A weakly dispersive and fully nonlinear wave model for the simulation of nearshore wave transformation and overtopping

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Context and objectives

- **Protection of coastal infrastructures** (including nuclear power plants) **and areas** against **flooding hazards** due to waves.

- **Coastal protections** designed from **average overtopping discharges.**

- Average overtopping discharges estimated with **empirical formulas** (e.g. EurOtop, 2018)
  - **well suited for simple configurations**, invariant alongshore,
  - can be **insufficient** for **complex sea states** and/or **complex nearshore bathymetry and breakwater geometry.**

→ Develop and validate a **numerical <u>phase-resolving</u> model** for simulating

1. **irregular wave transformation** in the nearshore zone,
2. **wave breaking** (and associated effects, e.g. wave setup),
3. **average overtopping discharges** over coastal protections.

Coastal nuclear power plant

Wave overtopping

1HD wave model

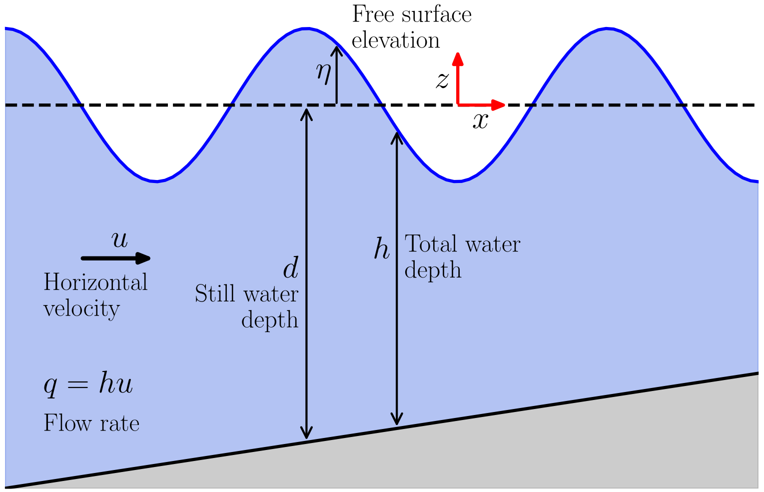
- Mathematical model: **enhanced Serre-Green-Naghdi (eSGN)** equations (Bonneton *et al.,* 2011)
- Decoupled form, separating the **hydrostatic NLSWE** and the **dispersive** part (Kazolea *et al.*, 2023):

$$h_t + q_x = 0$$

$$q_t + (hu^2)_x + gh\eta_x = \Phi$$

$$\Phi = \Psi + \frac{gh}{\alpha}\eta_x$$

$$(I + \alpha\mathcal{T})\Psi = -\frac{gh}{\alpha}\eta_x - Q + D_{wb}$$



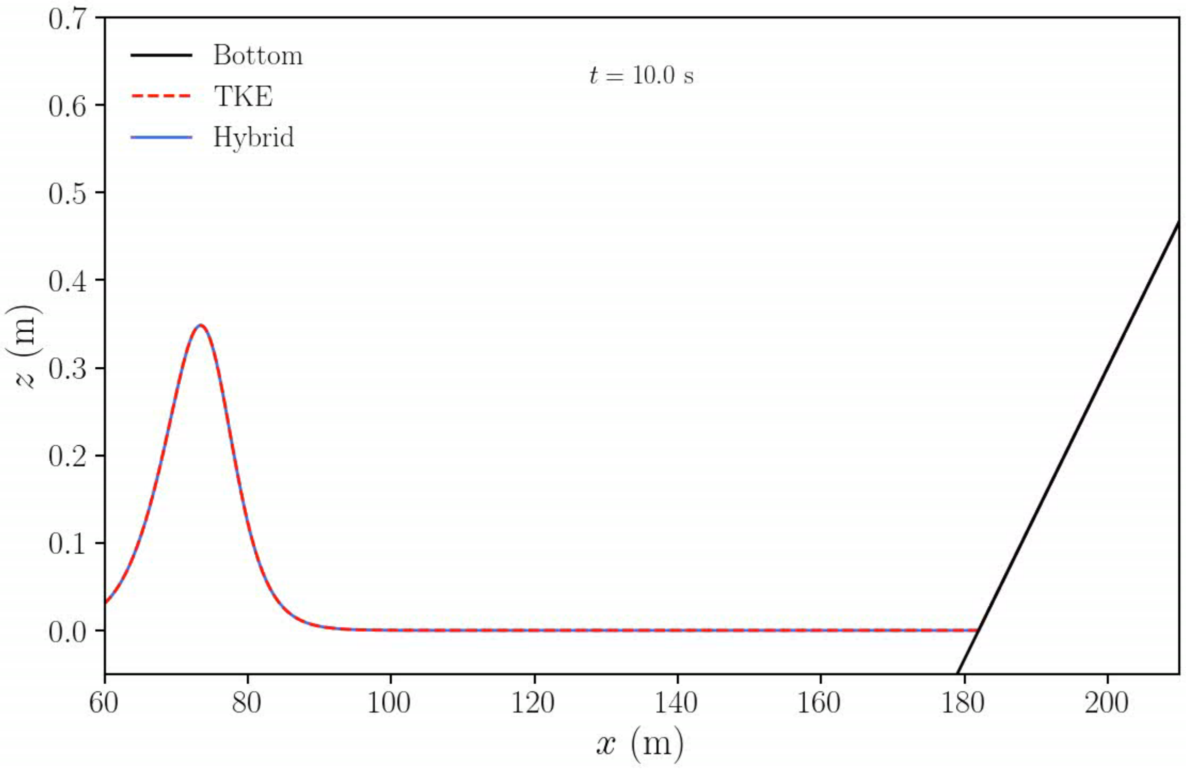
Definition of the flow variables

- Two **wave-breaking** modelling strategies tested:
  1) neglect dispersion (Φ) to model breaking waves as **shocks** with the NLSWE (=> **hybrid** approach),
  2) add the energy dissipation term $D_{wb} = (2\nu_T hu_x)_x$ with ν_T computed with a one-equation turbulence model on the turbulent kinetic energy (**TKE**).

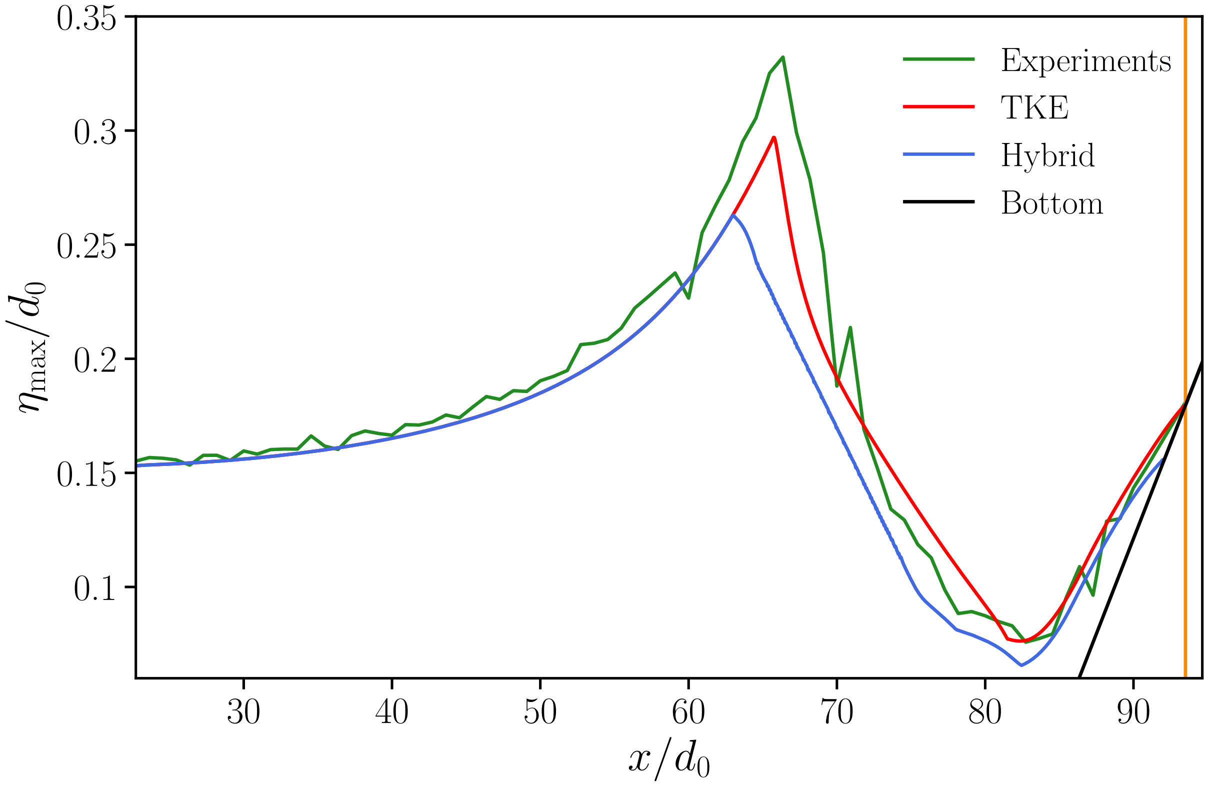
- Numerical methods:
  - NLSWE solved with **3rd or 4th order finite volume (FV) MUSCL** schemes,
  - Dispersive terms discretised with **2nd order finite difference (FD)** scheme.

Comparison of the two wave-breaking approaches

- Experiments of **solitary wave** propagation, shoaling and breaking from **Hsiao *et al.* (2008)**
- **Decrease in amplitude** well captured with both approaches
- **Oscillations** produced by the **hybrid** approach, prevents use of fine meshes → **TKE favoured for further applications**



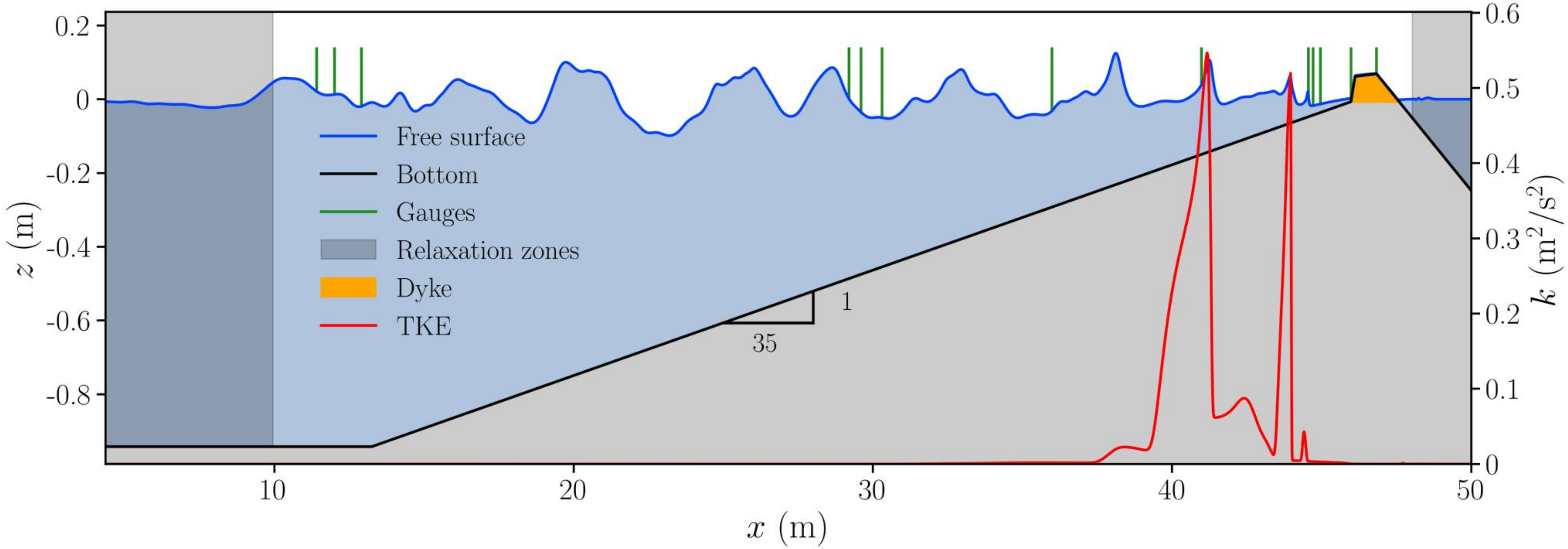
Shoaling and breaking of a solitary wave (a/d_0 = 0.152) with both breaking approaches



Spatial evolution of the solitary wave amplitude (a/d_0 = 0.152) in the experiments and with both breaking approaches

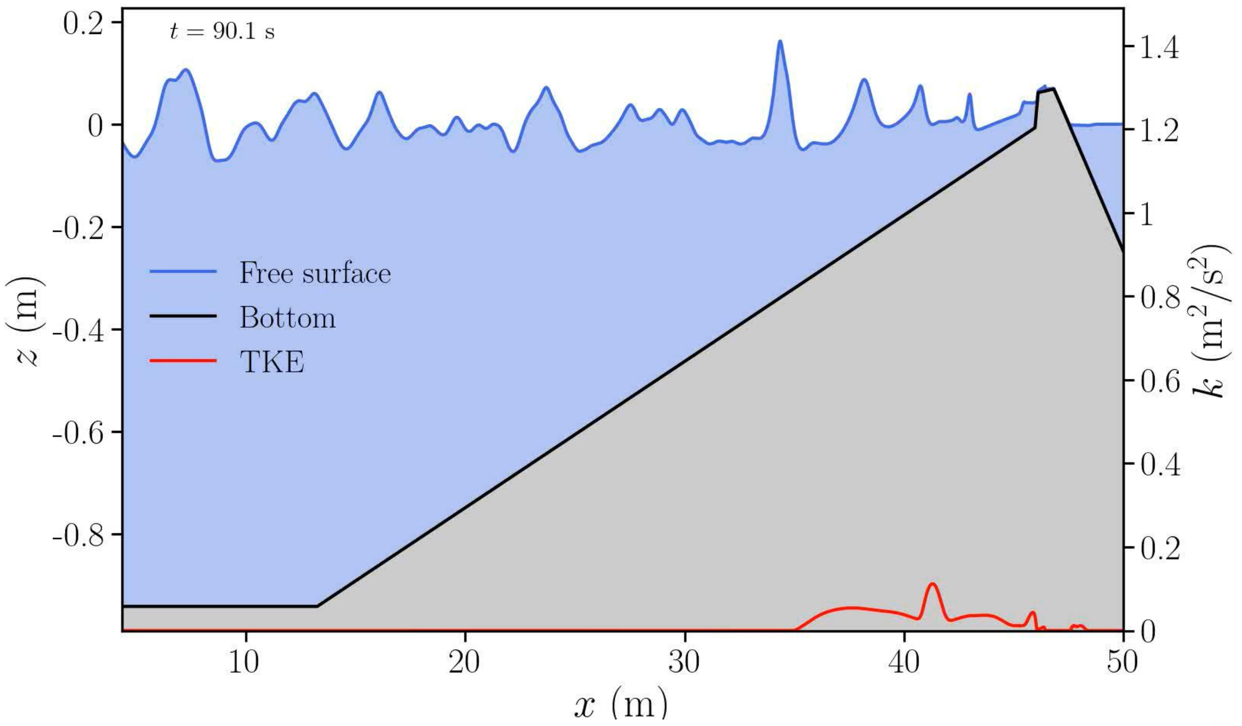
The wave overtopping experiments *(Altomare et al., 2016; Suzuki et al., 2017)*

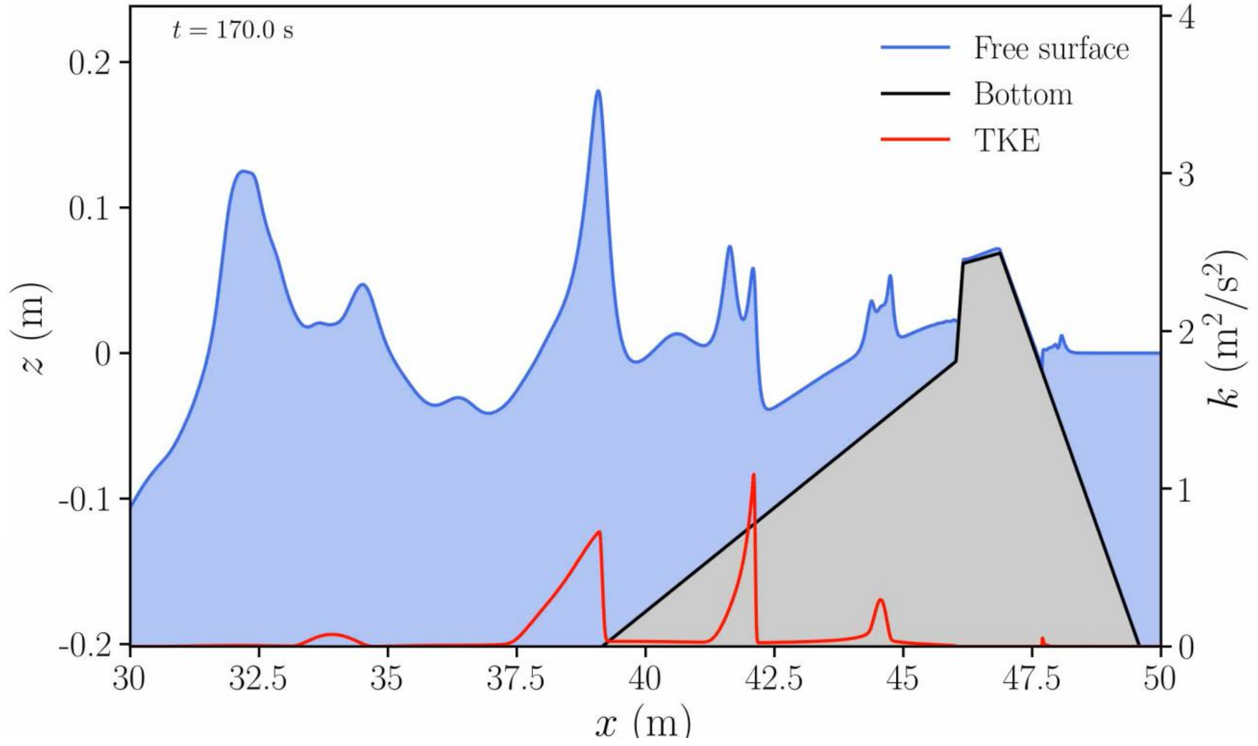
- **Overtopping experiments** in a wave flume by **Flanders Hydraulics, Belgium**
- **3 dyke slopes**: 1:2, 1:3 or 1:6
- **Foreshore slope 1:35** → significant wave transformations and intense breaking
- Irregular sea states with $H_{m0} \in [11, 25]$ cm and $T_p \in [2, 2.5]$ s
- **111 trials**, all simulated over the full duration ≈ 40 min, i.e. **more than 1000 waves**, with the **TKE** breaking model .



Example of the setup for one case, and visualisation of wave breaking (TKE)

Visualisation of wave breaking and wave overtopping

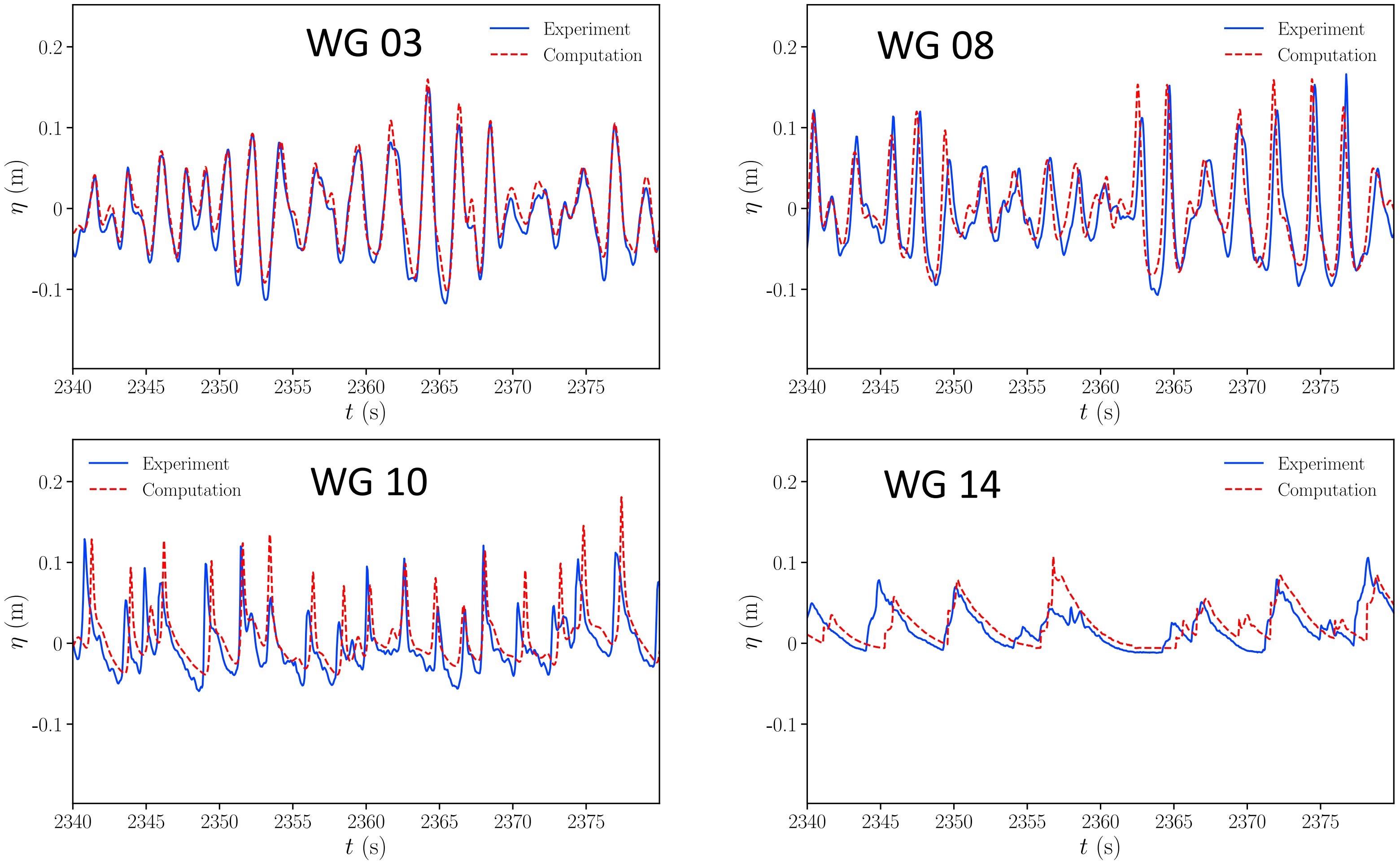




Free surface profiles

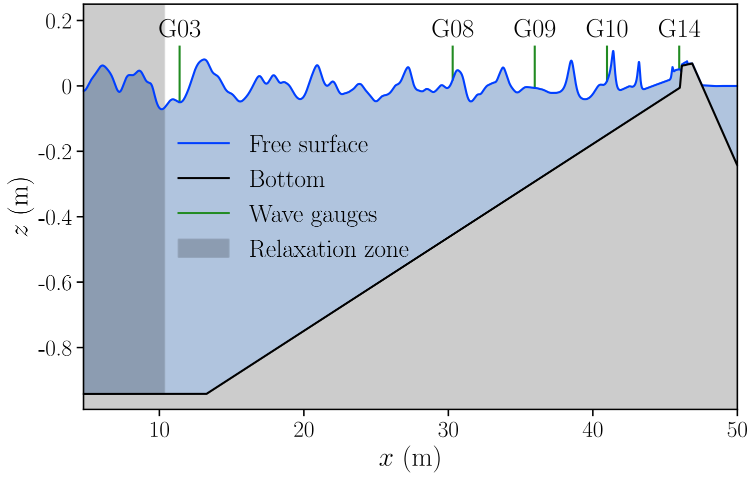
- Mean index of agreement d_r (Willmott *et al.*, 2012) for the 111 cases:

  0.85 for the deep-water gauges (**very good**), **0.64** for all gauges (**reasonable**)



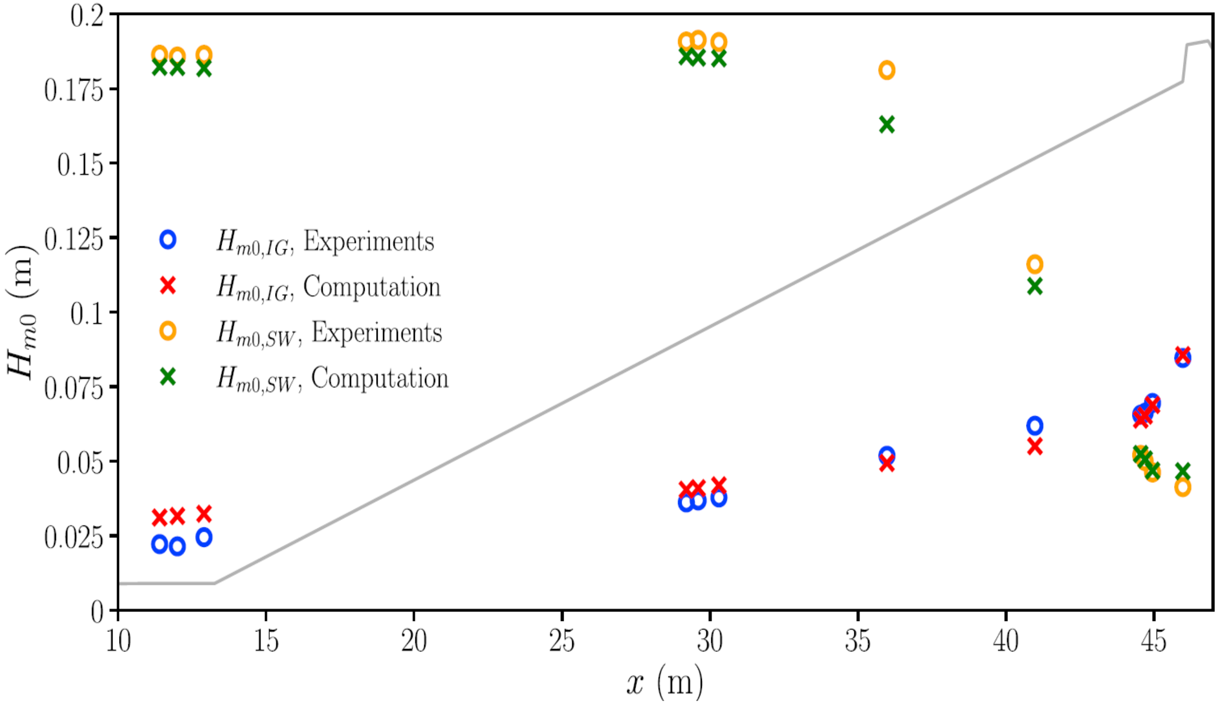
Free surface profiles in the experiment and the simulation in 5 locations

Slope 1:2, h = 0.942 m, $H_{m0} = 0.189$ m, $T_p = 2.13$ s

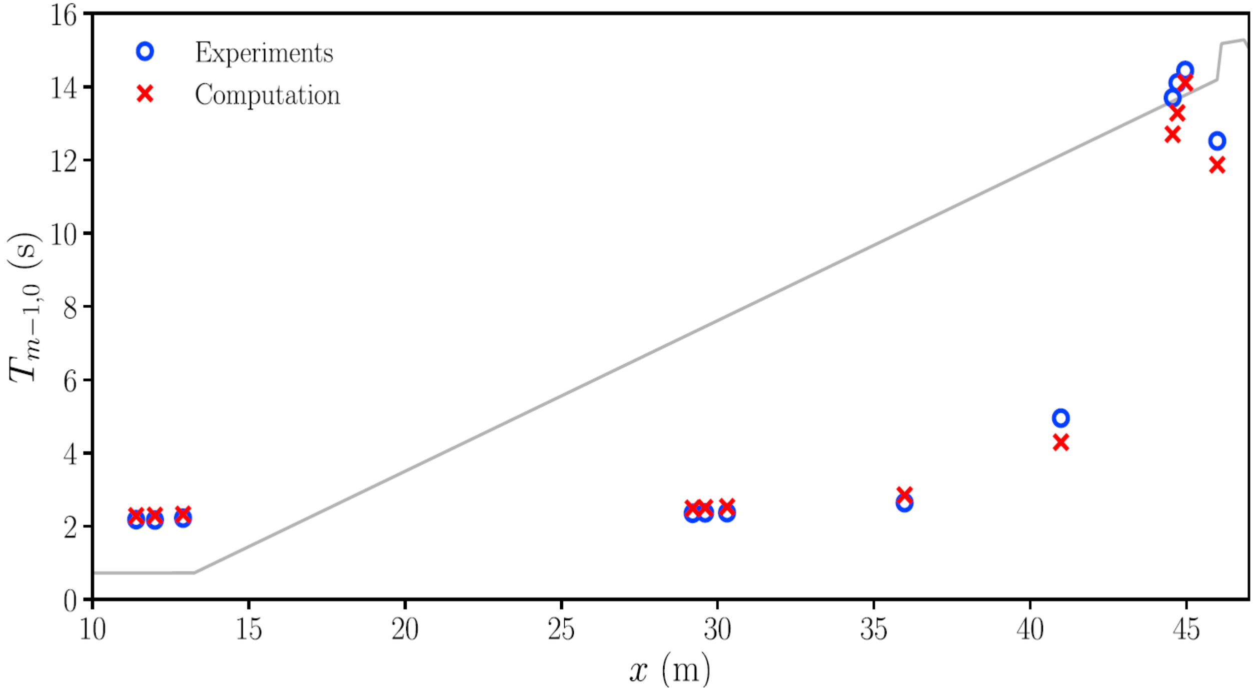


Position of the gauges

Significant wave height and energy period



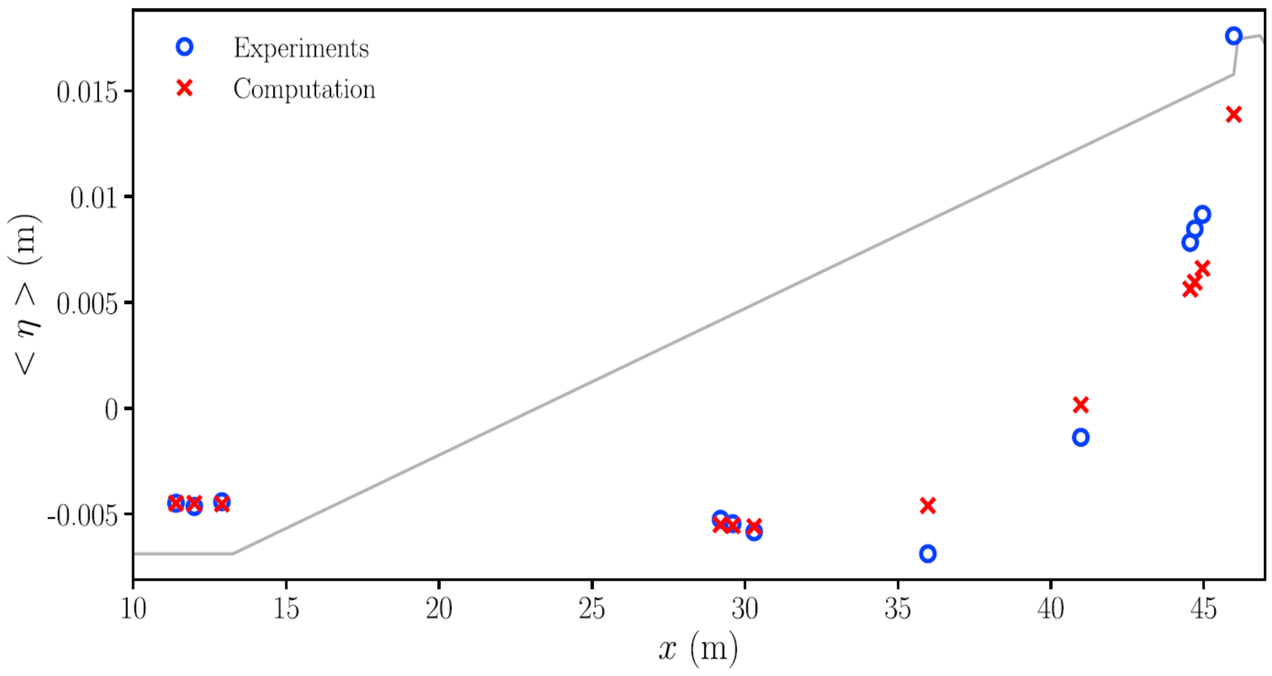
Spatial evolution of the **infragravity** and **short waves significant wave heights H_{m0}** in the experiment and the simulation



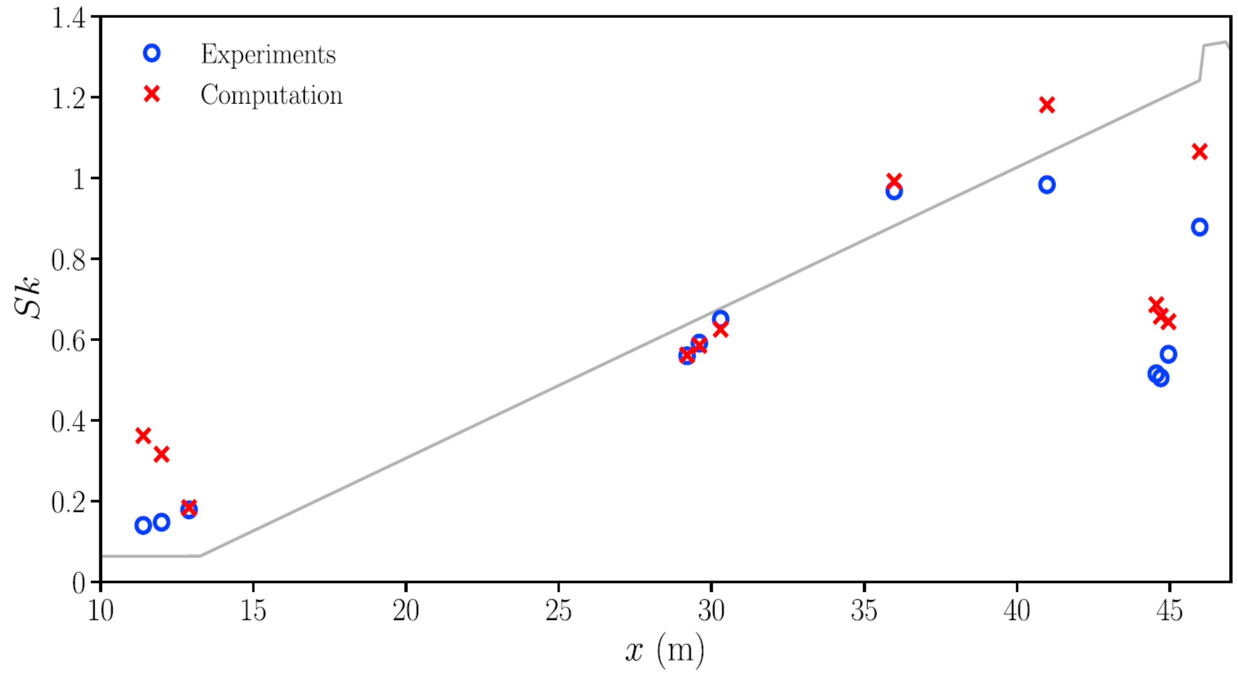
Spatial evolution of the **energy period $T_{m-1,0}$** in the experiment and the simulation

Case with slope 1:2, h = 0.942 m, $H_{m0} = 0.189$ m, $T_p = 2.13$ s

Wave setup and skewness



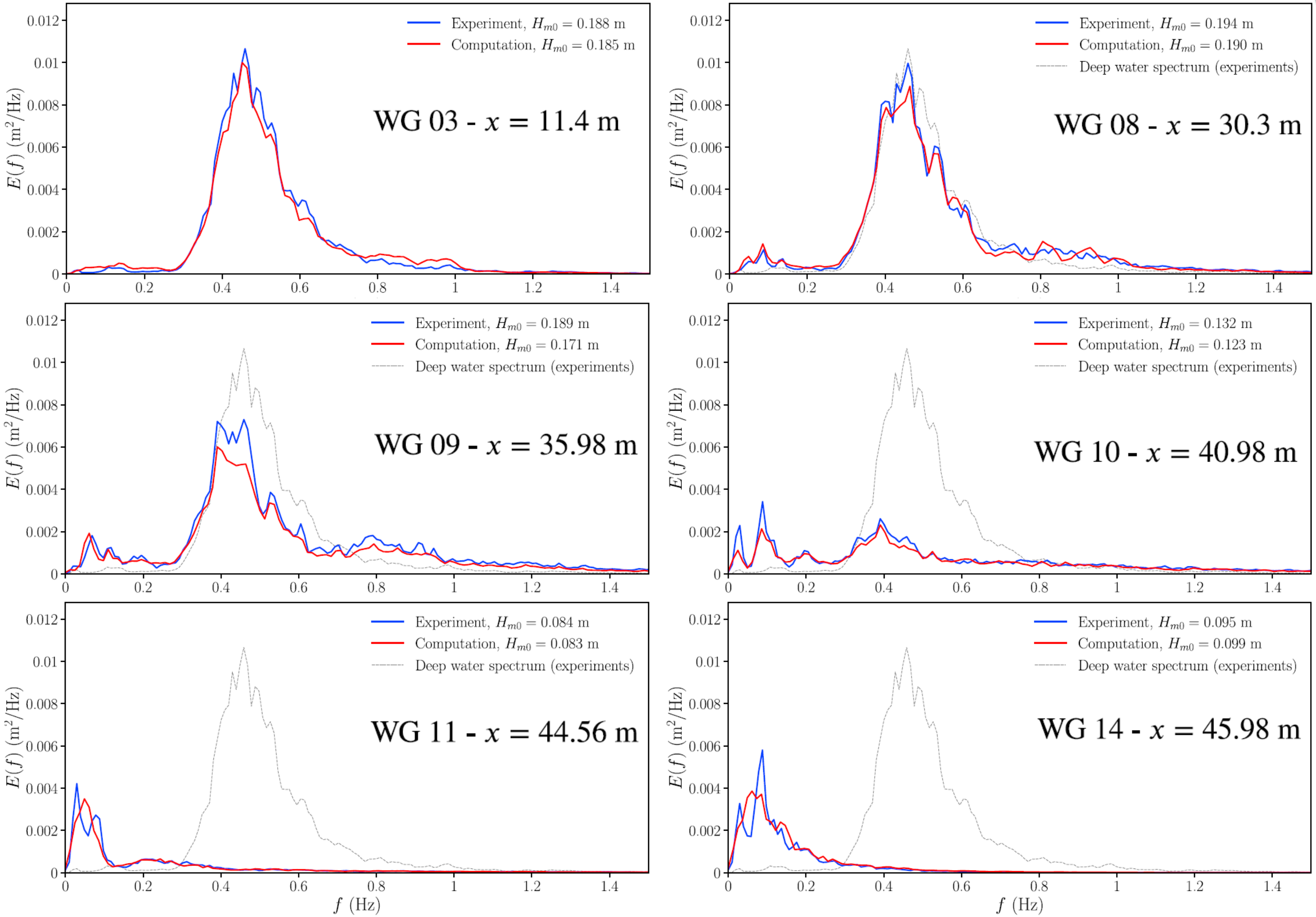
Spatial evolution of the **mean sea level (wave-set-up / set-down)** in the experiment and the simulation



Spatial evolution of the **wave skewness** in the experiment and the simulation

Case with slope 1:2, h = 0.942 m, $H_{m0} = 0.189$ m, $T_p = 2.13$ s

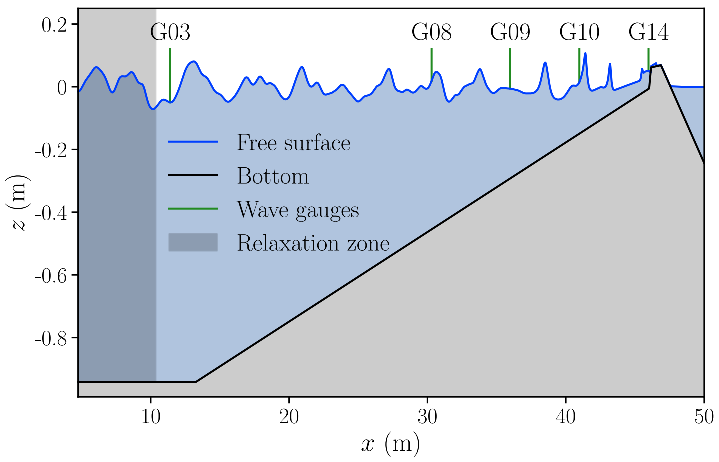
Total wave spectra



Total wave spectra in the experiment and the simulation at 6 locations.

Slope 1:2, h = 0.942 m,

H_{m0} = 0.189 m, T_p = 2.13 s



Grid convergence on the average discharge

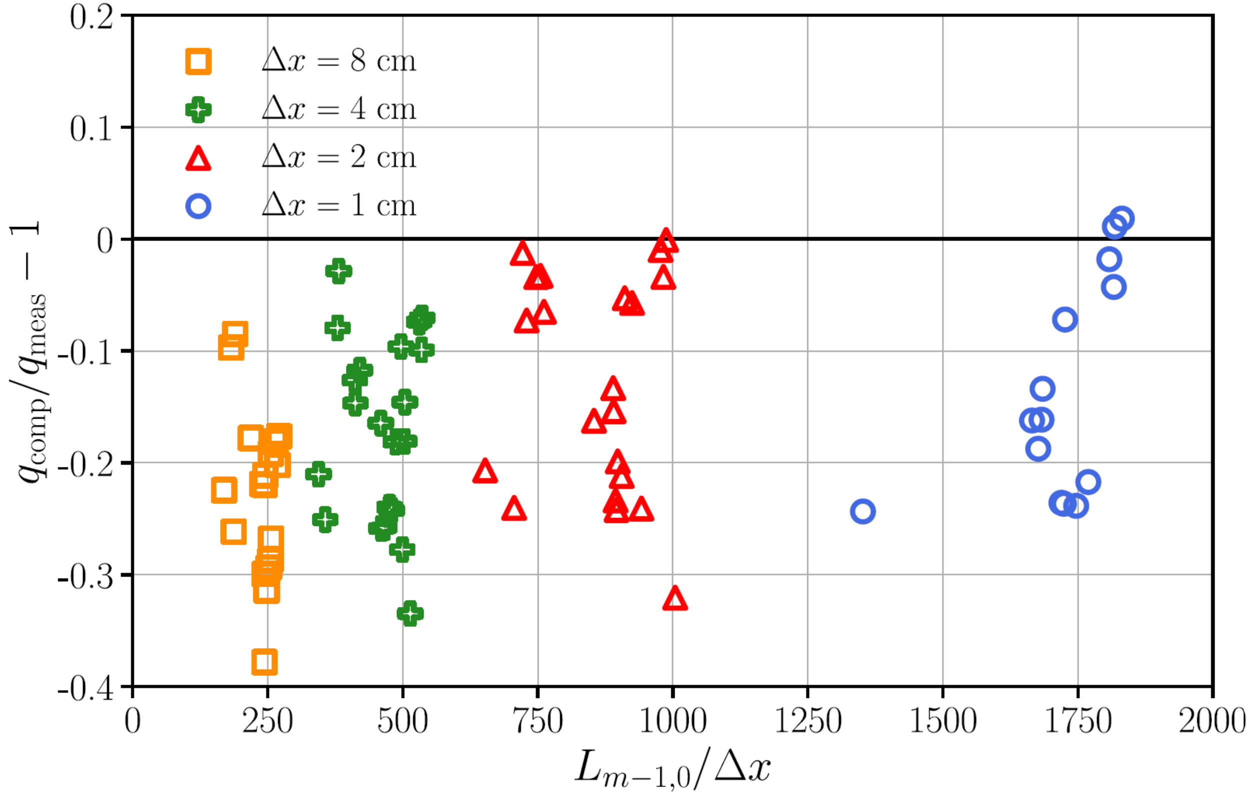
- Representative wavelength of the incident waves at the toe of the dike:

$$L_{m-1,0} = T_{m-1,0}\sqrt{g(d + H_{m0})}$$

- No instabilities when using fine grid sizes

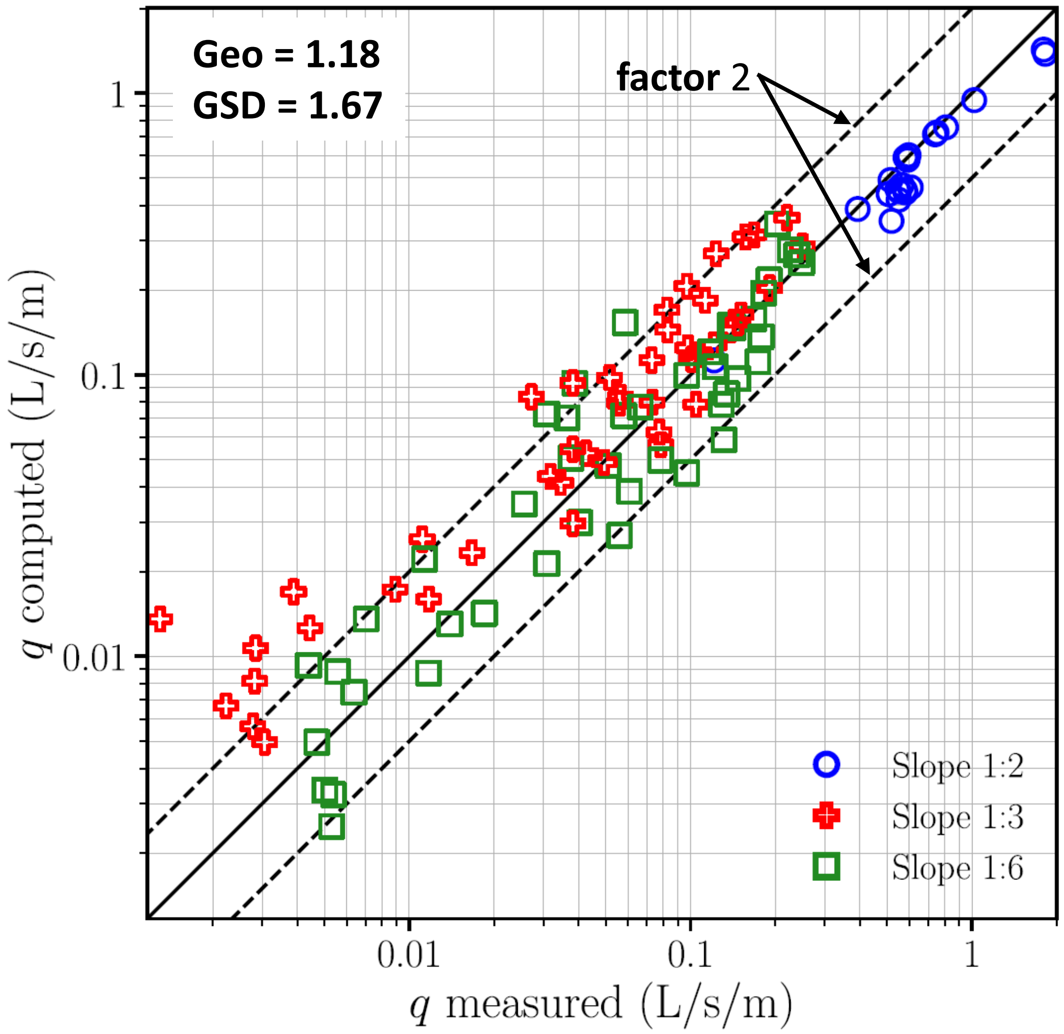
- **Computational time:**
  1000 – 1400 waves (40 min)
  CFL = 0.3
  - $\Delta x = 2$ cm: ≈ 3 hours
  - $\Delta x = 1$ cm: ≈ 13 hours

  on <u>one core</u> of an Intel Xeon 6140 processor



Relative error on the average discharge for decreasing grid sizes for the 21 cases with dike slope 1:2

Average overtopping discharges (all 111 tests – log scale)



(a) Datasets 00-025 and 13-116 (111 tests in total), plotted using a logarithmic scale. The dashed lines indicate overestimation or underestimation by a factor of 2.

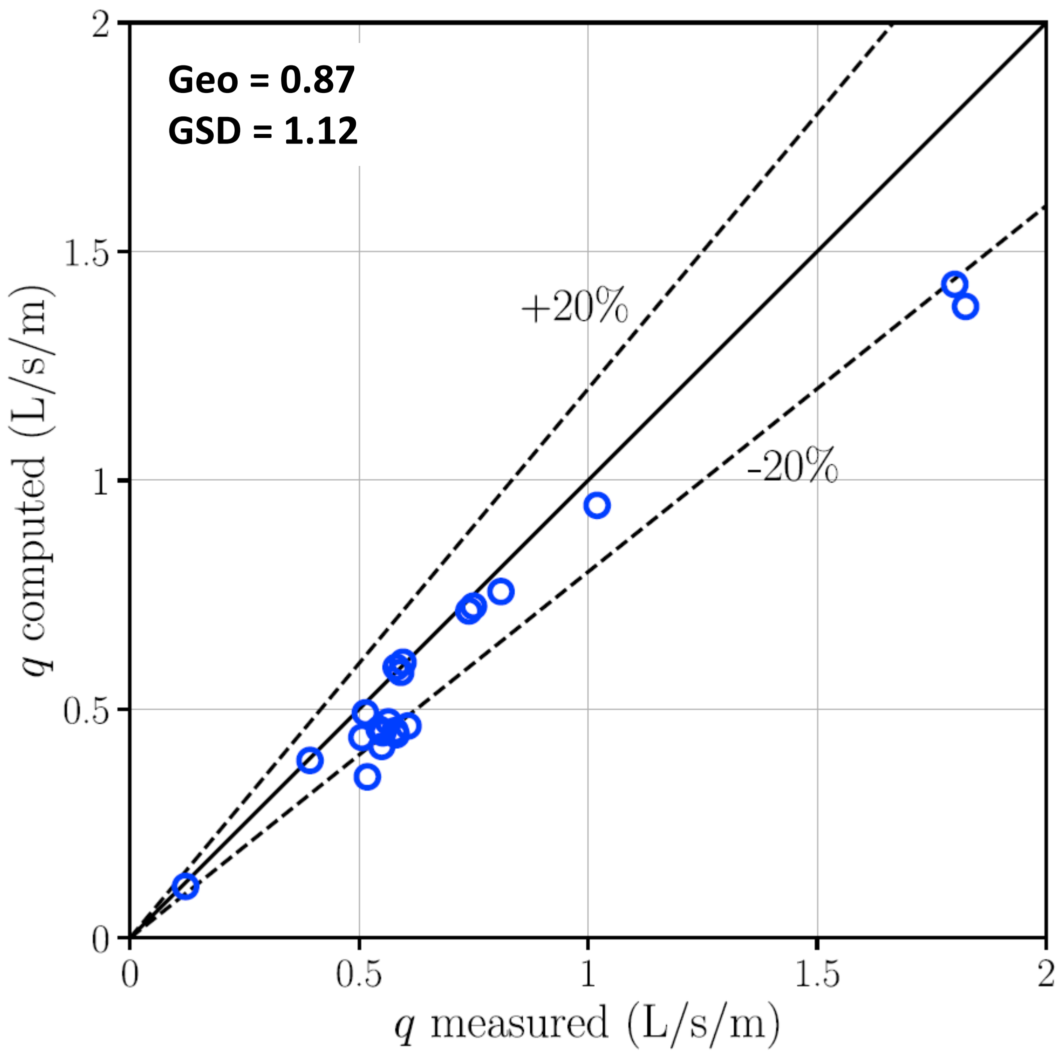
$$\mathrm{Geo} = \exp\left(\overline{\ln\left(\frac{q_{\mathrm{comp}}}{q_{\mathrm{meas}}}\right)}\right)$$

$$\mathrm{GSD} = \exp\left(\sqrt{\overline{\ln\left(\frac{q_{\mathrm{comp}}}{q_{\mathrm{meas}}}\right)^2} - \ln(\mathrm{Geo})^2}\right)$$

Dataset	N	Relative error (%)	Geo	GSD
00–025	21	13	0.87	1.12
13–116	90	67	1.27	1.72
All	111	57	1.18	1.67

- Good agreement on the average discharge over 3 orders of magnitude
- Mean absolute relative error = 57 %
- Higher scatter for lower discharges

Average overtopping discharges (21 tests of dataset 00-025 – linear scale)



(b) Dataset 00-025 only (21 tests), plotted using a linear scale. The dashed lines indicate overestimation or underestimation by 20%.

$$\mathrm{Geo} = \exp\left(\overline{\ln\left(\frac{q_{\mathrm{comp}}}{q_{\mathrm{meas}}}\right)}\right)$$

$$\mathrm{GSD} = \exp\left(\sqrt{\overline{\ln\left(\frac{q_{\mathrm{comp}}}{q_{\mathrm{meas}}}\right)^2} - \ln\left(\mathrm{Geo}\right)^2}\right)$$

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All	111	57	1.18	1.67

- For cases with slope 1:2 (O), **mean error = 13 %**

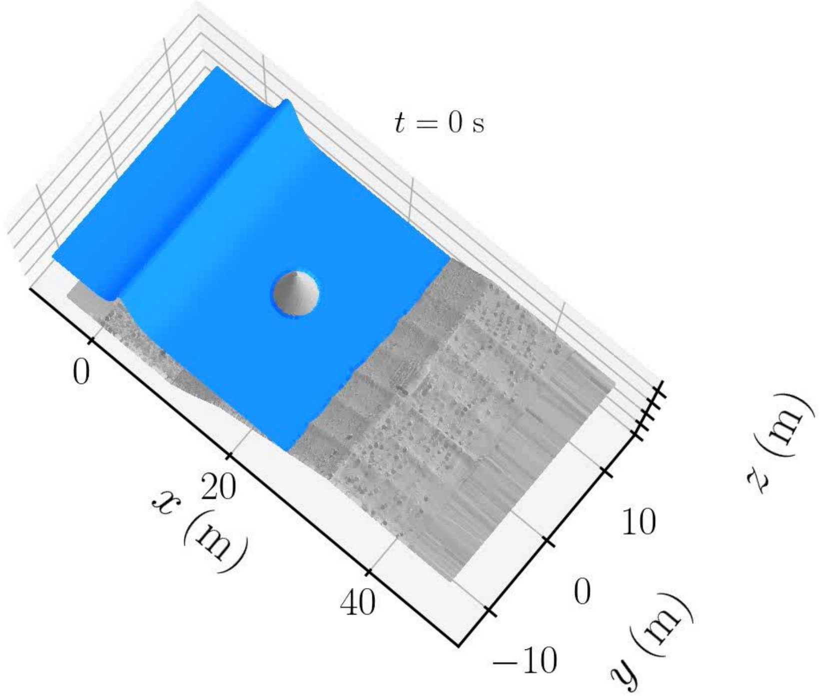
Conclusions and future work

Conclusions

- Validation of our **depth-averaged** eSGN wave model for simulating the transformations of **irregular sea states** due to **shoaling and intense breaking.**
- **Experimental validation** for the **overtopping** of **smooth dikes** by **irregular breaking waves** (111 trials with over 1000 waves each)
- **Reasonable computational time** (a few hours with one processor) → potential industrial applications

Ongoing and future work

- **Development of a 2HD model** and **experimental validation**
- **Application** to **realistic cases** in **2HD**
- Overtopping of **porous structures**
- Simulation of cases with **vertical sea-walls**



For all details and other validation cases, see:

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Numerical modelling of nearshore wave transformation, breaking and overtopping of coastal protections with the enhanced Serre–Green–Naghdi equations

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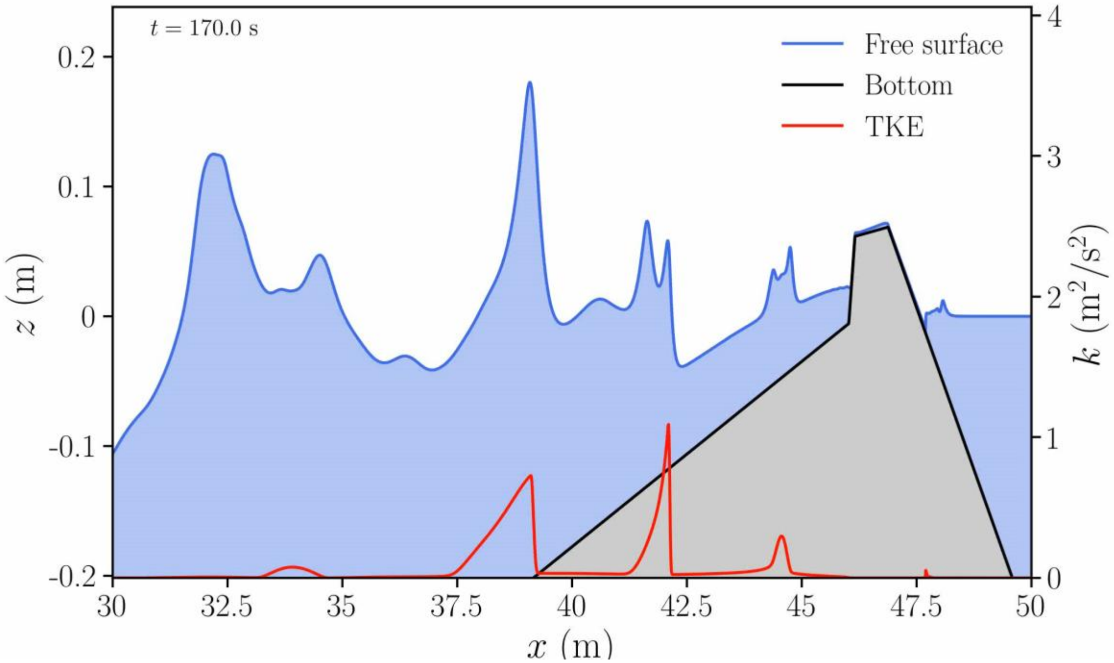


Thank you for your attention!

We thank Flanders Hydraulics and Pr. Tomohiro Suzuki (now at KU Leuven) for sharing the data from the overtopping experiments

Complete eSGN model of Bonneton *et al.* (2011)

- Continuity eq.: $h_t + q_x = 0$
- Momentum eq.: $q_t + (hu^2)_x + gh\eta_x = \Phi$
- Dispersive source term: $\Phi = \Psi + \dfrac{gh}{\alpha}\eta_x$
- Elliptic equation: $(I + \alpha\mathcal{T})\Psi = -\dfrac{gh}{\alpha}\eta_x - Q$

$$Q(u) = 2h^2h_xu_x^2 + \tfrac{4}{3}h^3u_xu_{xx} - h^2d_xu_x^2 - h^2d_{xx}uu_x - \left[h_xd_{xx} + \tfrac{1}{2}hd_{xxx} - d_xd_{xx}\right]hu^2$$
$$\mathcal{T}(v) = \left[\frac{1}{3}(h_x^2 + hh_{xx}) + d_x^2 - h_xd_x - \frac{1}{2}hd_{xx}\right]v - \tfrac{1}{3}hh_xv_x - \frac{1}{3}h^2v_{xx}$$

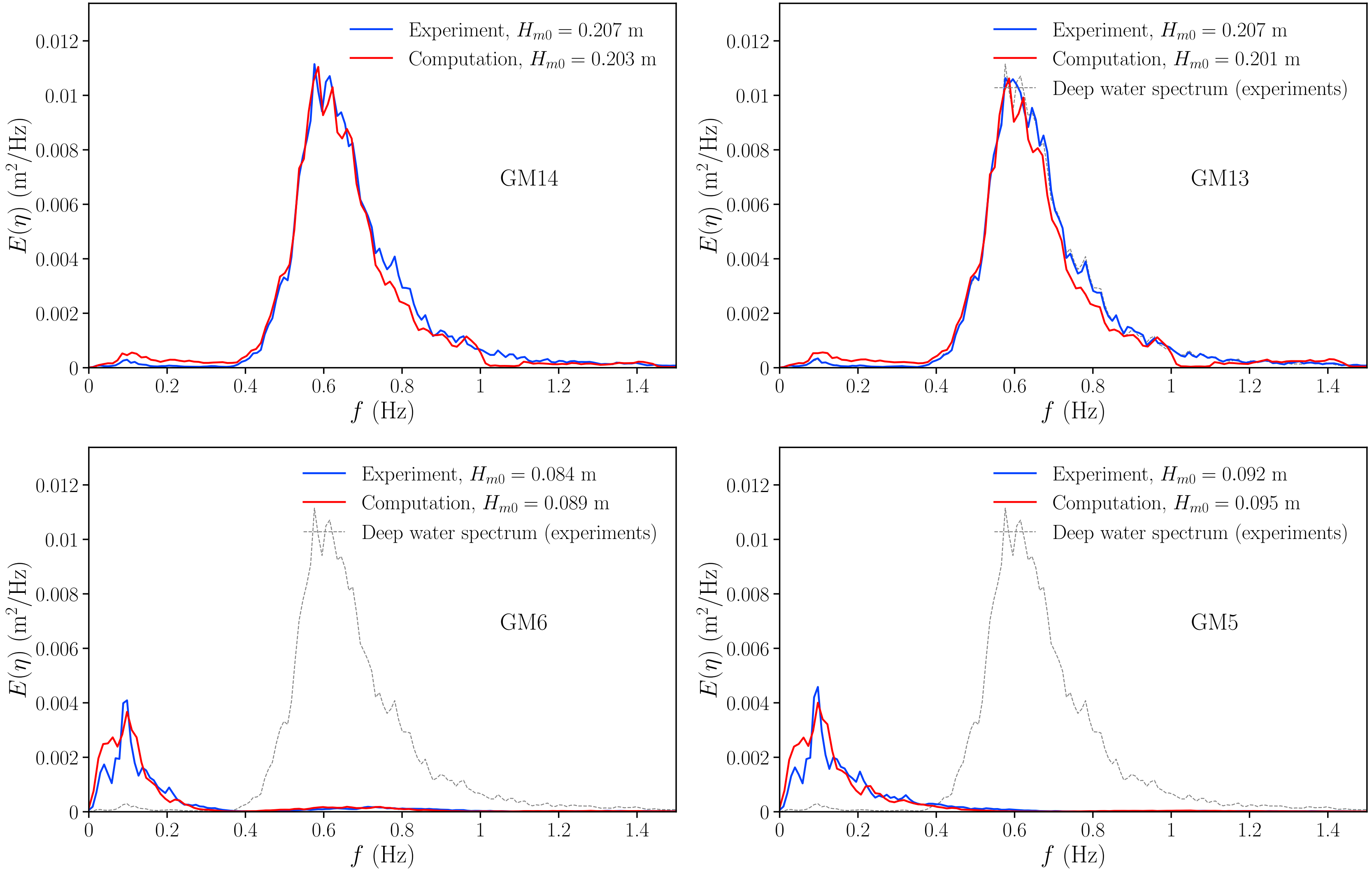
Wave breaking model

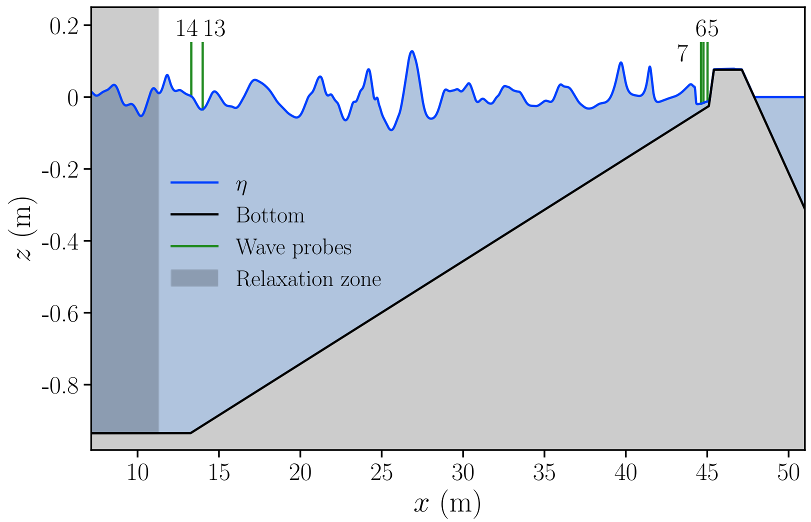
- Turbulent viscosity: $\nu_T = C_\nu \sqrt{k} l_T, \; l_T = \kappa h$

- Transport equation for the TKE k: $(hk)_t + (huk)_x = h\nu_T k_{xx} + B\dfrac{hl_T^2}{\sqrt{C_\nu^3}}|u_z(z=\eta)|^3 - hC_\nu^3 \dfrac{k^{3/2}}{l_T}$

Appendix – Total wave spectra for one case with slope 1/3

LABORATOIRE D'HYDRAULIQUE SAINT-VENANT



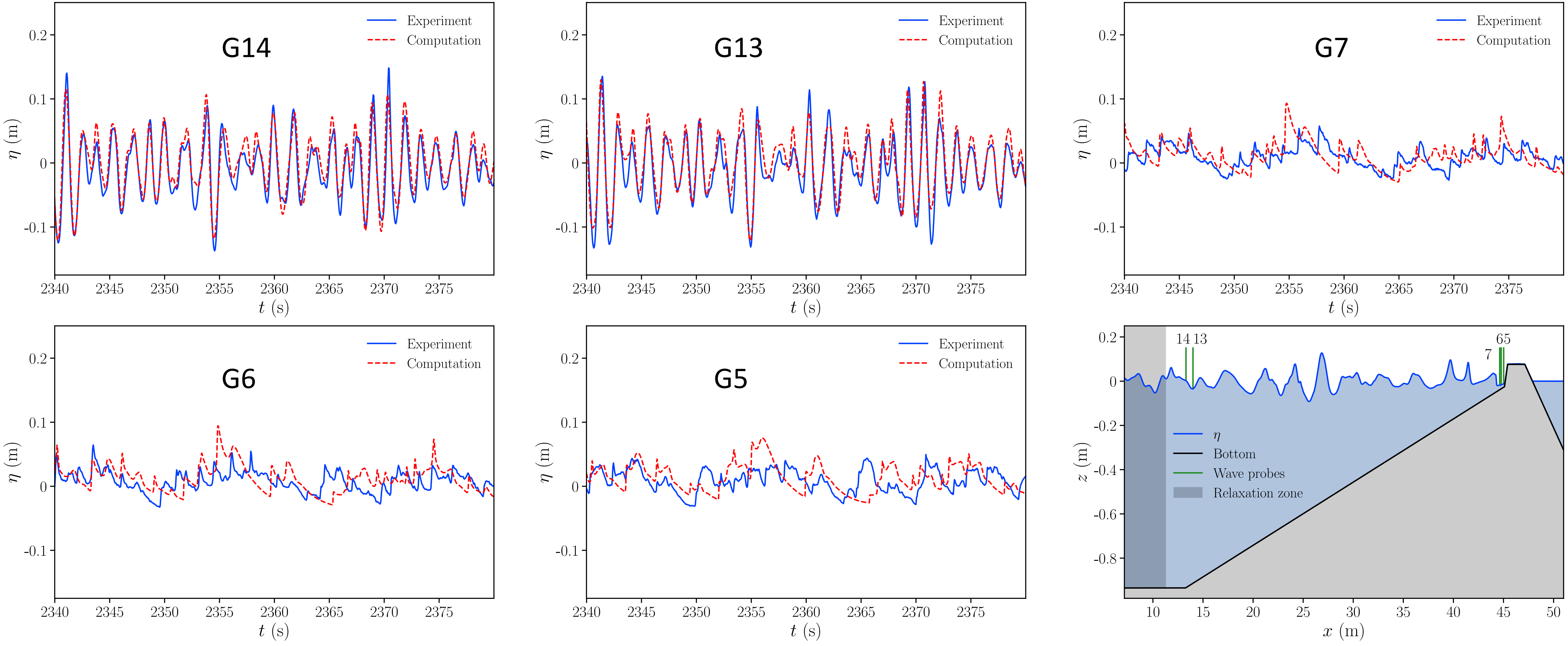


Total wave spectra in the experiment and the simulation in 5 locations

Slope 1/3, water depth = 0.935 m, H_{m0} = 0.201 m, T_p = 1.7 s

Position of the gauges

Appendix – Free surface profiles for one case with slope 1/3



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