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Subgrid-scale Storm Surge Coastal Flood Modelling in ADCIRC

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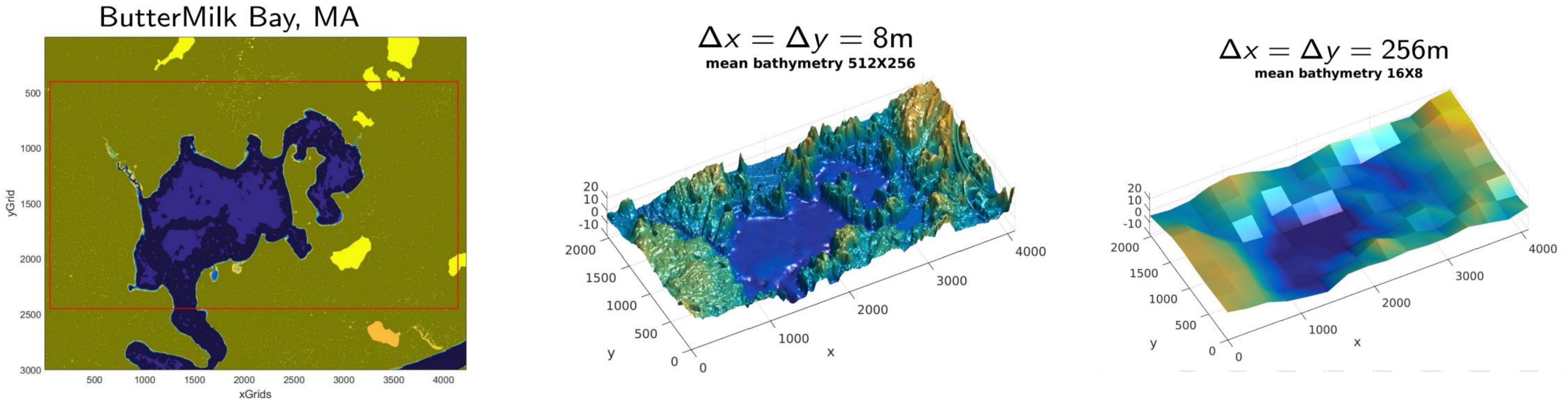
Motivation

- Representation of a bottom is important in obtaining accurate solution for hydrodynamic simulations.
- The bottom profile is typically approximated on the grid level.
- A large amount of topographical detail could be lost in the low-resolution case.



- Casulli (2009) devises a subgrid wetting / drying algorithm using a porosity function to ensure the positivity of the water column and determine the partial filling of cells from the subgrid bathymetry.

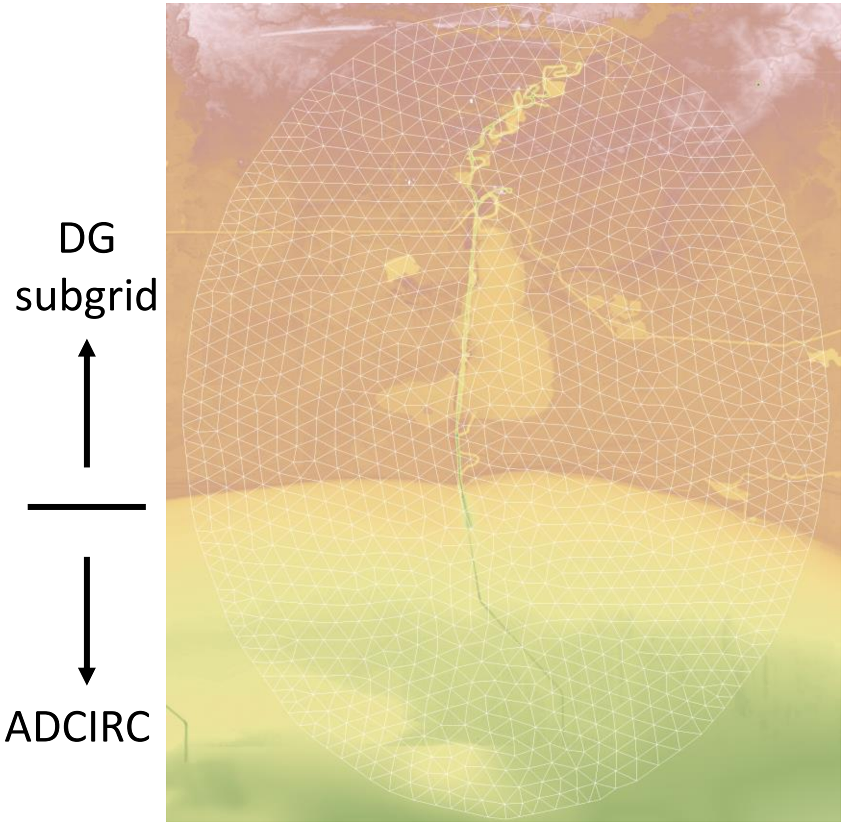
Motivation (cont'd)

- The ADCIRC hydrodynamic model has successfully been used in many tide and storm-surge applications.

Goal

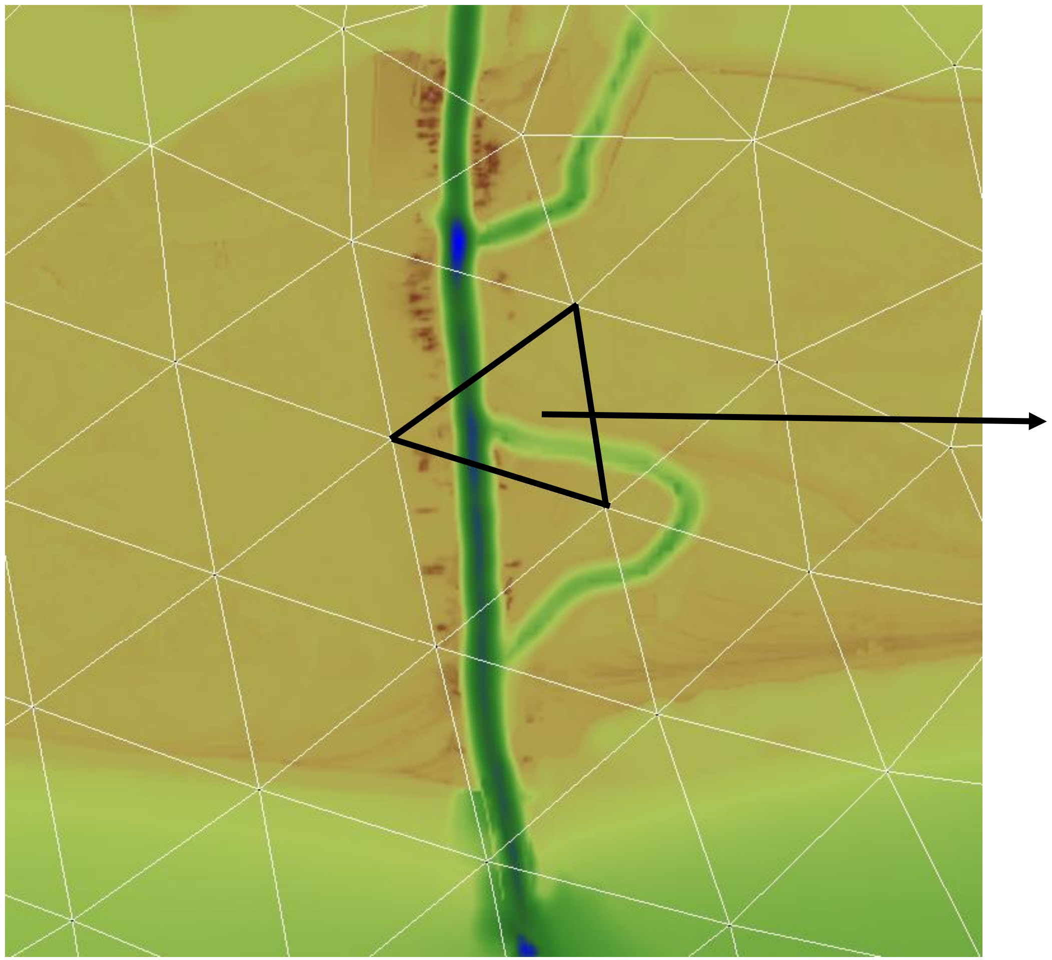
- Selective spatial application of subgrid correction
- This work pursues a combination of
  - Standard ADCIRC for deep water ~ coast
  - A locally mass conservative method for coast ~ floodplain resolving subgrid-scale topo/bathy

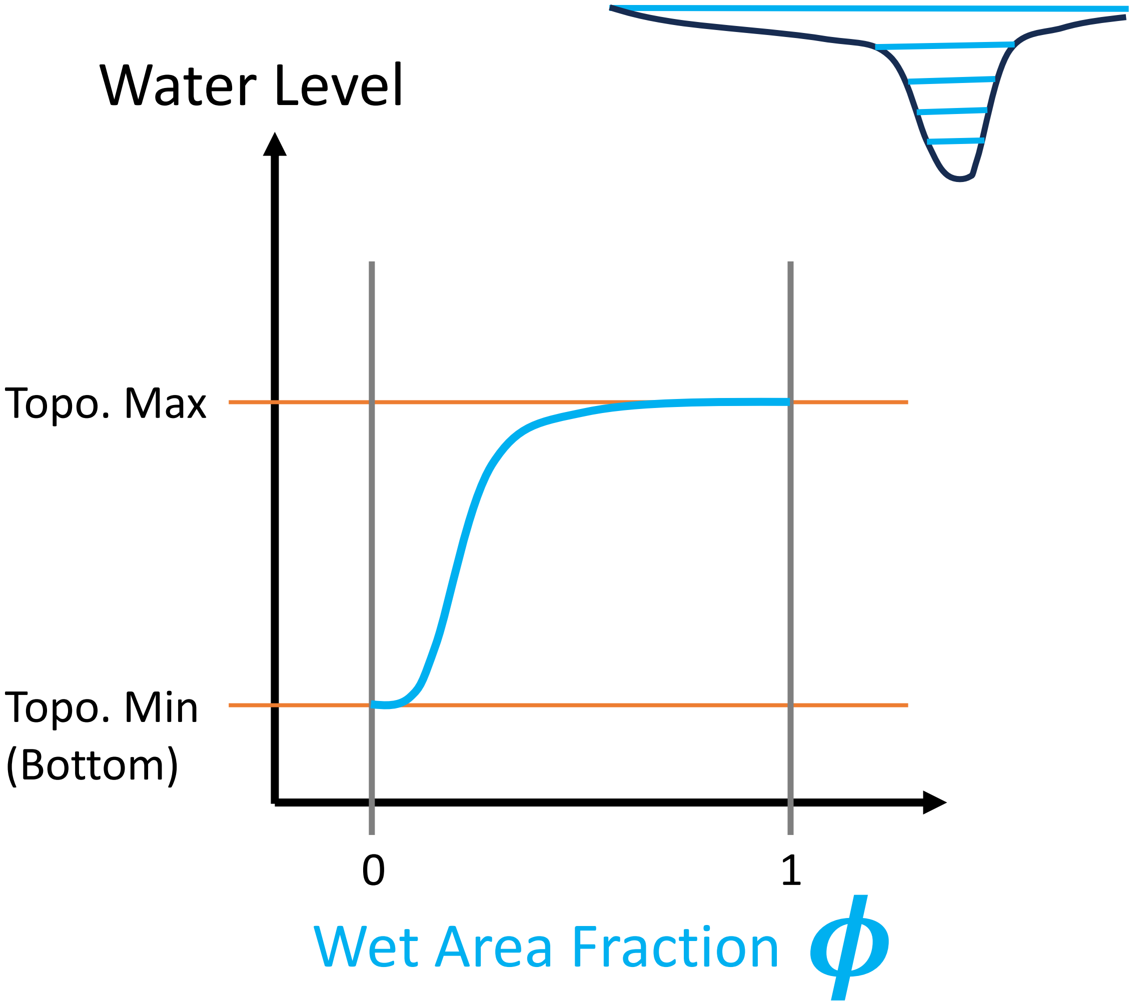
- The choice of this work for the locally conservative method
  - The discontinuous Galerkin (DG) method with subgrid correction



Subgrid-scale Bathymetry Profile as ϕ

DEM + Unstructured mesh





→ Lookup tables are created for each element and each edge.

Averaged 2D Shallow Water Equations (SWE)

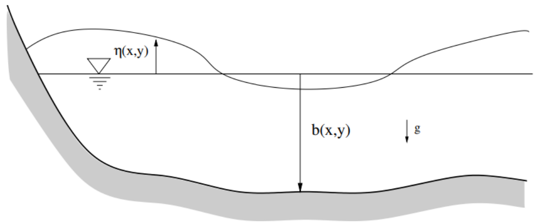
☞ Governing equations[1]: consider *partial filling and varying bottom roughness*

$$\frac{\partial H}{\partial t} + \nabla \cdot (\boldsymbol{u} H) = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + f\widehat{\mathbf{k}} \times \boldsymbol{u} = -g\nabla\eta - \frac{\phi}{H} c_f |\boldsymbol{u}|\boldsymbol{u} - \frac{1}{\rho_0}\nabla P_A + \frac{\phi}{H}\frac{\rho_a}{\rho_0} c_d |\boldsymbol{w}|\boldsymbol{w}$$

η = surface elevation
H = area-averaged total water depth
$= \frac{1}{|A_G|} \int_{A_G} \max(0, \eta + b(\boldsymbol{x}))d\boldsymbol{x}$
$\phi = \frac{|A_w|}{|A_G|}$ wet area fraction
c_f = effective bottom friction coefficient
$\boldsymbol{u}$ = grid-scale depth-avg velocity

A_G = 'averaging area'
$A_w = \{\boldsymbol{x} \in A_G \mid \eta + b(\boldsymbol{x}) > 0\}$
P_A = atmospheric pressure
$\boldsymbol{w}$ = 10m wind
$b(x, y)$ = bathymetric depth



[1] Kennedy et al., Ocean Modelling, 2019

Spatial Discretization: Discontinuous Galerkin FEM

- Favorable properties of DG FEM
  - Conserves mass locally
  - Stable and accurate for a larger range of the Froude number
  - Orthogonality in unstructured mesh is not required

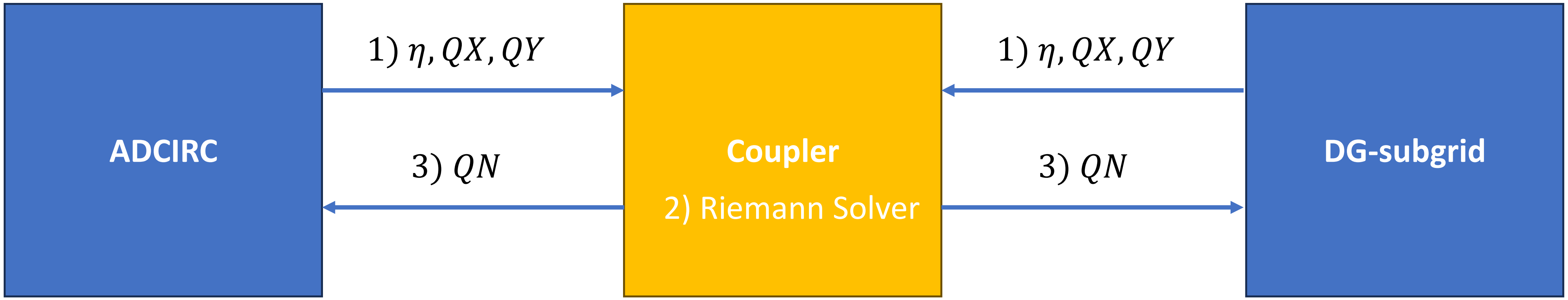
- G. Fu's formulation[1]
  - Gives the well-balanced property
  - Has a high affinity with the governing equations with subgrid corrections

- Other specs in the discretization design
  - Piece-wise constant in space ← Requirement from subgrid
  - Forward Euler in time ← For computational efficiency

[1] G. Fu, Journal of Scientific Computing, 2022.

Coupling ADCIRC and DG-subgrid

- Two models are coupled through mass flux boundary conditions in the following steps:
    1) The states on both sides are shared with a coupler program.
    2) The coupler program computes mass flux using a Riemann solver.
    3) The computed mass flux is sent back to two models.



- The communications are processed through MPI.

Test 1: Parabolic Bowl Problem [Thacker, 1981]

- Frictionless rotating basin of paraboloid shape:

$$b(x,y) = h_0\left(1 - \frac{r^2}{L^2}\right)$$

where $r = \sqrt{x^2+y^2}$, $L = Const.$

- Initial condition:

$$\boldsymbol{u}(\boldsymbol{x},0) = \frac{1}{2C_0}\left[f(\sqrt{1-C^2} - C_0)\right](-y,x)$$

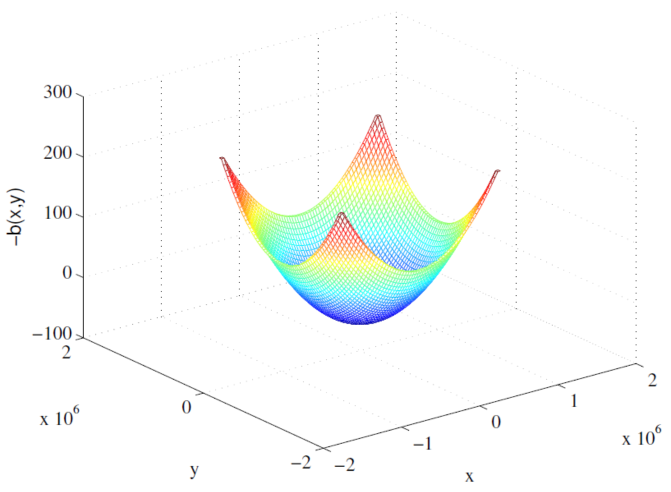
$$H(\boldsymbol{x},0) = h_0\left[\frac{\sqrt{1-C^2}}{C_0} - \frac{r^2}{L^2}\left(\frac{1+C}{(C_0}\right)\right]$$

$$\eta(\boldsymbol{x},0) = H(\boldsymbol{x},0) - b(\boldsymbol{x})$$

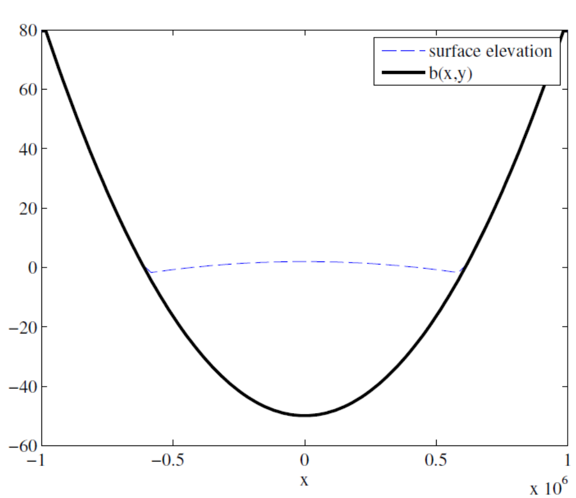
- $C = \frac{(h_0+\zeta_0)^2 - h_0^2}{(h_0+\zeta_0)^2 + h_0^2}, \zeta_0 = \eta(\mathbf{0},0),$
  $C_0 = 1 - C$

- $L = \sqrt{\frac{8gh_0}{\omega^2 - f^2}}$ for a given ω

Bed profile

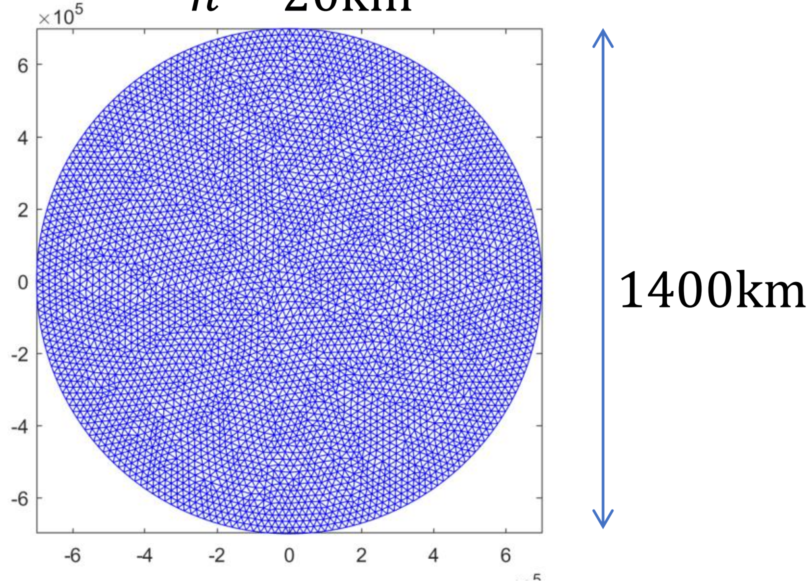


Initial surface elevation

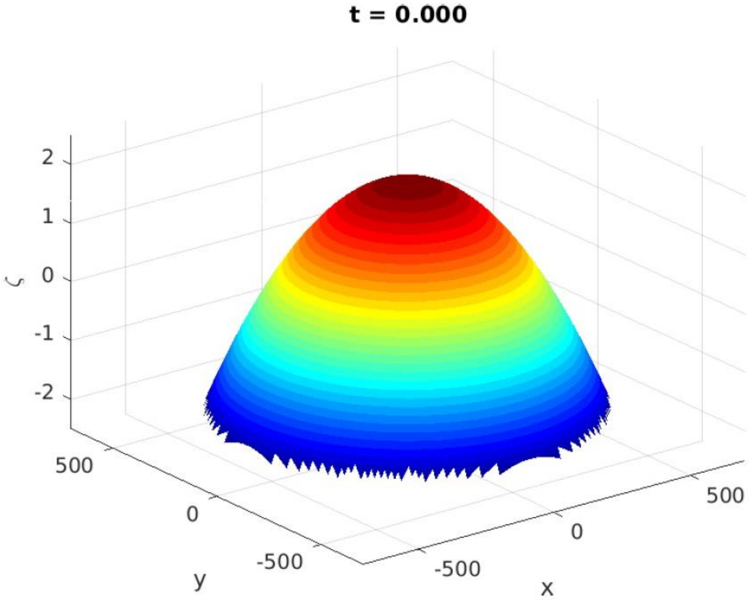


Test 1: Configurations

$h \sim 20\text{km}$



Exact Initial Surface Elevation



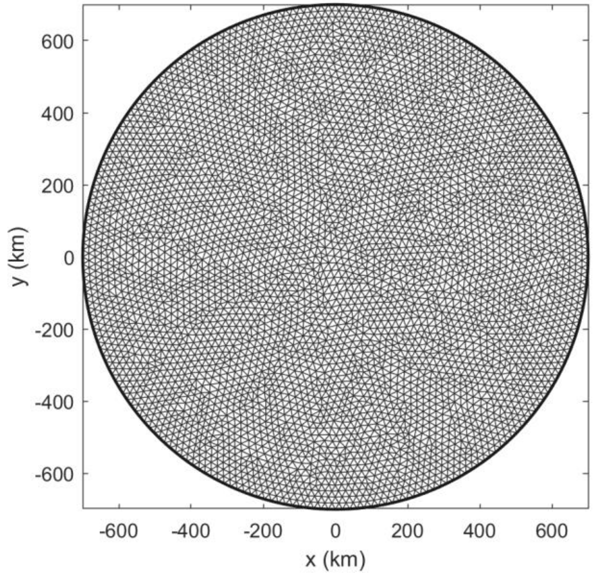
PARAMETERS: $h_0 = 50\text{m}$, $\zeta_0 = 2\text{m}$, $f = 1.03 \times 10^{-3}$, $\omega = 2\pi/12$ (hr^{-1})

NUMERICAL DETAILS:

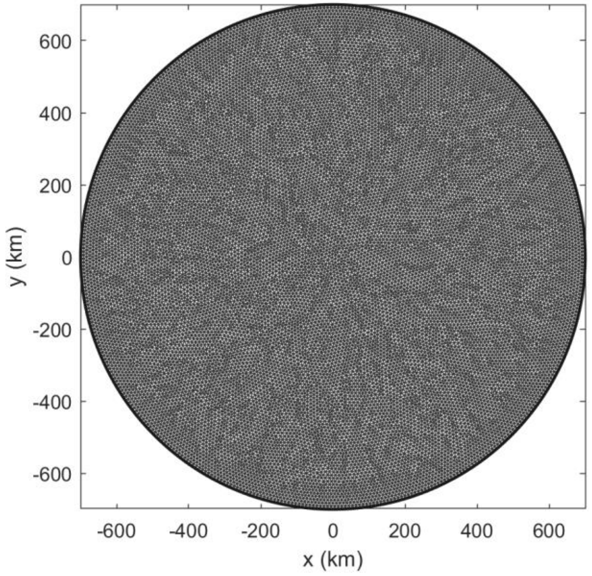
- h = 20 (Δt = 200), 10 (Δt = 100), and 5km (Δt = 50s)
- Subgrid scale hs = 1 km.

Test 1: Unstructured Mesh

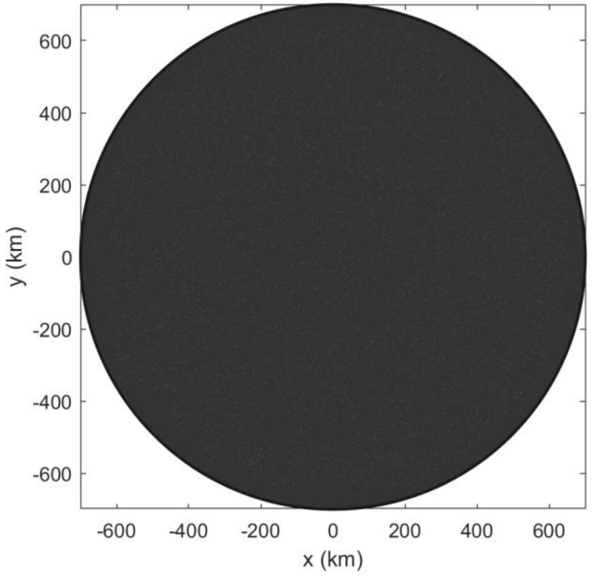
$h \sim 20\text{km}$



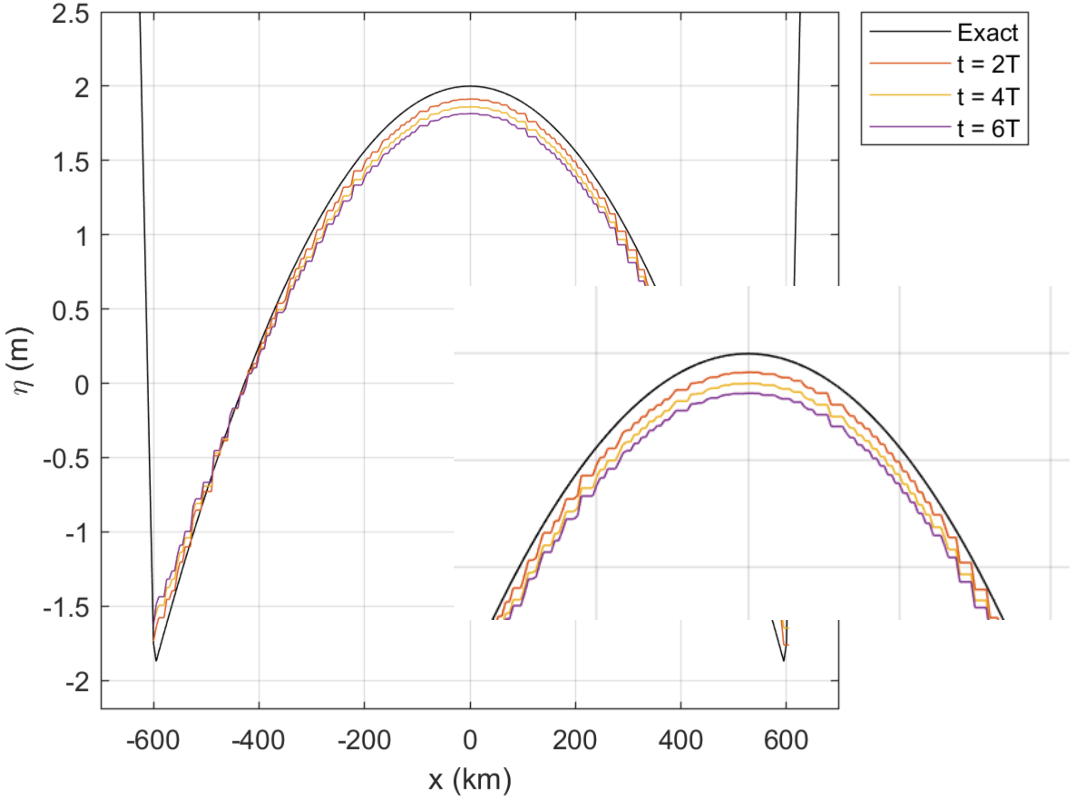
$h \sim 10\text{km}$

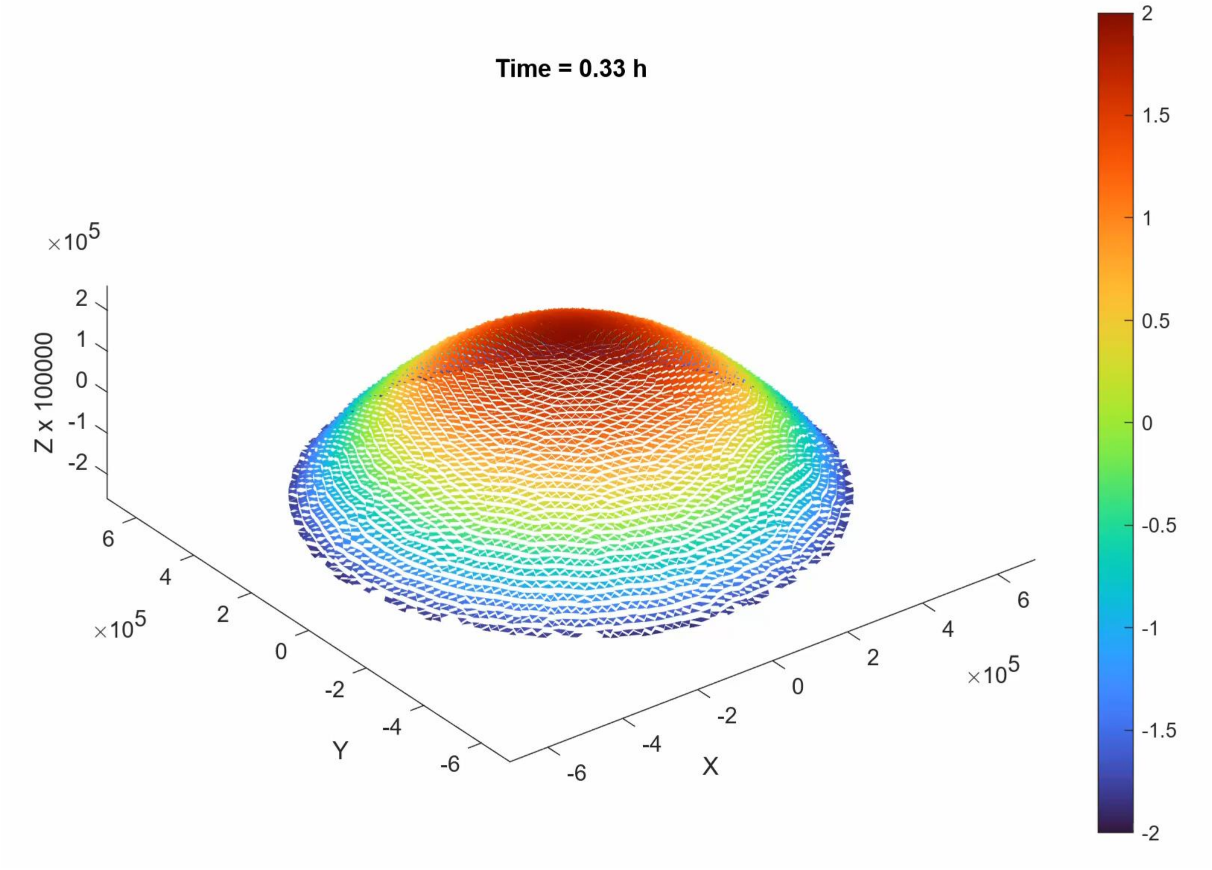


$h \sim 5\text{km}$

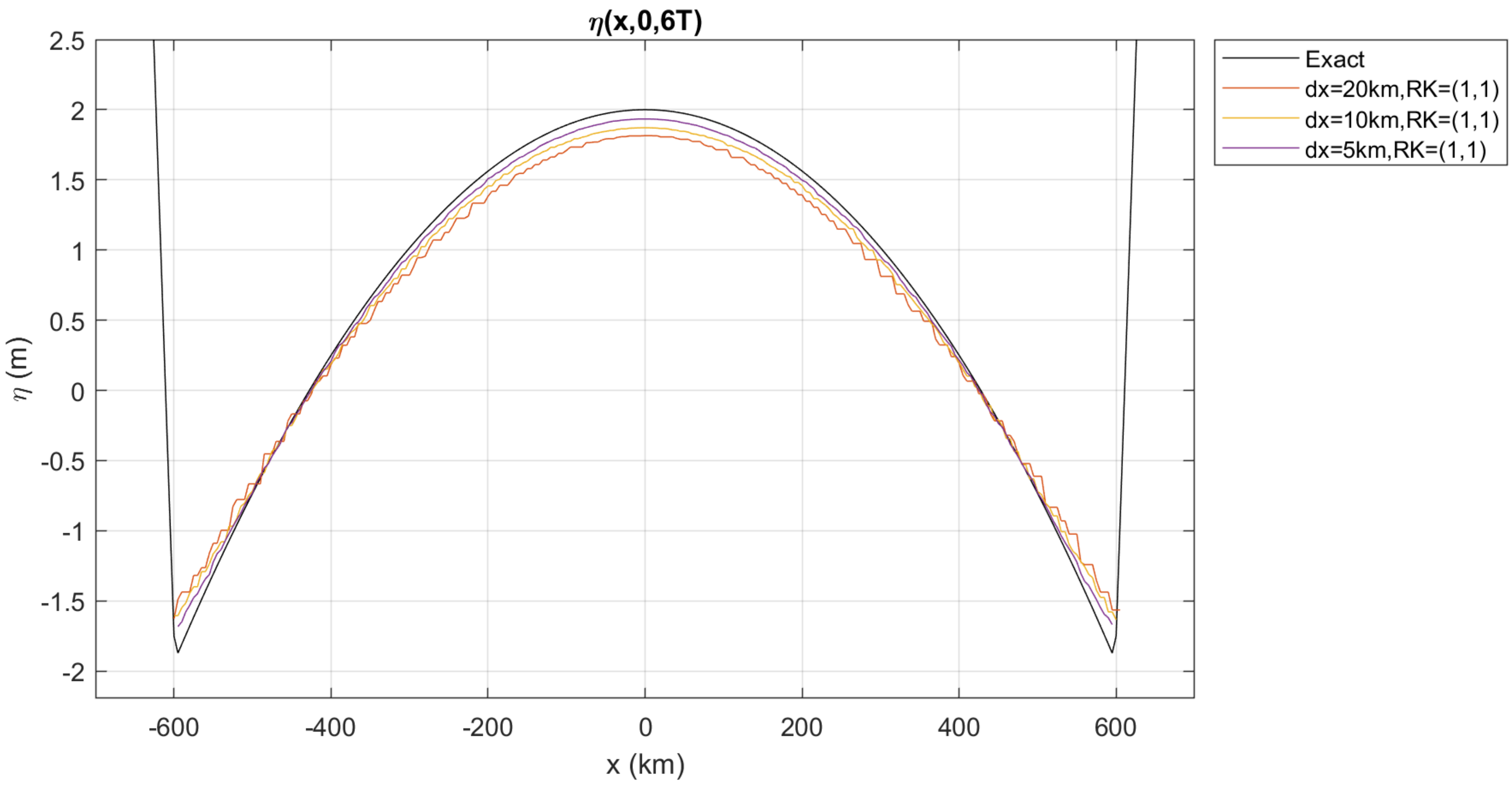


Test 1: Results, *h* ~20km (DG-subgrid, no coupling)

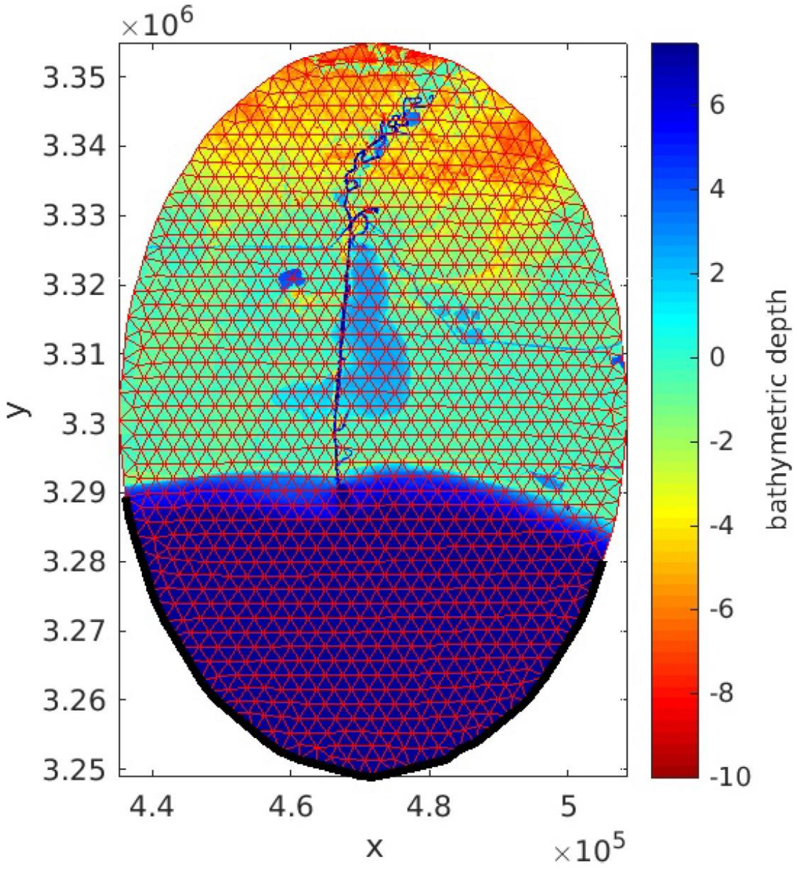




Test 1: Results, *h* ~20, 10, and 5km (DG-subgrid, no coupling)

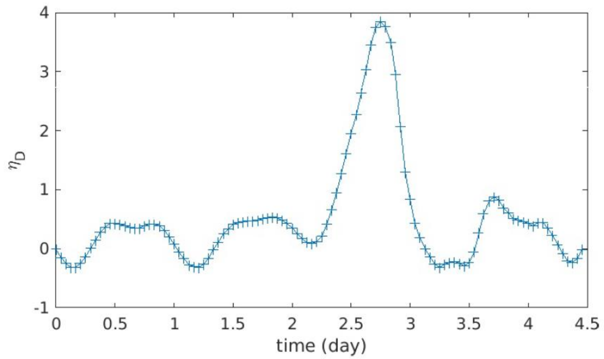


Test 2: Calcasieu Lake, Hurricane Rita

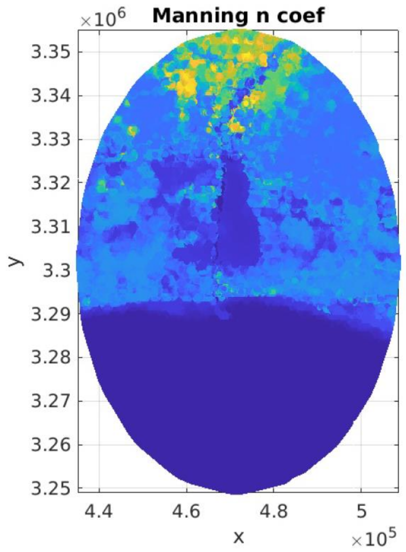


- $h \sim 2500\mathrm{m},\ N_{el} = 2370,\ N_{ed} = 3605$

- Elevation forcing

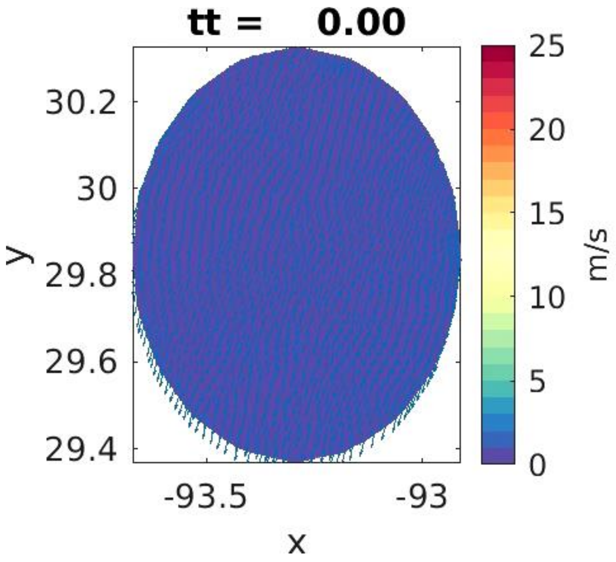


- Manning n value

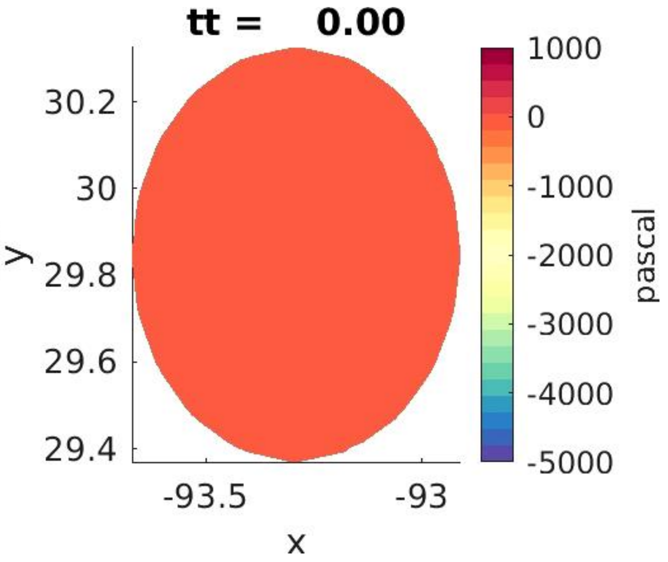


Test 2: Calcasieu Lake, Hurricane Rita

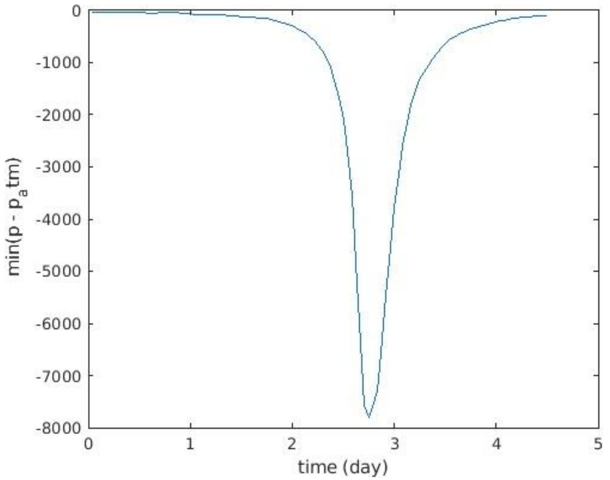
Wind



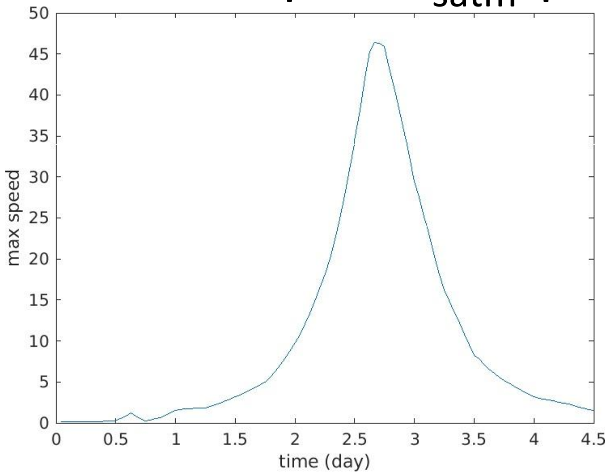
P - P_{atm}



max | ***w*** |



min | P - P_{satm} |

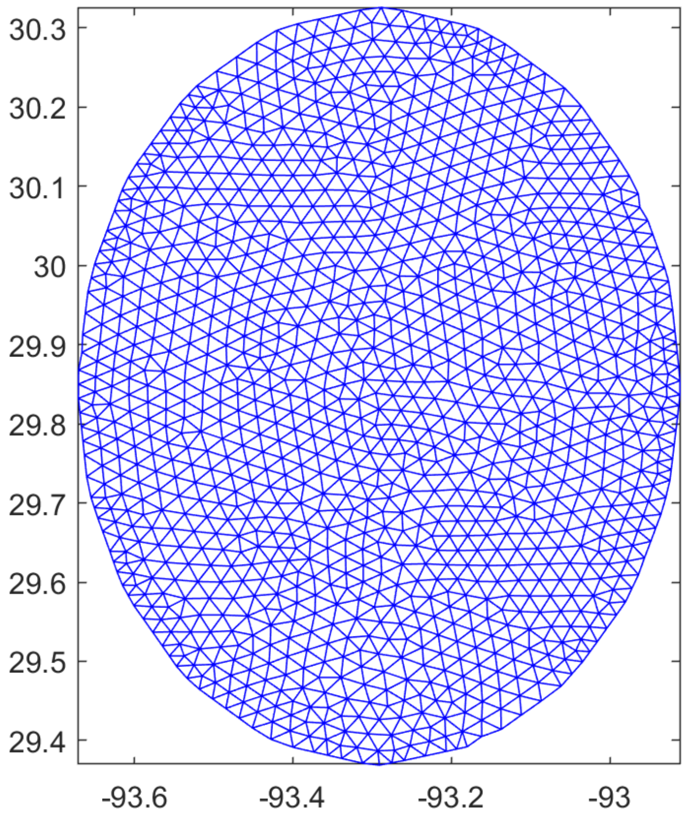


Test 2: Unstructured Mesh

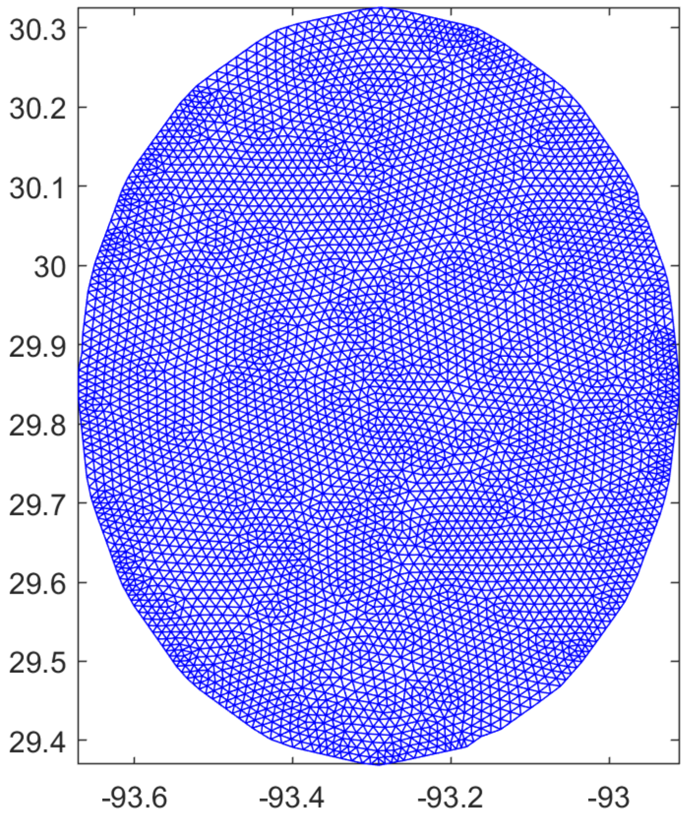
Coupled ADCIRC and DG-subgrid

ADCIRC (reference)

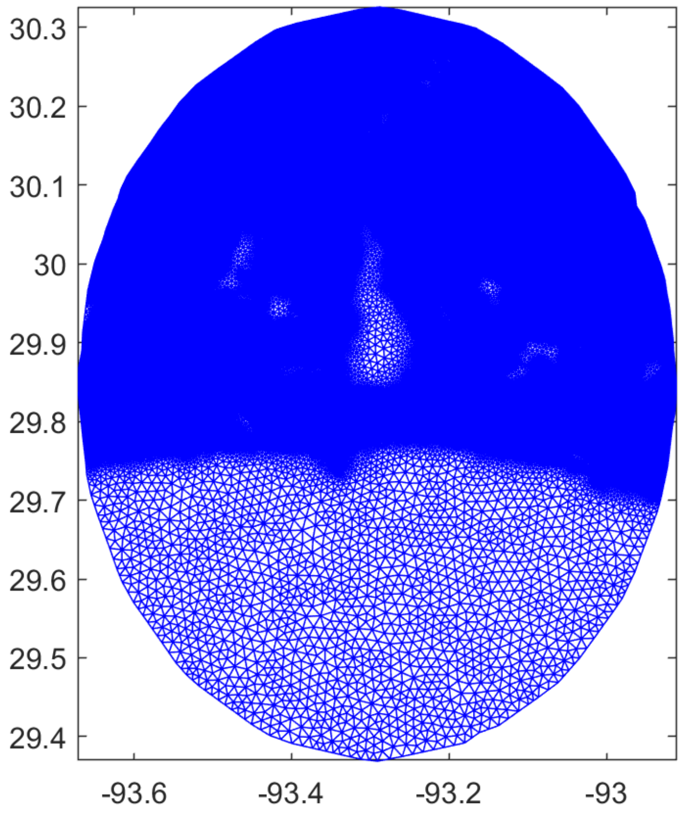
Coarse
h ~2.5km, 1.2K nodes



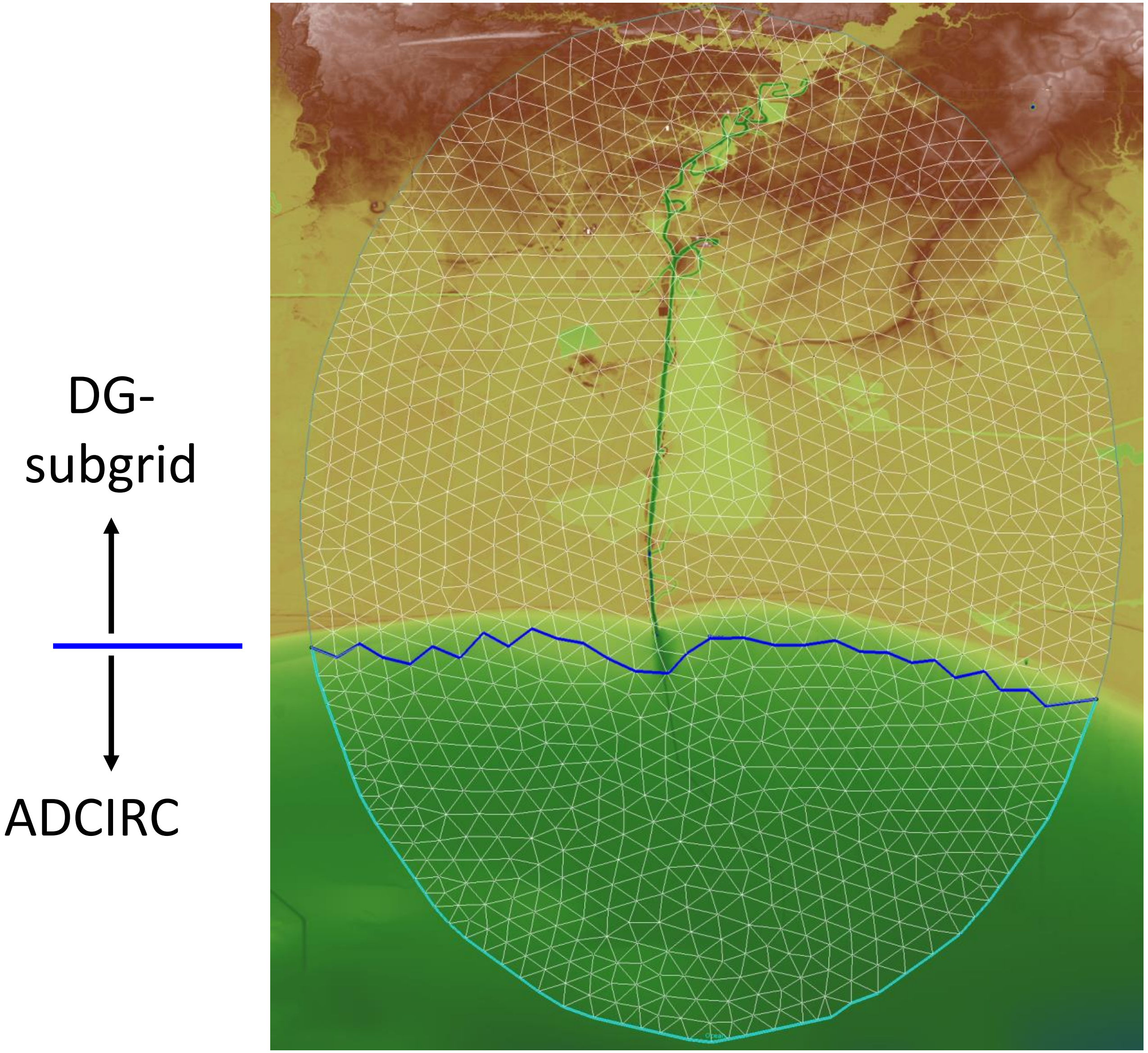
Refine1, 4.8K nodes
h ~1.25km, 1.2K nodes



High res, 4.8K nodes
h ~1.25km, 1.2K nodes



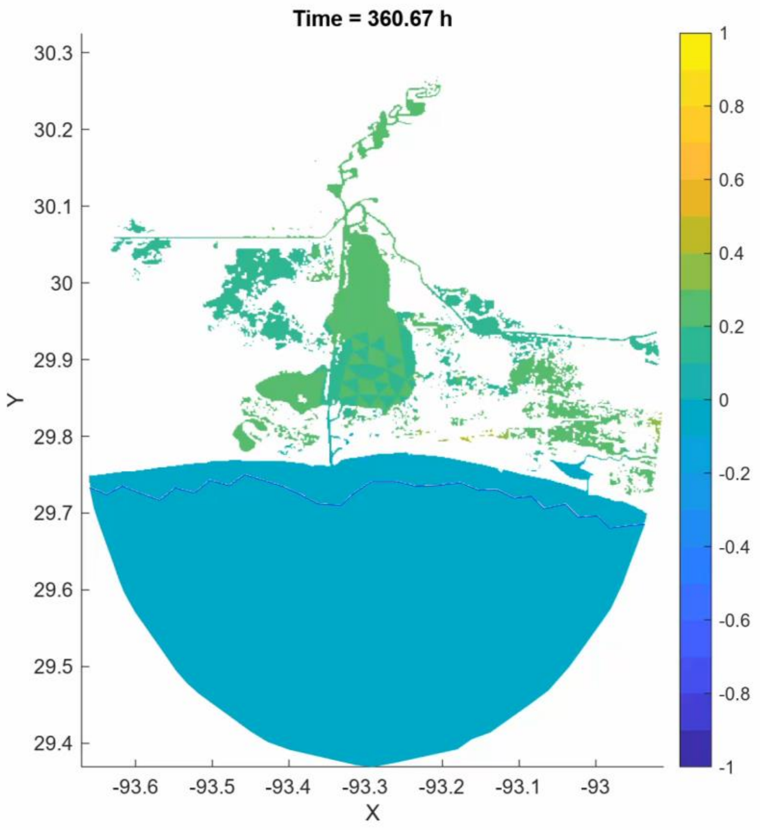
Test 2: ADCIRC and DG-subgrid Coupling



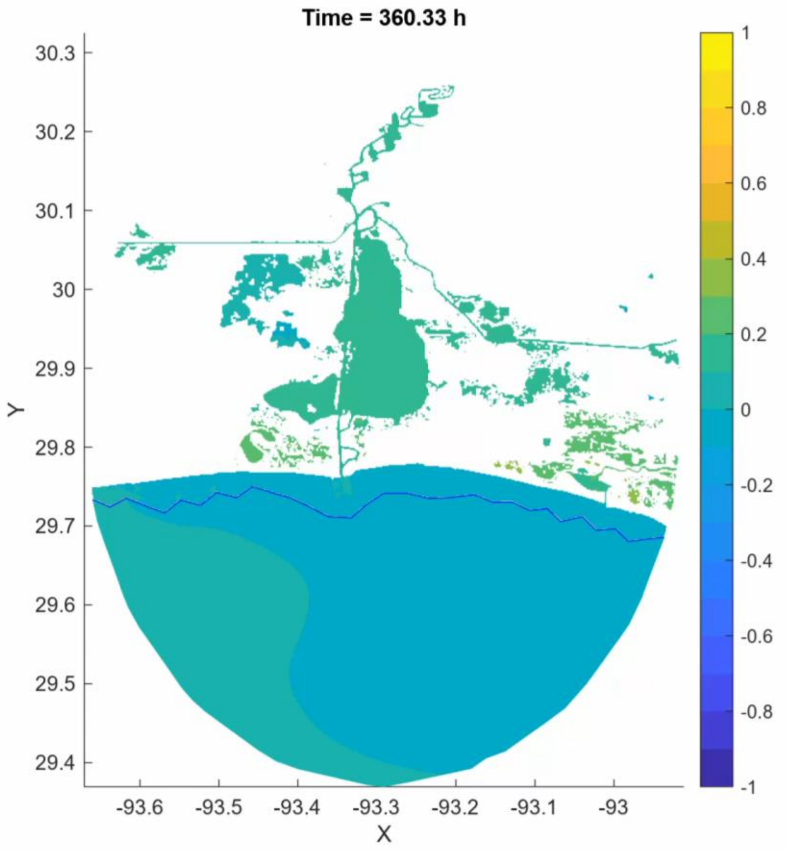
Test 2: Results

Coupled ADCIRC and DG-subgrid

Coarse, *h* ~2.5km

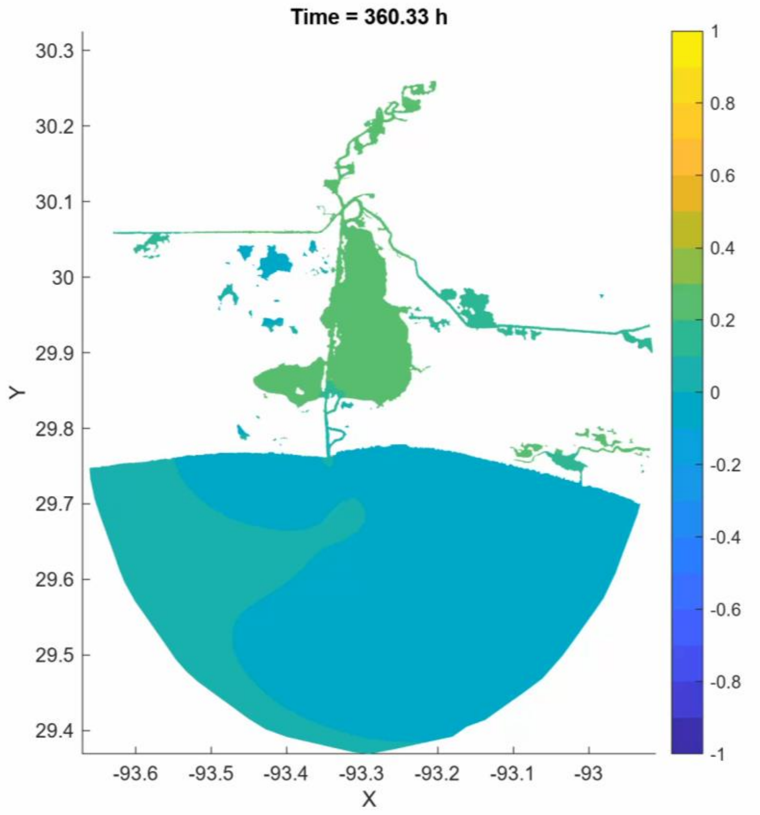


Refine1, *h* ~1.25km



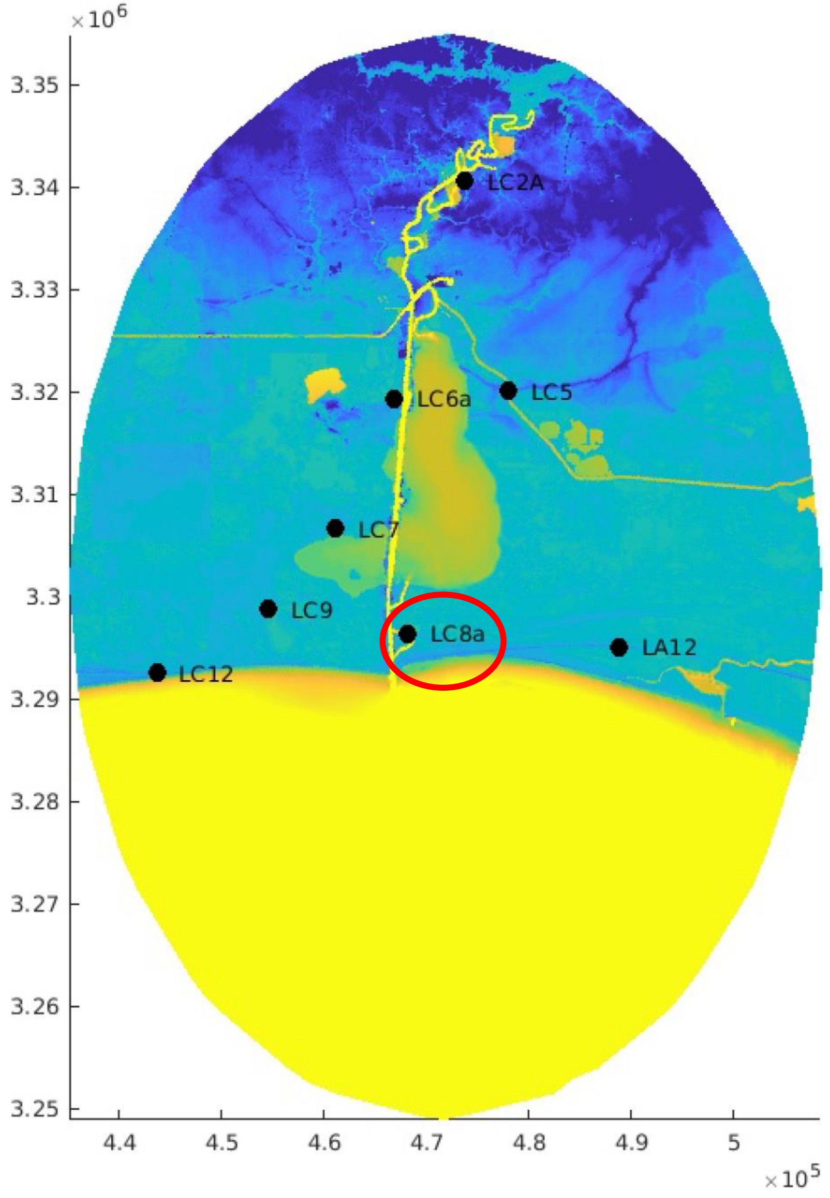
ADCIRC (reference)

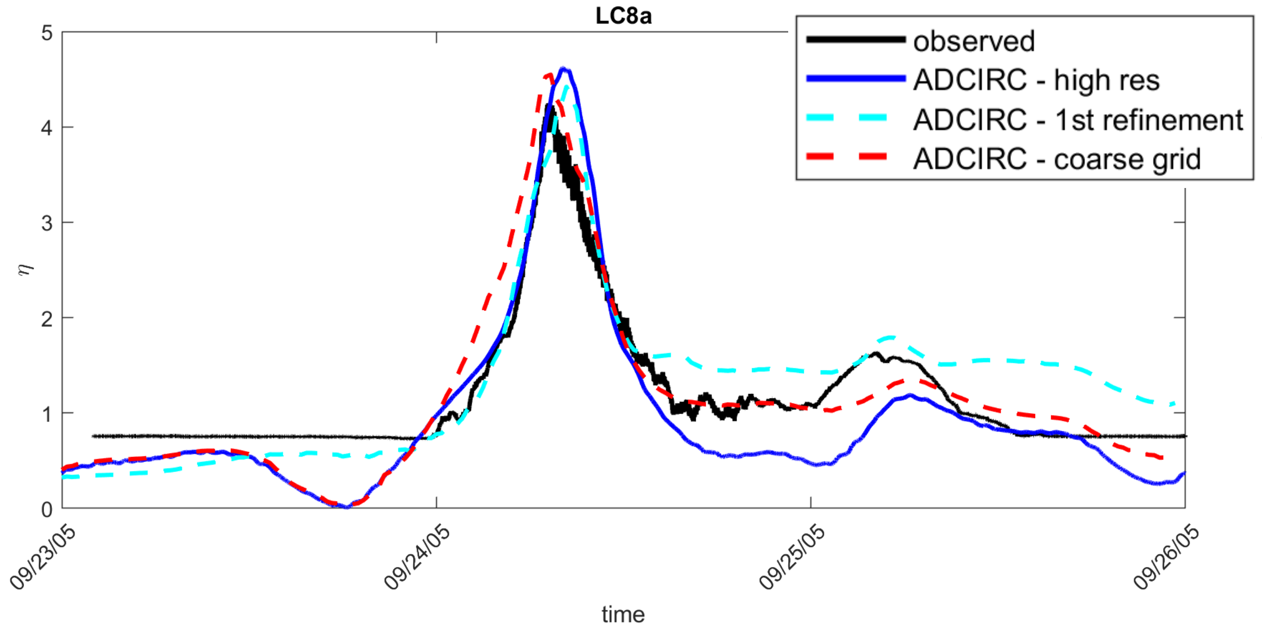
High res, *h* ~1.25km



Post processed using DEM

Test 2: Results vs USGS Station Observations





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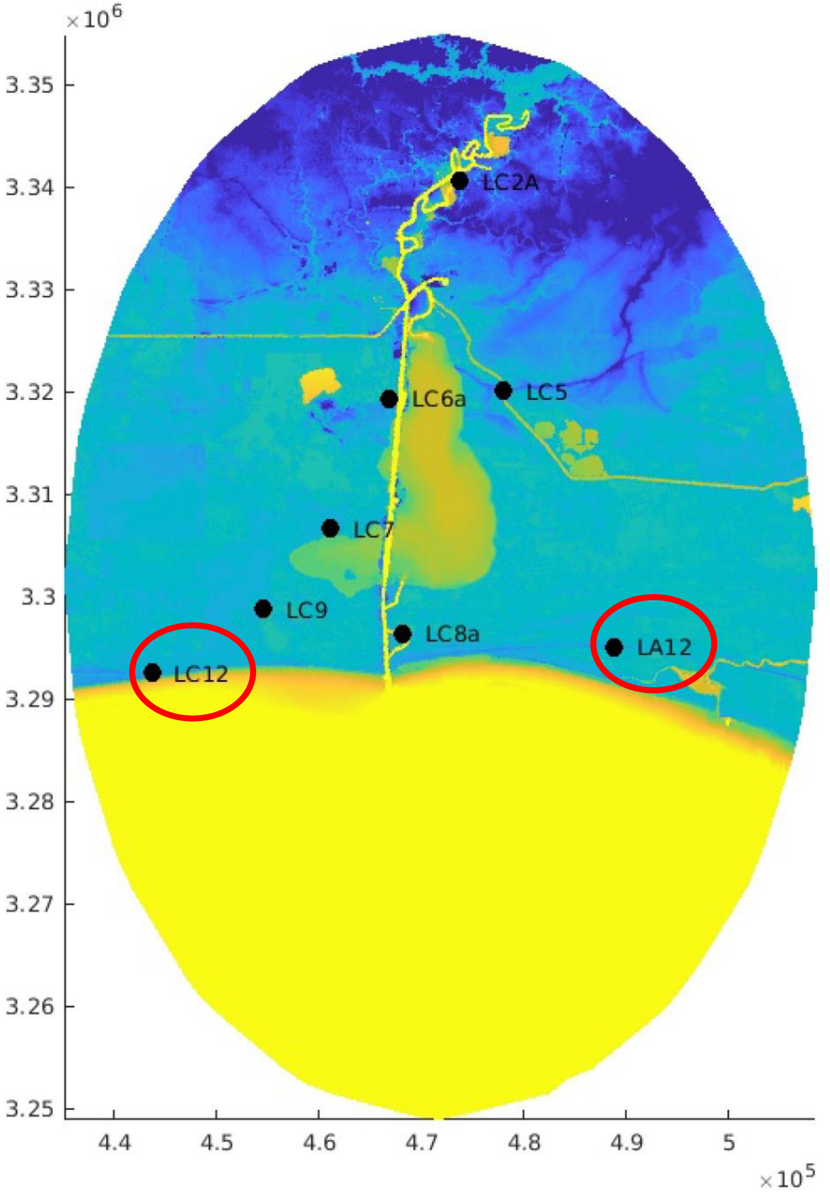

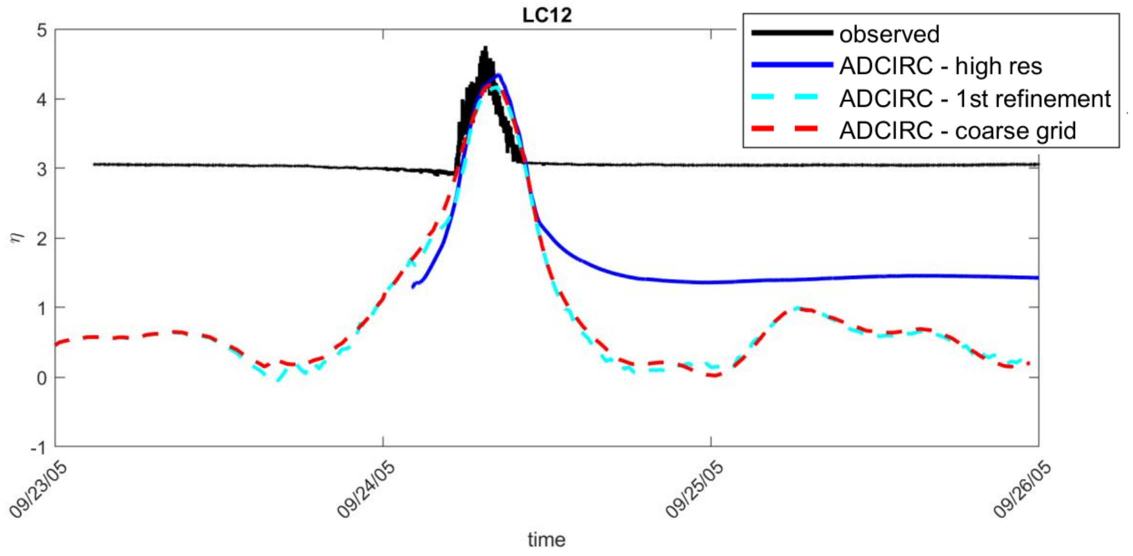

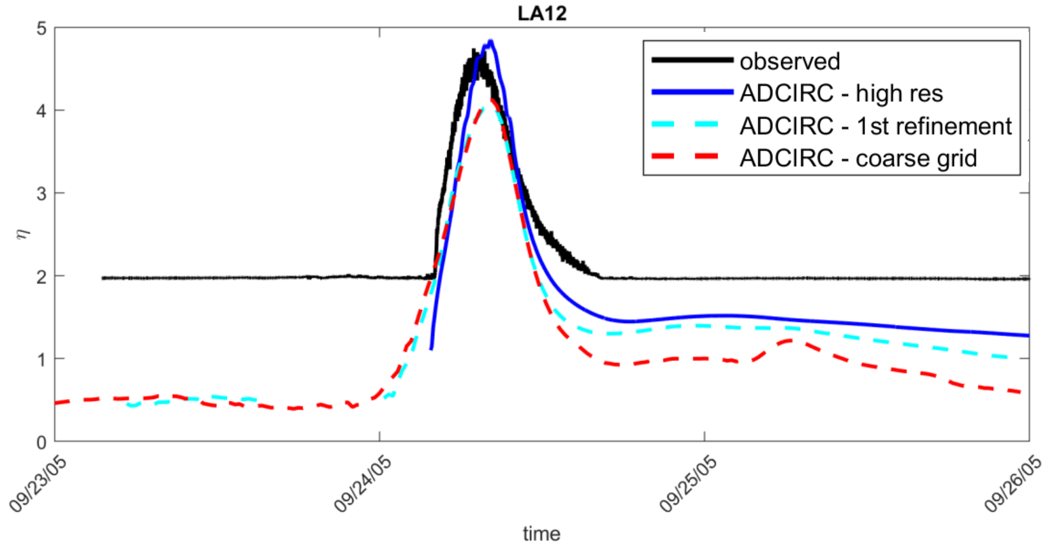

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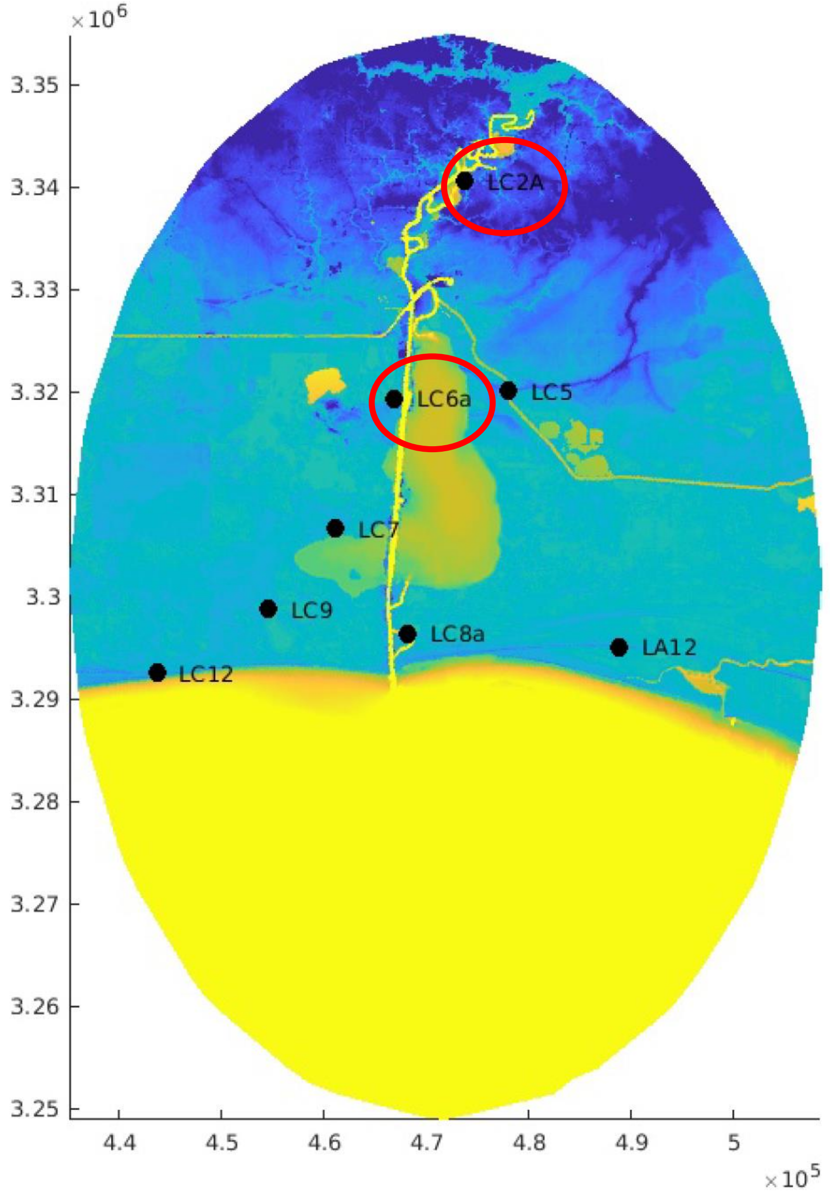
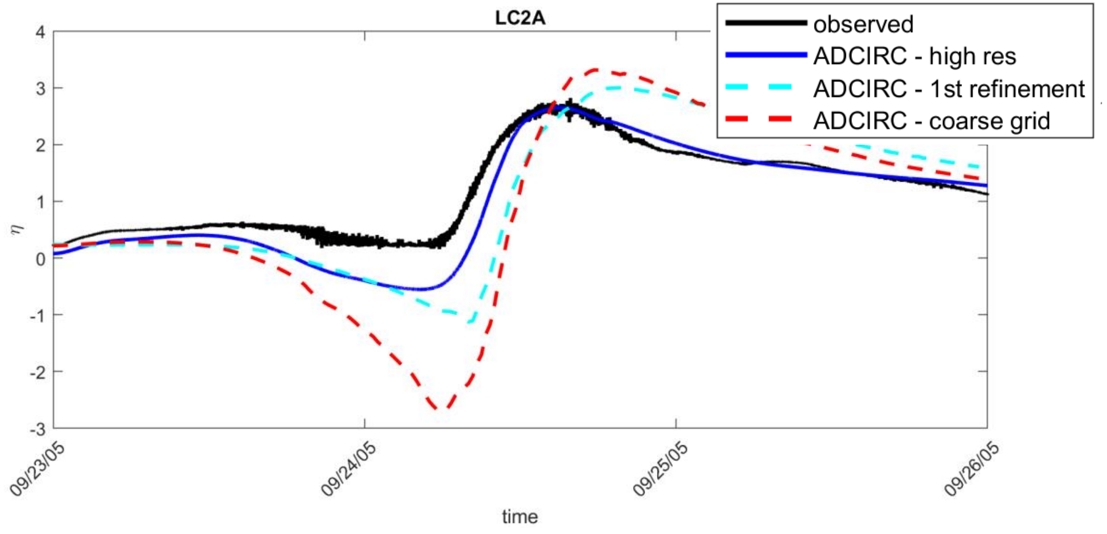
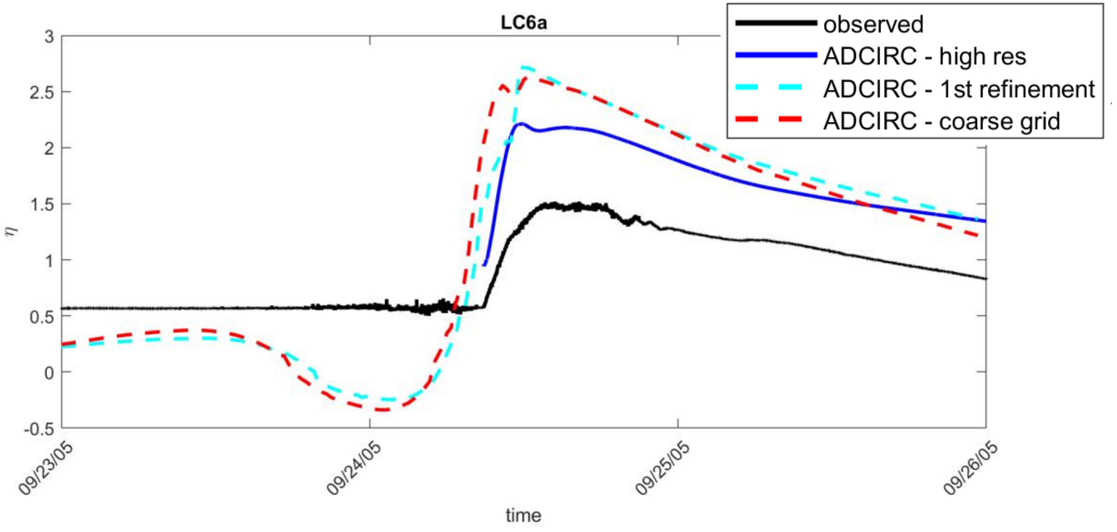




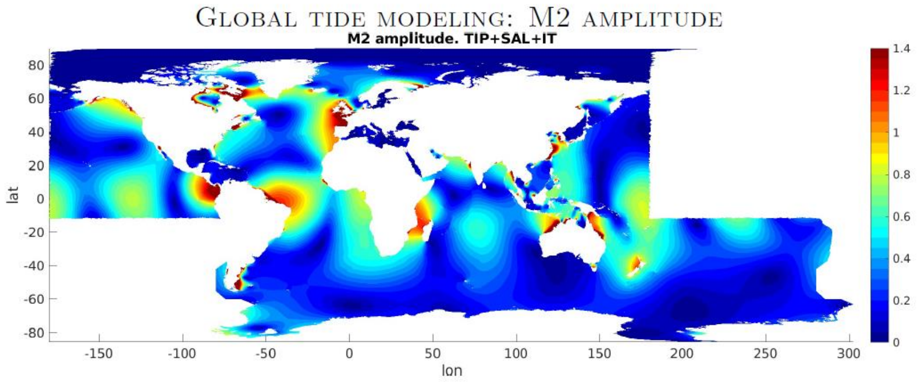


Summary

- A new subgrid model discretization is proposed with Fu’s DG formulation.
- A method to couple CG-FEM (ADCIRC) and DG-subgrid is proposed.
- The proposed methods are validated in comparisons with exact solutions and storm surge observations during Hurricane Rita.

Future Work

- Improvement of solution accuracy
- Application to larger-sale realistic problems



Thank you.

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