Subgrid Correction of Storm Surge Modeling in Orthogonal Curvilinear Coordinates

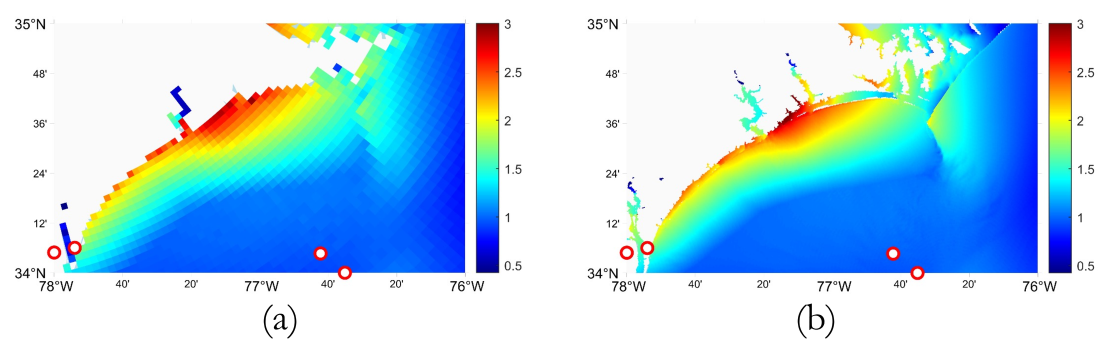
Haoran Sun[1]*, Damrongsak Wirasaet[1], Yuepeng Li[2], Andrew Kennedy[1], Diogo Bolster[1]
[1]University of Notre Dame, Civil and Environmental Engineering and Earth Sciences
[2]Florida International University, International Hurricane Research Center

Contact: hsun4@nd.edu

Introduction

Background

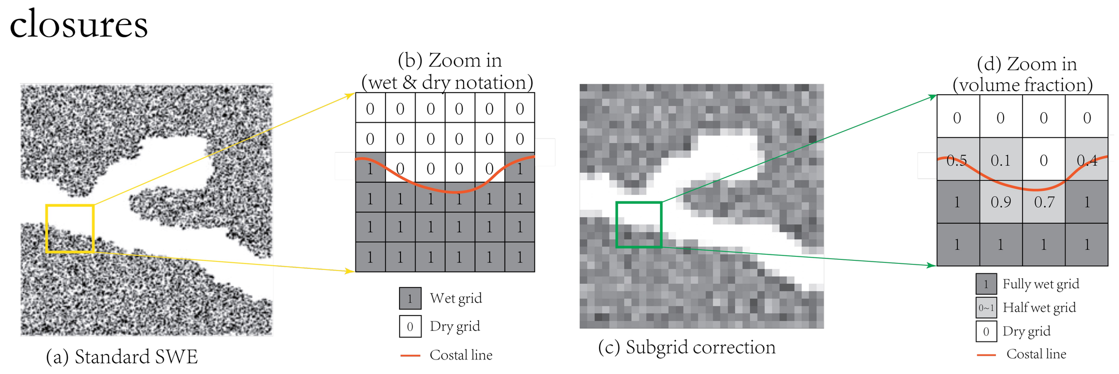
Detailed resolution of flow pathways and barriers is critical for storm surge modeling, however, resolution often comes with significant computational costs for numerical models, posing challenges within restricted forecast runtime windows.



Maximum Surface Elevation for Hurricane Florence 2018 at Onslow Bay, NC (a) Coarse grid (~4.3 km) (b) Fine grid simulation (~0.5 km)

Subgrid Correction[1]

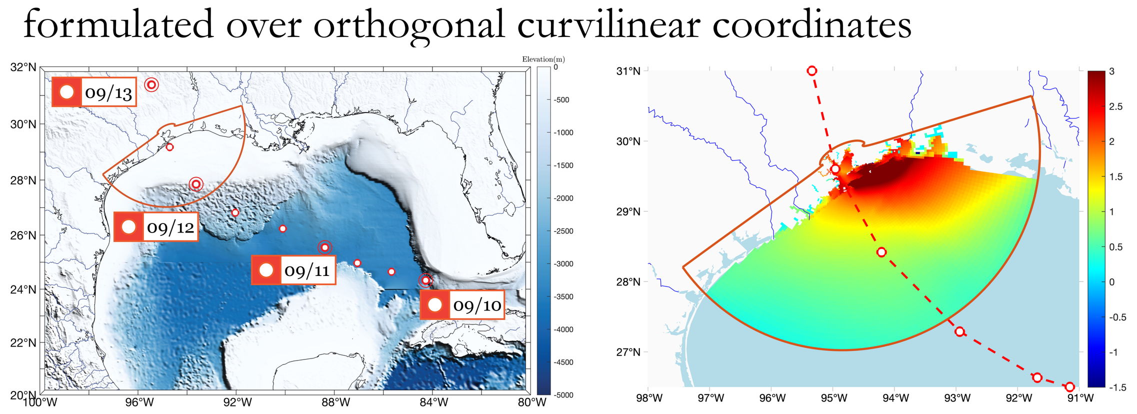
Subgrid approaches offer a means to integrate high-resolution information into coarse-grid simulations using a variety of closures



Wetting and drying methods (a) Standard method (c) Subgrid method

Coastal and Estuarine Storm Tide (CEST)[2]

The CEST surge model based on shallow water equations formulated over orthogonal curvilinear coordinates



Example case for Ike (a) Hurricane setup (b) Simulation result for maximum surface elevation

Governing equations[1]

Upscaled form of 2D non-conservative shallow water equations in orthogonal curvilinear coordinates become

- Mass equation

$$\frac{\partial H}{\partial t}+\frac{1}{h_1h_2}\left[\frac{\partial(Hh_2u)}{\partial q_1}+\frac{\partial(Hh_1v)}{\partial q_2}\right]=0$$

- Momentum equations

$$\frac{\partial u}{\partial t}+\frac{1}{Hh_1h_2}\left(\frac{\partial Hh_2uu}{\partial q_1}+\frac{\partial Hh_1uv}{\partial q_2}-u\frac{\partial Hh_2u}{\partial q_1}-u\frac{\partial Hh_1v}{\partial q_2}\right)=\frac{1}{h_1h_2}\left(v^2\frac{\partial h_2}{\partial q_1}-uv\frac{\partial h_1}{\partial q_2}\right)-\frac{g}{h_1}\frac{\partial}{\partial q_1}\left(\eta+\frac{\Delta P_a}{\rho}\right)+fv-\frac{\phi\tau_B^{q_1}}{H}+\frac{\phi\tau_W^{q_1}}{\rho H}+\frac{\nu}{h_1^2}\frac{\partial^2u}{\partial q_1^2}+\frac{\nu}{h_2^2}\frac{\partial^2u}{\partial q_2^2}$$

$$\frac{\partial v}{\partial t}+\frac{1}{Hh_1h_2}\left(\frac{\partial Hh_2uv}{\partial q_1}+\frac{\partial Hh_1v^2}{\partial q_2}-h_2v\frac{\partial Hu}{\partial q_1}-h_1v\frac{\partial Hv}{\partial q_2}\right)=\frac{1}{h_1h_2}\left(u^2\frac{\partial h_1}{\partial q_2}-uv\frac{\partial h_2}{\partial q_1}\right)-\frac{g}{h_2}\frac{\partial}{\partial q_2}\left(\eta+\frac{\Delta P_a}{\rho}\right)-fu-\frac{\phi\tau_B^{q_2}}{H}+\frac{\phi\tau_W^{q_2}}{\rho H}+\frac{\nu}{h_1^2}\frac{\partial^2v}{\partial q_1^2}+\frac{\nu}{h_2^2}\frac{\partial^2v}{\partial q_2^2}$$

Subgrid Correction for SWE[1]

- Wet fraction

$$\phi(\eta)=\frac{|\Omega_W|}{|\Omega|},\quad \Omega_W=\{q'\in\Omega\mid\eta+b(q')>0\}$$

- Water height coefficient

$$H(q,t)=\frac{1}{|\Omega|}\int_\Omega \max[0,\eta+b(q')]dq'$$

- Bottom friction coefficient

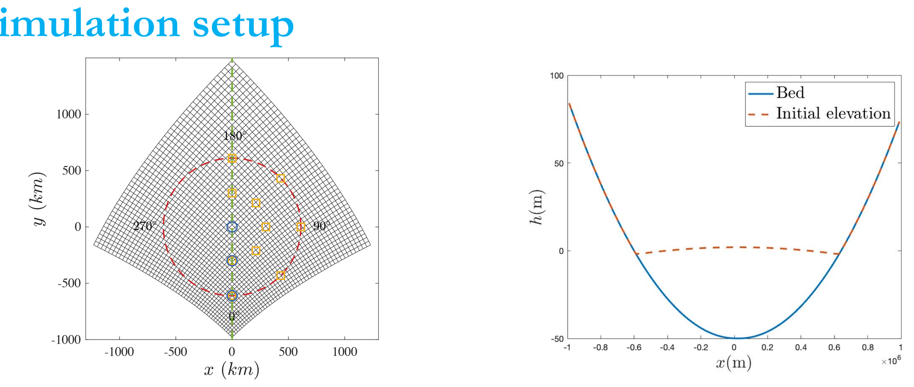
$$c_{M,f}=\frac{1}{|\Omega_W|}\frac{\left[\int_\Omega \max(0,\eta+b(q'))dq'\right]^3}{\left[\int_\Omega \max(0,\eta+b(q'))^{\frac{3}{2}}\sqrt{1/c_f}dq'\right]^2}$$

Test Cases and Results

Parabolic Bowl

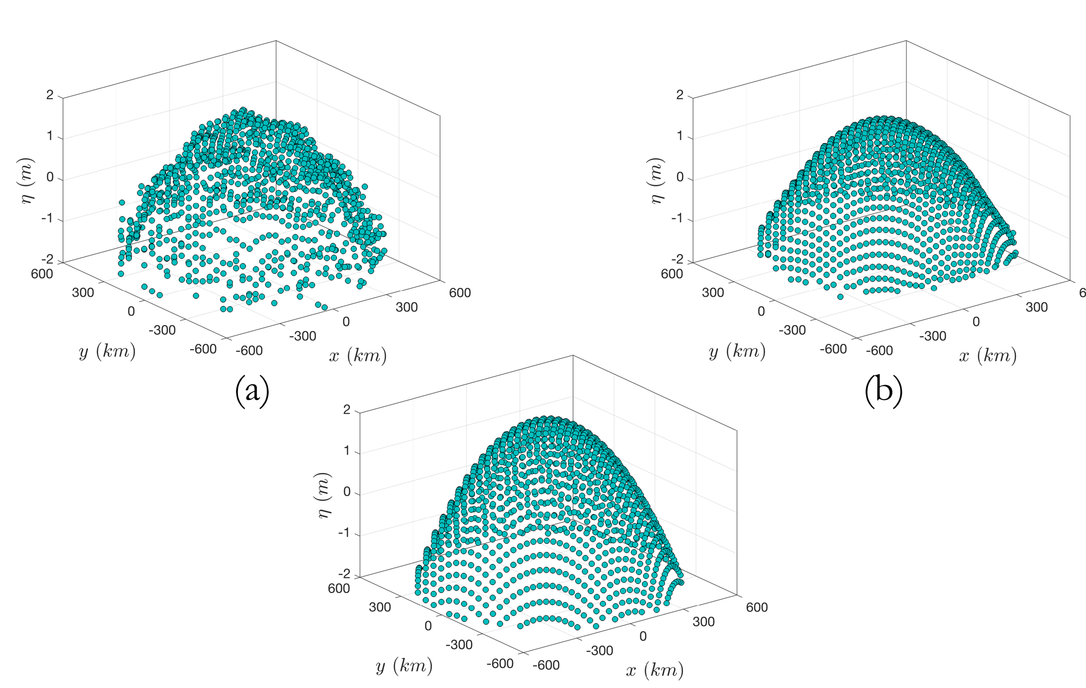
This test case involves oscillatory flow with moving shorelines in a frictionless, paraboloid rotating basin

Simulation setup



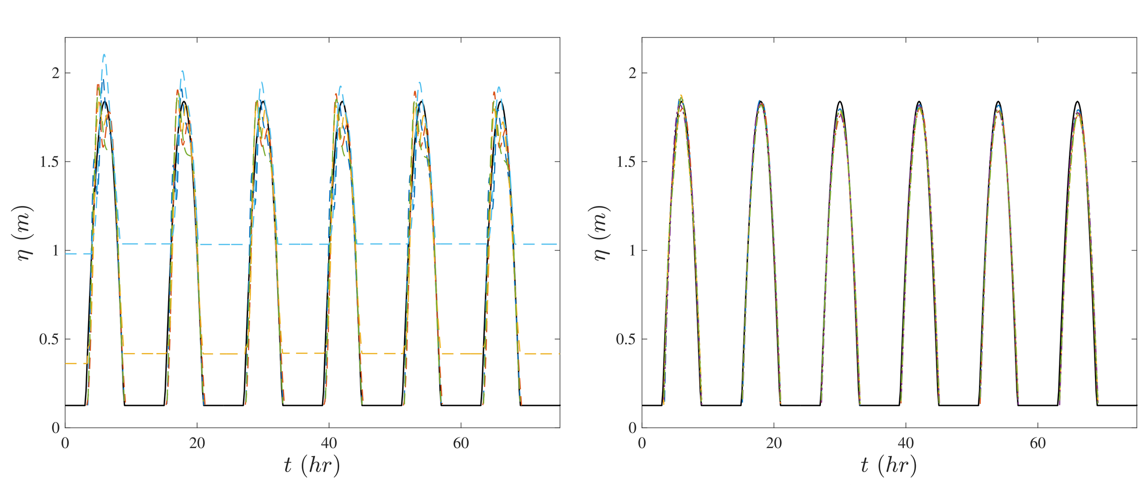
(a) Bathymetry of bowl case in OCCS with three stational locations
(b) Initial condition at x=0m

Coarse-resolution



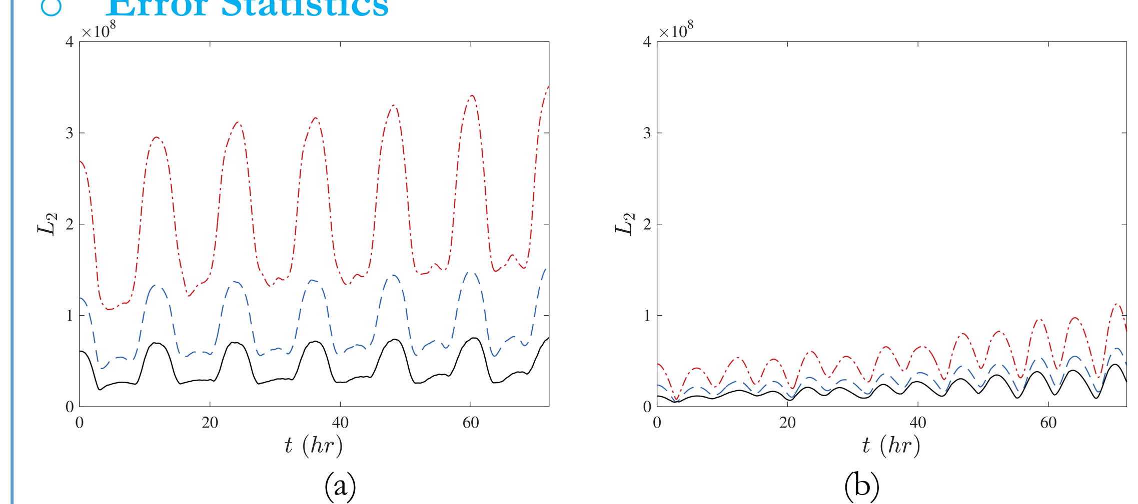
Surface elevation at t = 6T (a) Conventional method with mesh 50x50 (b) subgrid method with mesh 50x50 (c) Analytical solution

Fine-resolution



Comparison of surface elevation time series for angle = 0 (- - -), 45 (- - -), 90 (- - -), 135 (- - -), 180 (- - -) with analytical solution (—) at r = 610 km: (a) Conventional and (b) Subgrid method

Error Statistics



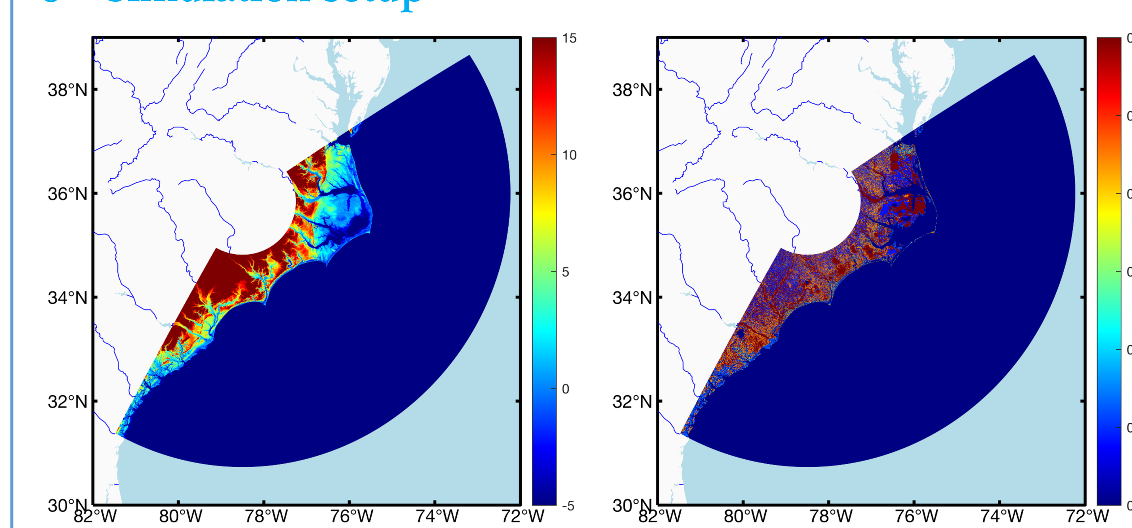
Comparison of L_2 error time series for surface elevation on 50 × 50 (- · - ·), 100 × 100 (- - -), and 200 × 200 (—) mesh: (a) Conventional and (b) Subgrid method

Grid	$\frac{1}{N}\sum_{i=1}^{N}\frac{\|E(s_{q,conv};t_i)\|_{L_2}}{\|E(s_{q,sg};t_i)\|_{L_2}}$		
	$s=\eta$	$s=u$	$s=v$
50×50	4.36	2.17	2.31
100×100	3.52	1.81	1.82
200×200	2.65	1.47	1.35

Mean L_2-error ratios of the conventional scheme relative to the subgrid scheme over the entire length of simulations.

Hurricane Florence 2018

Simulation setup



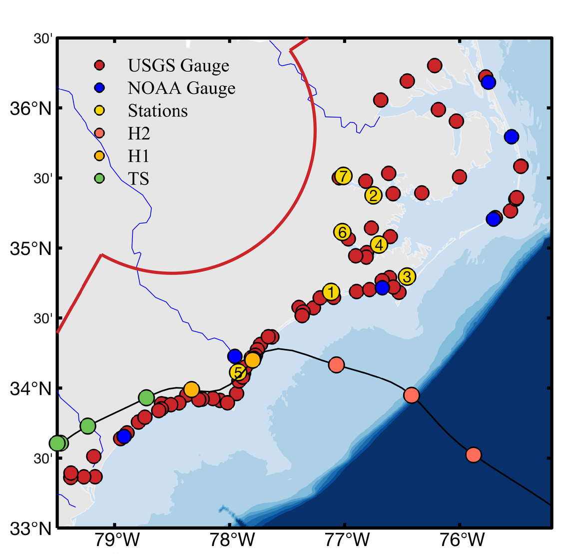
Input data represented by high-resolution subgrid resolution (mesh size 13567 × 22784) (a) Ground surface elevations (m relative to NAVD88) generated from NOAA Digital Coast with finest ~5m (b) Manning coefficients corresponding to different land cover based on National Land Cover Database 2019 with 30 m resolution

Grid ($q_1 \times q_2$)		106×178	212×356	424×712	13568×22784
	Minimum	1.698	0.849	0.419	0.00555
$h_1\Delta q_1$ (km)	Maximum	8.714	4.408	2.221	0.07826
	Average	4.298	2.165	1.087	0.03359
	Minimum	1.173	0.848	0.421	0.00714
$h_2\Delta q_2$ (km)	Maximum	8.717	4.408	2.219	0.07783
	Average	4.298	2.165	1.087	0.03359

Cell sizes for the grids to describe coastal NC, SC, and GA

Setups:

- *Time period*: Sep 12 to 17, 2018
- *Time step*: 30 s
- *Open BCs*: Clamp
- *Initial elevation*: 0.4237
- *Wind data*: WRF
- *Station information*: 88 stations

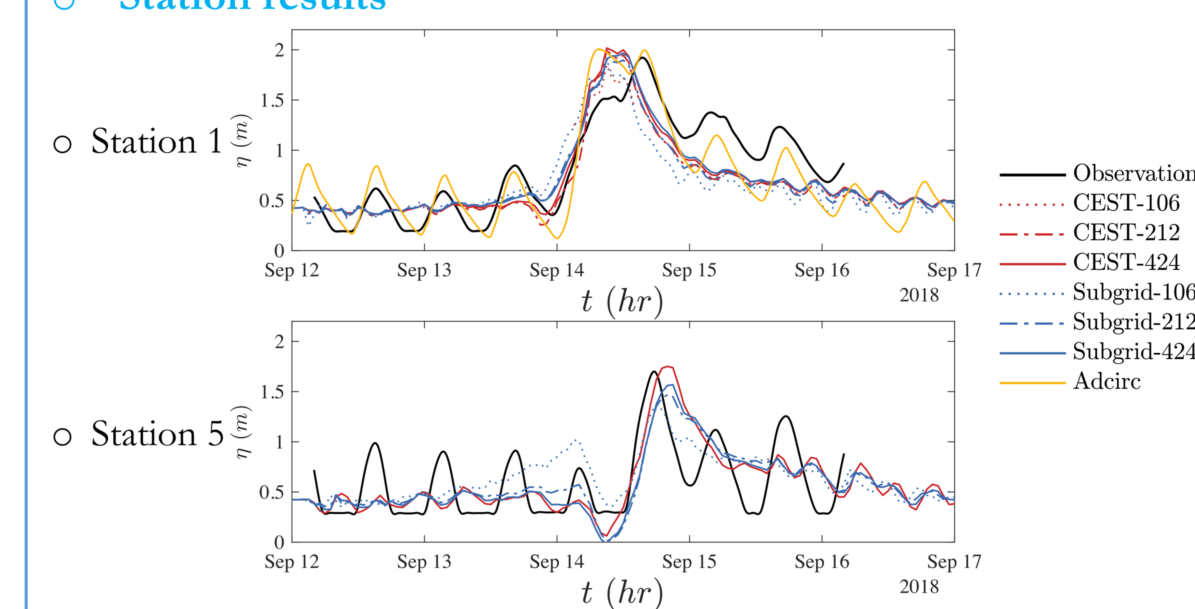


Gauge locations and best track for Florence (2018)

Station results

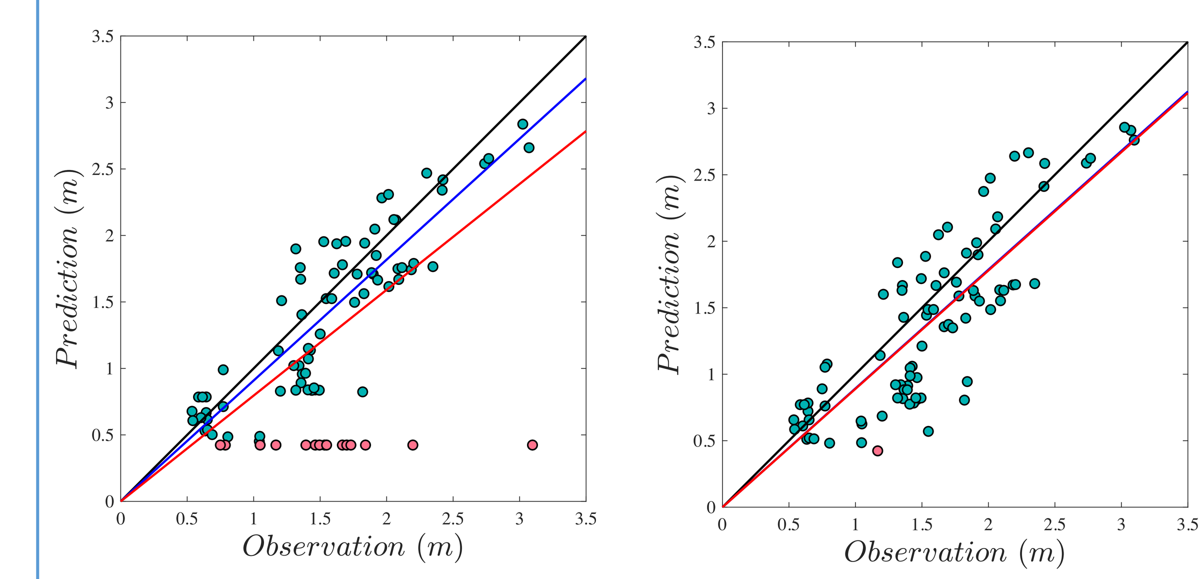
- Station 1

- Station 5



Comparison of surface elevation time series at stations

Error Statistics



Peak water level comparison between observations and coarse grid simulations: (a) Conventional and (b) Subgrid method

Grid($q_1 \times q_2$)	Simulation	Dry	E_{RMS} (m)	R^2_{all}	R^2_{wet}	a_{all}	a_{wet}
106×178	Conventional	15	0.6078	0.2093	0.7087	0.7957	0.9090
	Subgrid	1	0.4017	0.6277	0.6335	0.8901	0.8931
212×356	Conventional	18	0.6831	-0.0151	0.6757	0.7584	0.9084
	Subgrid	1	0.3861	0.6629	0.6691	0.9051	0.9020
424×712	Conventional	13	0.5790	0.2474	0.7095	0.8081	0.9077
	Subgrid	2	0.4192	0.6030	0.6366	0.8732	0.8826

Error statistics for predictions of peak water levels. Dry: the number of "Dry" stations; R^2: Root-mean square errors; a: Best fit slope

Summary

- ✓ A subgrid correction is developed in CEST model
- ✓ Verification using idealized non-trivial test case with analytical solution.
- ✓ Validation using hurricane-induced storm surge processes with observation data

References

[1] Kennedy, A.B., Wirasaet, D., Begmohammadi, A., Sherman, T., Bolster, D. and Dietrich, J.C., 2019. Subgrid theory for storm surge modeling. *Ocean Modelling*, 144, p.101491.
[2] Li, Y., Chen, Q., Kelly, D.M. and Zhang, K., 2021. Hurricane Irma simulation at South Florida using the parallel CEST Model. *Frontiers in Climate*, p.79.

Acknowledgments

This research was supported by National Science Foundation, USA grants ICER 1664040 and 1664037