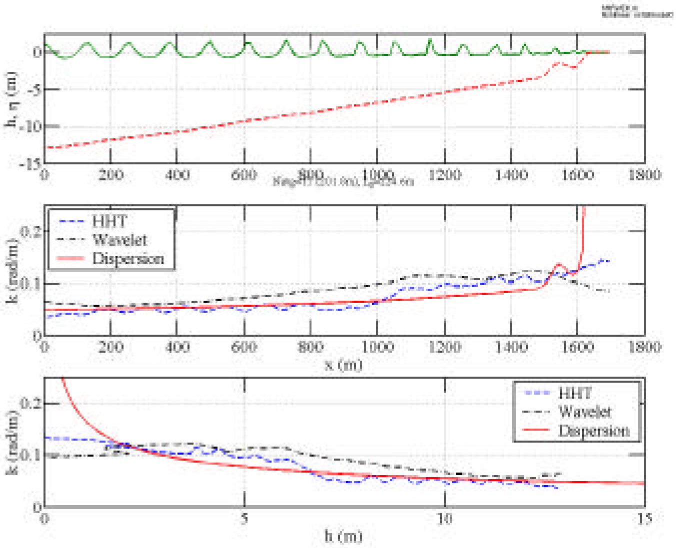

Fig. 5. Running average of the dispersion result. (a) Waveform and bathymetry. (b) Spatial evolution of the peak wavenumber. (c) Peak wavenumber vs. water depth.