ON THE PREDICTION OF EXTREME WAVE CREST HEIGHTS

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1. INTRODUCTION

An important parameter regarding structural safety is the height from the still water level (accounting for tide and storm surge) to the lowest deck level of the platform. Standard practise, at least for structures at the Norwegian Continental Shelf, is to require that this height is larger than the wave crest height occurring with an annual probability of occurrence of 10^{-4} after accounting properly for the increase in crest height due to wave-structure interaction. For floating structures this requirement may be rather difficult (or, rather, very costly) to meet and for such cases the requirement is to design the structure such that it can take the impact forces caused by these events with merely local damage, i.e. the impact event is not to escalate into a catastrophic failure. For floating structures it is of course the relative wave motion that is of concern, but the methodology used herein for the undis turbed wave crests may also be applied for the relative wave motion.

In harsh weather areas, the 10^{-4}-probability [(1)] undisturbed crest height may well be in the order of 20-25m. The water level variations due to tide and surge may in open, deep water areas typically be in the order of +/- 2m, i.e. the height level reached by the wave crest is completely dominated by the wave crest height itself. Depending on the transparency of the structure under consideration, wave-structure interactions may also add to the air gap requirement. For a transparent jacket structure, the crest height amplification may be rather small, while for a gravity based concrete structure the crest height increase may be larger – say up to 20-30% of the incoming crest height. Irrespective of structure, however, the most important quantity to estimate correctly, will be the incoming undisturbed wave crest height. Finally, the effects of tide and surge can be added by sufficient accuracy by means of rather simple statistical methods. If important, model testing may be required to address the effects of sea structure interactions. Based on the above discussion, we will in this paper focus on predicting estimates for the 10^{-4}-probability crest height for a Northern North Sea location. This will be done using various methods in order to indicate possible inherent differences between the methods.

2. TARGET QUANTITIES

Within offshore rules, the characteristic response, x_c, to be used in various limit state checks is usually chosen as the response value corresponding to an annual exceedance probability of q, where $q=10^{-2}$ for the ultimate limit state and $q=10^{-4}$ for the accidental limit state. An estimated extreme response value, x_e may be exceeded in a broad range of different extreme sea states. Each of the sea states is typically characterized by the significant wave height and the spectral peak period. Denoting the annual probability of exceeding x_e in the sea state characterized by a significant wave height, h_{si}, and a spectral peak period, t_{pj} by q_{ij}, the annual probability of exceeding x_e reads:

$$q = \sum_i \sum_j q_{ij} \tag{1a}$$

In order to adopt x_e as a proper estimate for the 10^{-2}-probability response or the 10^{-4}-probability response, , i.e. $x_c=x_e$, the sum has to equal 10^{-2} or 10^{-4}, respectively. **__The important message by Eq. (1a), is therefore that there is not a one-to-one correspondence between the annual probability of exceeding a given sea state and the annual probability of the expected maximum response (or maximum crest height) of that sea state.__** The expected maximum wave crest height of the 10^{-4}-probability storm is obviously a severe crest height, but its annual exceedance probability is significantly larger than 10^{-4}. The annual exceedance probability for a given sea state, q_{ij}, is given as the product of two terms, the probability of exceeding x_e during a T-hour realization of the sea state and annual probability of experiencing a T-hour realization of the sea state. In practise T=3hours is commonly adopted. Introducing these components Eq. (1a) can be written:

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(1): In this paper the notation q-probability value denotes the value corresponding to an annual exceedance probability of q.

$$q = \sum_i \sum_i q(x_e \mid h_{si}, t_{pj})\, p(h_{si}, t_{pj}) \qquad (1b)$$

It is seen from Eq. (1b) that a consistent extreme response prediction in view of rule requirements consist of a short term problem, i.e. the exceedance probability within stationary sea states, and a long term problem, i.e. the long term probabilities of the particular sea states. No matter which method is used, a minimum requirement to the method is that it, exact or with good approximation, is able to combine these two problems, i.e. a long term response analysis is in principle required. Such an analysis can be performed by considering all sea states, ***"all sea states approach"***, Battjes(1970), Nordenstrøm(1971), or it may merely include sea states corresponding to storms exceeding some threshold, ***"storm based approach"***, Jahns and Wheeler(1972), Haring and Heideman(1978), Tromans and Vanderschuren(1995) .

In closing this chapter on target quantities, we will discuss the importance of the predicted extreme sea state characteristics. Over the years much focus has been devoted to how to predict accurate estimates of the significant wave height. It should be stressed, however, that from a structural design point of view, the q-probability significant wave height in itself is not of much concern. The structure will not fail as a consequence of a significant wave height, it will possibly fail if its final capacity is exceeded by an extreme individual response maximum. In ensuring that the annual probability for such a catastrophic scenario is sufficiently low, the method actually adopted should handle the convolution of the short term and the long term problem consistently.

Over the last 2-3 decades, there has been an apparently everlasting discussion on whether one should adopt an ***"all sea state approach"*** , or a ***"storm based approach"*** for extreme value predictions. However, regarding a prediction of response extremes, the choice of method is not the most crucial element. Both classes of methods should, if being properly implemented, yield reasonable estimates for the target quantities, namely the q-probability response extremes. If, on the other hand, the main purpose is to predict consistent estimates of the extreme storm peak significant wave height, the storm approach should be favoured. The latter approach may also be more convenient for prediction of response extremes in areas where the long term sea condition are of a typical two population nature.

3 AVAILABLE METHODS

3.1 Introductory Remarks

As stated above, an important requirement to an adequate analysis method is that the short term (conditional) exceedance probabilities are consistently accumulated into a resulting long term (marginal) exceedance probability. For illustrative purposes, the undisturbed crest height is selected as the response quantity to be considered and below we will first discuss the short term modelling of this quantity.

3.2 Short Term Modelling

Provided that the surface elevation process, $\Xi(t)$, can be modelled as a reasonably narrow banded stationary Gaussian process with zero mean and a variance, s_{Ξ}^2, the height of the global crests, C, (i.e. largest maximum between adjacent zero-up-crossings) is described by the Rayleigh distribution:

$$F_{C|H_sT_p}(c|h,t) = 1 - \exp\left\{-\frac{1}{2}\left(\frac{c}{s_{\Xi}}\right)^2\right\}; \quad c \geq 0 \qquad (2)$$

It is (and has been for some years) realized that the surface elevation process deviates significantly from the Gaussian assumption, i.e. the observed surface process is positively skewed with higher crests and shallower troughs than expected under the Gaussian assumption. An empirical correction to the Rayleigh model was suggested 3 decades ago by Jahns and Wheeler(1972). This model can be written:

$$F_{C|H_sT_p}(c|h,t,d)=1-\exp\left\{-8\left(\frac{c}{h}\right)^2\left[1-\beta_1\frac{c}{d}\left(\beta_2-\frac{c}{d}\right)\right]\right\} \tag{3}$$

d is water depth and β_1 and β_2 are empirical coefficients. β_1=4.37 and β_2=0.57 are recommended by Haring and Heideman (1978).

At present the most advanced surface model being available for routine work, is the full random second order Stoke process see e.g. Forristall (2000) and papers referred to therein. Based on a large number of second order simulations for various environmental conditions and water depths, a 2-parameter Weibull distribution is suggested as the short term model for crest heights;

$$F_{C|H_sT_1}(c|h,t_1,d)=1-\exp\left\{-\left(\frac{c}{\alpha_F\, h}\right)^{\beta_F}\right\} \tag{4}$$

where t_1 is the mean wave period calculated from the two first moments of the wave spectrum, k_1 is the wave number corresponding to the wave period t_1, and d is the water depth. The parameters, α_F and β_F, are expressed in terms of two parameters, a measure of steepness, s_1, and the Ursell number, Ur, which is a measure of the impact of water depth on the non-linearity of waves. These quantities read:

$$s_1=\frac{2\pi\, h}{g\, t_1^2} \quad \text{and} \quad Ur=\frac{h}{k_1^2 d^3} \tag{5}$$

For long crested sea, the expressions for α_F and β_F, read, Forristall(2000):

$$\alpha_F=0.3536+0.2892\, s_1+0.1060\, Ur \tag{6}$$

$$\beta_F=2-2.1597 s_1+0.0968 Ur^2 \tag{7}$$

Over the years, the extreme wave crest height has sometimes been estimated by first of all estimating the extreme wave height. Thereafter an estimate of the corresponding crest height is obtained by introducing the wave height into a deterministic 5^{th} order Stoke profile. This of course also required an estimate of the corresponding wave period and the water depth for the location under consideration. Commonly adopted models for the short term distribution of wave height are of Weibull type:

$$F_{H|H_sT_p}(h|h_s,t_p)=1-\exp\left\{-\left(\frac{h}{\alpha_H}\right)^{\beta_H}\right\} \tag{8}$$

Various parameterisations are:

Extremely narrow banded Gaussian sea (Rayleigh model): $\alpha_H=0.707 h_s$ and $\beta_H=2$ (9)

Narrow banded Gaussian sea (Naess Model, Naess (1985)): $\alpha_H=\frac{1}{2}\sqrt{1-r(\frac{T}{2})}\, h_s$ and $\beta_H=2$ (10)

Empirical model (Forristall model, Forristall (1978)): $\alpha_H=0.683 h_s$ and $\beta_H=2.13$ (11)

Where $r(T/2)=R(T/2)/\sigma_{\xi}$, $R(\ )$ is the auto correlation function of the wave process, and T is the dominant wave period. Depending on water depth and wave steepness, the crest height of the 5^{th} order Stokian wave profile is typically 58-62% of the wave height. Adopting for illustrative purposes C = 0.6 H, the corresponding crest height distribution can be obtained by transforming Eq. (8).

Assuming that the 10^{-4}–probability sea state (3-hour duration) for the area under consideration reads h_s=18m and t_p=17s, the distribution function for the 3-hour maximum crest height is shown for the various models in Figs. 1 and 2. The 3-hour extreme value distribution is obtained by raising the maxima distribution to the power equal to the number of maxima in 3 hours. Water depth is taken to be 150m, the mean wave period, t_1, used in connection with Eq. (4) is approximated by $0.79t_p$, and $\rho(T/2)$ is taken to be –0.73 corresponding approximately to a spectral peakedness factor of about 3.

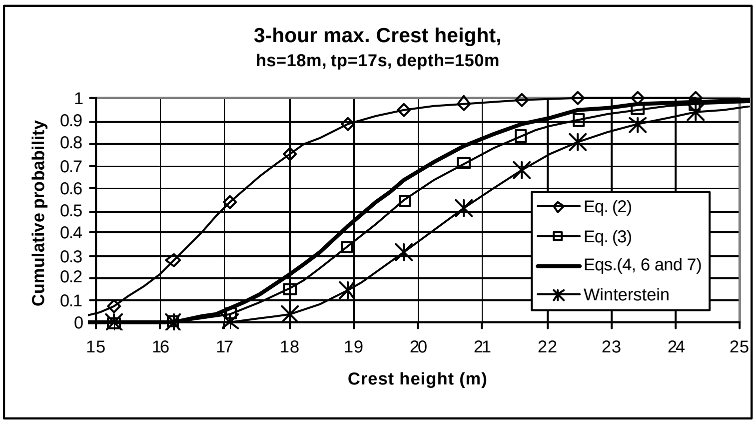


Fig. 1 3-hour max crest height from crest height models

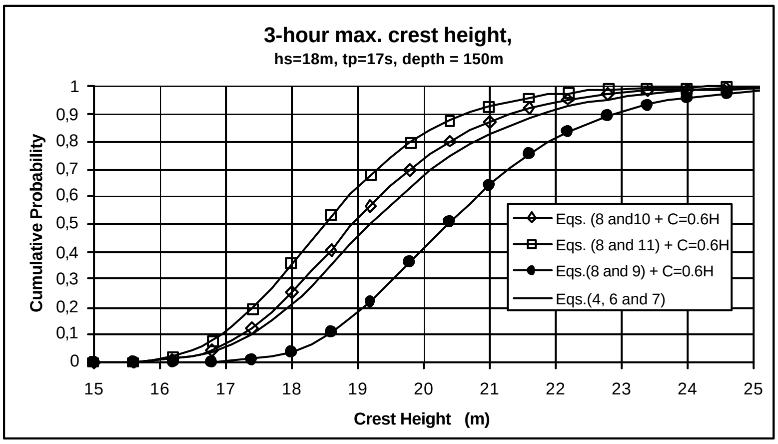


Fig. 2 3-hour max crest height from wave height models and 5^{th} order Stoke profile

It is seen from from Fig. 1, that for this particular depth, the Jahns and Wheeler model and the Forristall second order model nearly coincide. These models yield a significant larger extreme crest height than the Rayleigh model. In the figure we have also included the crest height distribution obtained using an approach suggested by Winterstein(1988), see e.g. Haver and Karunakaran(1998) for an application of this model as a crest height model. The latter model appears to be somewhat conservative as compared to the second order model of Forristall. It should, however, be stressed that regarding the most extreme waves, higher order effect may have a certain impact and, therefore, the Forristall model should be considered as lower bound model regarding extreme crest heights.

If the indirect approach is used, Fig. 2 shows that there is a tendency of underestimating the height of the 3-hour maximum crest height (assuming the Forristall model to be an adequate description), except if the pure Rayleigh is adopted as the wave height model.

3.3 All Sea States Long Term Approach (***"All sea states approach"***)

3.3.1 Long term modelling of sea conditions

Assuming that a short term sea state is reasonably well characterized by the significant wave height and spectral peak period, the long term wave climate is conveniently described by a joint probability density function for these characteristics. For the purpose of fitting the joint model to data, it is conveniently written:

$$f_{H_sT_p}(h_s,t_p) = f_{H_s}(h_s)\, f_{T_p|H_s}(t_p|h_s) \tag{12}$$

A joint model for long term response analysis is given by Haver and Nyhus(1986). The joint modelling is based on the following probabilistic models:

$$f_{H_s}(h_s) = \begin{cases} \dfrac{1}{\sqrt{2p}\; a\, h_s} \exp\left\{ -\dfrac{(\ln h_s - q)^2}{2a^2} \right\}; & h \le h \\[2ex] \dfrac{b}{r}\left(\dfrac{h_s}{r}\right)^{b-1} \exp\left\{ -\left(\dfrac{h_s}{r}\right)^{b} \right\}; & h > h \end{cases} \tag{13}$$

$$f_{T_p|H_s}(t_p|h_s) = \frac{1}{\sqrt{2p}\; s\, t_p} \exp\left\{ -\frac{(\ln t_p - m)^2}{2s^2} \right\} \tag{14}$$

where:

$$m = a_1 + a_2\, h_s^{a_3} \tag{15}$$

$$s^2 = b_1 + b_2 \exp\{-b_3\, h_s\} \tag{16}$$

The parameters of the hybrid model for the significant wave height are estimated as follows. At first the log-normal parameters, θ and α, are estimated from the marginal data. The parameters of the Weibull tail are then estimated by requiring the hybrid model to be continuous in both density function and distribution function at h_s = η. η is varied until a best possible fit is obtained. A scatter diagram for the Northern North Sea is given in Table A.1. The scatter diagram covers the years 1973 – 2001. The values of h_s and t_p represent ideally a pair of 20-minute average values every 3 hours. In practise a significant amount of data is missing and the 69428 simultaneous observations correspond to a data coverage of about 85%. Eq. (13) is fitted to the h_s data. The kji-square error normalized with respect to the corresponding number degrees of freedom is shown versus the shift point, η, in Fig. 3. At a reasonable acceptance level all models are rejected. This is mainly caused by inaccuracies for the lowest wave height classes, where a small error yields a very large contribution to the kji-square variable due to the very large number of data (69428). Here we will mainly use these results for indicating that a minimum kji-square error is achieved for η between 2.8 and 3, corresponding to a 10^{-2}-probability value for h_s between 14.1m and 14.9m. As our recommended model we will adopt the model for h=2.9m, which corresponds to a 10^{-2}-probability value of 14.5m. The adopted model is compared to the empirical model in Fig. 4. It should be pointed out that the 10^{-2}-probability value for this method is to be interpreted as the threshold which, in an accumulated sense, is expected to be exceeded for 3 hours during a 100-year period, i.e. the 10^{-2} probability event for this method is not necessarily a single event.

The parameters of the conditional distribution for T_p given H_s are taken from Johannessen and Nygaard (2000). All the parameters of the recommended joint omni-directional model is given in Table 1.

Table 1 Parameters for the joint model of H_s and T_p

Season	a	q	h	b	r	a_1	a_2	a_3	b_1	b_2	b_3
All-year	0.6565	0.77	2.90	2.691	1.503	1.134	0.892	0.225	0.005	0.120	0.455

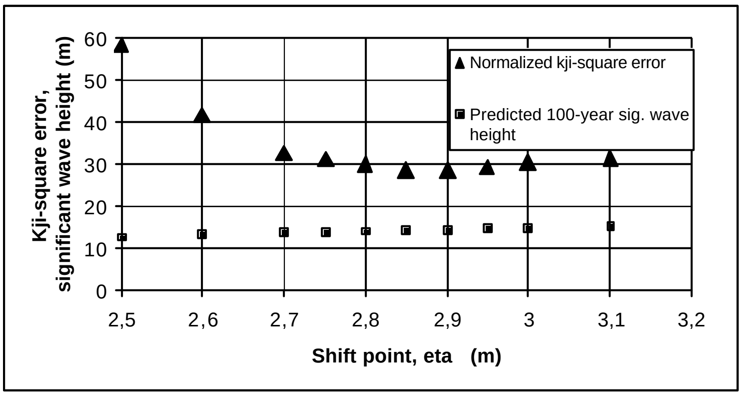


Fig. 3 Normalized fitting error versus the shift point

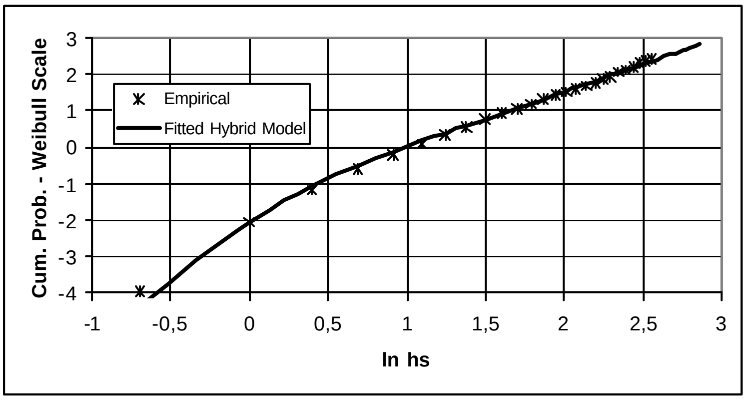


Fig. 4 Fitted marginal model for the significant wave height

3.3.2 Long term modelling of crest heights

<u>*All global maxima approach*</u>

In this approach we establish the distribution function for global maxima (i.e. largest maximum between adjacent zero-up-crossings) within a stationary sea state. Denoting the short term distribution function of crest heights by $F_{C|H_sT_p}(c|h_s,t_p)$, the long term distribution of crest heights read, see e.g. Battjes (1970):

$$F_C(c) = \frac{1}{\overline{n_0^+}} \int_h \int_t n_0^+(h,t) F_{C|H_sT_p}(c|h,t)\, f_{H_sT_p}(h,t)\, dt dh \tag{17}$$

where the long term mean zero-up-crossing frequency is given by:

$$\overline{n_0^+} = \int_h \int_t n_0^+(h,t)\, f_{H_sT_p}(h,t) dt\, dh \tag{18}$$

assuming that individual response maxima are statistically independent, the q-probability crest height is found by solving:

$$1 - F_C(c_q) = \frac{q}{365 \cdot 24 \cdot 3600 \cdot \overline{n_0^+}} \tag{19}$$

It is seen that we have not introduced any particular duration of the stationary sea states in connection with this approach. The reason for this is that since the all global maxima approach is adopted, it is the long term probability of the various combinations of the sea state characteristics that is of concern (this corresponds to the expected cumulative relative duration for the various combinations).

3-hour maximum approach

Since we are mainly interested in extremes. We may alternatively focus on merely the largest crest height during a 3-hour stationary period. Bearing in mind that most wave measurements correspond to a duration of 20 minutes, 1/3 hour would possibly be a more proper choice. A 3-hour duration is chosen herein since it is a typical choice in practical applications. The distribution function for the 3-hour maximum, is reasonably well approximated by raising the short term global maxima distribution to the power equal to the expected number of waves in 3 hours, $k^{(3h)} = 10800 \cdot n_0^+(h_s, t_p)$. Thus the long term distribution of the 3-hour maxima, $C^{(3h)}$, reads:

$$F_{C^{(3h)}}(c) = \int\int \left[F_{C|H_s T_p}(c|h,t)\right]^{k^{(3h)}} f_{H_s T_p}(h,t)\, dt\, dh \tag{20}$$

An estimate of the q-probability crest height is found by solving (2920 is the number of 3-hour events per year):

$$1 - F_{C^{(3h)}}(c_q) = \frac{q}{2920} \tag{21}$$

In order to illustrate how extreme values can be predicted using methods from the field of structural reliability analysis, i.e. FORM (First-Order-Reliability-Method) and SORM (Second-Order-Reliability-Method), the probability of exceeding a crest height threshold, $\tilde{c}$, in an arbitrary 3-hour sea state is written:

$$1 - F_{C^{(3h)}}(\tilde{c}) = \iiint_{c > \tilde{c}} f_{C^{(3h)}|H_s T_p}(c|h,t)\, f_{H_s T_p}(h,t)\, dc\, dt\, dh \tag{22}$$

For this particular case, the failure surface, $c = \tilde{c}$, is of a very simple form and the integral is easily solved numerically. However, the integral may alternatively be solved very efficiently using the above mention methods. The first step is to transform the problem into a variable space consisting of independent, standard Gaussian variables, see e.g. Madsen et al. (1986), i.e. $h_s \to u_1$, $t_p \to u_2$, and $c^{(3h)} \to u_3$. The failure surface is also transformed into the u-space. By a proper searching algorithm, the point on the failure surface being the nearest point to the origin is determined. This point is referred to as the design point, $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$, and it represents the most probable parameter combination as failure take place, i.e. as $c > \tilde{c}$. The distance to the design point, β, is given by the square root of the sum of squares of the design point coordinates. Under the assumption that the failure surface is approximated by a tangent plane in the design point (i.e the FORM approach is used), the exceedance probability of Eq. (22) simply reads $\Phi(-b)$. In order to determine the q-probability crest height, $\tilde{c}$ is varied until the estimated probability equals q/2920.

If one mainly is interested in predicting extremes corresponding to a given annual exceedance probability, it is more convenient to follow an inverse procedure referred to as IFORM, see e.g. Winterstein et al. (1993), Haver et al. (1998). Since the target "failure" probability is known, $p_f = q/2920$, one knows the distance, β, to the requested failure surface, e.g. β=5.367 for $q=10^{-4}$. This means that we, a priori, know that the 10^{-4}-probability crest height in the u-space will be located somewhere on the surface of a sphere with radius 5.367. The aim of the IFORM algorithm is to search this surface for the point being the tangent point for the failure surface in the u-space, i.e. the design point, $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$. Transforming this point back to the physical parameter space, the c-value corresponding to $\tilde{u}_3$ represents an estimate for the q-probability crest height.

For the crest height problem as formulated herein with few variables and explicit distribution functions for all variables available, the FORM approach does not represent a major improvement in computational efficiency. However, for more complicated problems involving for example a larger number of random variables this approach will often represent an efficient solution technique if one mainly is interesting in values corresponding to very low exceedance probabilities. As an alternative to obtaining the short term probabilistic structure of the crest heights by a full second order time domain simulation, a FORM approach is discussed by Tromans and

Vanderscuren (2002). Here we have used the Forristall crest height model and have thus avoided the time consuming second order simulations.

3.4 Peak over Threshold Long Term Approach (***"storm based approach"***)

3.4.1 Introductory Remarks

As long term extremes are our main concern, an alternative approach is to consider the storm maximum crest height as our "short term" quantity. Short term is in this connection put in quotation marks since the storm itself is a non-stationary event, which, however, can be approximated by a sequence of stationary events. Regarding examples of this approach reference is made to e.g. Jahns and Wheeler (1972), Haring and Heideman (1978) and Tromans and Vanderscuren (1995). The basic idea by this approach is to establish the distribution function of the largest wave or response during an arbitrary storm. Over the years this target has been approached in various ways. A common approach has been to merely account for the observed storms. Each observed storm has been given the probability $1/n_s$, where n_s is the number of storms, and there is zero probability for more severe storms than those included in the observed sample. This may yield reasonable estimates for the q-probability extremes provided the storm sample is very large and that a reasonable amount of very extreme storms are included in the sample. Tromans and Vanderschuren (1995) recommend that one should account for non-observed events when establishing the distribution function for the storm maximum response of an arbitrary storm. The way they solve this is that the storm maximum response is described conditionally upon the most probable largest storm maximum. A probabilistic model is fitted to the storm sample of most probable largest storm response maxima. The long term distribution of storm maximum response (i.e. the distribution of the largest value in an arbitrary storm) is obtained by convoluting the conditional distribution of storm maximum response given the most probable largest storm maximum response with the long term distribution of most probable storm maximum. In this way the effect of non-observed storms is accounted for, and, in principle, the approach is brought closer to the long term analyses approaches reviewed in Ch. 3.3. For realistic sizes of the storm samples, it is important to account for non-observed extreme events. We will illustrate this in the end of this chapter.

In the following the investigation is restricted to the wave crest height. In principle, we will assume that the distribution function of the storm maximum crest height, $C^{(s)}$, is reasonably well defined if we know the storm maximum significant wave height, $H_{s,sp}$, the simultaneously occurring spectral peak period, $T_{p,sp}$, and the duration, Δ, of the part of the storm exceeding, say, 80% of the storm maximum significant wave height. Utilizing the suggestions in Tromans and Vanderscuren(1995), we may possibly represent the resulting effect of these three characteristics into a single characteristic, namely the most probable largest storm crest height, $\tilde{c}^{(s)}$. Regarding the wave crest height, however, we think a reasonable accurate conditional distribution can be obtained by simply conditioning the storm maximum crest height on the storm maximum significant wave height. Thus the long term distribution of storm maximum crest height reads:

$$F_{C^{(s)}}(c) = \int_h F_{C^{(s)}|H_{s,sp}}(c|h)\, f_{H_{s,sp}}(h)\, dh \tag{23}$$

3.4.2 Distribution of storm maximum significant wave height

Storm maximum significant wave heights for Northern North Sea storms exceeding 10m significant wave height are given in Table A.2. A probabilistic model commonly adopted in connection with peak-over-threshold assessments is the Generalized Pareto model, see e.g. Naess and Clausen(2002):

$$F_{H_{s,sp}}(h) = 1 - \left(1 + c_P \frac{(h-h_0)}{q_P}\right)_+^{-1/c_P} \quad \rightarrow \quad 1 - \exp\left\{-\frac{(h-h_0)}{q_p}\right\} \;\; if \;\; c_P \rightarrow 0 \tag{24}$$

The notion $(z)_+$ means max(0, z). $\theta_P > 0$ is the scale parameter and c_P is the shape parameter. As shown in Eq. (24), the Generalized Pareto distribution will approach the Exponential distribution as the shape parameter approaches zero. Moment estimators for the Pareto parameters read, Naess and Clausen(2002):

$$q_P = 0.5E\left[H_{s,sp} - h_0\right]\left\{1 + \left(\frac{E\left[H_{s,sp} - h_0\right]}{S\left[H_{s,sp} - h_0\right]}\right)^2\right\} \quad \text{and} \quad c_P = 0.5\left\{1 - \left(\frac{E\left[H_{s,sp} - h_0\right]}{S\left[H_{s,sp} - h_0\right]}\right)^2\right\} \tag{25}$$

Using these, we find θ_P = 0.919 and c_P = -0.130. The fitted Pareto model is compared with the empirical distribution in Fig.5. The fitted exponential, c_P = 0 and θ_P=0.813 model is shown in the same figure. Both models show a reasonable fit to the data, but they differ very much as we enter into the range of the most interesting extremes. With an average number storms equal to 1.375 per year, the 10^{-2} annual probability level corresponds to 4.92 in the vertical axis of Fig. 5, while the 10^{-4} probability level corresponds to 9.53. A 10^{-4} probability significant wave height of about 15m seems rather low for the Northern North Sea. If we suggest that the Exponential model is the true model, we may simulate realizations for a sample of 33 storms. For this generated sample (from an exponential model), we may estimate the Pareto parameters. By repeating the simulation 50 times we obtain a set of 50 estimates for the Pareto parameters. Based on this, a 90% interval for c_P is about (–0.25, 0.15). Since our estimated value for the actual storm sample yield a value well within this interval, we will adopt the Exponential distribution as our model for storm maximum significant wave height. This corresponds to a 10^{-2}-probability significant wave height of 14.0m and a 10^{-4}-probability significant wave height of 17.8m.

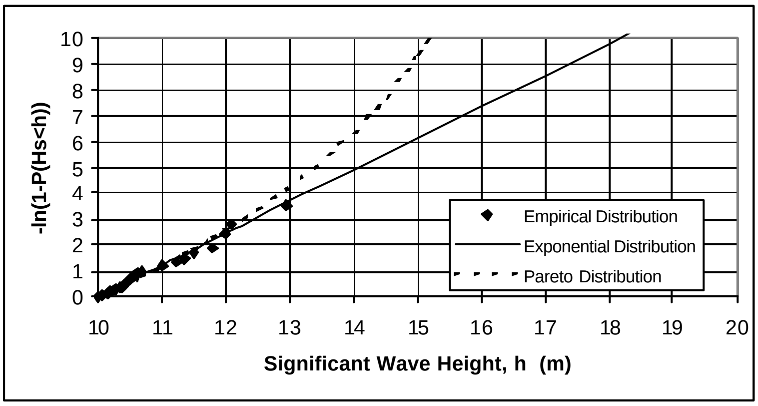


Fig. 5 Fitted distribution for storm maximum significant wave height, Northern North Sea 1973 - 1997

In order to indicate the adequacy of the extremes predicted above, annual extremes for the same period are considered. The data are shown in Table A.3. A Gumbel model is fitted to the data by the method of moments and compared to the empirical distribution in Fig.6. The 10^{-2}-probability level corresponds to 4.6 in the Gumbel scale, while 10^{-4}-probability level corresponds to 9.21. The fitted Gumbel model seems to yield a reasonable fit to the data, and it suggests a 10^{-2}-probability significant wave height of about 14m, while the 10^{-4}-proability value reads about 18m. These results support the selection of the exponential model for the storm maximum significant wave height. It should be mentioned that the higher order statistical moments, skewness and kurtosis, of the annual extreme value sample are lower than expected for a Gumbel variable. The Gumbel model is an asymptotic result and by introducing a generalized Gumbel model, see e.g. Winterstein and Haver (1991), one may possibly obtain somewhat lower extreme values for the annual extreme value approach.

3.4.3 Distribution function of storm maximum crest height

For each stationary part of the storm, the crest heights are assumed to follow the Forristall crest height model, Eqs. (4,6 and 7). As a consequence of that, it is likely that the storm maximum crest height is reasonably well modeled by the Gumbel distribution:

$$F_{C^{(s)}|H_{s,sp}}(c|h) = \exp\left\{-\exp\left\{-\frac{c - a_C(h)}{b_C(h)}\right\}\right\} \tag{26}$$

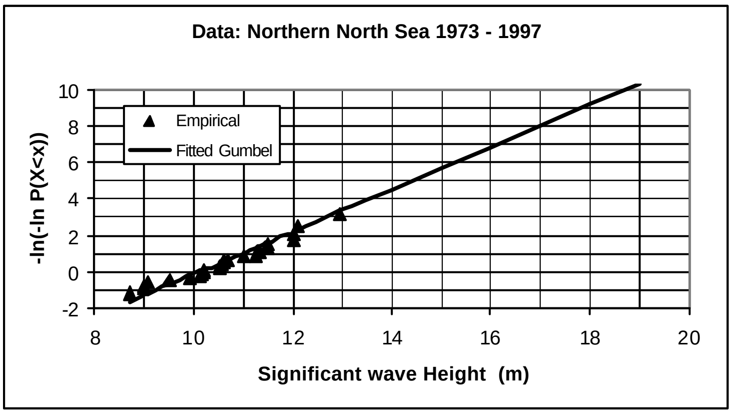


Fig. 6 Annual extreme value distribution of significant wave height

The Gumbel parameters are modeled as functions of the storm maximum significant wave height only. This will not be perfectly fulfilled.

Provided that the mean, $m_{C^{(s)}|H_{s,sp}}$, and standard deviation, $s_{C^{(s)}|H_{s,sp}}$, of the storm maximum crest height are available, the Gumbel parameters read:

$$b_C = \frac{s_{C^{(s)}|H_{s,sp}}}{1.28255} \quad \text{and} \quad a_C = m_{C^{(s)}|H_{s,sp}} - 0.57722\, b_C \tag{27}$$

Herein samples of storm maximum crest heights are generated by means of Monte Carlo simulations. For each storm 100 simulations of the 3-hour maximum crest height are generated for each stationary 3-hour period of the observed storm event. From these data sets we obtain 100 generated realizations of the storm maximum crest height where all stationary storm periods are properly accounted for. Based on these realizations, the mean and standard deviation are calculated and the corresponding Gumbel parameters are estimated by Eq. (27). The estimated Gumbel parameters are shown versus storm peak significant wave height in Fig. 7. A linear regression line is determined for both parameters and these expressions, given in Fig. 7, are used in combination with Eq. (26). It is seen that a certain amount of scatter is neglected when using these expressions. The correlation coefficient between α_C and h is found to be 0.85, while the correlation between β_C and h is lower and reads 0.6. If the scatter is caused by inherent variability, the consequence of neglecting it is that the estimated extreme values may be slightly on the low side.

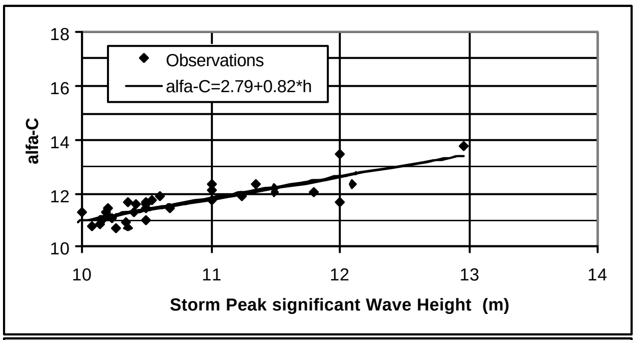
A simple investigation of the sensitivity to this scatter is done by considering the estimated parameters uncorrelated and determine the long term distribution of crest heights using mean parameter values +/- one standard deviations when introducing Eq. (26) into Eq. (23). This indicated that until more careful studies are carried out, 0.5m should be added to the estimated extremes in order to account for the scatter around the regression lines.

The long term distribution of storm maximum crest height is now obtained by introducing Eqs. (24 and 26) into Eq. (23). The q-probability crest height is then found by solving:

$$1 - F_{C^{(s)}}(c_q) = \frac{q}{\bar{r}} \tag{28}$$

where $\bar{r}$ is the expected number of storms per year. For the selected storm criterion, $h_{s,sp} > 10$m, $\bar{r} = 1.375$.

The effect of including non-observed storm events is indicated in Fig. 8. The curve referred to as "Empirical" is obtained by merely including the observed storm events. The probability of occurrence for each storm is equal to the inverse number of events. The curve referred to as "Extrapolated" is obtained by extrapolating the storm maximum significant wave height distribution beyond the range covered by measurements. At the 10^{-2}-probability level (4.92 at vertical axis) the effect is about 0.3m, while for the 10^{-4}-probability level (9.53 at vertical axis) the effect is slightly more than 1m.



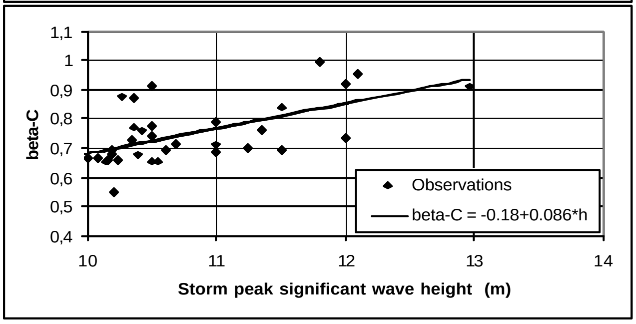


Fig. 7 Gumbel parameters for $C^{(s)}$ versus storm maximum h_s

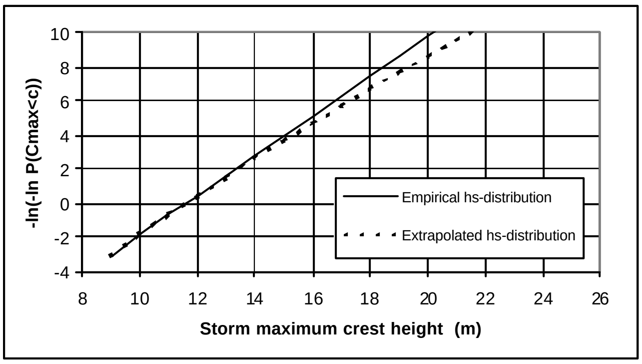


Fig. 8 Long term distribution of storm maximum crest height

3.5 Environmental Contour Line Approach

The long term distribution of crest heights, Eq. (17 or 20), is obtained as a weighted sum of the short term distribution for all possible sea states. One may, however, come up with reasonable estimates for the long term extremes by means of short term statistics by utilizing the so called environmental contour lines for significant wave height, H_s, and spectral peak period, T_p.

Contour lines for the joint model presented in Ch. 3.3.1 are shown in Fig. 9. Regarding exceedance of a specified crest height level, i.e. wave-deck impact problems, it is the point with the highest significant wave height that is the most unfavourable location along the contour lines. If we at first assume that the distribution of the largest crest height in a given 3-hour sea state is extremely narrow, then we can neglect the randomness of the conditional extreme value and simply adopt the median (or the mean value) as a characteristic value. In that case we will know that the q-probability crest height is expected to be realized in the most unfavourable q-probability sea state (which for this problem will be the sea state with the highest significant wave height). The q-probability crest height can then be approximated by:

$$\hat{c}_q = F^{-1}_{C^{(3h)}|H_s T_p}(0.50|(h_s, t_p)_q) \tag{29}$$

where $(h_s, t_p)_q$ denotes the most unfavourable parameter combination (in view of the problem under consideration) along the q-probability contour line. The conditional distribution for $C^{(3h)}$ is found by raising Eq. (4) to the power equal to the number of waves during a 3-hour period of this storm event.

In practice one cannot neglect the short term variability of the 3-hour extreme value. The correct approach to account for this variability is of course to carry out some sort of a long term analysis. One can, however, approximate the long term results by selecting a higher fractile as the representative 3-hour characteristic, see e.g. Winterstein et al. (1993) for a more thorough discussion. In Haver et al. (1998), a fractile of about 0.85 is recommended regarding the estimation of 10^{-2}-probability crest heights from the 10^{-2}-probability environmental contour line. It is reasonable to expect that as the 10^{-4}-probability crest height is to be estimated from the 10^{-4}-probability contour line, the relative importance of the short term variability is somewhat increased. As a consequence it is likely that a somewhat higher fractile should be used. Assuming a fractile level of 0.90 to be adequate, a proper estimate of the 10^{-4}-probability crest height is given by:

$$\hat{c}_{10^{-4}} = F^{-1}_{C^{(3h)}|H_s T_p}(0.90|(h_s, t_p)_{10^{-4}}) \tag{30}$$

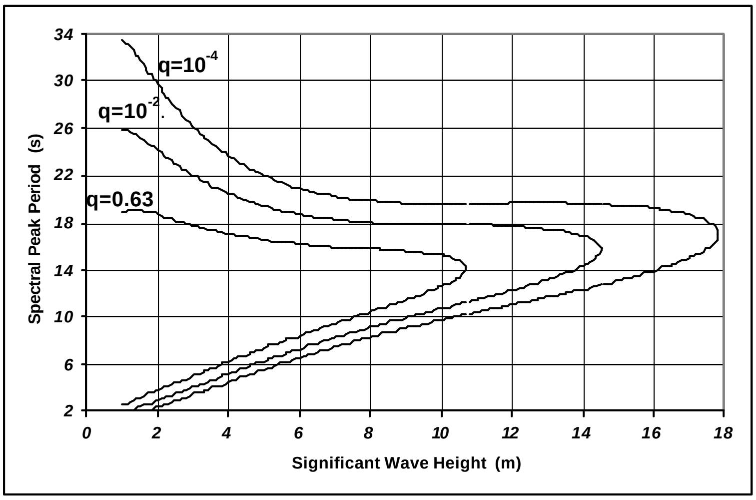


Fig. 9 Environmental contour lines for the Northern North Sea (q=0.63 corresponds to 1-year return period)

The 10^{-4}-probability sea state is given by h_s=18m (or, more correct, 17.8m) and t_p=17s. 3-hour extreme value distributions for this sea state are shown in Fig. 1. Assuming that the 90% percentile is a reasonable short term characteristic for estimating the 10^{-4}-probability crest height, the figure suggests that this quantity is around 22m if emphasis is given to the results provided by the Forristall crest height model. For wave problems, the environmental contour line principle does not represent any major improvement since it is rather straight forward to do a full long term analysis. For complicated response problems where extensive model testing or time domain simulations are necessary, however, it represents a convenient tool for an approximate “long term” response analysis.

4. PREDICTED EXTREME WAVE CREST HEIGHTS IN THE NORTHERN NORTH SEA

4.1 Results

Using the methods described in the previous chapter, the estimated 10^{-2}- and 10^{-4}-probability crest heights are given in Table 2. The corresponding 10^{-2}- and 10^{-4}-probability significant wave heights are also given in the table.

Table 2 Predicted extreme wave characteristics for the Northern North Sea (The reference to Figs. 1 and 2 is for the 10^{-4} case, corresponding figures were produced for the 10^{-2} case.)

Approach	Reference	Sig. wave height (m)		Crest height (m)	
		10^{-2}	10^{-4}	10^{-2}	10^{-4}
All sea states approach	Eq. (17 and 19)	14.5	17.9	**17.2**	**21.8**
	Eq. (22 and 21), IFORM	"	"	**16.8**	**21.7**
Storm data approach	Eq. (23 and 28)	14.0	17.8	**16.1**	**21.0**
	Assumed correction for scatter shown in Fig.7	-	-	**16.6**	**21.5**
Contour lines, 90% fractile	Fig. 1, Forristall Crest Model	14.5	17.9	**17.0**	**21.8**
	Fig. 1, Rayleigh Crest Model	"	"	15.4	19.0
	Fig. 2, Forristall Height Model + 60% Crest	"	"	16.6	20.6
	Fig. 2, Rayleigh Height Model + 60% Crest	"	"	18.5	22.9
Contour lines, 50% fractile	Fig. 1, Forristall Crest Model	"	"	15.4	19.2
	Fig. 1, Rayleigh Crest Model	"	"	13.8	17.0
	Fig. 2, Forristall Height Model + 60% Crest	"	"	15.0	18.5
	Fig. 2, Rayleigh Height Model + 60% Crest	"	"	16.5	20.3

4.2 Discussion

The all sea state approach and the storm data approach indicate a similar severity regarding the significant wave height. Too much emphasis should not be given to this comparison since the target quantities are not the same. The all sea state approach predicts the threshold, h_{sq}, which is expected to be exceeded for 3 hours (accumulated sense) per 1/q years, while the storm approach predicts the maximum significant wave height of the storm corresponding to an annual probability of exceedance of q. Furthermore, both approaches will be associated with uncertainties, both regarding choice of probabilistic models and the fitting of these models to data. It should also be remembered that the fitted Generalized Pareto distribution gave much lower extremes than the model finally chosen. It may well be that one should have adopted a lower threshold giving a much larger storm sample and thereby obtain somewhat more robust estimates for the Pareto parameters.

It is more interesting to compare the predicted extreme crest heights. Before correcting for the scatter around the regression lines in Fig. 7, the storm based approach yield estimates about 5% lower than the all data approach. This may well be a true difference. In both methods extreme value predictions are used assuming statistical independence between crest heights considered. This assumption is possibly closer to being fulfilled by the storm based approach and, consequently, one may expect that the all sea state approach should give slightly conservative extreme value estimates. However, one should be careful in utilizing this difference because by correcting the extremes for the above mentioned scatter increases the predictions to more or less the same level as predicted by the all sea state approach. This correction is rather approximate, and further work on the distribution function of the maximum storm crest height is recommended. In that connection one should account for the fact the significant wave height most likely will fluctuate somewhat during the 3-hour storm steps.

The contour line results suggest clearly that it is important to adopt a higher fractile than the median if long term extremes are to be estimated by a short term consideration. The importance of an accurate crest height model is also clearly demonstrated.

Since the most extreme waves typically will be somewhat affected by terms of order 3 and 4, the results obtained using second order models should be considered a slightly on the low side, i.e. for the Northern North Sea a the 10^{-4}-probability crest height should not be taken smaller than 22m.

It should also be pointed out that these predictions do **not** cover possible transient and strongly non-linear wave phenomena referred to as "freak waves". If such events take place, the corresponding crest height may exceed the predictions above by several meters. See e.g. Haver and Andersen (2000) and Olagnon and Athanassoulis(2000) for a discussion on the existence of such events.

5. ACKNOWLEDGMENTS

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Appendix Data Tables

Table A.1 Scatter diagram Northern North Sea, 1973 – 2001. Values given for hs and tp are upper class limits.

h_s (m	t_p (s)																		
			5	6	7	8	9	10	11	12	13	14	15	1	1	1	1	2	>
0.5	18	15	123	113	110	390	260	91	38	42	32	3	19	13	9	1	3	2	7
1.0	16	49	675	433	589	1442	1802	959	273	344	125	33	64	29	13	1	7	1	6
1.5	5	32	417	893	1107	1486	2757	1786	636	731	299	121	92	43	18	10	5	2	13
2.0	1	0	102	741	1290	1496	2575	1968	780	868	492	200	116	51	31	8	4	4	8
2.5	0	0	9	256	969	1303	2045	1892	803	941	484	181	157	58	23	19	5	1	8
3.0	0	0	1	45	438	1029	1702	1898	705	957	560	218	196	92	40	11	4	2	5
3.5	0	0	1	4	124	650	1169	1701	647	865	456	237	162	100	36	12	6	1	5
4.0	0	0	2	0	33	270	780	1369	573	868	427	193	157	91	51	13	3	0	1
4.5	0	0	0	0	3	90	459	1017	466	761	380	127	137	86	31	23	6	5	0
5.0	0	0	0	0	0	15	228	647	408	737	354	119	96	50	32	18	2	4	1
5.5	0	0	0	0	0	2	68	337	363	580	283	94	92	31	24	10	6	2	0
6.0	0	0	0	0	0	1	20	166	221	418	307	63	76	24	13	9	4	0	0
6.5	0	0	0	0	0	0	5	50	140	260	257	59	49	20	12	4	2	2	2
7.0	0	0	0	0	0	0	0	23	90	180	193	41	53	20	5	3	3	0	0
7.5	0	0	0	0	0	0	0	6	25	93	121	45	46	17	5	5	0	1	0
8.0	0	0	0	0	0	0	0	3	14	50	84	26	47	11	6	0	1	0	0
8.5	0	0	0	0	0	0	0	0	7	25	45	23	25	20	8	0	0	0	0
9.0	0	0	0	0	0	0	0	1	2	12	30	22	20	19	0	0	0	0	0
9.5	0	0	0	0	0	0	0	0	1	2	20	21	14	7	1	1	0	1	0
10.0	0	0	0	0	0	0	0	0	0	2	5	4	21	6	2	0	0	0	0
10.5	0	0	0	0	0	0	0	0	0	3	4	8	9	12	2	0	0	0	0
11.0	0	0	0	0	0	0	0	0	0	0	2	0	4	3	1	0	1	0	0
11.5	0	0	0	0	0	0	0	0	0	0	2	1	2	3	0	0	0	0	0
12.0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	0	0	0	0
12.5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Table A.2 Peak values for the significant wave height of storms exceeding 10m (1973-1997)

Storm No.	Storm dat	Max h (m)	Storn No.	Storm date	Max h (m)	Storm N	Storm dat	Max h (m)
1	6/11-1973	11.8	12	24/11-1981	10.15	23	25/12-1990	10.5
2	12/1-1974	12.1	13	6/1-1983	10.42	24	18/10-1991	10.0
3	7/12-1975	10.19	14	9/1-1983	10.54	25	1/1-1992	11.5
4	15/1-1976	10.14	15	31/10-1983	10.18	26	4/1-1993	12.0
5	20/1-1976	10.4	16	29/3-1985	10.69	27	17/1-1993	11.0
6	31/3-1977	10.61	17	28/2-1988	10.20	28	1/1-1995	10.5
7	19/3-1978	11.35	18	22/12-1988	12.96	29	5/1-1995	11.5
8	5/12-1979	10.36	19	29/1-1989	10.08	30	20/1-1995	10.5
9	13/12-1979	10.24	20	15/2-1989	10.35	31	31/1-1995	11.0
10	4/1-1980	10.56	21	18/2-1989	10.27	32	12/3-1996	11.0
11	15/1-1981	11.24	22	12/12-1990	10.5	33	17/2-1997	12.0

Table A.3 Annual maximum significant wave height 1973 – 1997.

Year	H_s (m)	Year	H_s (m)	Year	H_s (m)
1973/1974	12.1	1981/1982	10.15	1989/1990	9.0
1974/1975	8.7	1982/1983	10.54	1990/1991	10.50
1975/1976	10.19	1983/1984	10.18	1991/1992	11.50
1976/1977	10.61	1984/1985	10.69	1992/1993	12.0
1977/1978	11.75	1985/1986	9.93	1993/1994	9.0
1978/1979	9.06	1986/1987	9.51	1994/1995	11.5
1979/1980	10.56	1987/1988	10.2	1995/1996	11.0
1980/1981	11.24	1988/1989	12.96	1996/1997	12.0