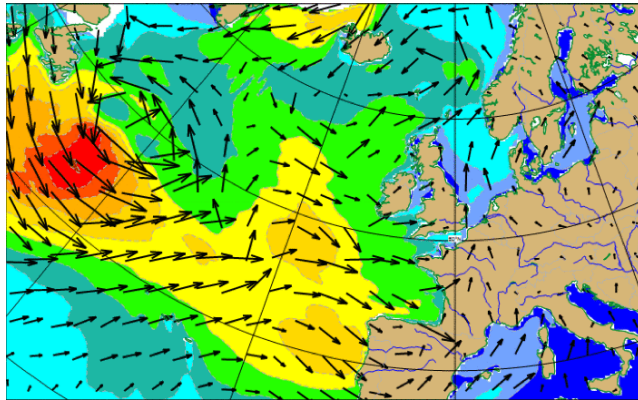


# Wave-by-wave forecasting: What do we need?

Alexis MERIGAUD and Paolino TONA

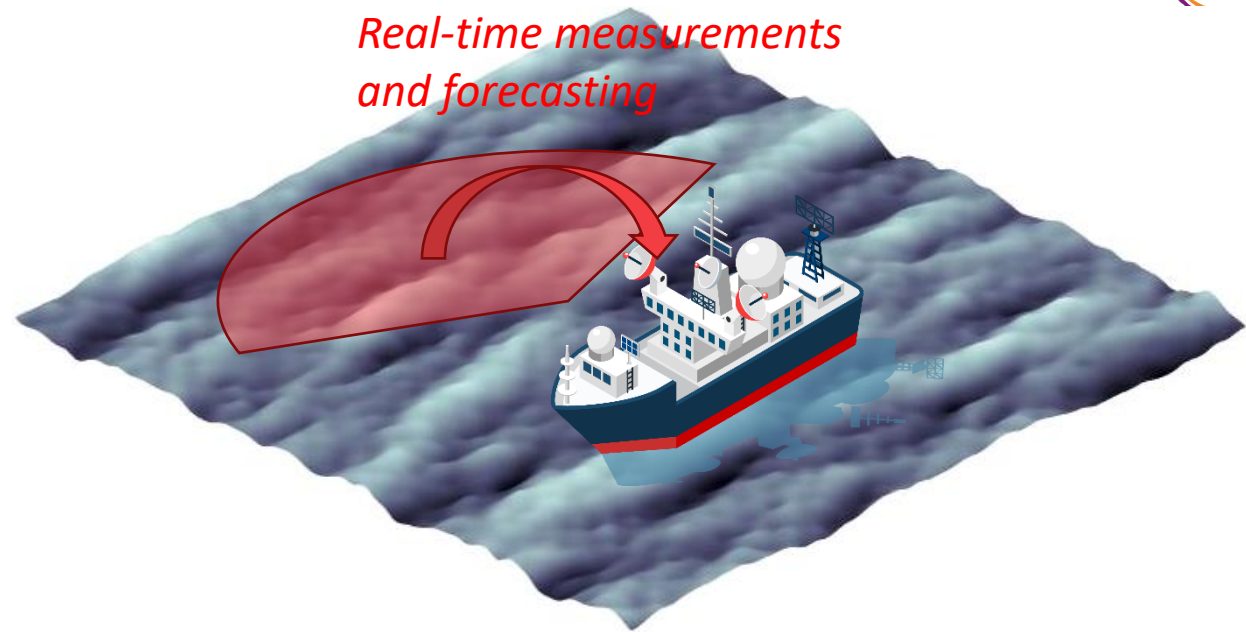


# Wave-by-wave forecasting



Source: ECMWF

Met-ocean forecasts ( $T > 1h$ )



Wave-by-wave forecasts ( $T \approx 1 \text{ min}$ )

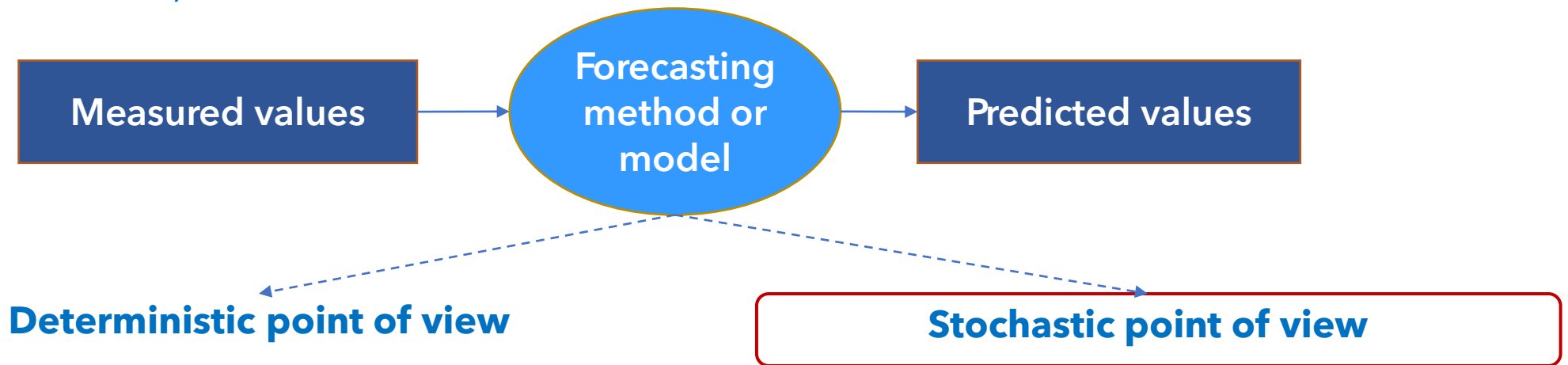
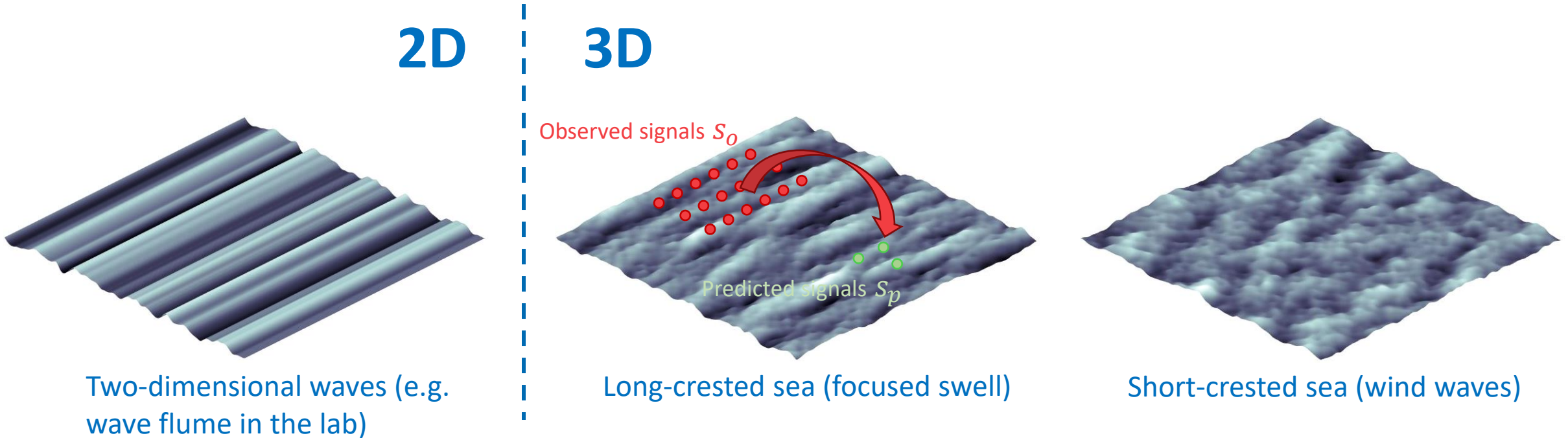
## -> Model-predictive control:

- For motion compensation, dynamic positioning, ship stabilisation, FOWT control, wave energy device control, etc.

## -> Decision support system:

- Through quiescent wave run or dangerous wave train prediction,
- For lifting and drilling operations, transfers and ship-to-ship operations, ROV launch and recovery, helicopter landing, etc.

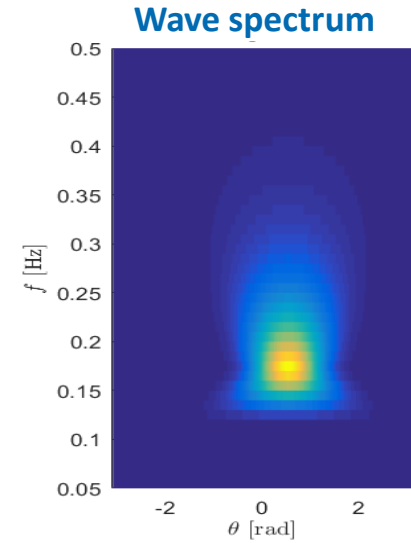
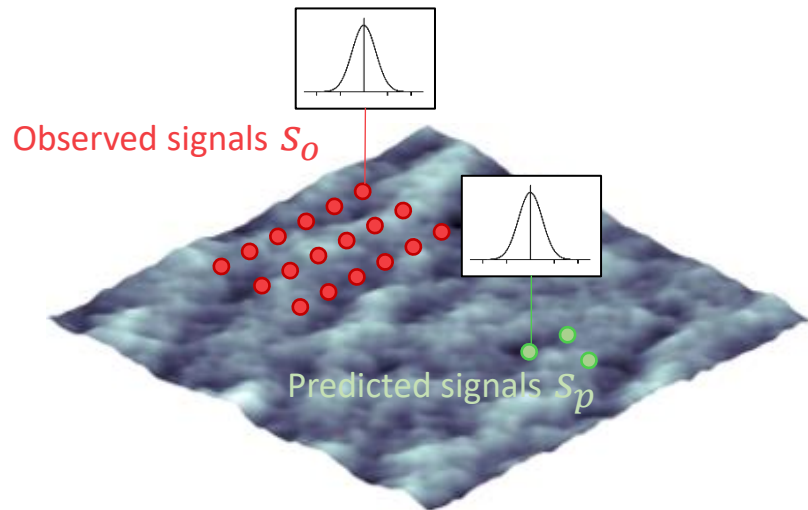
# Wave-by-wave forecasting paradigms



# A stochastic view on wave forecasting

## Classical Gaussian wave description

Over our small scales in space and time, the wave elevation is a homogeneous, stationary, Gaussian random field



Wiener-Khinchine relation

$$\begin{pmatrix} S_o \\ S_p \end{pmatrix} \sim N \left( 0, \begin{pmatrix} R_{oo} & R_{op} \\ R_{po} & R_{pp} \end{pmatrix} \right)$$

$$P = R_{po} R_{oo}^{-1}$$

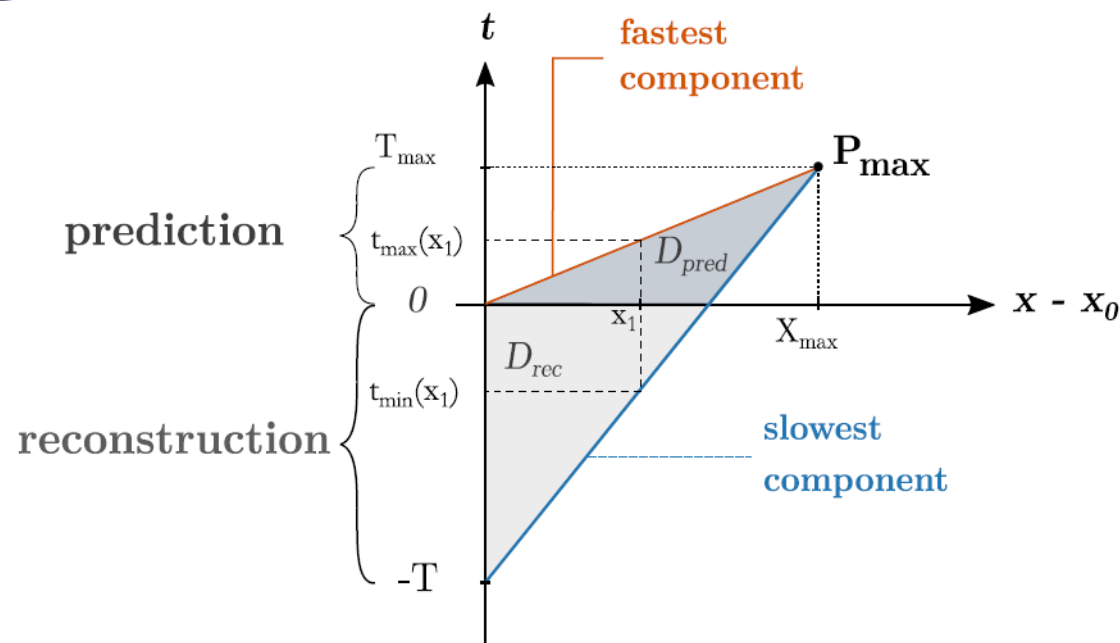
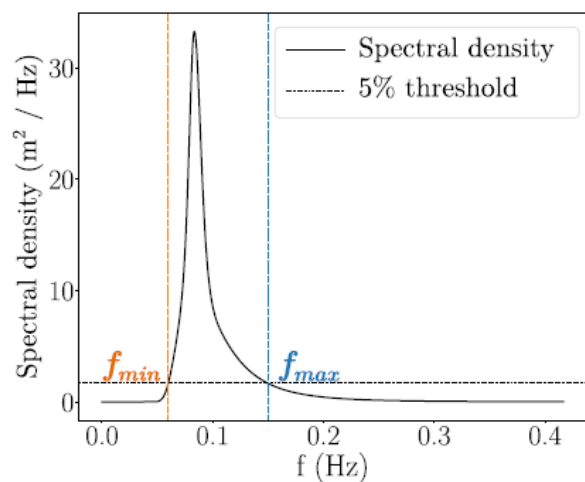
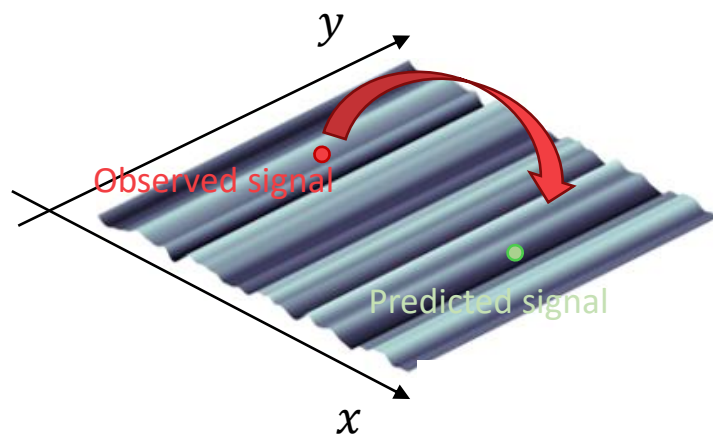
Optimal prediction  $\tilde{s}_p = P S_o$   
with error  $\epsilon = s_p - \tilde{s}_p$

$$R_{\epsilon\epsilon} = R_{pp}^{-1} - R_{po} R_{oo}^{-1} R_{op}$$

Optimal prediction error characterisation

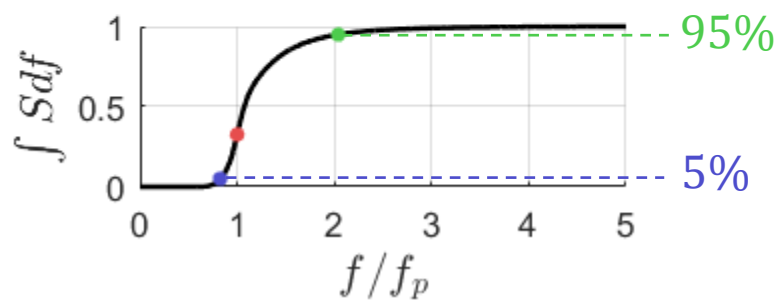
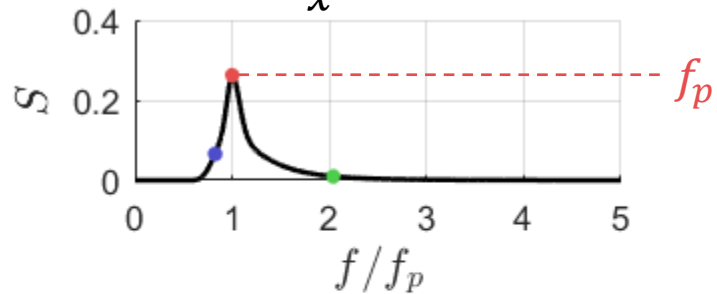
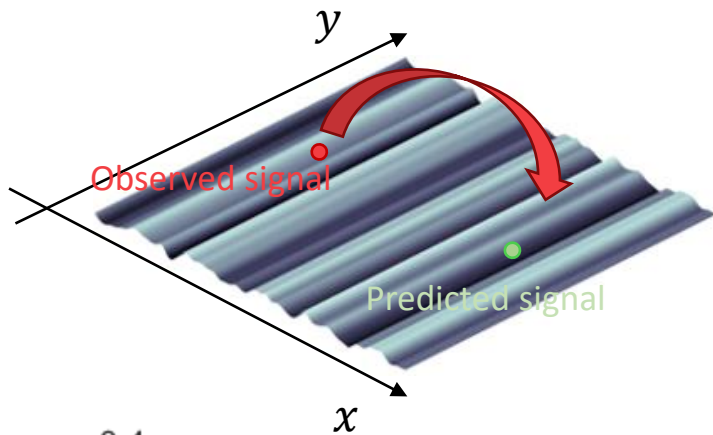
Mérigaud and Ringwood, IEEE JOE, 2018  
Mérigaud and Tona, OMAE 2022

# Deterministic prediction zone in 2D

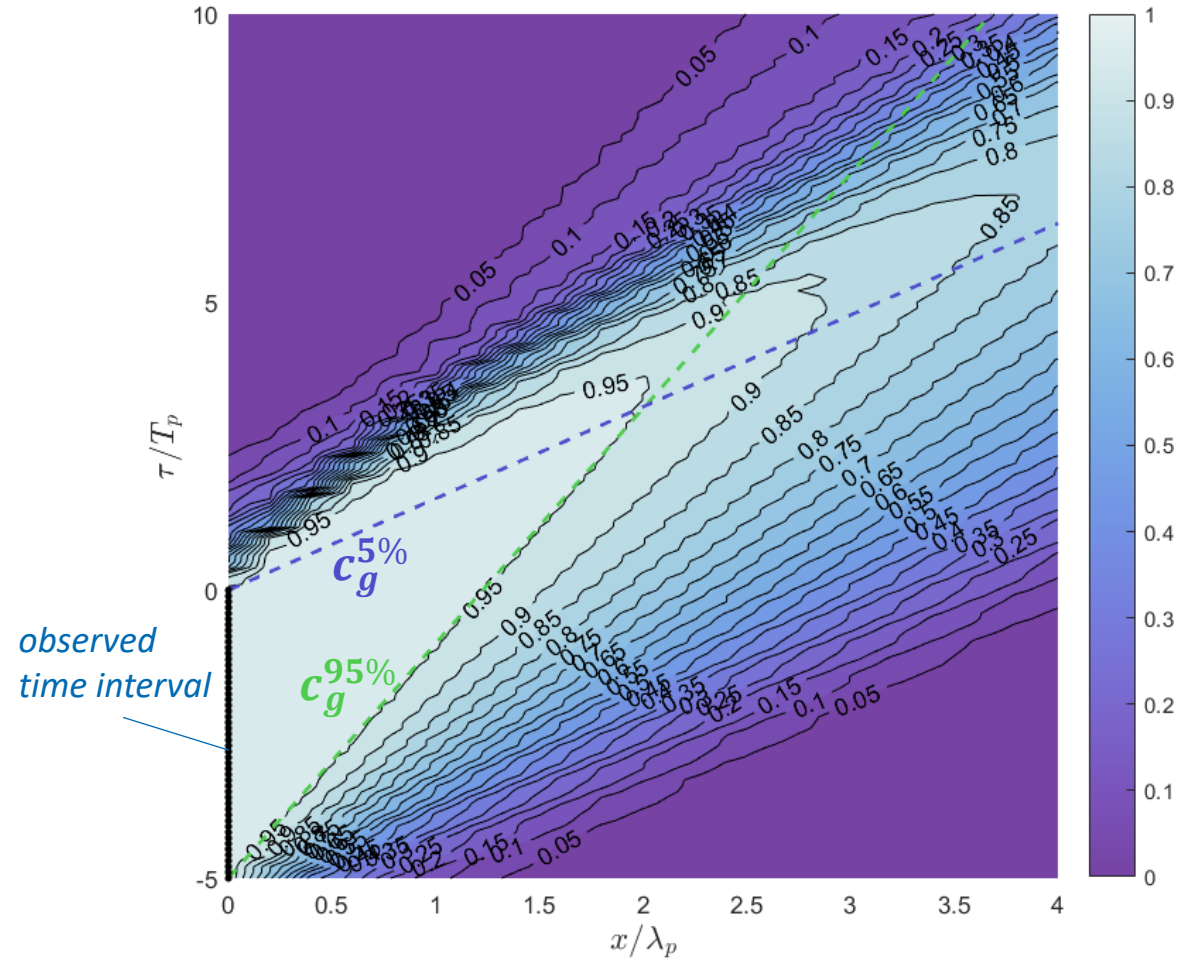


Huchet et al, Ocean Eng. 2023

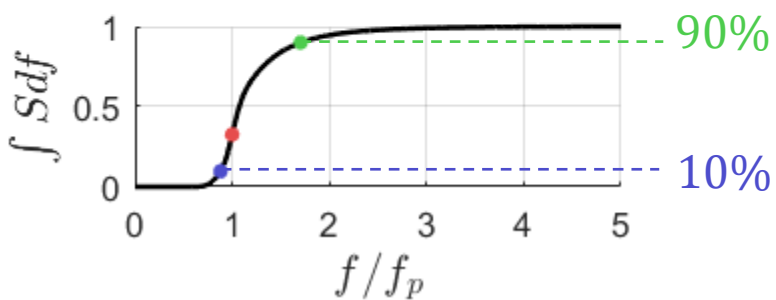
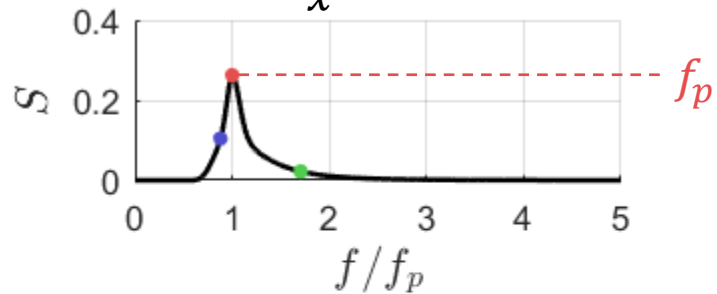
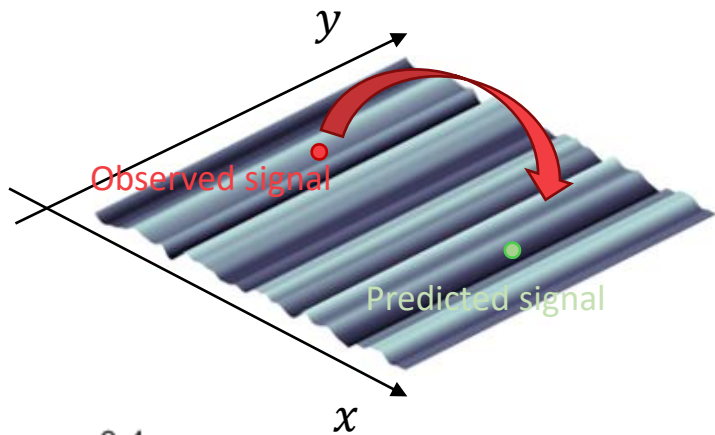
# Probabilistic prediction zone in 2D



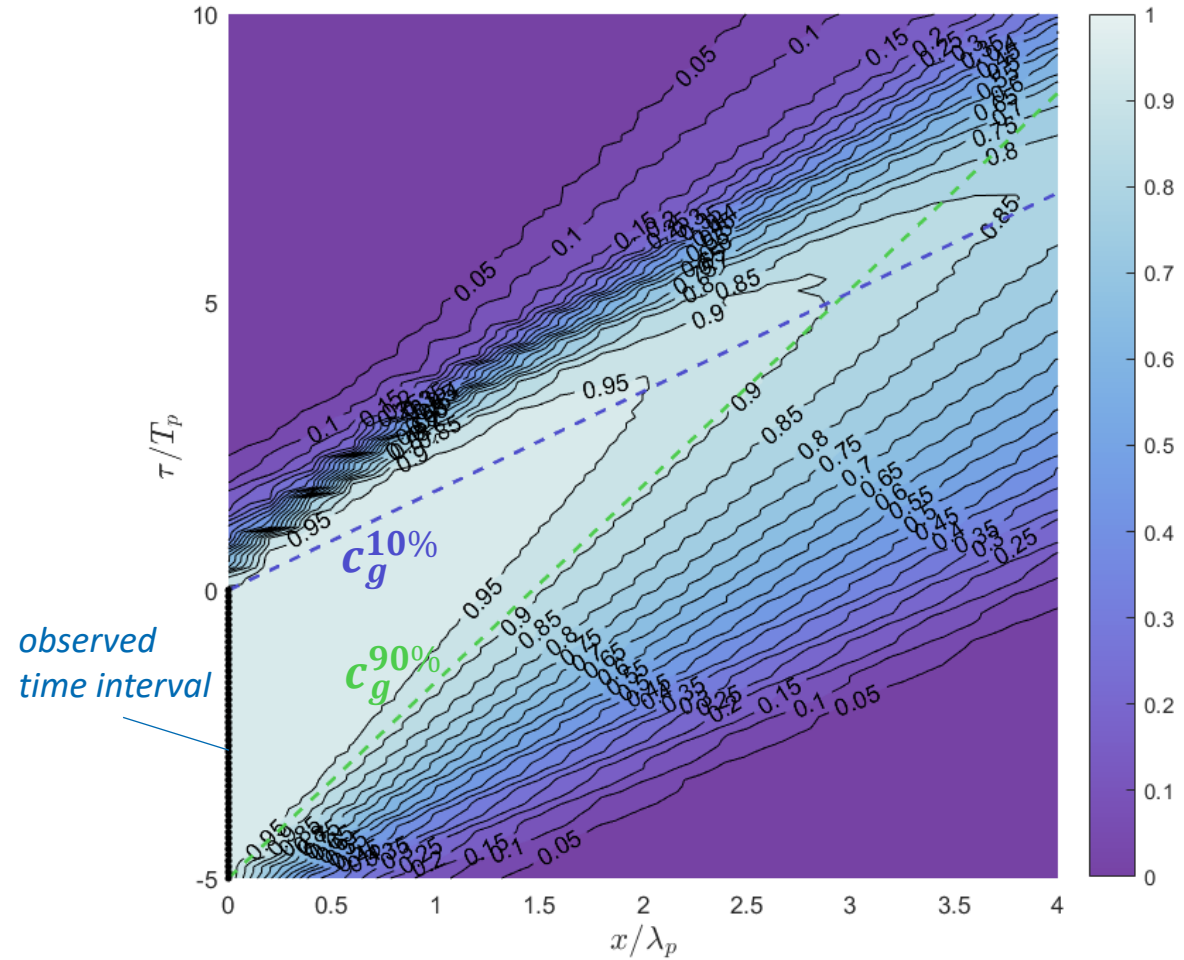
Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$



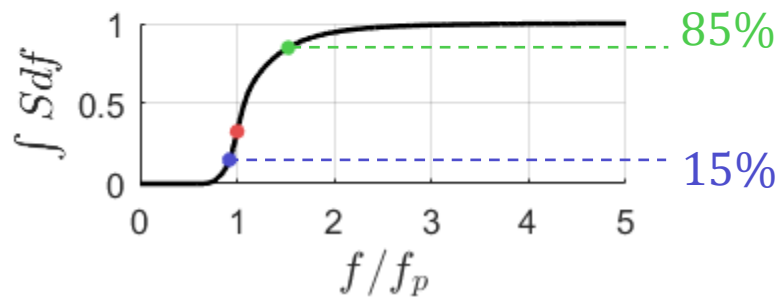
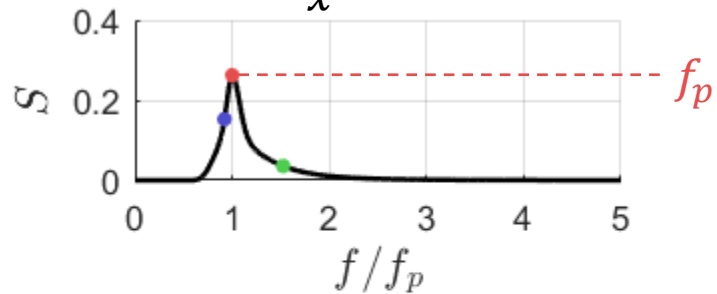
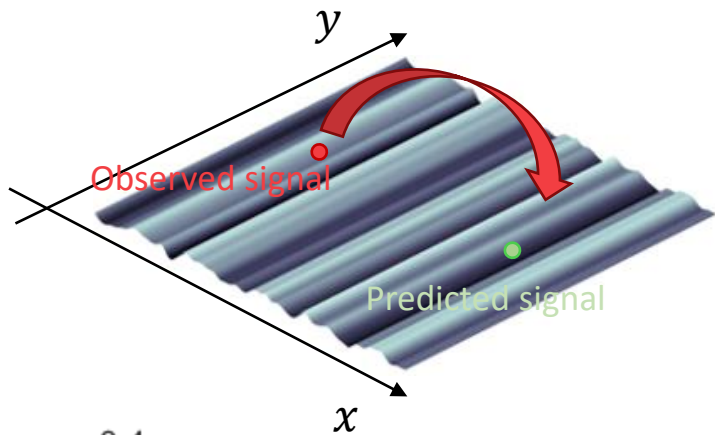
# Probabilistic prediction zone in 2D



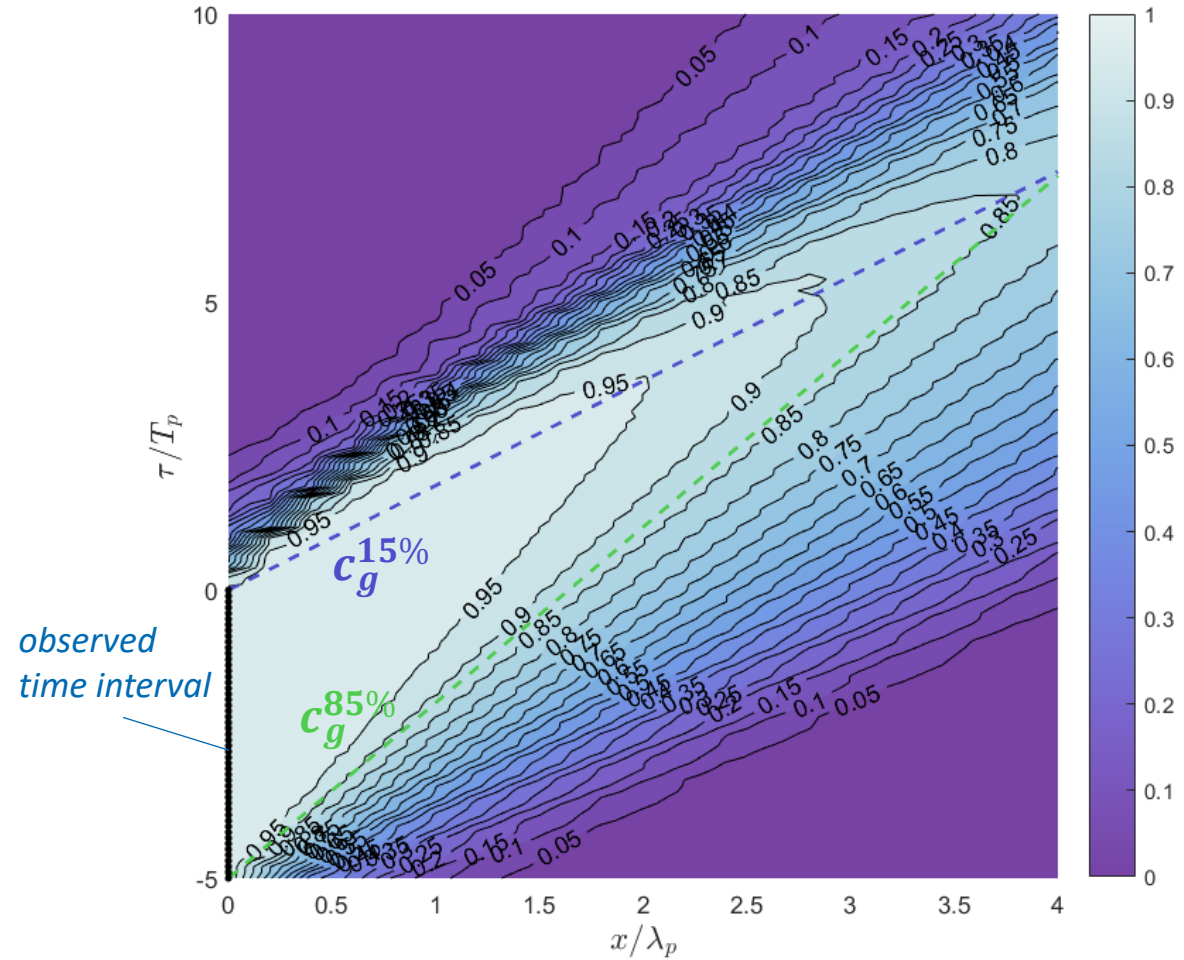
Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$



# Probabilistic prediction zone in 2D

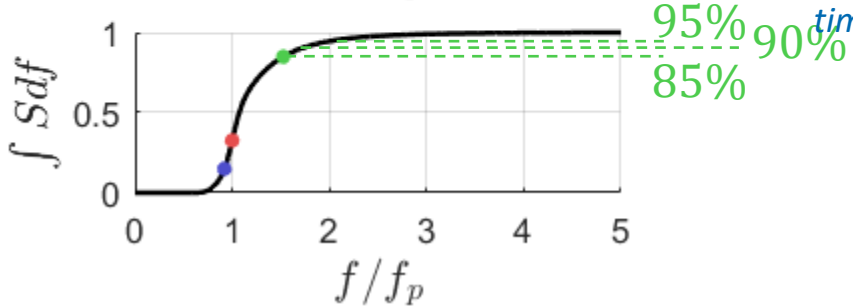
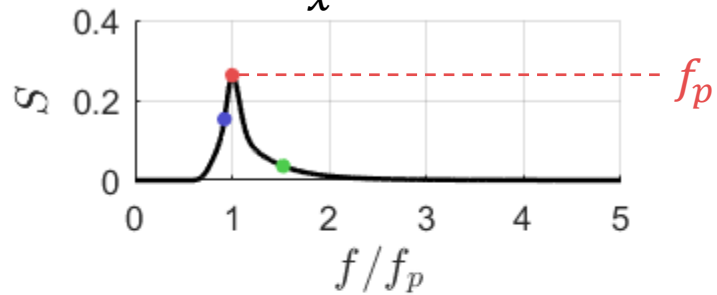
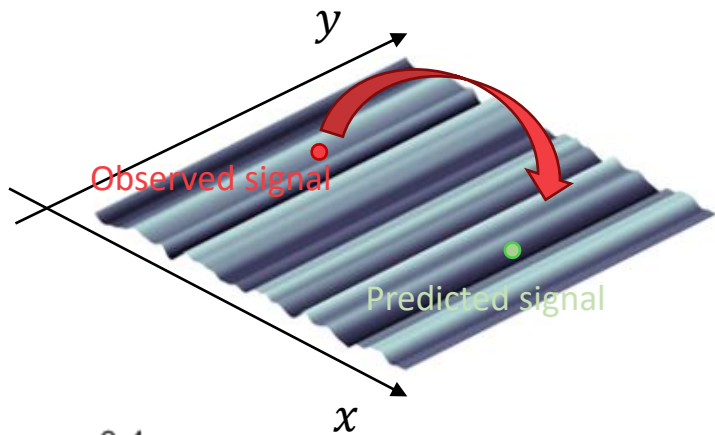


Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$

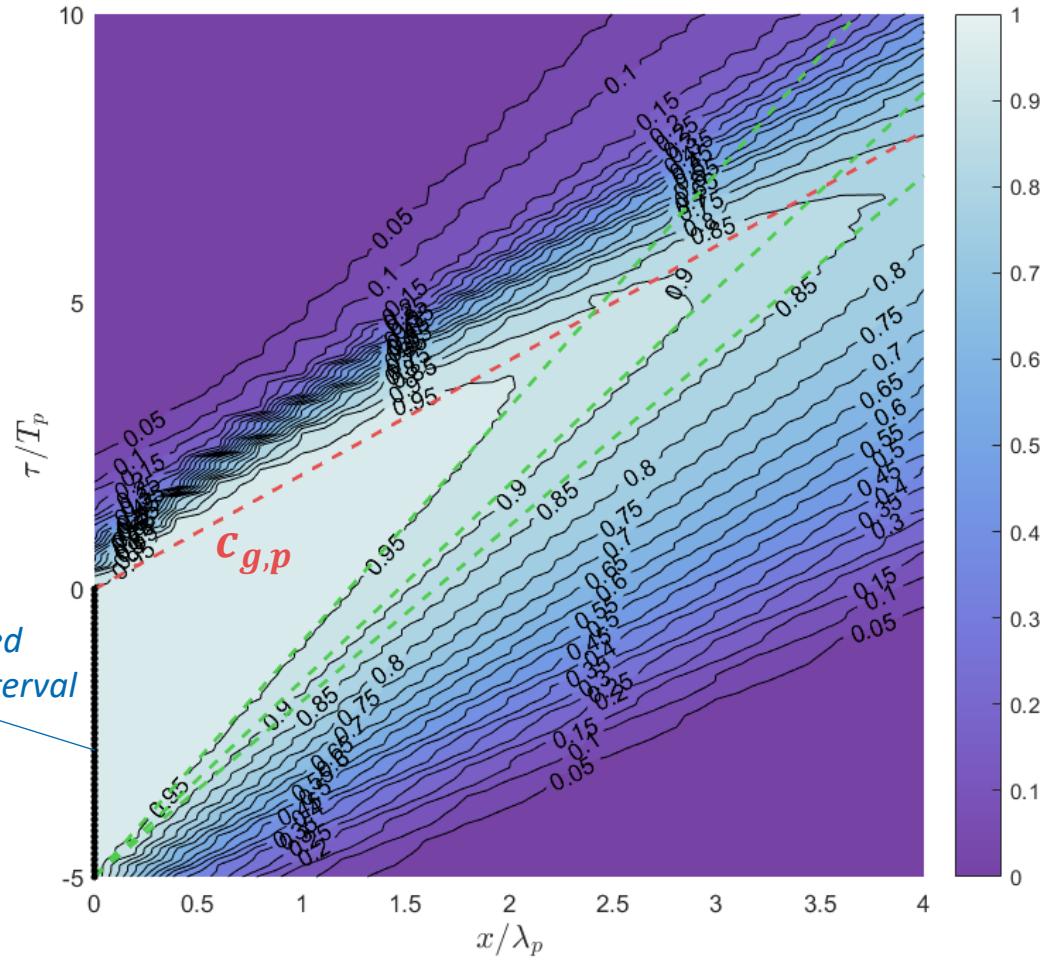




# Probabilistic prediction zone in 2D



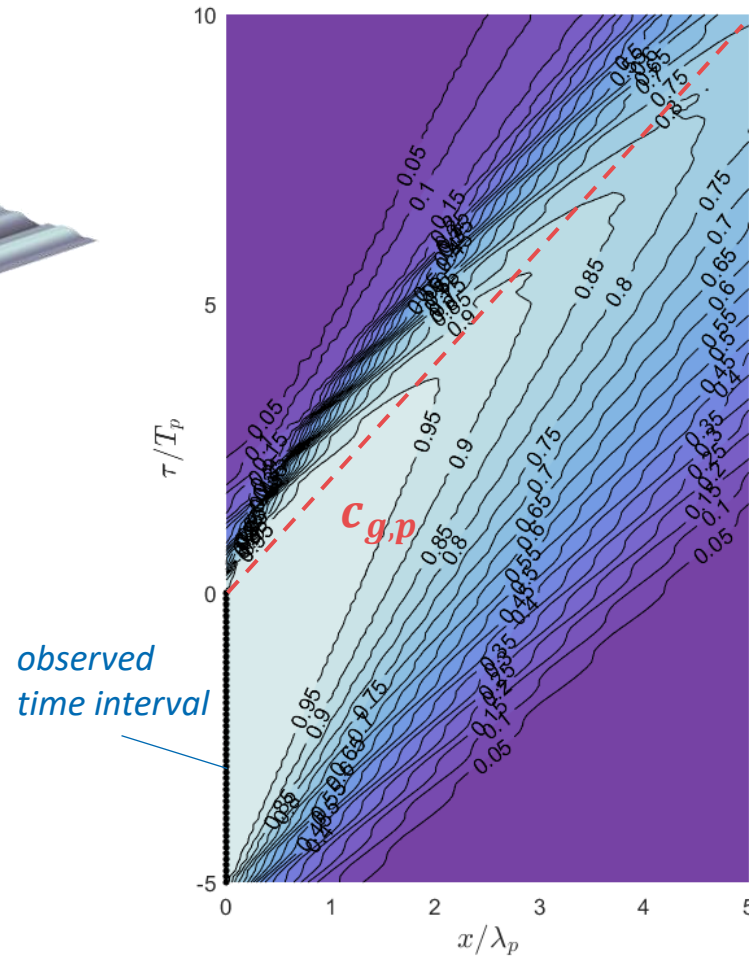
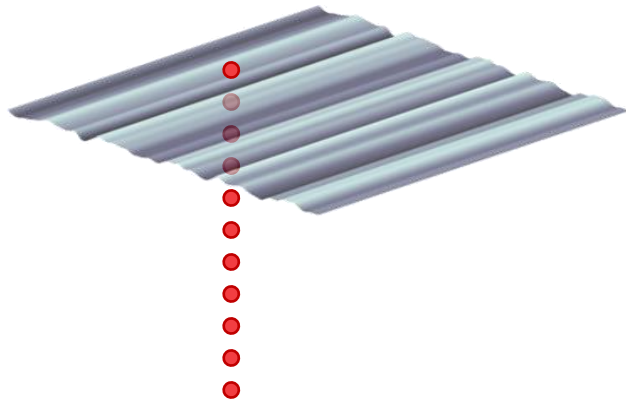
Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle s_p^2 \rangle}$



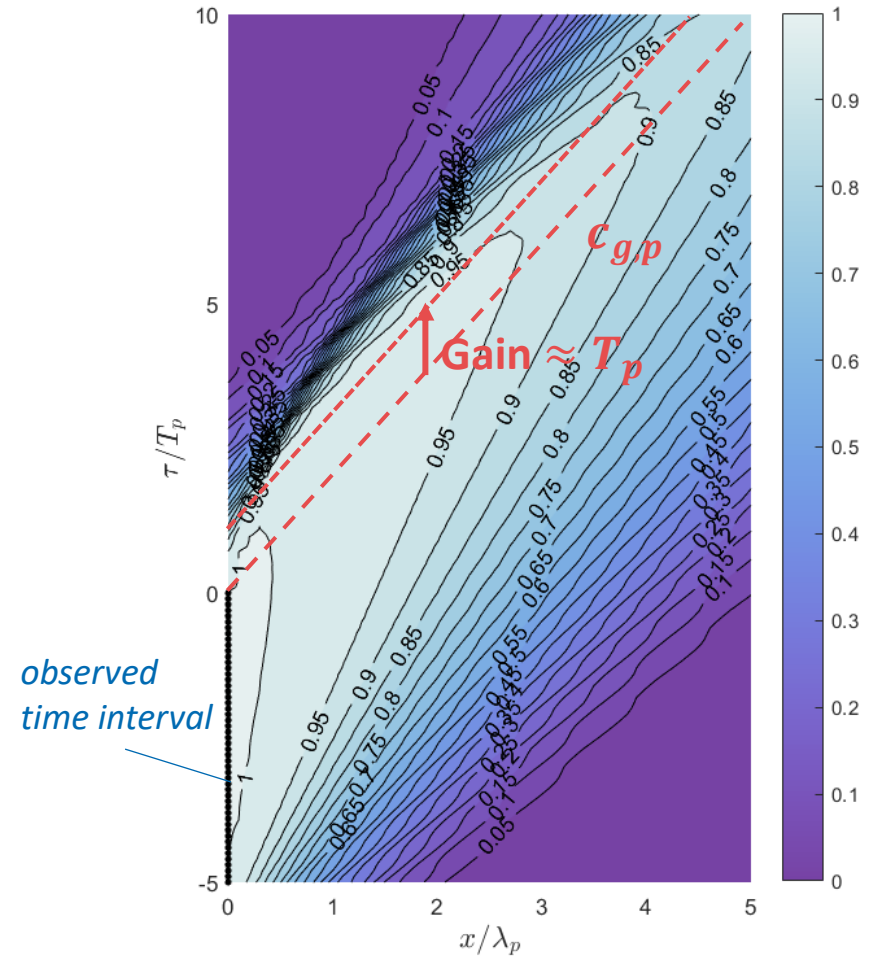
observed time interval

# What about water column measurements ?

Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle s_p^2 \rangle}$



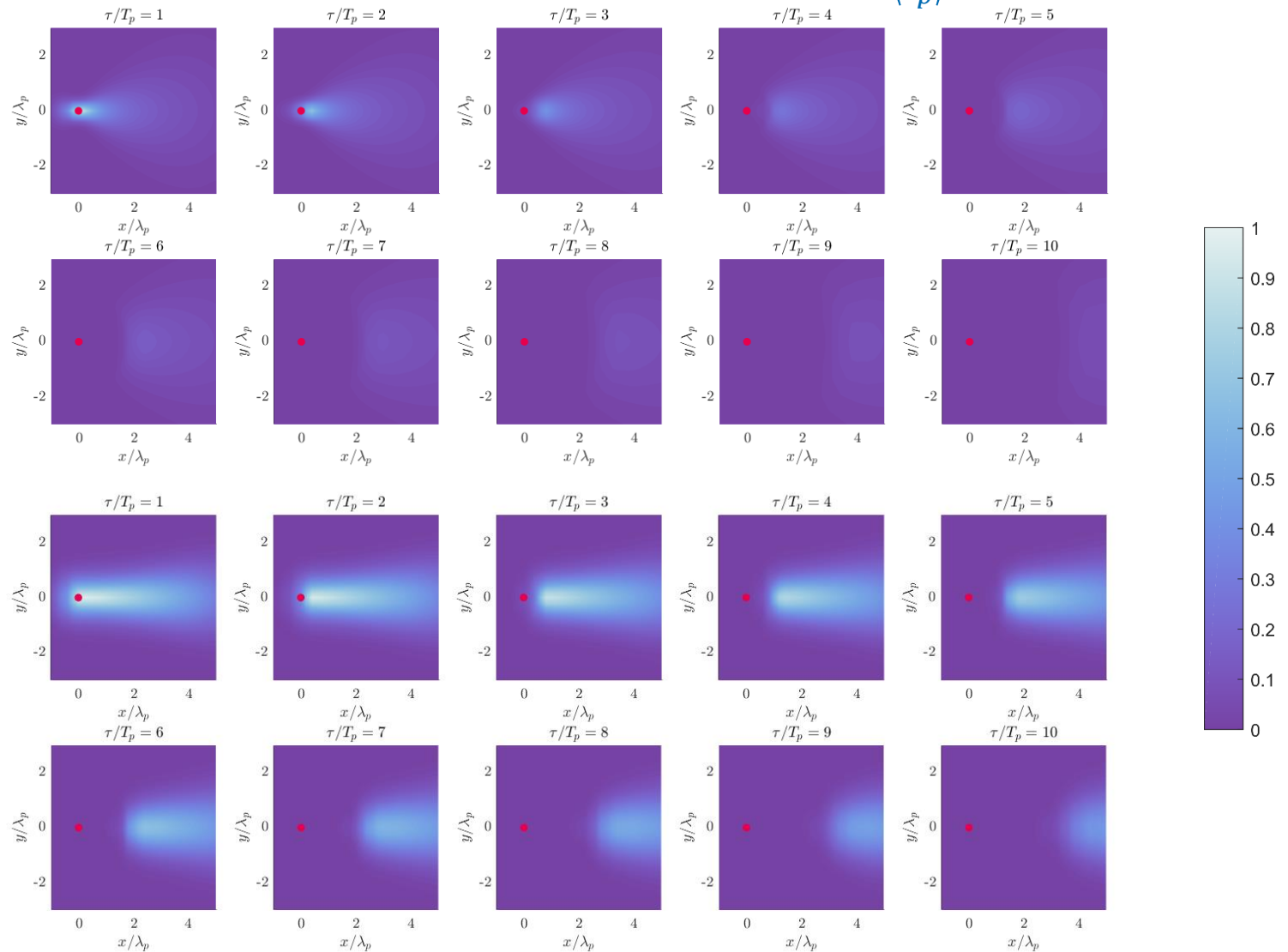
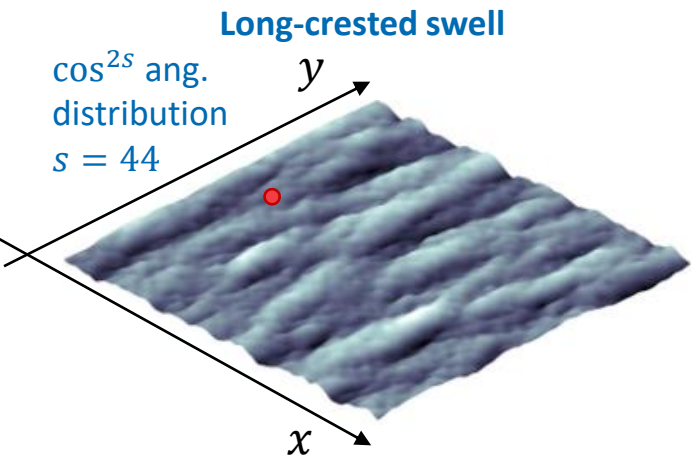
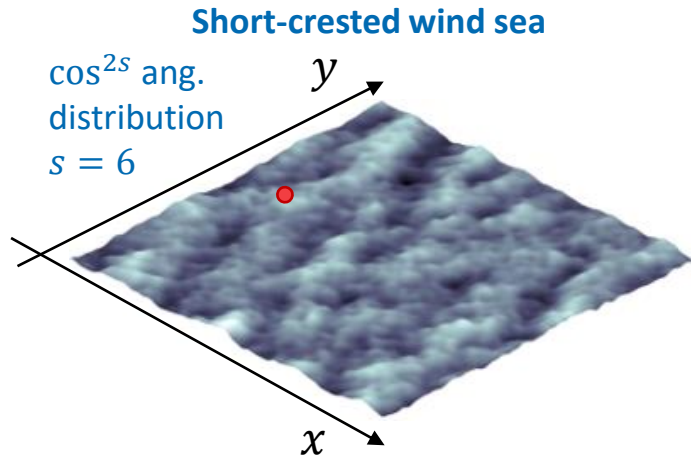
Measuring  $\eta$  only



Measuring  $\eta$  + the pressure along the water column

# Probabilistic prediction zone in 3D

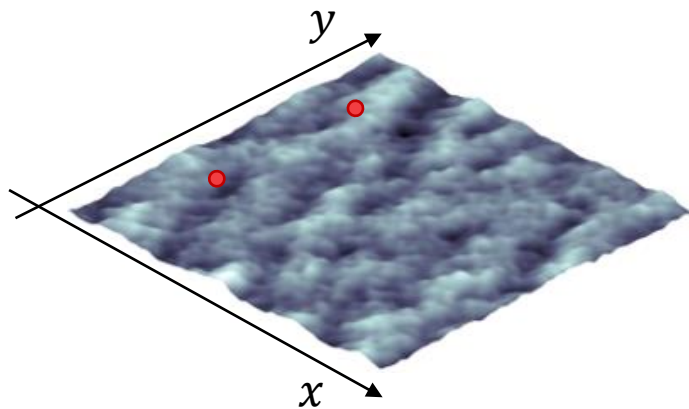
Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$



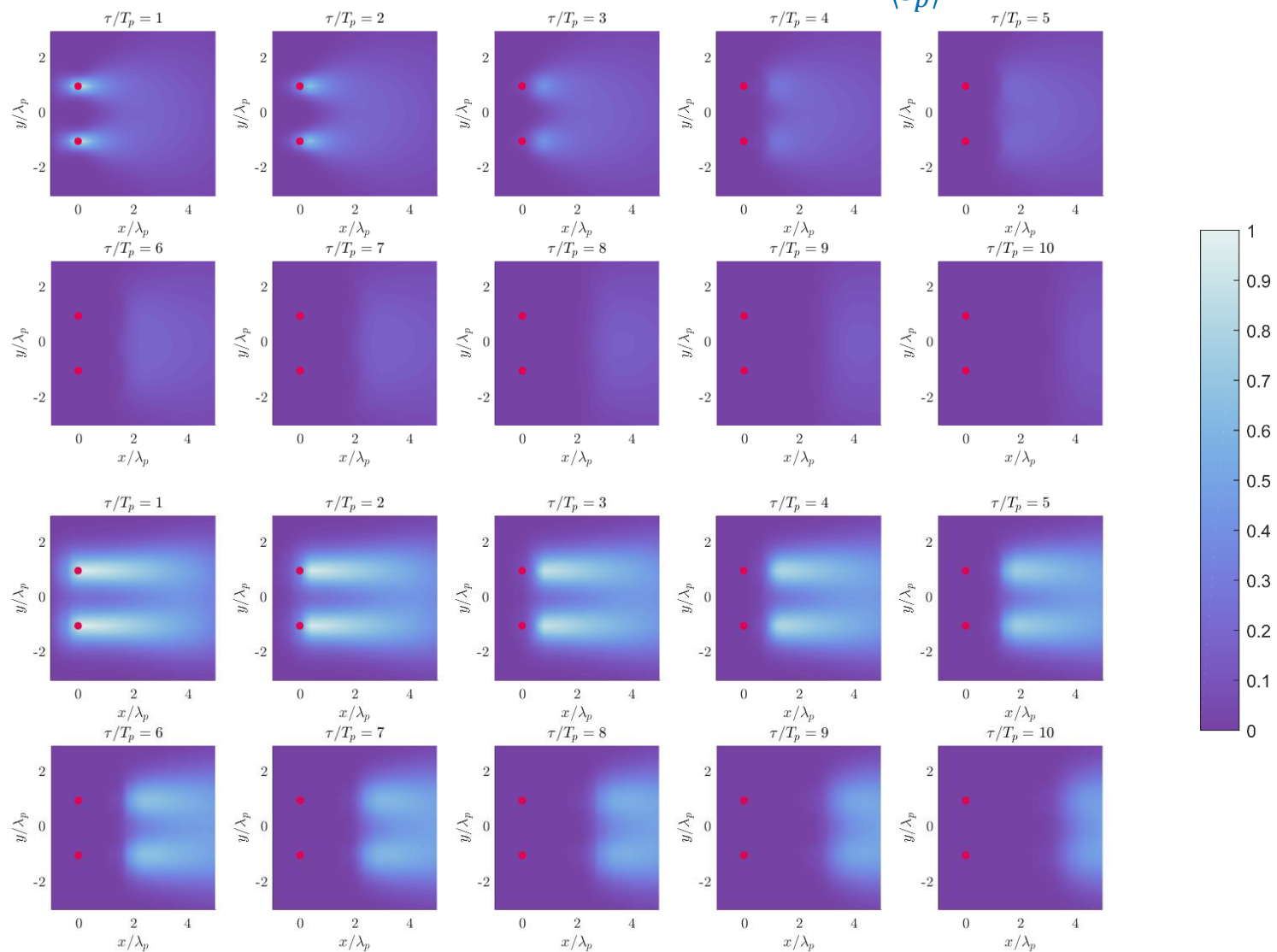
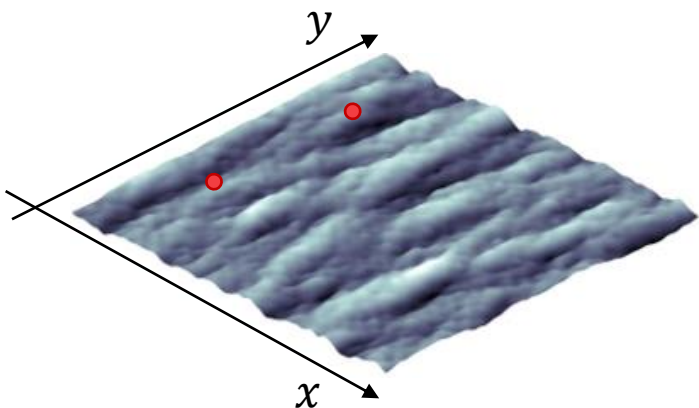
# Probabilistic prediction zone in 3D

Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$

Short-crested wind sea



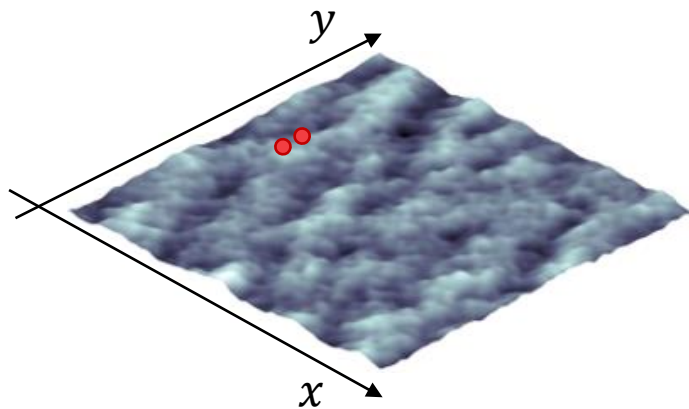
Long-crested swell



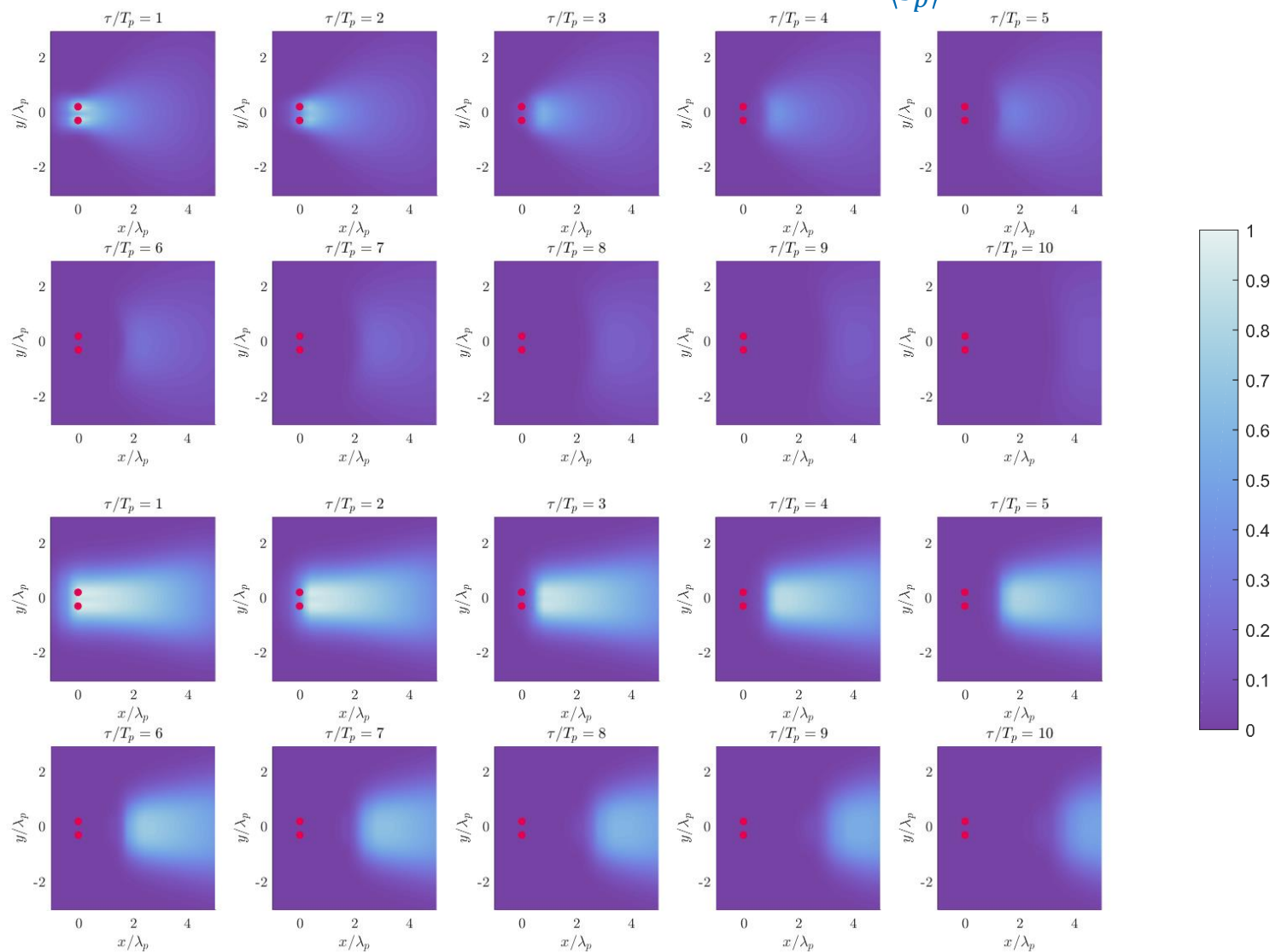
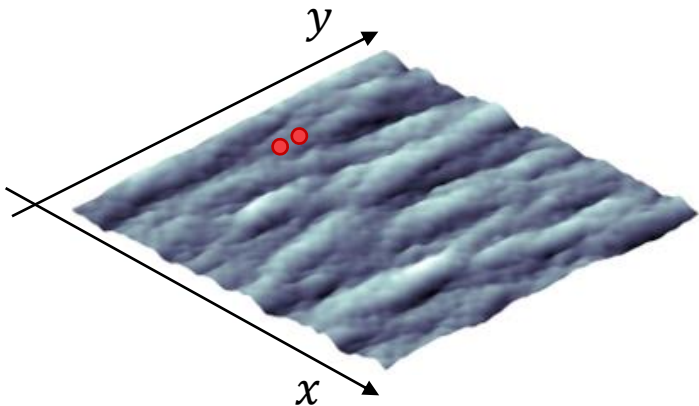
# Probabilistic prediction zone in 3D

Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$

Short-crested wind sea



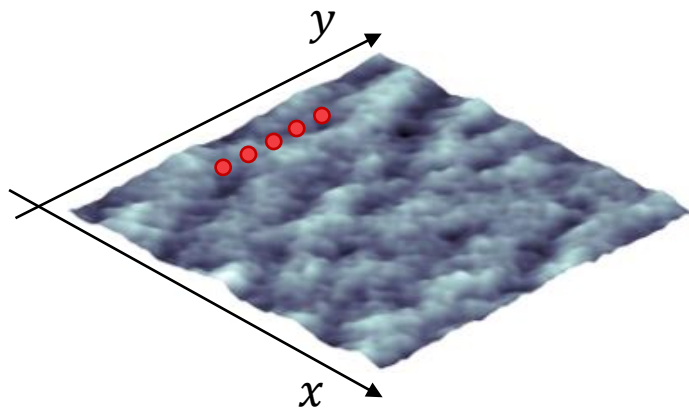
Long-crested swell



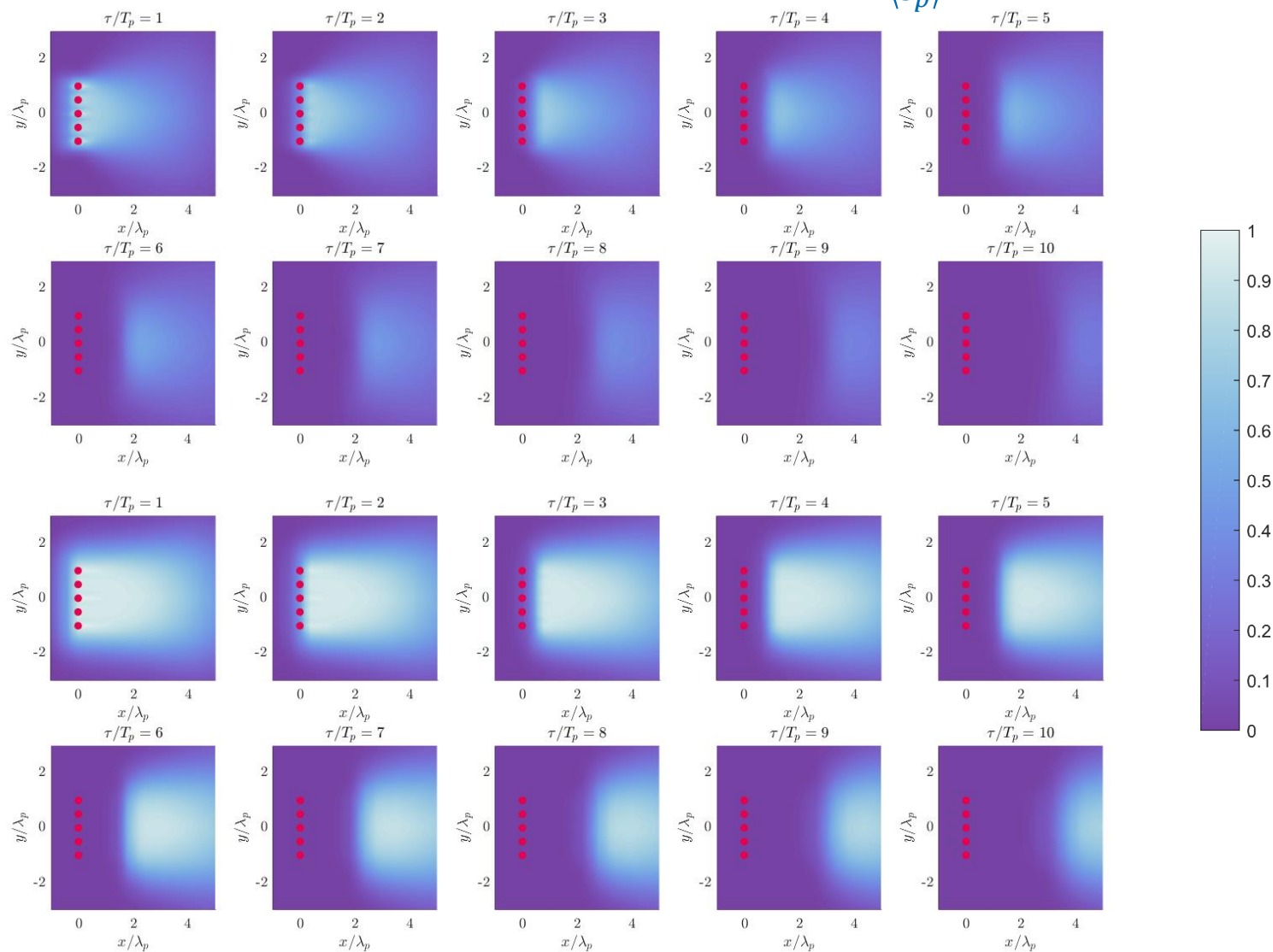
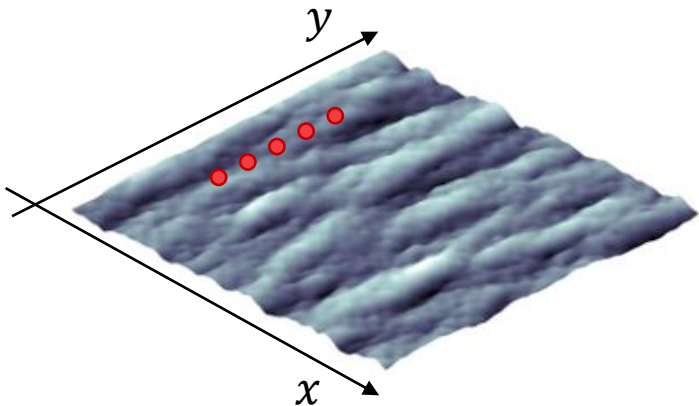
# Probabilistic prediction zone in 3D

Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$

Short-crested wind sea



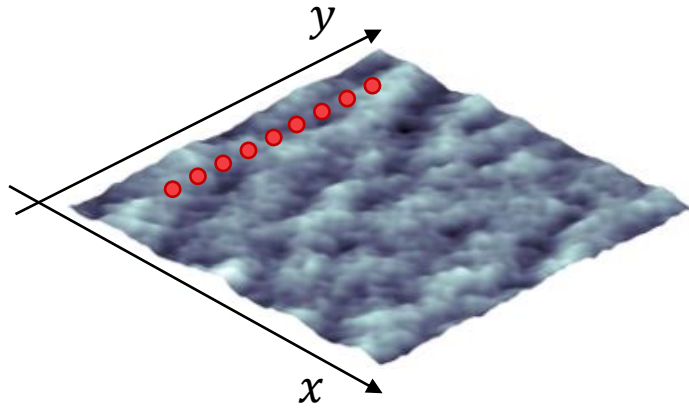
Long-crested swell



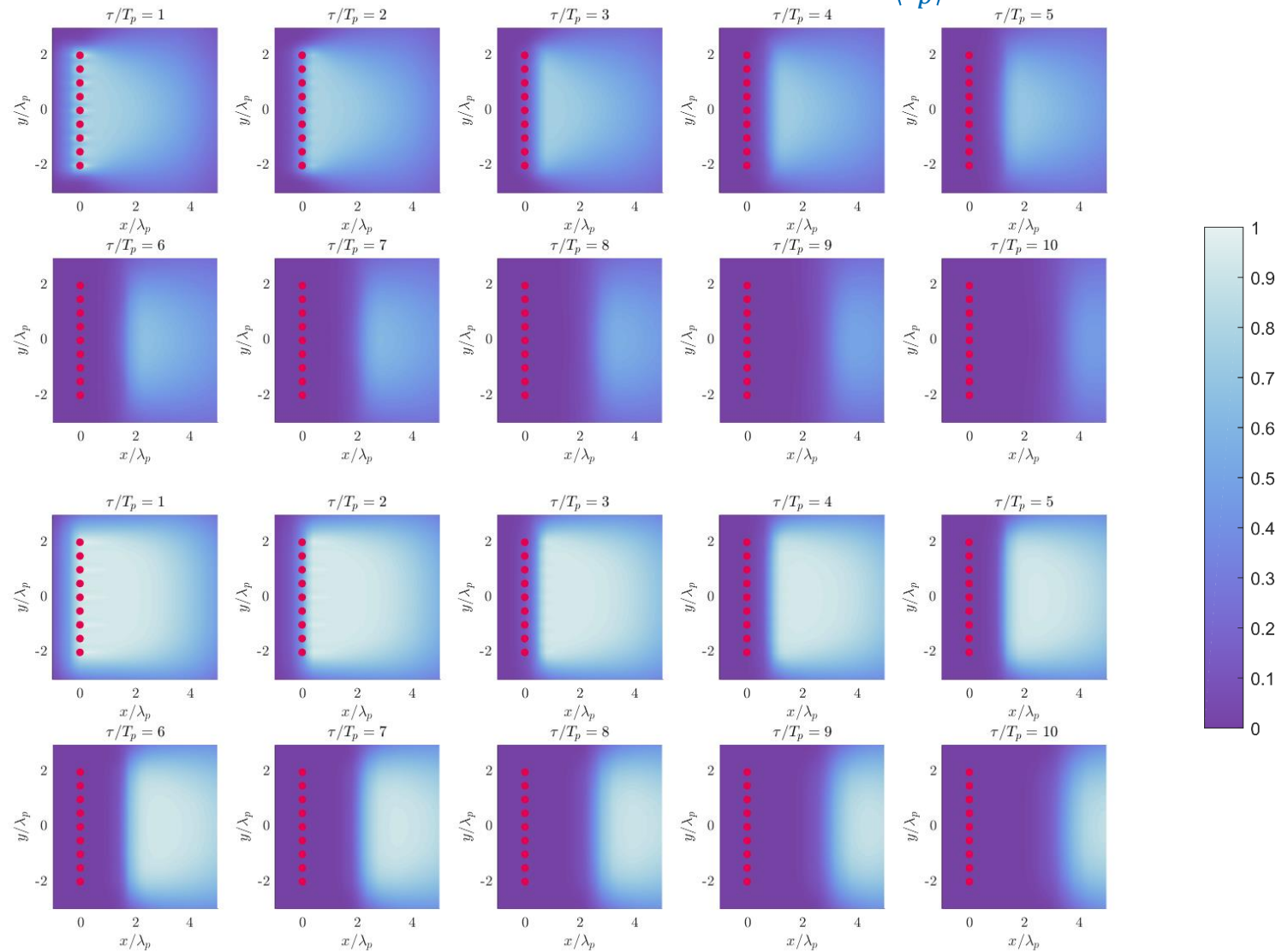
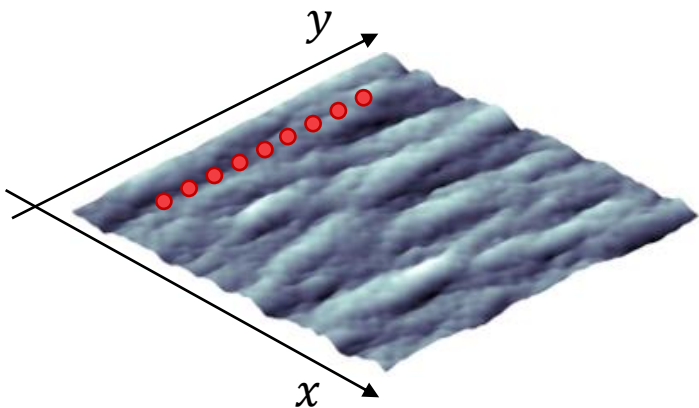
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Ratio of predicted energy  $r = 1 - \frac{\langle \epsilon^2 \rangle}{\langle S_p^2 \rangle}$

Short-crested wind sea

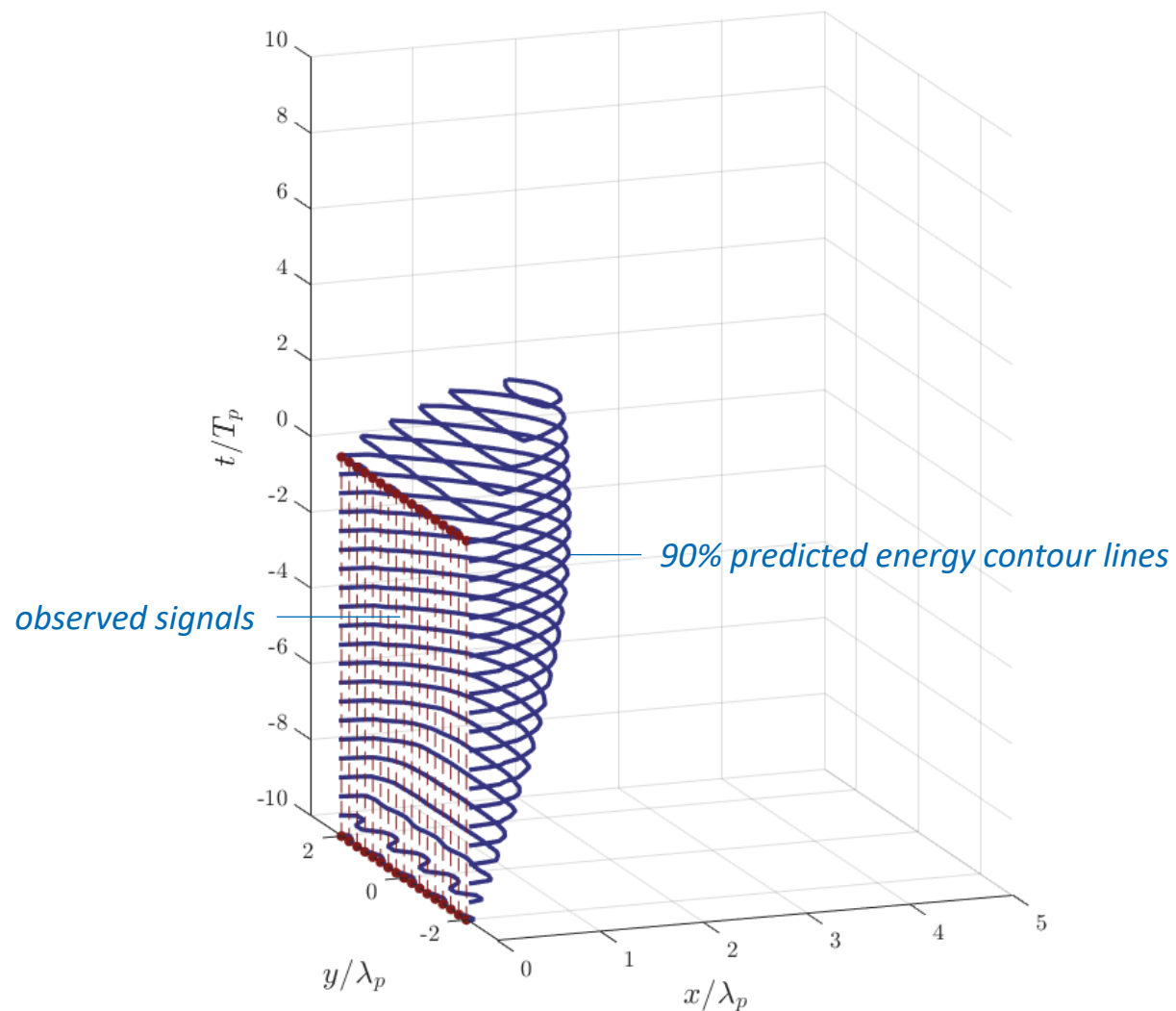
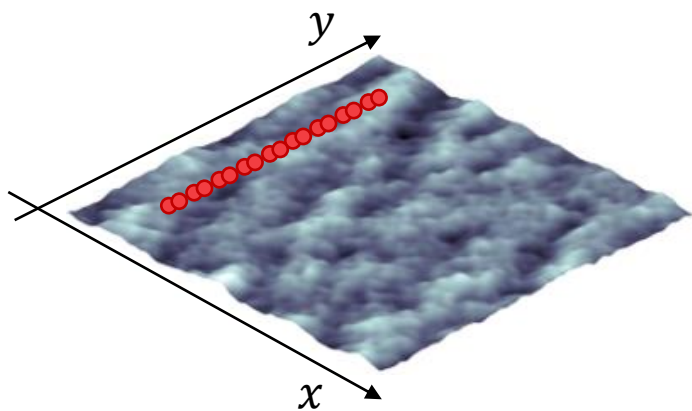


Long-crested swell



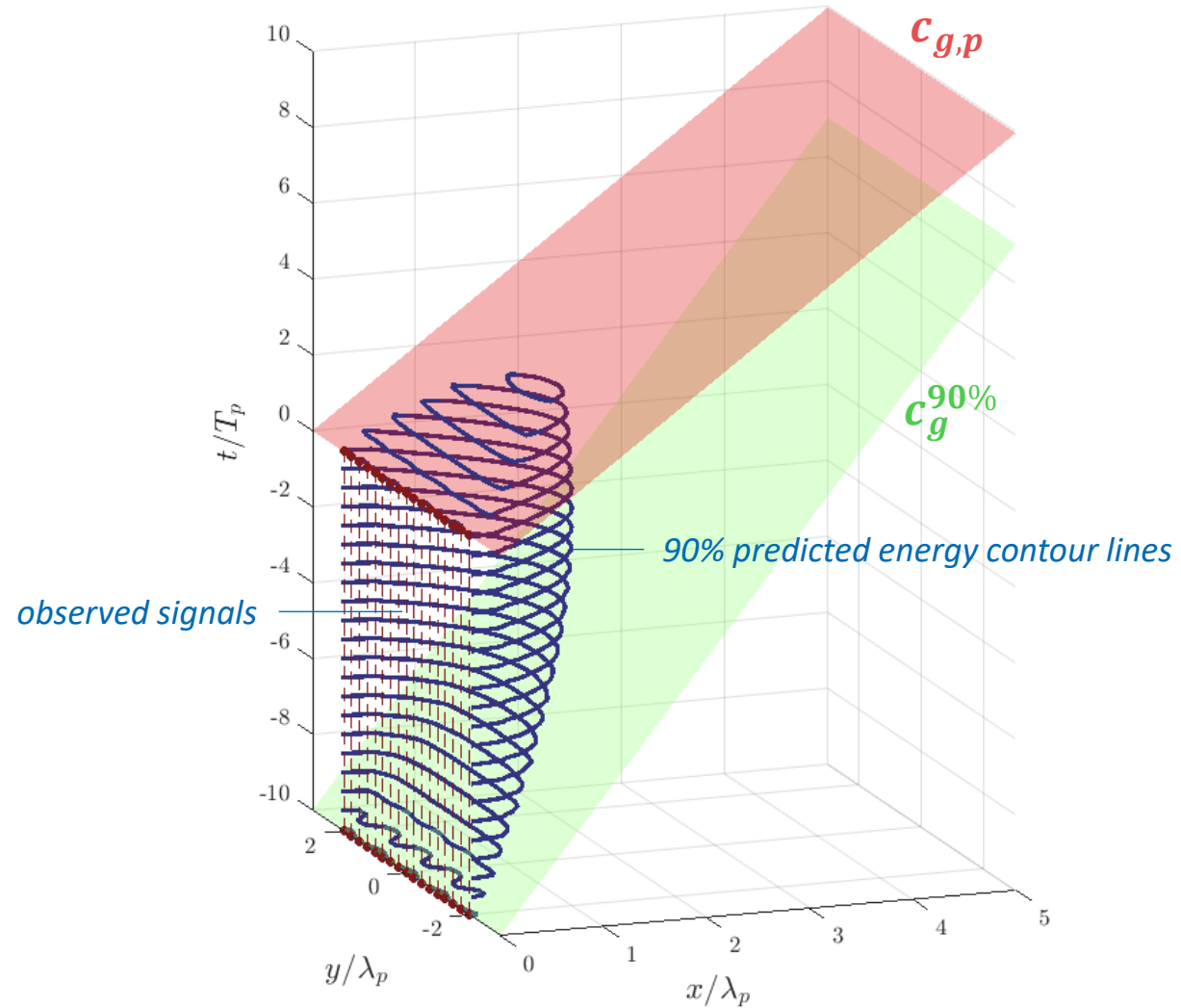
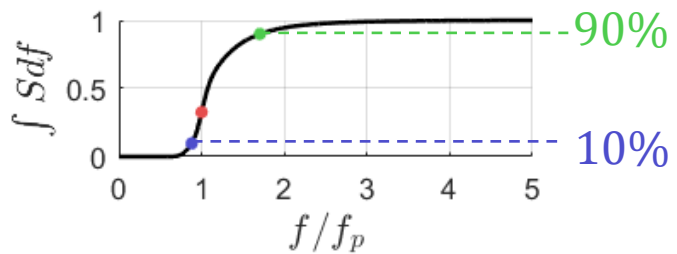
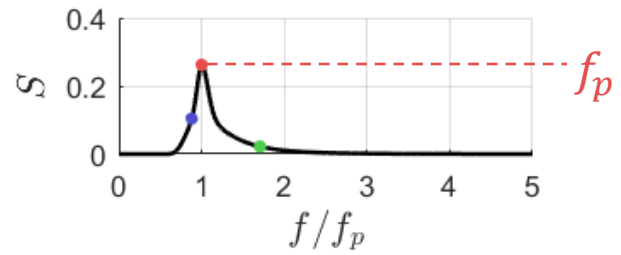
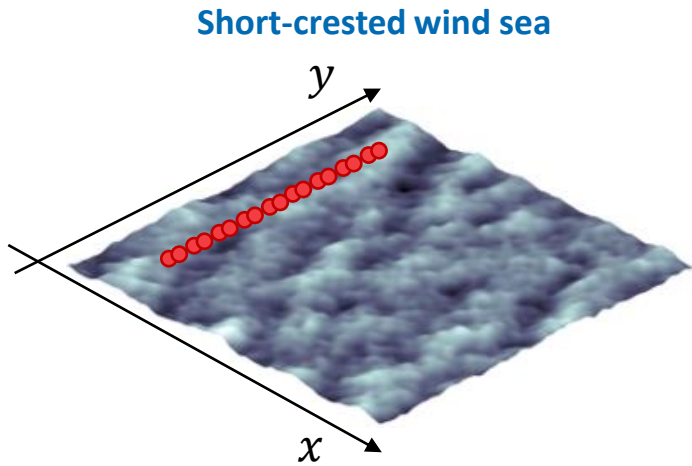
# Probabilistic prediction zone in 3D

Short-crested wind sea

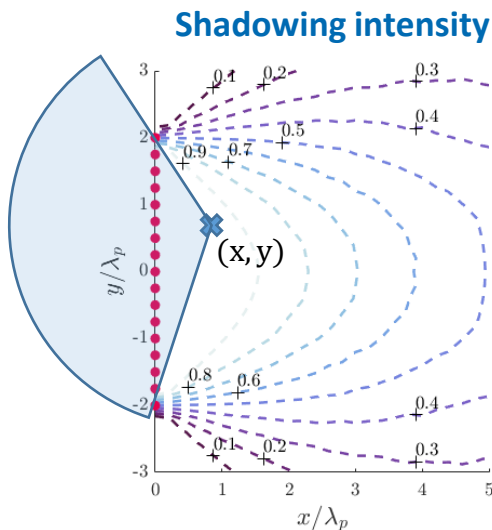
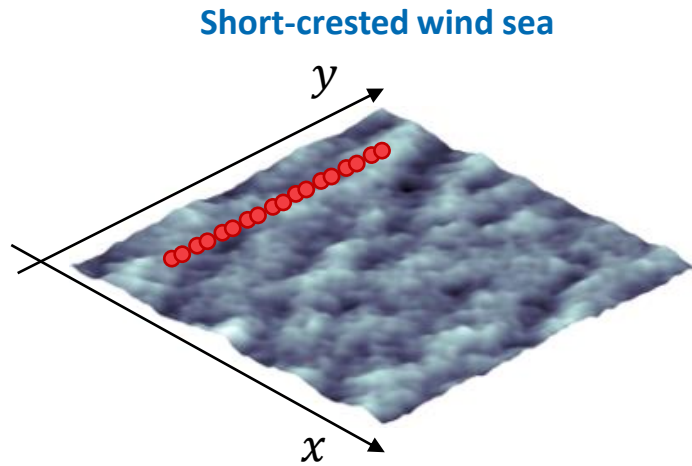




# Probabilistic prediction zone in 3D

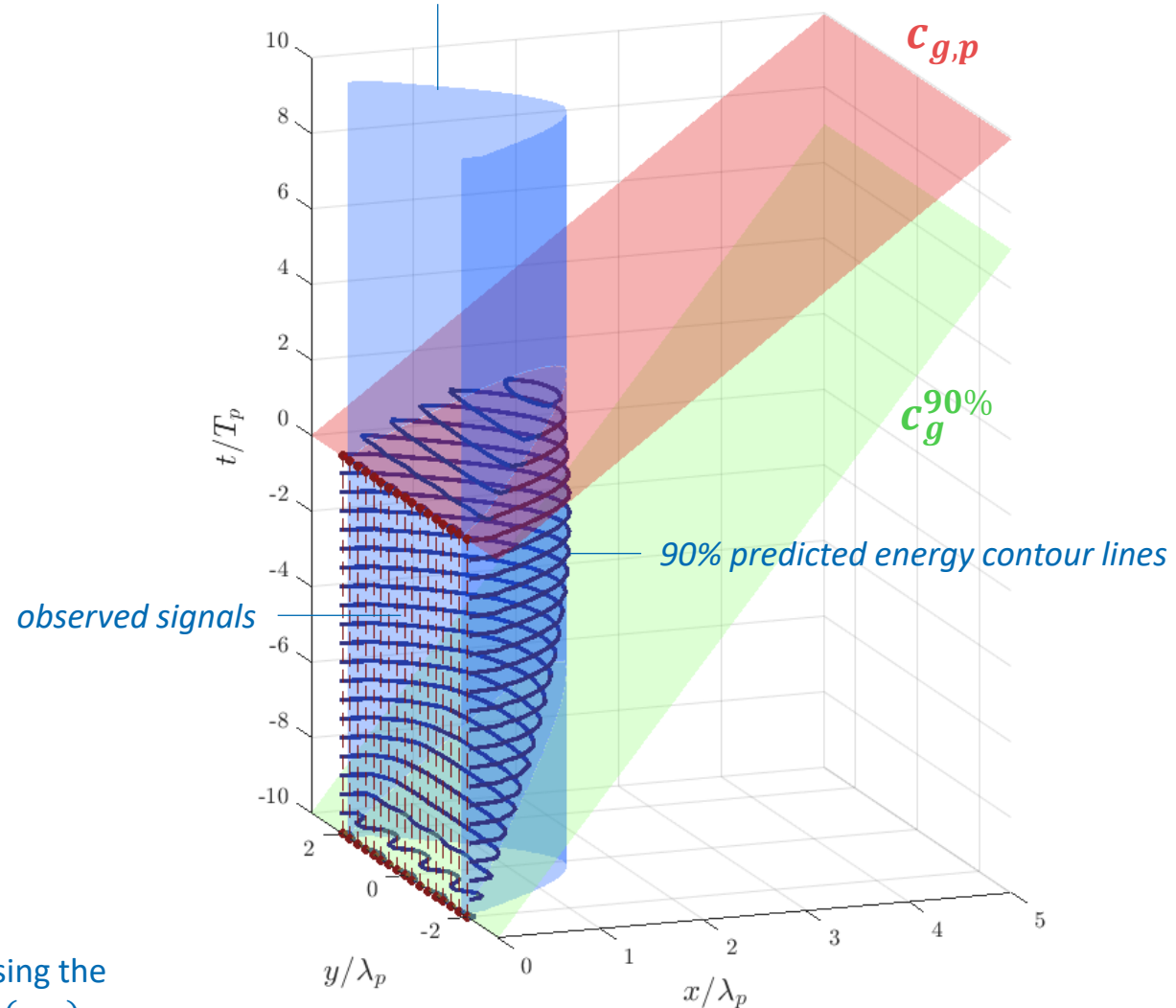


# Probabilistic prediction zone in 3D

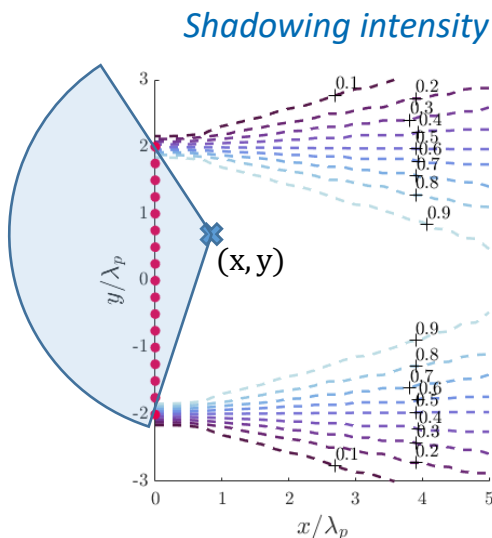
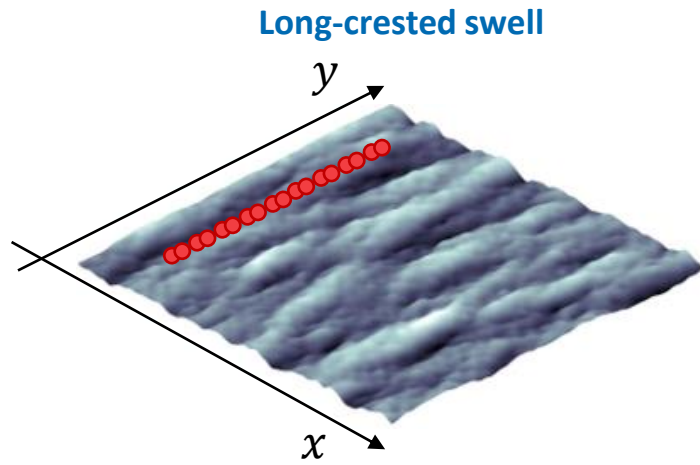


fraction of directional wave energy crossing the observation array before reaching point (x, y)

90% shadowing intensity contour surface

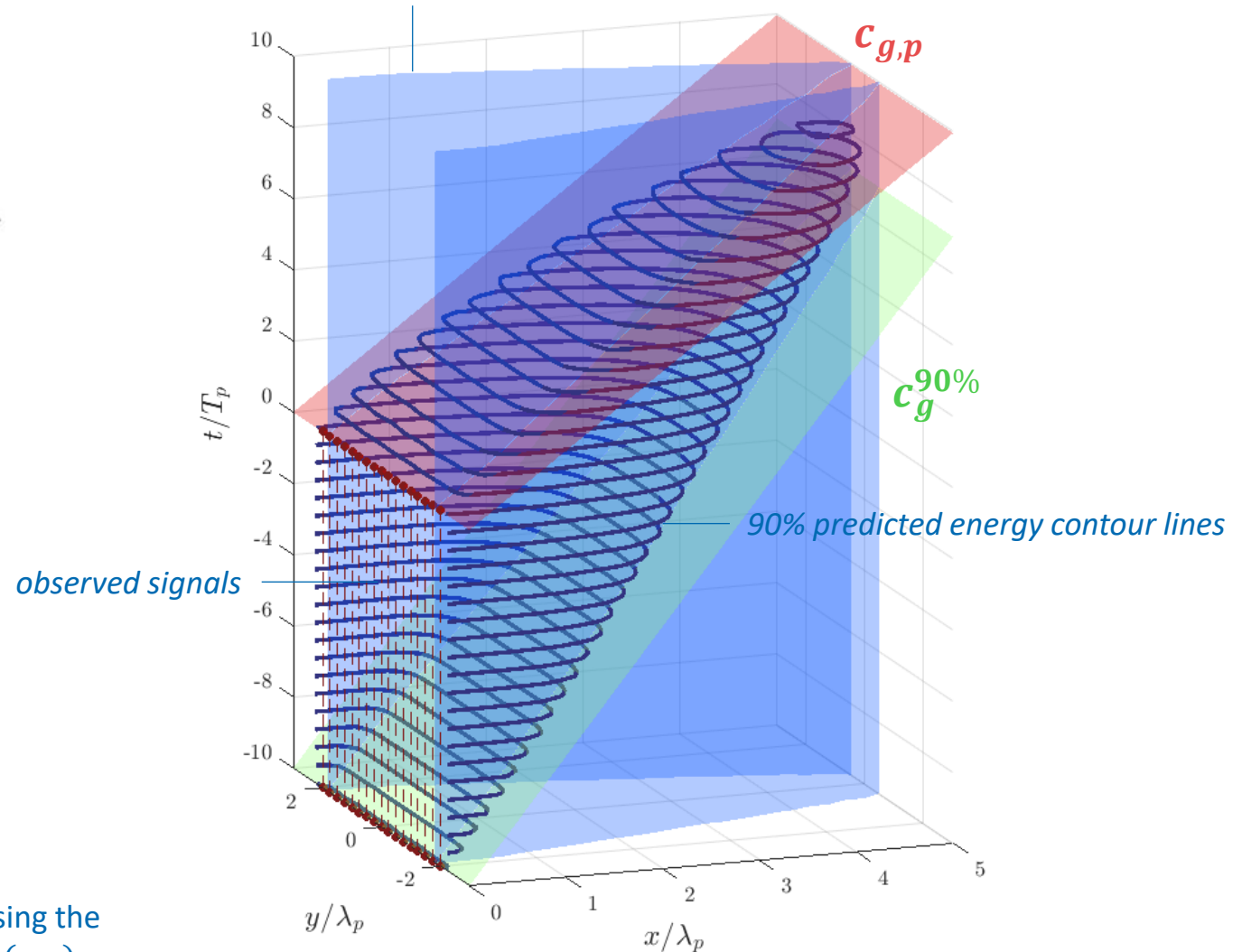


# Probabilistic prediction zone in 3D



fraction of directional wave energy crossing the observation array before reaching point  $(x, y)$

90% shadowing intensity contour surface



# Conclusions

- There is no « deterministic prediction zone », strictly speaking.
- In 2D, the shape of probabilistic prediction zones, defined by desired error thresholds, can be geometrically approximated from two group velocities:
  - An energy cutoff threshold for the minimum group velocity
  - The peak frequency for the maximum group velocity
- In 2D, the forecasting horizon can be extended indefinitely.
- In 3D, arrays of closely-spaced ( $\approx \frac{\lambda_p}{2}$ ) observation points are required to obtain appreciable prediction zones.
- In 3D, the min and max group velocities of the 2D case provide upper and lower bounds to the prediction zones.
- In 3D, the « shadow intensity », cast by the array, provides an envelope that further limits the extent of the prediction zones.

alexis.merigaud@ifpen.fr

paolino.tona@ifpen.fr

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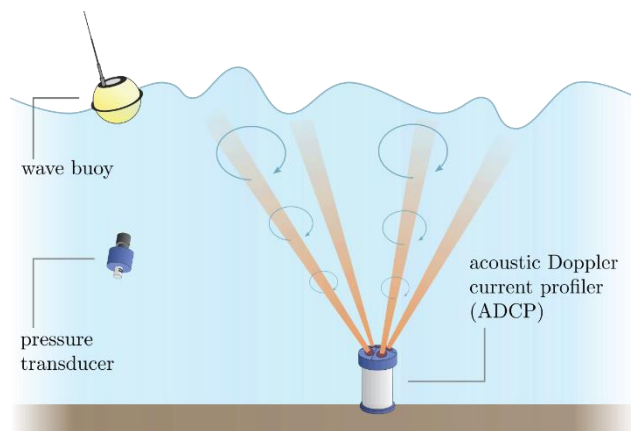
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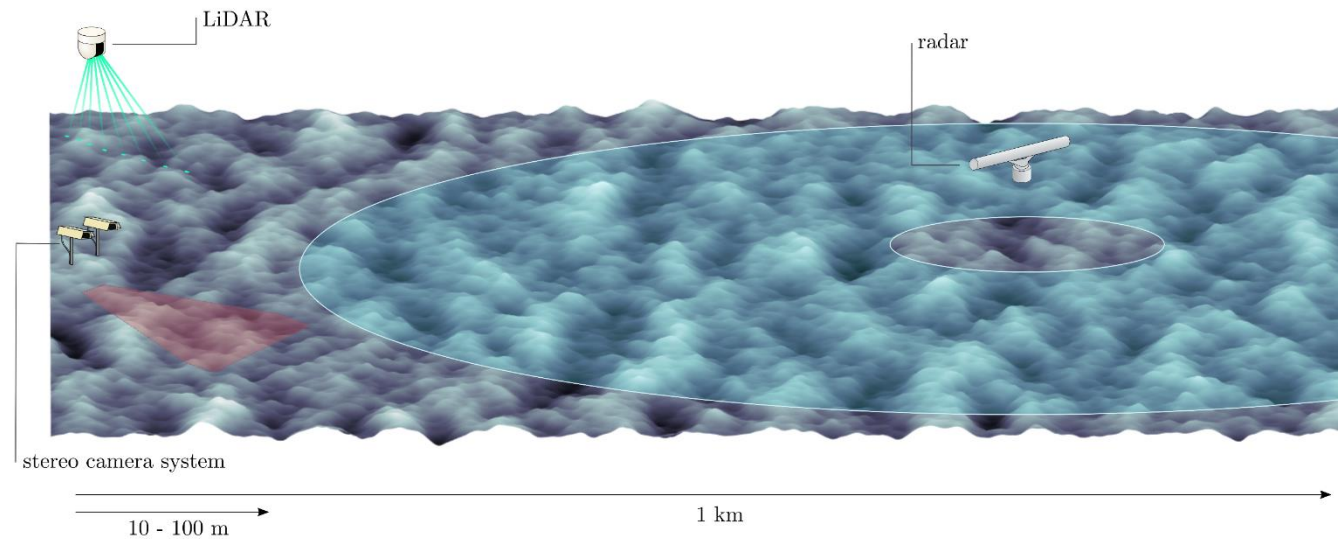


# Wave-by-wave forecasting paradigms

*Point measurements*



*Distributed measurements*



# Other observation layouts

