HARMONIC DISTORTION IN STORM WAVES AND CONSEQUENCES FOR EXTREME CREST HEIGHTS

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1. INTRODUCTION

The distribution of heights of waves and heights of crests above mean water level are two of the principal measures of interest to coastal and offshore engineers. Generally these measures are treated as stochastic variables in which the underlying assumption is that the surface elevation has a gaussian or quasi-gaussian probability distribution. Nonetheless, there is general agreement that a wind driven sea is positively skewed having crests that are sharper than troughs in general. Indeed this is entirely consonant with visual evidence. Accelerometer buoys, on the other hand, generally report negligible skewness of the sea surface but this is a consequence of the coupling of the buoy's horizontal motion with its measurement of vertical acceleration (Longuet-Higgins, 1986; Magnusson et al., 1999). Here we examine the relationship between skewness and the intensity of wind forcing of storm seas. We ask the question "does the skewness arise principally from the harmonic distortion of the large waves near the peak of the spectrum or are the contributions from the smaller equilibrium range waves important?" We obtain the "phase-averaged" shape of the large waves in a storm sea and show that their harmonic content corresponds to that of the Stokes infinite wave train.

2. DATA DESCRIPTION

The sources of data were carefully chosen to avoid the introduction of artifacts associated with the measurement method and yet to cover a range of storm waves in the open sea, in lakes and in laboratory tanks. In these three situations the waves are successively more strongly forced by the wind as measured by the parameter termed inverse wave age U/cp.

The open sea data are from the "Flare South" laser on the Ekofisk platform complex in the central North Sea. These data were sampled at 2 Hz and are continuous (except for a 2.5 second gap every 20 minutes to write date and time) for 3 hours from 1998, December, 27, 0000 GMT. The lake data are from the research tower of the National Water Research Institute (Canada) near the western end of Lake Ontario (Donelan et al., 1985). The wave gauges were 4.8 mm diameter capacitance wires 6m long and held in 50 kgm tension. There were 14 such gauges arranged in a Mills cross of 28 m x 20 m extent providing several independent estimates of the skewness. These gauges were sampled at 5 Hz. The laboratory tank data was obtained from two wind-wave tanks at the National Water Research Institute. Data from the larger (100 m long) tank were obtained with 1.1 mm diameter wave gauges at several fetches and sampled at 10 Hz. Data from the smaller tank (32 m long) were measured with a non-intrusive optical technique (Donelan and Plant, 2002) and were sampled at 1000 Hz.

3. RESULTS

The “overshoot effect” in wind generated waves refers to the observed behaviour of the spectral density at a particular frequency (or wavenumber) as the spectral peak passes through that frequency (or wavenumber). The published observations from both field and laboratory that show this effect are all for fetch-limited conditions (Barnett and Wilkerson, 1967; Barnett and Sutherland, 1968; Hasselmann et al., 1973) but recent measurements, in duration-limited growth laboratory conditions, also exhibit this effect (Donelan and Plant, 2002). Figure 1 (from Barnett and Sutherland, 1968) illustrate the “overshoot effect” and indicate enhancement of the peak of about a factor of two. Donelan et al. (1985) have demonstrated that the degree of enhancement is well correlated with the degree of wind forcing at the peak, represented by the inverse wave age, U/c_p. Thus, we are led to the conclusion that waves of a given frequency or wave length are steepest when they occupy the peak position in a wind-generated sea, and that the average steepness of these waves depends on the wind forcing, U/c_p.

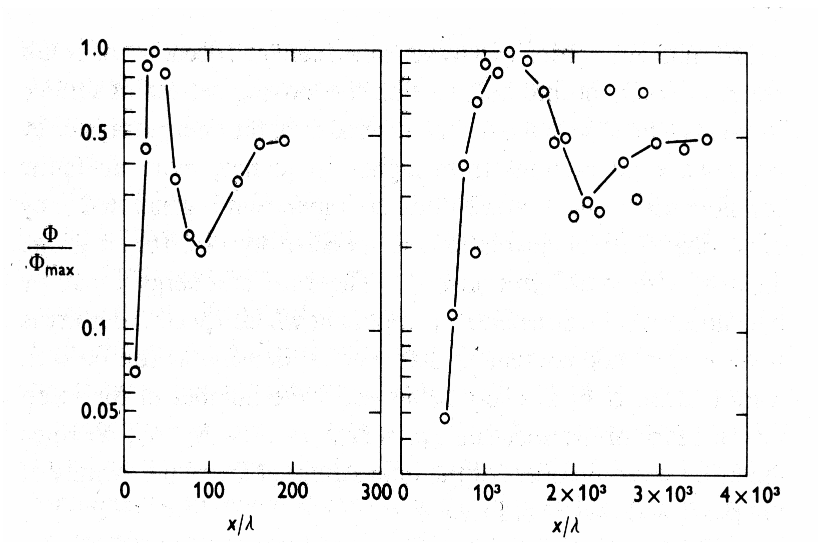

Fig. 1. The spectral density at fixed frequency as a function of fetch, measured by Barnett and Sutherland (1968), normalized with respect to the maximum spectral density attained. The diagram on the left represents laboratory measurements at a frequency of 4.0 Hz; on the right are oceanic measurements at a frequency of 0.127 Hz. (After Phillips, 1977).

3.1 Skewness

As gravity waves become steeper their profile becomes distorted with a tendency towards sharper crests and flatter troughs characteristic of the classical shape of the Stokes infinite wave train. Such a profile has a non-zero third moment, i.e. the skewness, $(h - <h>)^3 / s_h^3 > 0$, where $h = h(x,t)$ is the surface elevation, $<h>$ its average value and s_h is its standard deviation. In figure 2 we illustrate the dependence of the overall skewness on U/c_p. The data used in this figure are the optical measurements at short fetch in a laboratory tank. The wind was slowly and uniformly increased from 0 to 6 m/s (measured at 10 cm height) over 1800 seconds. The rate of increase of the wind is sufficiently slow that the waves are effectively fetch-limited and not duration-limited. In the left panel of figure 2 the dependence on skewness with U/c_p is shown. The points having skewness values below 0.3 are in the region of the wind threshold below which no waves can grow (Donelan and Pierson, 1987; Donelan and Plant, 2002). Once the waves start growing in these strong forcing conditions they grow quickly until they are limited by intermittent breaking. In this region the skewness and U/c_p are strongly positively correlated. Magnusson and Donelan (2000) have shown a clear relation between the inverse wave age and steepness of the forward face of storm waves at the Ekofisk site.

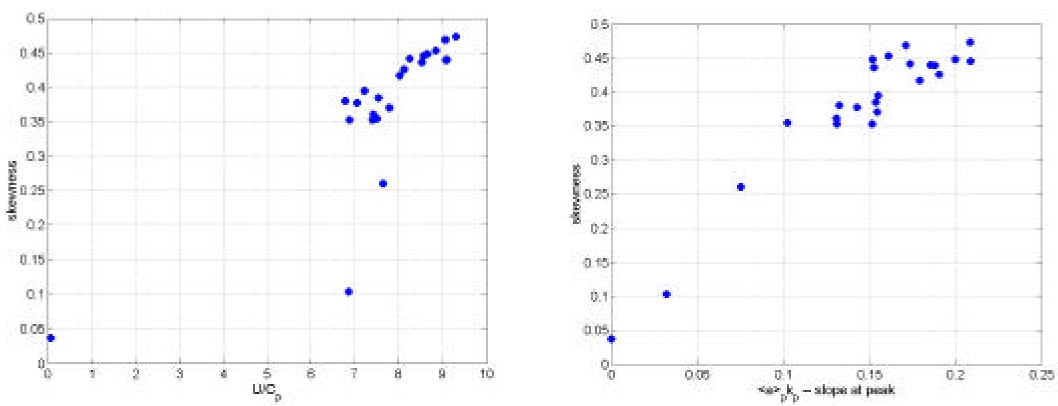

Figure 2. Measured skewness for waves at fetch of 14.3 m in a laboratory tank of 32 m length. The wind speed was slowly increased from 0 to 6 m/s (at 10 m height) over 1800 secs. The panel on the left indicates skewness versus the ratio of wind speed to phase speed of the waves at the spectral peak, U/c_p or the inverse wave age. The panel on the right graphs skewness against the mean slope of the waves near the spectral peak, $<a_p>k_p$, where k_p is the wave number corresponding to the peak frequency and $<a_p>$ is the square root of the area under the frequency spectrum in the peak region $(0.5 f_p < f < 1.5 f_p)$.

In the right panel the skewness is plotted against average slope, $<a_p>k_p$ of the waves near the spectral peak, where k_p is the wave number corresponding to the peak frequency and $<a_p>$ is the square root of the area under the frequency spectrum in the peak region $\left(0.5 f_p < f < 1.5 f_p\right)$. The excellent correspondence in this figure supports the view that it is the slope of the large waves near the peak that determine the skewness of the surface elevation.

3.2 Phase averaged shape

In order to examine the shape of the large waves in an average sense we employ the technique of phase averaging. The procedure we use is as follows: 1) the time series of surface elevation $h(t)$ is band-pass filtered to admit frequencies, f in the range $0.5 f_p < f < 1.5 f_p$, yielding the narrow band surface elevation, $h_f(t)$; 2) h_f is Hilbert transformed to yield local phase and envelope amplitude; 3) for a given range of envelope amplitude in every 10 degree phase bin the original raw time series $h(t)$ is averaged yielding the mean and standard deviation of the surface elevation at 36 points along a complete period. These phase averaged profiles are illustrated in figure 3a,b for the two extreme data: in figure 3a is the data set from the small wave tank in which the waves were a few centimeters high, figure 3b shows the waves from Ekofisk that exceed 10m in height. Both data sets were obtained with (different) non-intrusive laser measuring systems. They both show consistent increases in skewness with increasing envelope amplitude. The Stokes-like distortion is more apparent when the figures are inverted – perhaps because one expects to see sharpened crests and flattened troughs in water wave profiles.

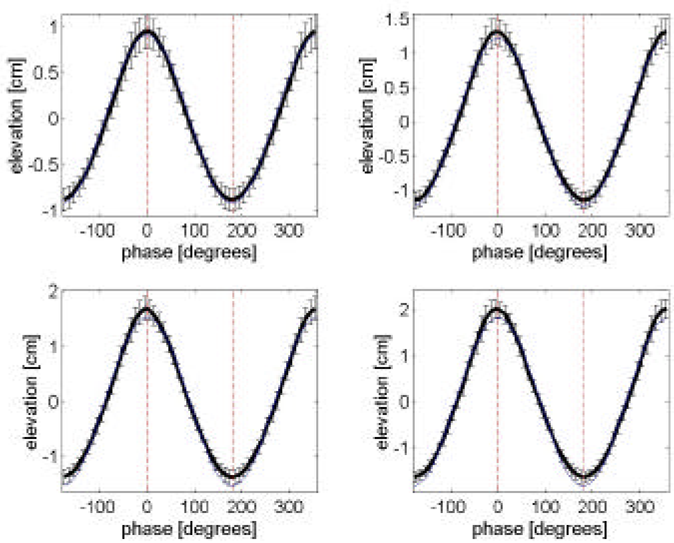

Figure 3a. Waves measured in a 32 m wind-wave tank at 14.3 m fetch. Heavy line: phase averaged surface elevation on the phases of waves in the range of $0.5\ f_p < f < 1.5 f_p$. The panels (a), (b), (c), (d) are averages where the envelope of these waves is (0.7, 1.0, 1.3, 1.6 $\pm$ 0.15) times the mean envelope. The error bars shown are ± 1 standard deviation of the variability of the estimates in each 10 degree bin. Light line: sinusoid having the period and amplitude of the fundamental of the heavy line.

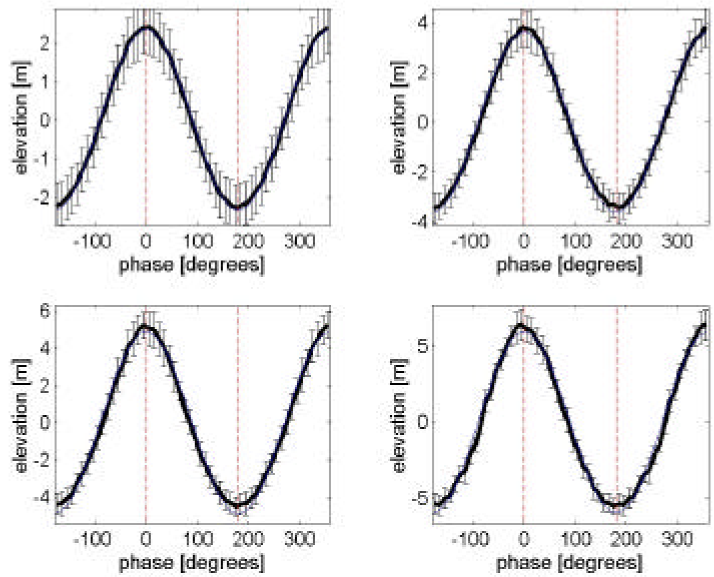

Figure 3b. Waves measured on the Ekofisk platform in the central North Sea. Heavy line: phase averaged surface elevation on the phases of waves in the range of $0.5\ f_p < f < 1.5\, f_p$. The panels (a), (b), (c), (d) are averages where the envelope of these waves is (0.7, 1.2, 1.7, 2.2 $\pm$ 0.25) times the mean envelope. The error bars shown are ± 1 standard deviation of the variability of the estimates in each 10 degree bin. Light line: sinusoid having the period and amplitude of the fundamental of the heavy line.

3.3 Harmonic balance

Various attempts to deal with the statistics of nonlinear wind-driven waves (Tayfun, 1986; Srokosz and Longuet-Higgins, 1986) have recourse to a Stokes-like shape of waves in a narrow band approximation to the observed spectrum. Although the Stokes (1880) perturbation expansion applies, strictly speaking, to an infinite monochromatic train of unidirectional waves, the similarity of these phase averaged profiles to the Stokes theoretical form invites further exploration of their shapes. In figure 4 we compare the ratio of 2nd to 1st (fundamental) harmonic (abscissa) to the ratio of 3rd harmonic to 1st (ordinate). The symbols represent 4 envelope steepnesses for each of the 4 data sets (different symbols for each data set). The Stokes ratios are indicated by the dashed line. All but the smallest waves (from the 32 m wind-wave tank) appear to belong to the same population and the trend is in agreement with the Stokes form although displaced somewhat from it. Any noise or statistical variability will affect the higher harmonics (lower energy) more and therefore will tend to increase the ordinate values above the abscissa. The points from the small tank are well above. These latter were obtained with waves of peak frequency 2.5 Hz so that their 3rd harmonics (7.5 Hz) may be influenced by surface tension (Hui and Tenti, 1982). Apart from these very short waves, the harmonic balance in the phase averaged waves near the spectral peak appear to be consonant with the Stokes form.

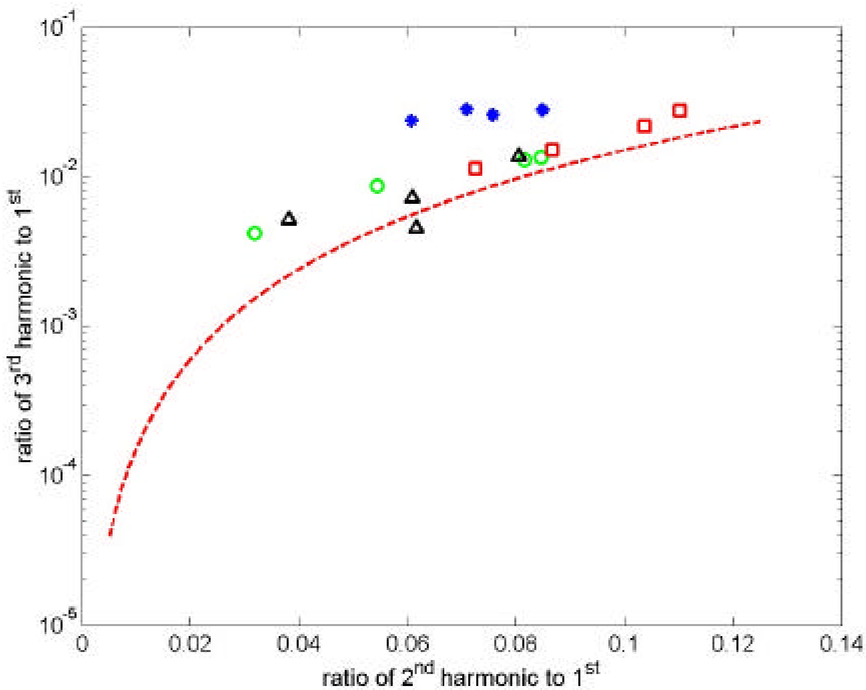

Figure 4. Ratio of 3rd harmonic to 1st harmonic (ordinate) versus ratio of 2nd harmonic to 1st harmonic (abscissa). The harmonics are obtained from the phase averaging procedure illustrated in figure 3. The first harmonic is the fundamental period of the wave, f_p and harmonics 2, 3 correspond to $2f_p, 3f_p$. The symbols correspond to the four different experiments: (*) 32 m wind-wave tank; (?) 100 m wind-wave tank; (△) Lake Ontario Tower; (O) Ekofisk oil platform in the central North Sea. The dashed line is the Stokes wave train.

4. DISCUSSION AND CONCLUSIONS

We have demonstrated that the waves that contribute to the enhanced spectral peak in strongly forced seas are, on average, skewed – with sharpened crests and flattened troughs – and the degree of harmonic distortion is in general agreement with that of the Stokes wave train. In this context it is interesting to see how the skewness is affected by higher frequencies. Figure 5 shows the skewness of the elevation signals after all frequencies above $f_{cut\text{-}off}$ have been removed. The first three panels are for the three cases of larger waves, while the fourth corresponds to a Stokes wave train of slope, ak = 0.4. The light line in the last panel is for the steady fourth order Stokes wave train of fundamental frequency, f_p and the heavy line is obtained by allowing a 17% Gaussian perturbation of the fundamental frequency every two periods. All cases, including the simulation of panel (d), indicate that the skewness is effectively determined by frequencies below five times the peak frequency, and the rapid drop-off, as the 2nd harmonic is discarded, is characteristic of all.

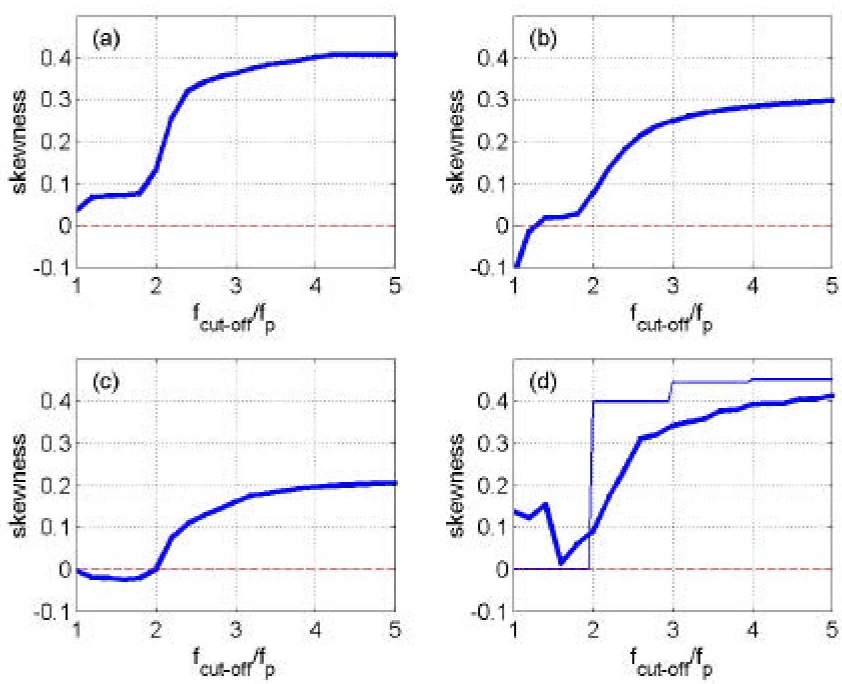

Figure 5. The dependence of skewness on high frequency cut-off. Panels (a), (b), (c) are data from: 100 m wind-wave tank; Lake Ontario Tower; Ekofisk oil platform in Central North Sea respectively. In panel (d) the light line is for a Stokes wave train and the heavy line is for a Stokes wave train in which the fundamental frequency is randomly adjusted with a standard deviation of $0.17\ f_p$.

We have discussed: the overshoot phenomenon with its attendant enhanced peak, the phase averaged shape of the large waves, and the dependence of skewness on the spectral peak and its harmonics. Taken all together these suggest that strongly wind-driven seas may be described by large waves (near the peak frequency) in various stages of nonlinear distortion, with attendant bound harmonics, skewness and random small waves superposed. The statistics of grouping of large waves will thus be largely dependent on the bandwidth of the spectral peak with only mild and random adjustments due to the spectral tail. The statistics of crest heights and trough depths will diverge as the seas are more strongly forced and the spectrum becomes more sharply peaked.

5. ACKNOWLEDGMENT

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