Machine learning approximation for nonlinear wave-wave interactions in spectral wave models

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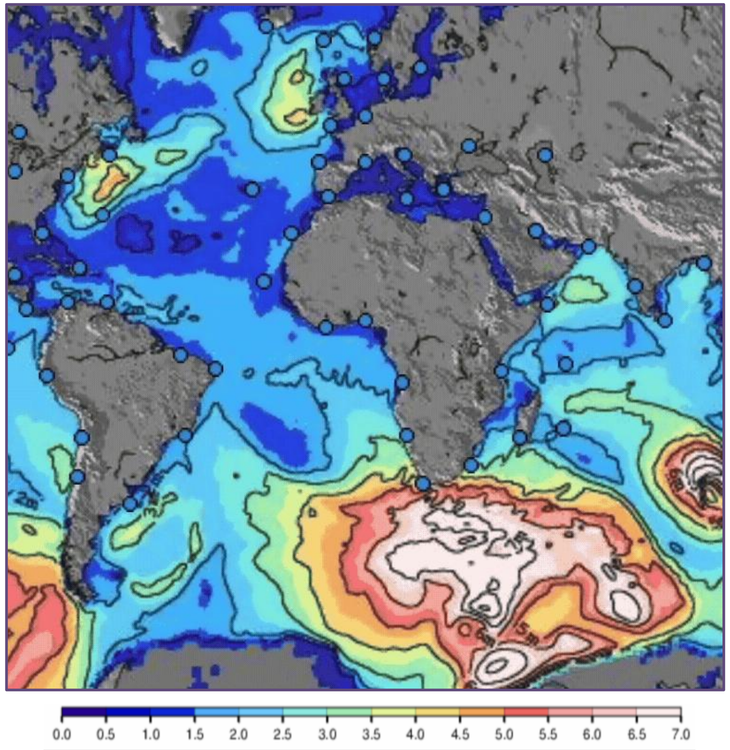
Ocean wave models

Spectral wave models can be used to simulate or forecast wave conditions, providing operational or design guidance

Statistical wave information



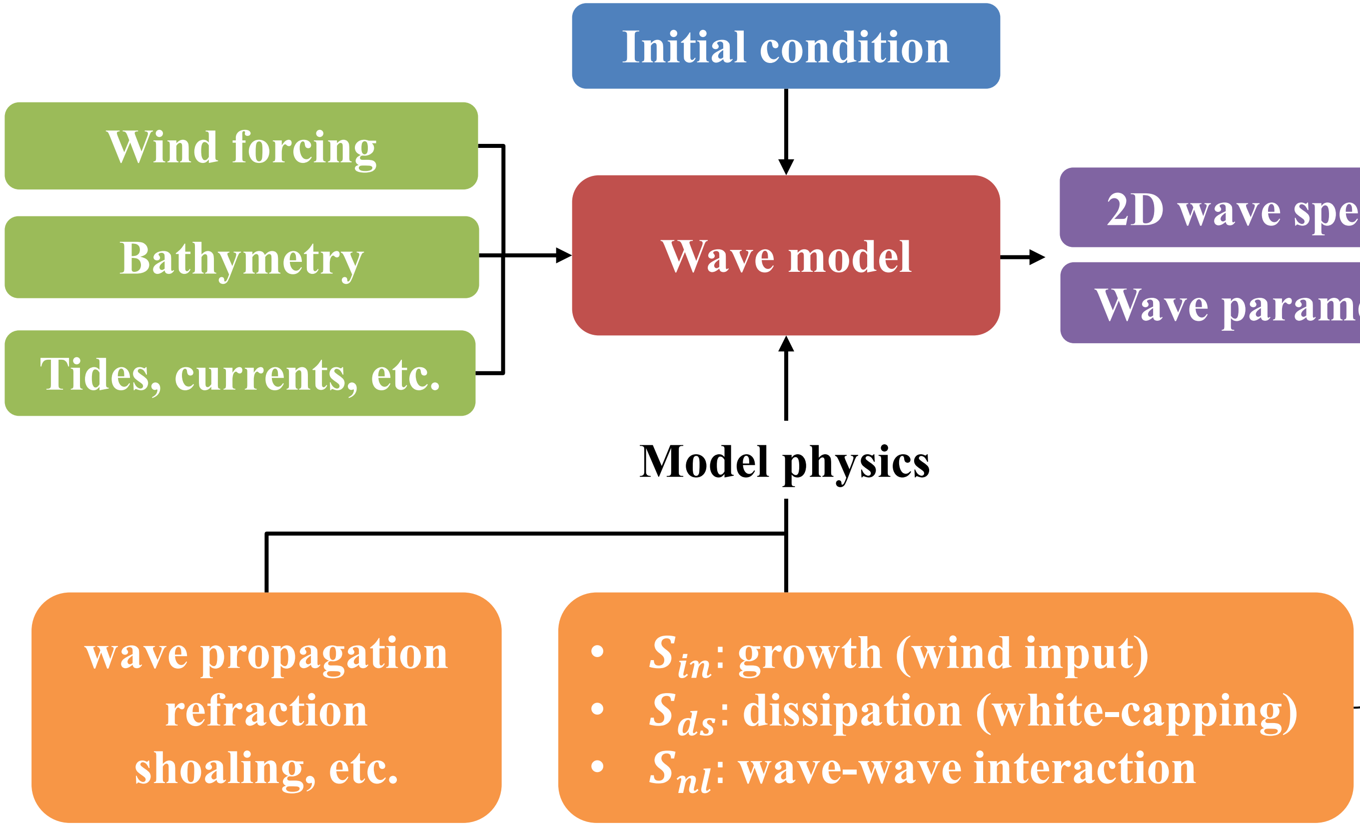
Significant wave height [m]

- Safe installation/maintenance
- Optimal ship routing

- Coastal protection
- Offshore structure design

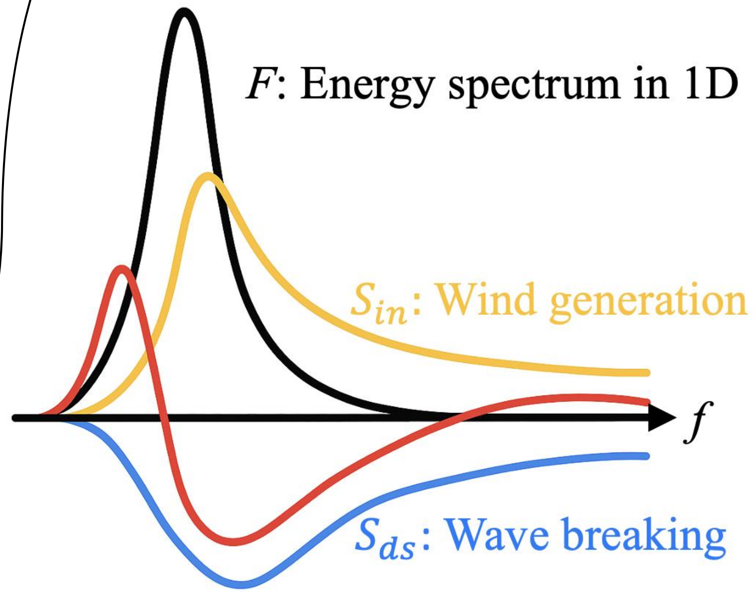
- Recreational activities

Ocean wave models



Energy balance equation:

$$\frac{\partial F}{\partial t} = S_{in} + S_{ds} + S_{nl} + \cdots$$



S_{nl}: nonlinear wave-wave interaction
(redistribute energy, stabilize wave growth)

Nonlinear wave-wave interactions

Resonant exchange of energy, momentum and action among 4 spectral components (K. Hasselmann, 1962)

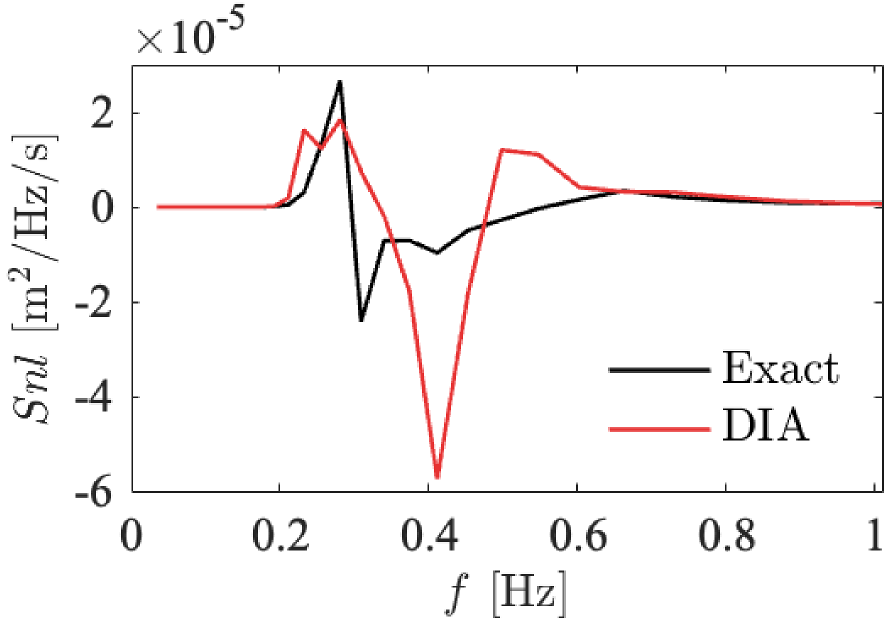
$$\frac{\partial n_1}{\partial t} = \iiint G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)\big(n_1 n_3 (n_4 + n_2) - n_2 n_4 (n_1 + n_3)\big)\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)\delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)\, d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4$$

- Very time consuming, 6D integral (~10^4 times than other parts)
- Not feasible in operational wave model

Current operational models use simplified approximation:

Discrete Interaction Approximation (DIA) (K. Hasselmann, 1985)

- Fast but introduces many deficiencies

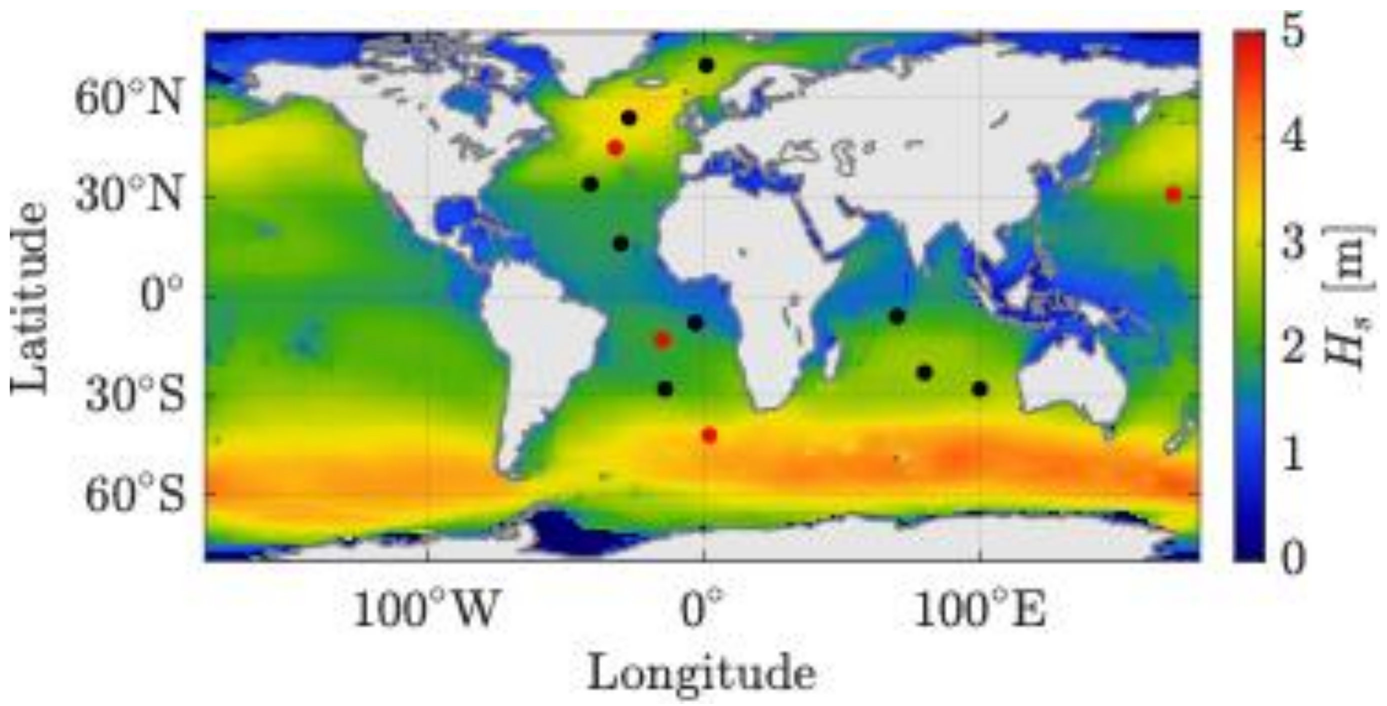


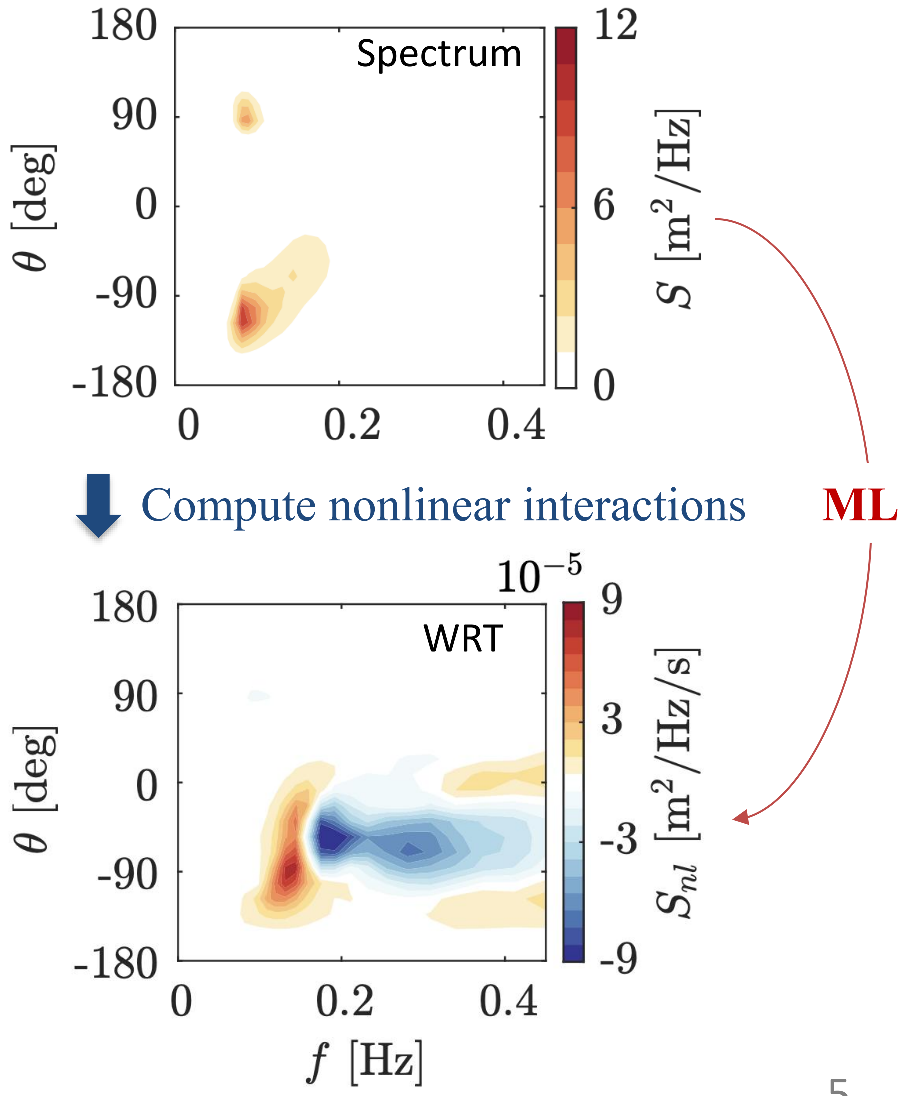
1. Computationally efficient

2. Generalizes well for arbitrary wave spectra

3. Integrates stably into wave models

ERA5 reanalysis data





- **Training data:** 8 years, 2016–2023 (210,384 spectra)
- **Testing data:** 1 year in 2024 (11,712 spectra)

Method

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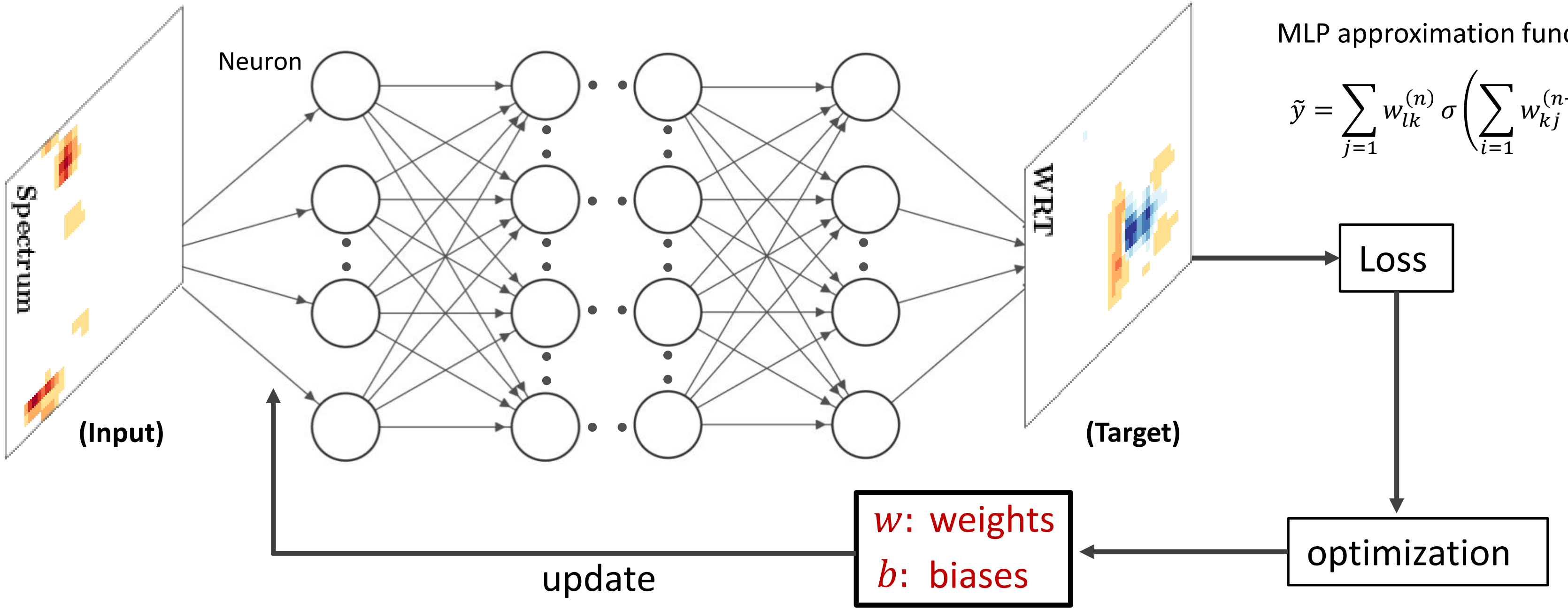
Multilayer perceptron (MLP)

- 2001 • Krasnopolsky et al.: MLP based on separable mathematical basis functions
- 2005 • Tolman: MLP based on Empirical Orthogonal Functions
- 2008, 2009 • Tolman/ Krasnopolsky et al.: Quality control mechanism
- 2009, 2014 • Wahle et al./ Puscasa et al.: MLP direct mapping



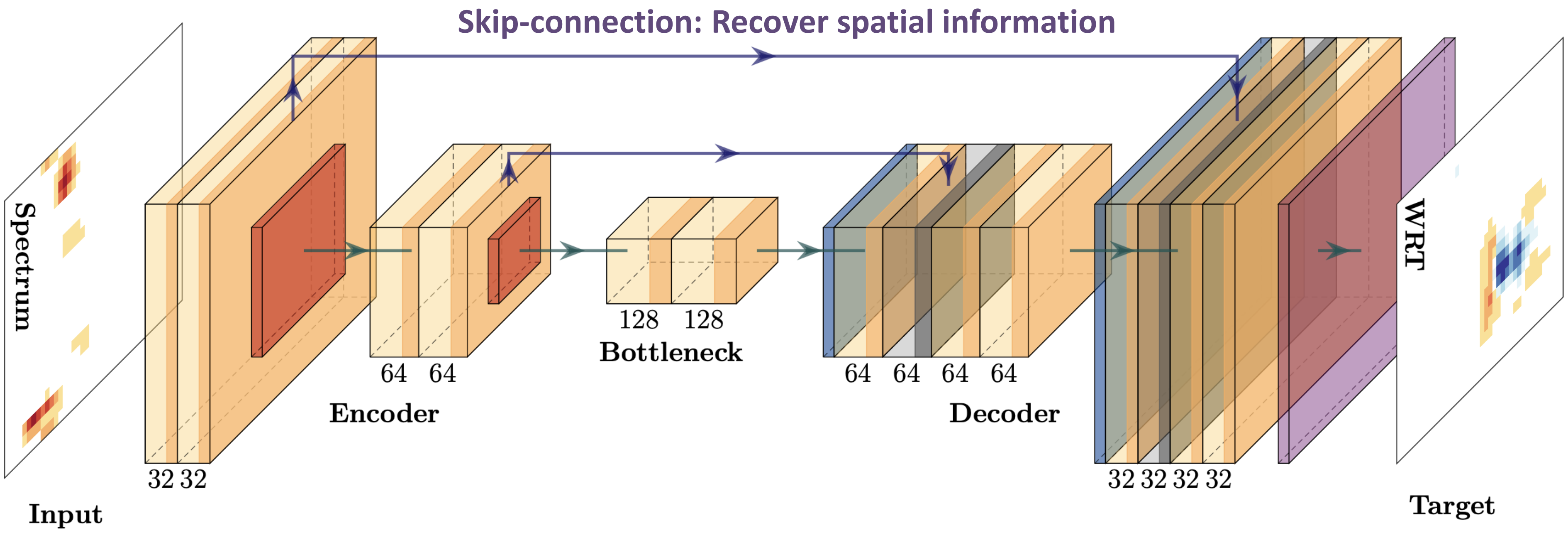
MLP approximation function:

$$\tilde{y} = \sum_{j=1} w_{lk}^{(n)} \sigma\left(\sum_{i=1} w_{kj}^{(n-1)} \sigma\left(\dots\sigma\left(\sum_{i=1} w_{ji}^{(1)} x_i + b_i^{(1)}\right)\dots\right) + b_k^{(n-1)}\right) + b_l^{(n)}$$

- **Limited generalization**
- **Unstable wave growth**

Method

Fully convolutional encoder–decoder architecture (U-Net)



- convolution
  - detect input features
- maxpooling
  - captures abstract info
- up-convolution
  - up-samples feature maps

Quantile-based Loss function for 95% uncertainty:

predictions uncertainty

$$\mathcal{L}(y, \tilde{y}, v) = \rho_{0.025}(y, \tilde{y} - v) + \rho_{0.975}(y, \tilde{y} + v)$$

$$\rho_q(y, \hat{y}) = \begin{cases} q \cdot (y - \hat{y}) & \text{if } y \geq \hat{y}, \\ (q-1) \cdot (y - \hat{y}) & \text{if } y < \hat{y}, \end{cases}$$

Method

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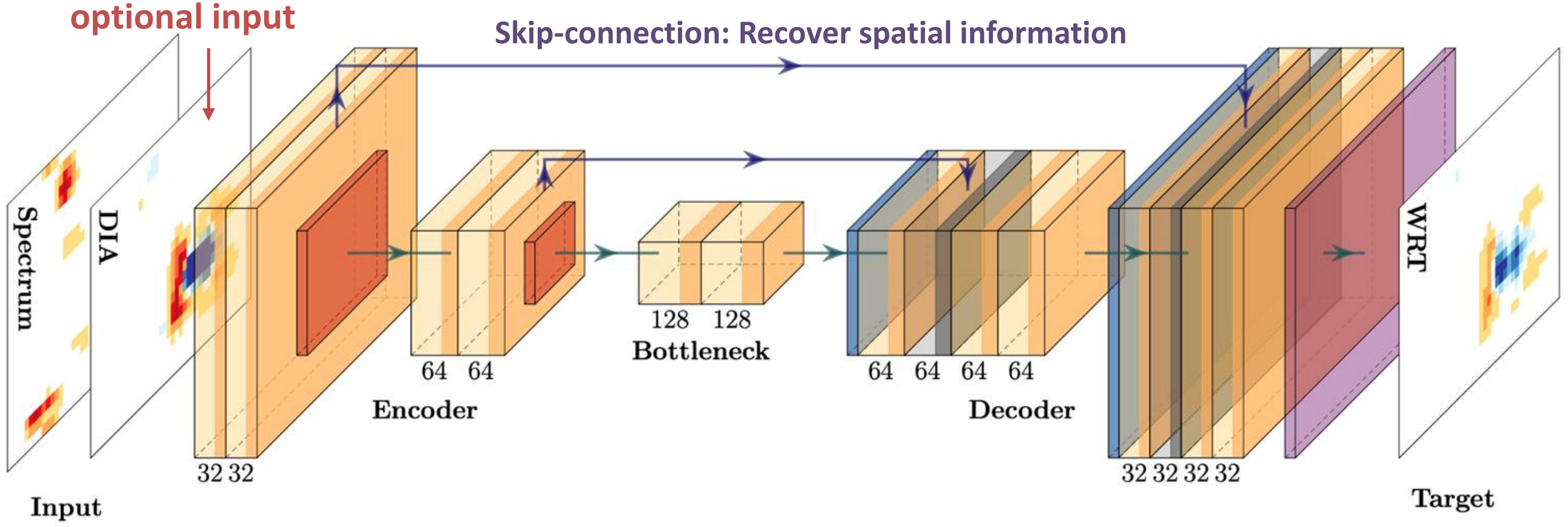

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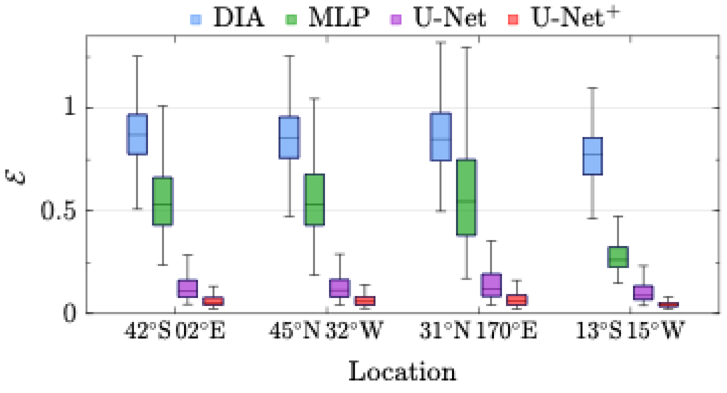
ERA5 test cases

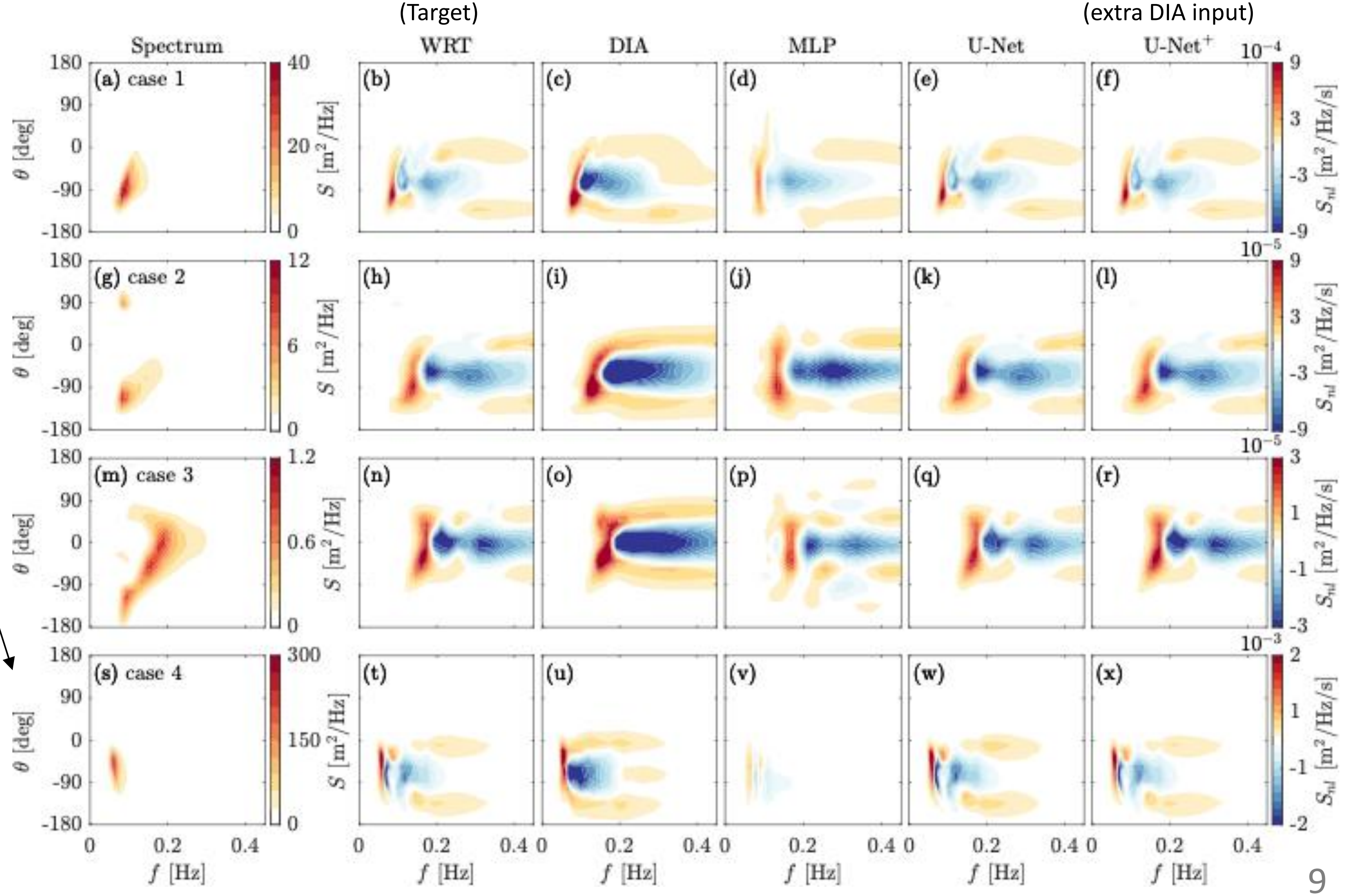
Method	DIA	MLP	U-Net	U-Net^{+}	WRT
Relative time	1.0	2.0	4.3	5.5	102.3

1. Single modal spectrum
2. Bi-modal spectrum
3. Multi-modal spectrum
4. Extreme wave conditions ($H_s = 11.5$ m)

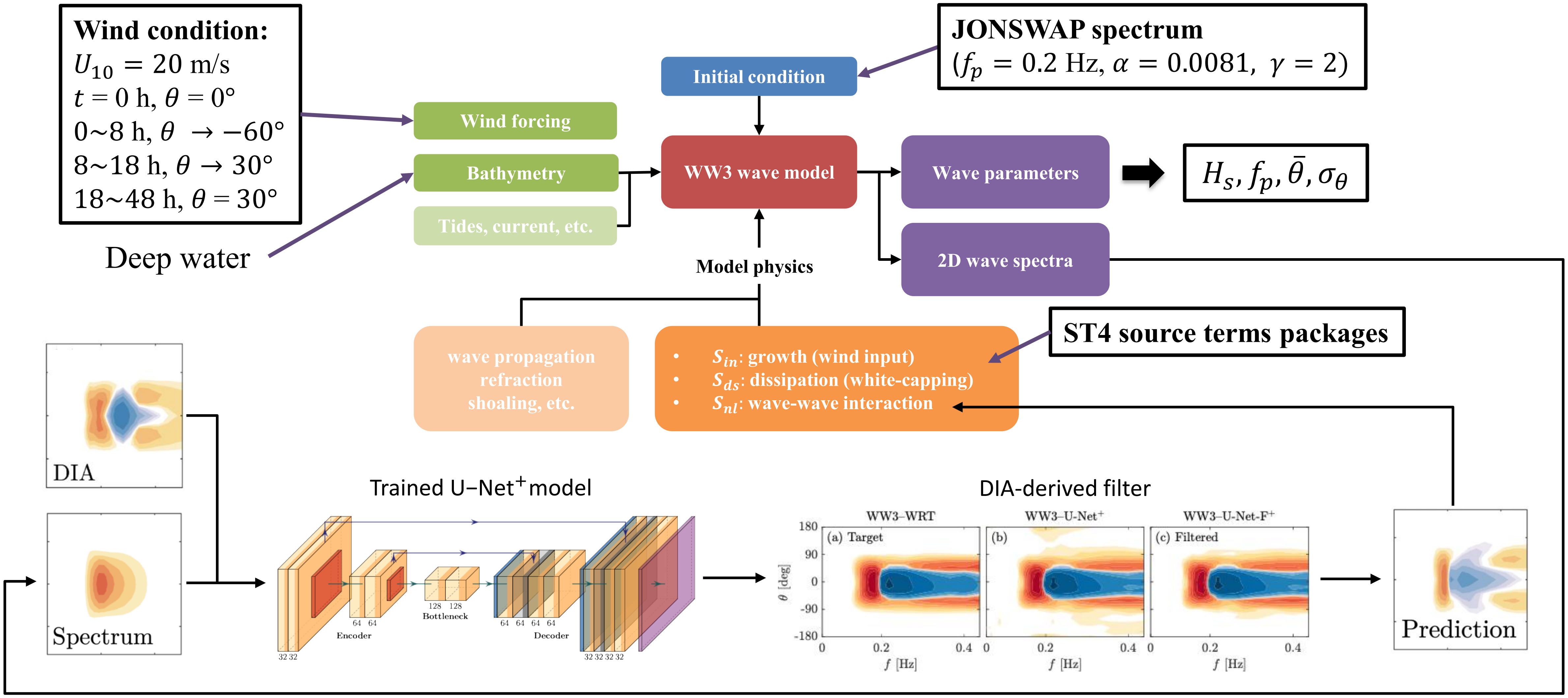
rare extreme event:

159 of 210,384 training cases for $H_s \geq 10$ m (0.08%)



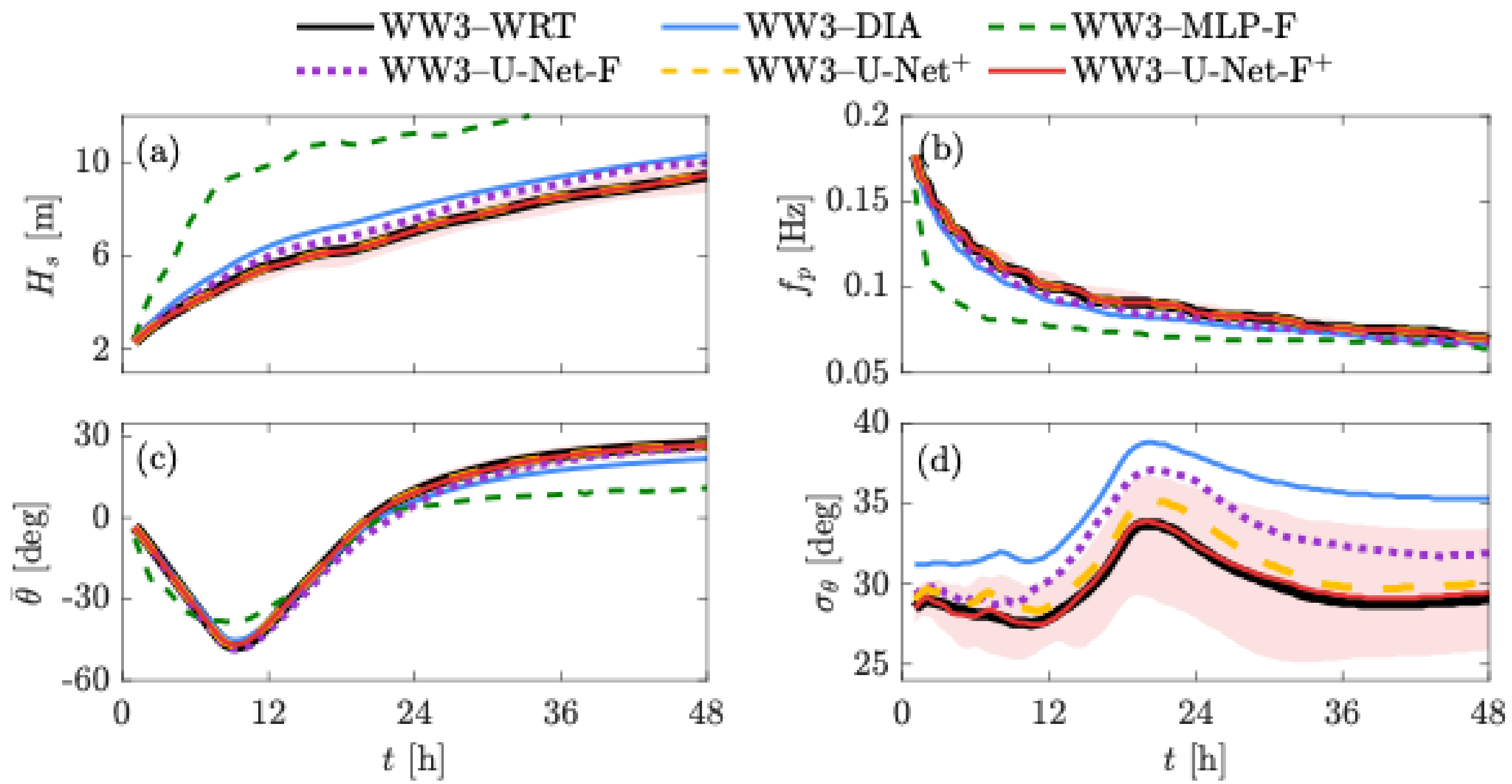


Wave growth test cases

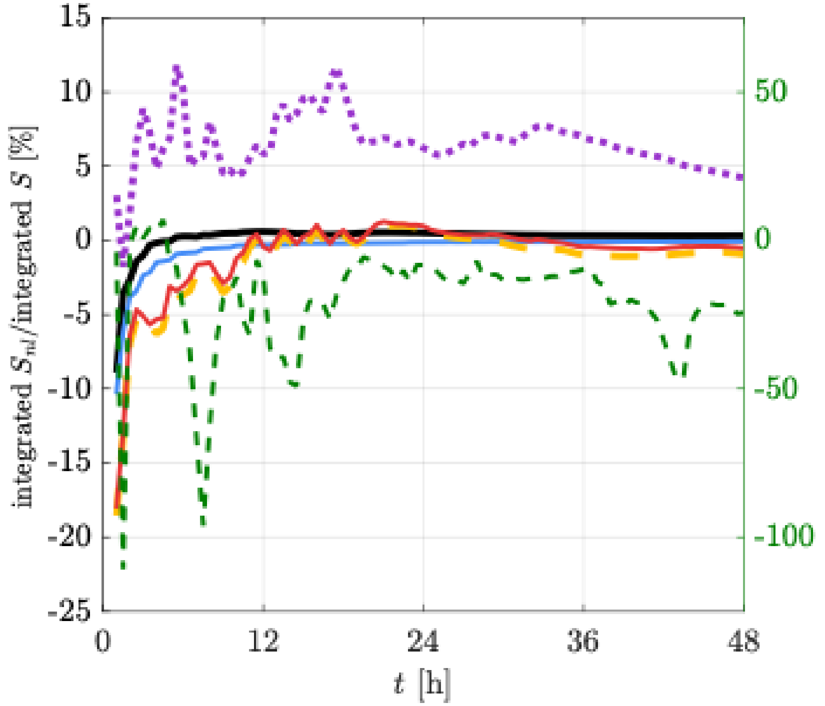


Wave growth test cases

Time evolution of key wave parameters over a 48-hour wind-forced simulation



Energy conservation



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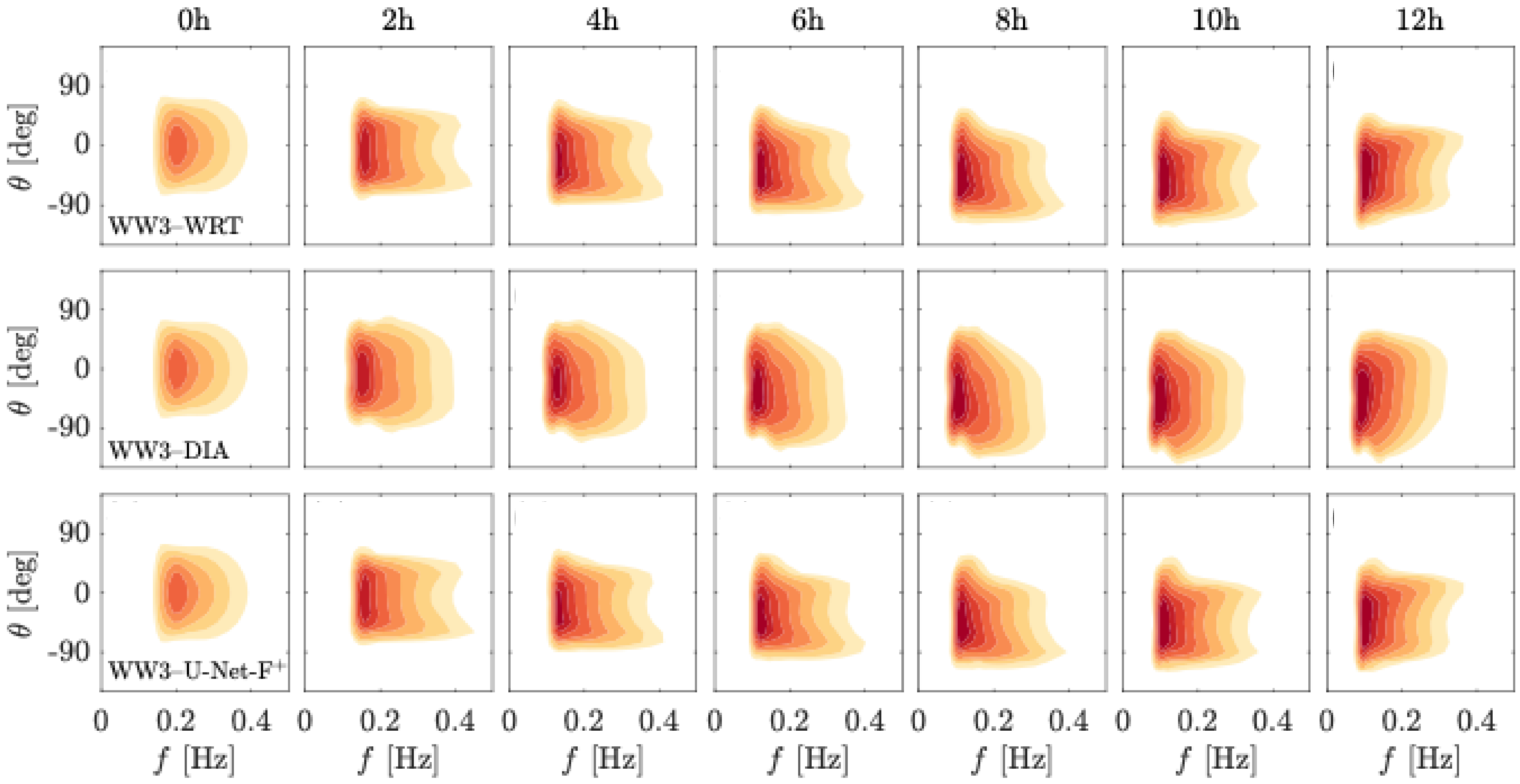

Summary

- The proposed ML model achieves **16 × higher accuracy** than DIA,
  up to **20 × faster** than WRT.

- The proposed model demonstrates excellent agreement for both test cases:
  (1) Generalize well for the ERA5 test dataset with unseen geographic locations
  (2) Stable wave growth for the model integration scheme

Limitations/ Future work:

- Extend to shallow water conditions/ Global simulation
- Limited to the fixed frequency–direction resolution
- Model interpretability

Thank you!

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