UPPER LIMIT ON WAVE HEIGHT IN DYNAMIC FETCH

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ABSTRACT

Wave steepness limits the growth of waves. Once waves reach a critical steepness, further transfer of energy causes them to break rather than to continue growing. When this principle is applied to Dynamic Fetch (DF) situations, it can define the limiting height resonant waves can achieve. Using basic wave equations it is shown here that this limiting height is a function of the forward speed of the fetch. This provides a useful tool for forecasters dealing with DF situations.

1. BASIC RELATIONSHIPS

In deep water, waves are defined by

$$C_p^2 = g\ L\ /\ 2B \tag{1}$$

$$C_p = L\ /\ T \tag{2}$$

$$C_g = C_p\ /\ 2 \tag{3}$$

and

$$L = gT^2\ /\ 2B. \tag{4}$$

With speeds in knots, lengths in feet, and period in seconds, these reduce to

$$C_p = 3.03\ T \tag{5}$$

$$C_g = 1.5\ T \tag{6}$$

and

$$L = 5.12\ T^2. \tag{7}$$

2. STEEPNESS

Wave steepness (S) is the ratio of height (H) to length (L). When S is expressed as a function of L, it is difficult to apply because L is not observed. But L is a function of period (T) so S can also be expressed as a function of T. Since H and T are both measured and reported, S can become more useful to the operational forecaster. More specifically, knowing the steepness limit can allow a forecaster to assess the potential severity of a Dynamic Fetch situation by putting an upper limit on the height of resonant waves.

Steepness is defined as

$$S = H / L \tag{8}$$

Since L is a function of T we get

$$S = (2B/g)\ (H / T^2). \tag{9}$$

3. STEEPNESS LIMIT

When waves reach a critical steepness, they become unstable and will break. In breaking, they lose energy. According to Stokes theory, this limiting value for individual waves is 1/7, but for real ocean waves, it seldom reaches 1/10 [WMO 1998]. This limiting value is a constant, so we get

$$H_{lim} / L = K. \tag{10}$$

Substituting for L and rearranging gives

$$H_{lim} / T^2 = (K / 2B)\ g. \tag{11}$$

WH Buckley published a study [Buckley,1988] in which he found K / 2B empirically by reviewing thousands of spectra from archived buoy data. Using significant height and peak period he found that

$$H_{lim} / T_p^2 = 0.00776\ g \tag{12}$$

Thus we solve

$$K / 2B = 0.00776$$

to get

$$K = 0.049\ .1/20$$

Thus, with significant height and peak period, the real limiting steepness for ocean waves is 1/20.

4. DYNAMIC FETCH

Dynamic Fetch (DF) occurs when the generating area moves with the waves it generates. In a DF all waves moving in the same direction as the generating area will be amplified. But the waves experiencing the greatest amplification will be those whose group velocity is equal to the speed of the fetch. Since C_g is a function of period, DF results in selective amplification of waves having a specific period. This is a kind of resonance phenomenon based on the group velocity, so DF is sometimes called “group velocity quasi-resonance” [Dysthe, 1987]. In a DF, fetch and duration are effectively unlimited - until the fetch changes speed or direction, or until the waves reach a coast.

DF is usually associated with small intense systems. The sizes of such systems and the curvature of their isobars means that fetches are very short so wave growth is limited unless DF is involved. DF has been observed in hurricanes [Bigio, 1996] and in Polar Lows [Dysthe, 1987]. There appears to be no reason, though, why DF cannot be found in much larger extratropical systems. The only requirement is for a fetch area to move with the wave group it generates.

Since the group velocity of waves is proportional to their period, we can relate the speed at which a generating area moves to the peak period of the waves with which it would be resonant. With V in knots and T in seconds, the result is simply

$$V = C_g = 1.5\ T_{resonant}.$$

Using the steepness limit, we can now find the maximum height possible for the resonant period. This makes it possible to relate the speed with which a fetch is moving to the maximum possible height of resonant waves.

Suppose a fetch is moving with a speed V (knots). The resonant period is

$$T_{resonant} = V / 1.5 \qquad (13)$$

and the upper limit to the significant height of resonant waves is

$$H_{lim} = 0.00776\ g\ T_{resonant}^2 \qquad (14)$$

which is Buckley's Steepness Limit. Figure 1 shows how $T_{resonant}$ changes with V. Figure 2 shows how H_{lim} changes with V.

When H_{sig} is plotted against T_p , the limiting height shows up as an envelope. Figure 3 is a sample of such a plot - in this case it shows the distribution of 9 years of hourly data (1992-2001) from buoy 44141. The curved line is Buckley's Steepness Limit.

For V=10 knots, the resonant period is ~6.5 sec, and the upper limit to H_{sig} for those resonant waves is ~3.5 m. For V=25 knots, the resonant period is ~16.5 sec, and the upper limit to H_{sig} for those resonant waves is ~21 m. Period of resonant waves increases linearly with V, while significant height limit increases with the square of V. So doubling the speed at which the fetch moves doubles the resonant period, but quadruples the limiting height of resonant waves.

Note that this does not limit the height of all waves - only the height of resonant waves. A hurricane moving at 5 knots or less cannot produce resonant waves of more than 1m, but can still be producing storm waves of several metres. In other words, a slow-moving fetch will produce insignificant resonant waves, while a fast-moving fetch has the potential to produce enormous resonant waves.

In an operational setting, the forecaster can make a simple assessment considering only how the fetch is moving. If the fetch is not moving in the direction of the waves, there will be no resonant waves. If the fetch is moving in the same direction as the waves, but at 8 knots or less, there will be resonant waves, but they will not build beyond 2m (storm waves, of course, can be much higher). If the fetch is moving in the same direction as the waves, but at 20 knots or more, there will be resonant waves, and they could reach 14m or more.

So for a storm in which the fetch is moving slowly, resonant waves will be only a minor factor for the forecaster to consider. But for a storm in which the fetch is moving at 15 knots or more, resonant waves could become a major factor in the forecast.

5. MAXIMUM WAVEHEIGHT

This process helps a forecaster predict H_{sig}, and gives an upper limit for it. Factors for H_{max} / H_{sig} range between 1.5 and 1.9.

6. IN AN OPERATIONAL SETTING

If the fetch is moving in the same direction as the waves it is producing, the following procedure will help a forecaster determine whether (and to what degree) resonant waves will be a factor in the forecast.

A) What storm waves will be produced? Using whatever technique or nomogram is preferred, predict H_{sig} and T_p for this fetch area. Disregard DF at this stage.

B) Determine $T_{resonant}$ corresponding to V from Figure 1 (or from equation 13). In the case of a tropical cyclone, the first guess for V could be the forward speed of the pressure centre.

C) Is T_p (from step A) within 1 second of $T_{resonant}$ (from step 2)? If so, resonance will amplify those waves in which the storm energy is already concentrated. The greater the difference between T_p and $T_{resonant}$, the less of a factor resonance will be. If the difference is 1 second or less, determine H_{lim} corresponding to V from Figure 2.

D) Is DF important? Compare H_{lim} (from step C) to H_{sig} (step 1). If $H_{lim} > H_{sig}$, resonance IS a factor. In that case, predicted height will be greater than H_{sig}, but less than H_{lim}.

E) After step D, it will have been determined whether resonant waves will be a factor. If so, predicted waveheight will be > H_{sig} but < H_{lim}. How much greater than H_{sig} will depend on the circumstances and will be for the forecaster to determine.

7. CONCLUSION

This procedure will not tell a forecaster what waveheight to predict. But it will give a quick way to assess the risk of resonant waves, and an upper limit to the significant height of resonant waves. The rest is still up to the forecaster.

8. LIST OF VARIABLES

C_g	group velocity
C_p	phase velocity
g	acceleration of gravity
H_{lim}	limit of significant waveheight
H_{max}	maximum waveheight
H_{sig}	significant waveheight
K	a constant
L	wave length
S	steepness
T	period
T_p	peak period
$T_{resonant}$	resonant period
V	speed at which fetch is moving

9. REFERENCES

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EXAMPLES

1. A tropical cyclone is moving in a straight line at 12 knots. To the right of the track, the fetch is 100 naut mi long, and the mean wind in the fetch is 60 knots. Bretschneider gives H_{sig} = 8m, T_p = 11 sec. Groen-Dorrenstein (G-D) gives H_{sig} = 10m, T_p = 11 sec. From Figure 1, resonant period is 8 sec. From Figure 2, limiting height for resonant waves is 5m. Resonance is present, but for waves that are not in this storm's energy peak. There will be some amplification, but your forecast significant height for storm waves will be only a little more than H_{sig}.

2. Same as 1, but system is moving at 16 knots. From Figure 1, resonant period is 11 sec. From Figure 2, limiting height for resonant waves is 9m, Resonance is present, and it applies to the waves that are in this storm's energy peak. But the steepness limit for 11-sec waves means that resonant waves will be about the same height as storm waves predicted by the nomograms. In this case, your forecast significant height for storm waves will be a little more than H_{sig}.

3. Same as 1, but system is moving at 22 knots. From Figure 1, resonant period is 14.5 sec. From Figure 2, limiting height for resonant waves is 16.5m. Again resonance is present, but for waves that are not in this storm's energy peak. Again there will be some amplification, and your forecast significant height for storm waves will be a little more than H_{sig}.

4. System is moving at 20 kts, fetch is 150 naut mi, and windspeed is 65 kts. Bretschneider gives H_{sig} = 11m, T_p = 12.5 sec. G-D gives H_{sig} = 13m, T_p = 13 sec. From Figure 1, resonant period is 13 sec. From Figure 2, limiting height for resonant waves is 14m, Resonance applies to the waves in this storm's energy peak, and your forecast significant height for storm waves will be about 14m.

5. Hurricane Luis [Bigio 1996] moved at 20-25 kts for about 24 hours, then accelerated. Resonant period was 14-17 sec (Figure 1). Buoys observed waves with T_p of 15-18 sec. Limiting height for resonant waves was 21m (Figure 2). Observed Hsig at 44141 reached 17.1m. Resonance was a factor here, and resonant waves did not reach their limit.

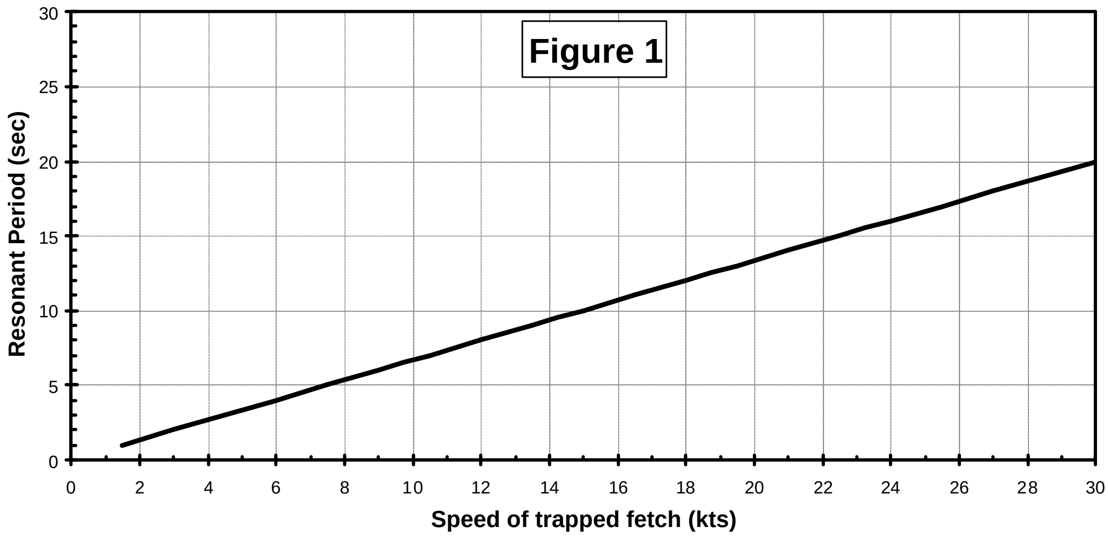
Figure 1
Resonant Period (sec)
Speed of trapped fetch (kts)
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

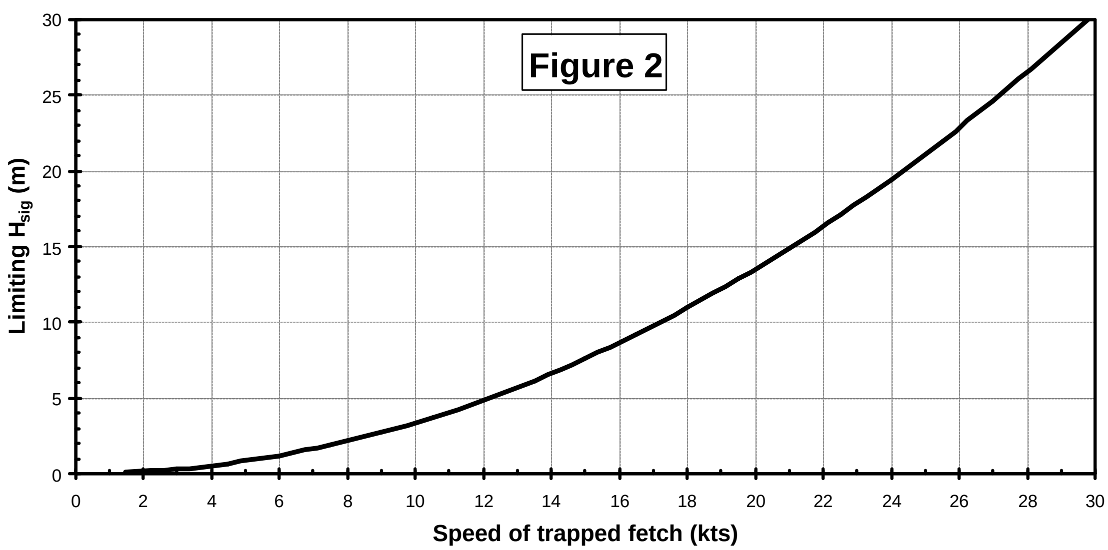
Figure 2
Limiting H_{sig} (m)
Speed of trapped fetch (kts)
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

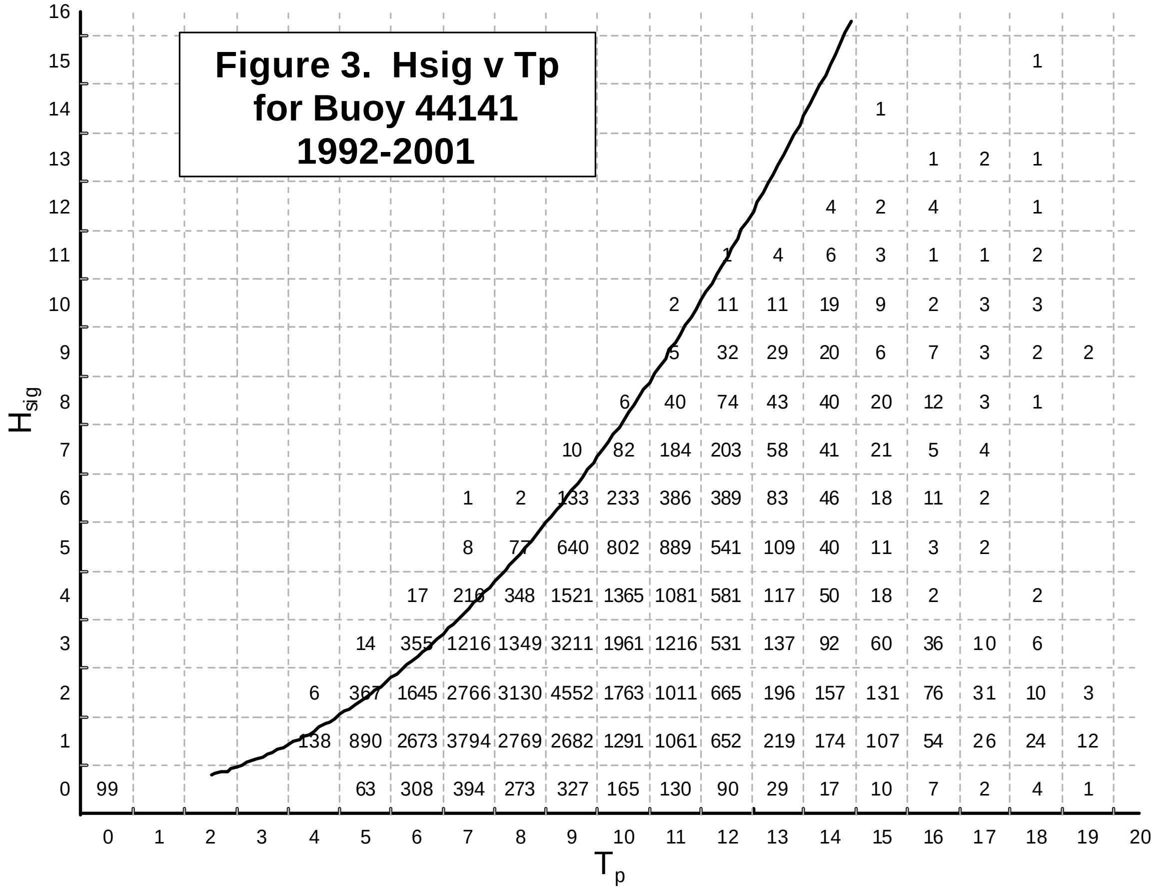

Figure 3. Hsig v Tp for Buoy 44141 1992-2001