Energy imbalance in random wave fields

Paolo Pezzutto

CNR-IRBIM

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  - **RANDOM**: ~~gaussian~~ $\rightarrow$ i.i.d. phases @$t = 0$

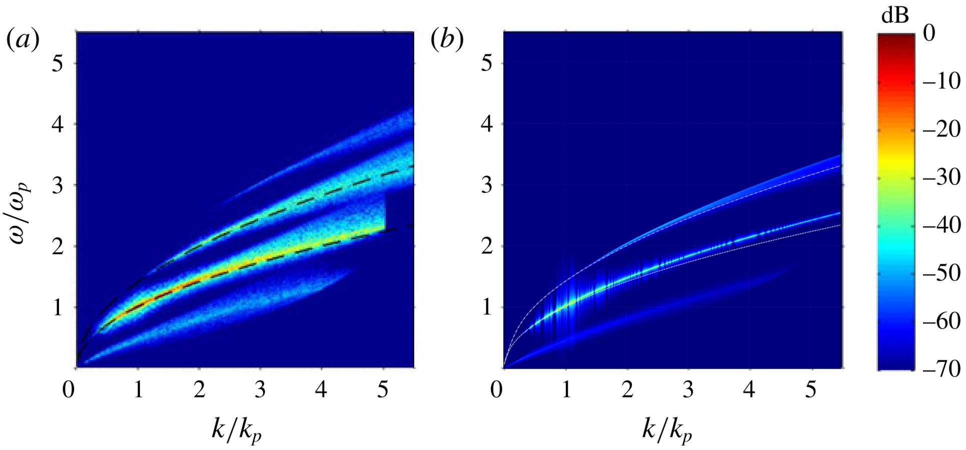
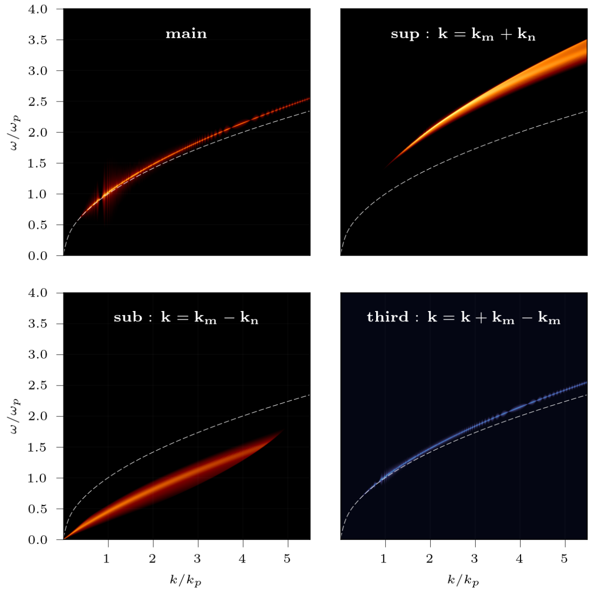
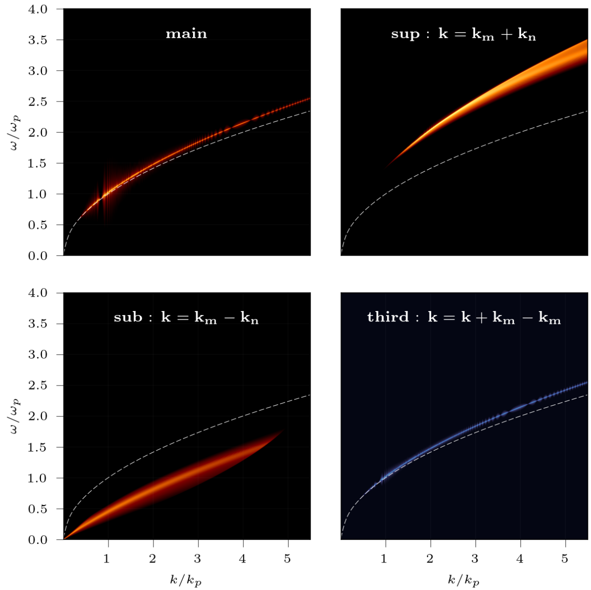


Figure: (a) Taklo et al. 2015, Zakharov - (b) Zakharov, “short-time” (RP)


main
sup : k = k_m + k_n
sub : k = k_m − k_n
third : k = k + k_m − k_m
ω/ω_p
k/k_p
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1
2
3
4
5



expected variance

$$\langle \eta^2 \rangle_\theta = \sum_{k_m} I_m + O(I^3)$$

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On some consequences of the canonical transformation in the Hamiltonian theory of water waves

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expected moment

$$\langle \mu_{r,s} \rangle_\theta = \sum_m k_m^r \, |\omega_m|^s \, I_m + \sum_{m,n} B_{m,n}^{(r,s)} I_m I_n$$

expected moment

$$\langle \mu_{r,s} \rangle_\theta = \sum_m k_m^r \, |\omega_m|^s \, I_m + \sum_{m,n} B_{m,n}^{(r,s)} I_m I_n$$

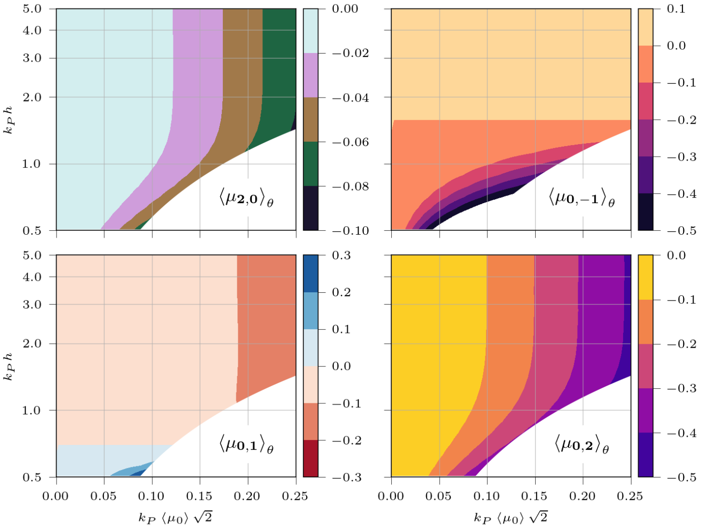

Figure: relative error (%) of linear *w.r.t.* $\langle \mu \rangle_\theta$

Example with 1D, JONSWAP "baseline"

variance

$$\langle \eta^2 \rangle_\theta = \sum_{k_m} I_m + O(I^3)$$

variance

$$\langle \eta^2 \rangle_\theta = \sum_{\mathbf{k}_m} I_m + O(I^3)$$

potential energy density

$$\langle \mathcal{P} \rangle_\theta = \frac{1}{2}\rho g \langle \eta^2 \rangle_\theta$$

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potential energy density

$$\langle \mathcal{P} \rangle_\theta = \frac{1}{2} \rho g \langle \eta^2 \rangle_\theta$$

kinetic energy density

$$\langle \mathcal{K} \rangle_\theta = \langle \mathcal{P} \rangle_\theta + \left(\rho g^2 \sum_{\mathbf{k}_m, \mathbf{k}_n} \frac{T^{m,n}_{m,n}}{\sigma_m \sigma_n} I_m I_n \right) + \frac{1}{2} \rho U^2 + \rho g \mathbf{U} \cdot \left(\sum_{\mathbf{k}_m} \frac{\mathbf{k}_m}{\sigma_m} I_m \right)$$

energy "imbalance"?

$$\langle \mathcal{K} \rangle_\theta - \langle \mathcal{P} \rangle_\theta = 2\rho g^2 \sum_{k_m, k_n} \frac{T_{m,n}^{m,n}}{\sigma_m \sigma_n} I_m I_n$$

Figure: relative surplus (%) of kinetic energy density *w.r.t.* potential energy density

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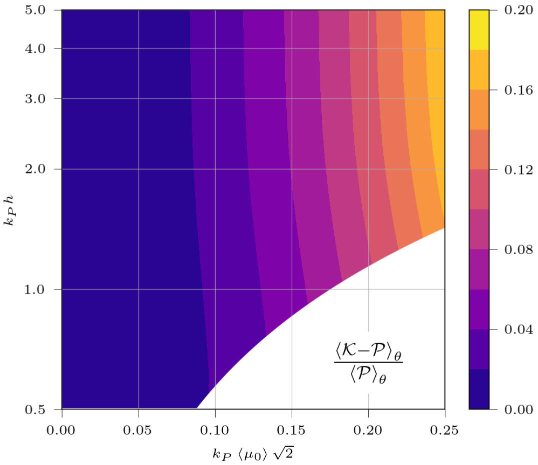

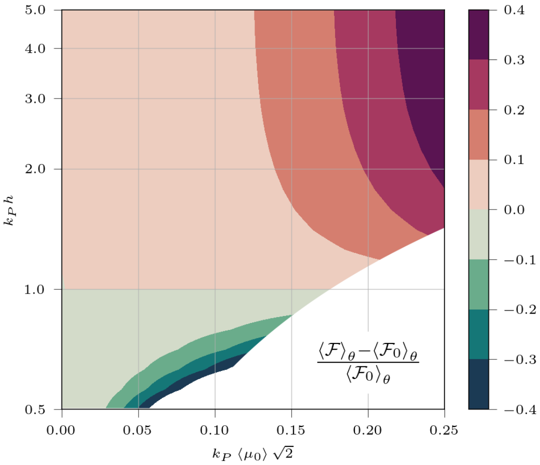

wave energy flux

$$\langle \mathcal{F} \rangle_\theta = \rho g \sum_m c_{g,m} I_m + \sum_{m,n} \mathscr{F}_{m,n} I_m I_n$$

Figure: relative surplus (%) of wave energy flux *w.r.t.* the "linear" value

Example with 1D, JONSWAP "baseline"