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The Role of Nonlinear Interactions in Spectral Evolution in Wind Waves at Different Scales

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Motivation

- **Major premise for 3G models:** A proper representation of the evolving spectra requires nonlinear interactions to contain as many degrees of freedom as the discretized spectrum being modeled.

- **Problems with 2G models:**

  1) They could not represent the nonlinear source term within their parameterizations,

  2) They had to be locally tuned for optimal performance.

Motivation

- **Today's 3G models are tuned holistically** for optimal global performance; however, this does not ensure local optimality. Consequently, the need for empirical coefficients has not diminished.

- The objective of 3G models is not the same with the initial premise which was that the spectrum had to be evolved into the proper shape.

(Same issues with 2G models)

- **Purpose:** To investigate the ability of current modeling methods to represent spectral evolution on different scales which is critical on capturing spectral shape.

  -Coupled modeling,

  -Remote sensing,

  -Naval operations,

  -Coastal processes.

Spectral Evolution on Different Scales

Method of testing is to investigate the following 3 scales. The previous comparisons were focused on static tests between the FBI and the DIA:

- Relaxation of the equilibrium range following a perturbation,
- Spectral evolution of the equilibrium range during an interval of constant winds,
- The evolution of spectral shape during transition to swell during propagation over long-distances.

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Full Boltzmann Integration (FBI)

Transfer Function* (Webb, 1978) *:

$$\mathrm{T}(\vec{k}_1,\vec{k}_3) = 2 \oint \mathrm{C}(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) \left|\frac{\partial \mathrm{W}}{\partial \mathrm{n}}\right|^{-1} \mathrm{H}\left(\left|\vec{k}_1 - \vec{k}_4\right| - \left|\vec{k}_1 - \vec{k}_3\right|\right) \mathrm{D}(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)\ \mathrm{ds}$$

D is a function consisting of triplets of action densities:

$$\mathrm{D}(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) = \mathrm{n}(\vec{k}_1)\ \mathrm{n}(\vec{k}_3)\ [\mathrm{n}(\vec{k}_4) - \mathrm{n}(\vec{k}_2)] + \mathrm{n}(\vec{k}_2)\ \mathrm{n}(\vec{k}_4)[\mathrm{n}(\vec{k}_3) - \mathrm{n}(\vec{k}_1)]$$

Resonance conditions: $\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$ $\qquad \omega_1 + \omega_2 = \omega_3 + \omega_4$

Using deep water dispersion:

$$Q + \vec{k}_2^{1/2} - \left(\left|\vec{P} - \vec{k}_2\right|\right)^{1/2} = 0$$

where

$$Q = \vec{k}_1^{1/2} - \vec{k}_3^{1/2} \ \ and \ \ \vec{P} = \vec{k}_1 - \vec{k}_3$$

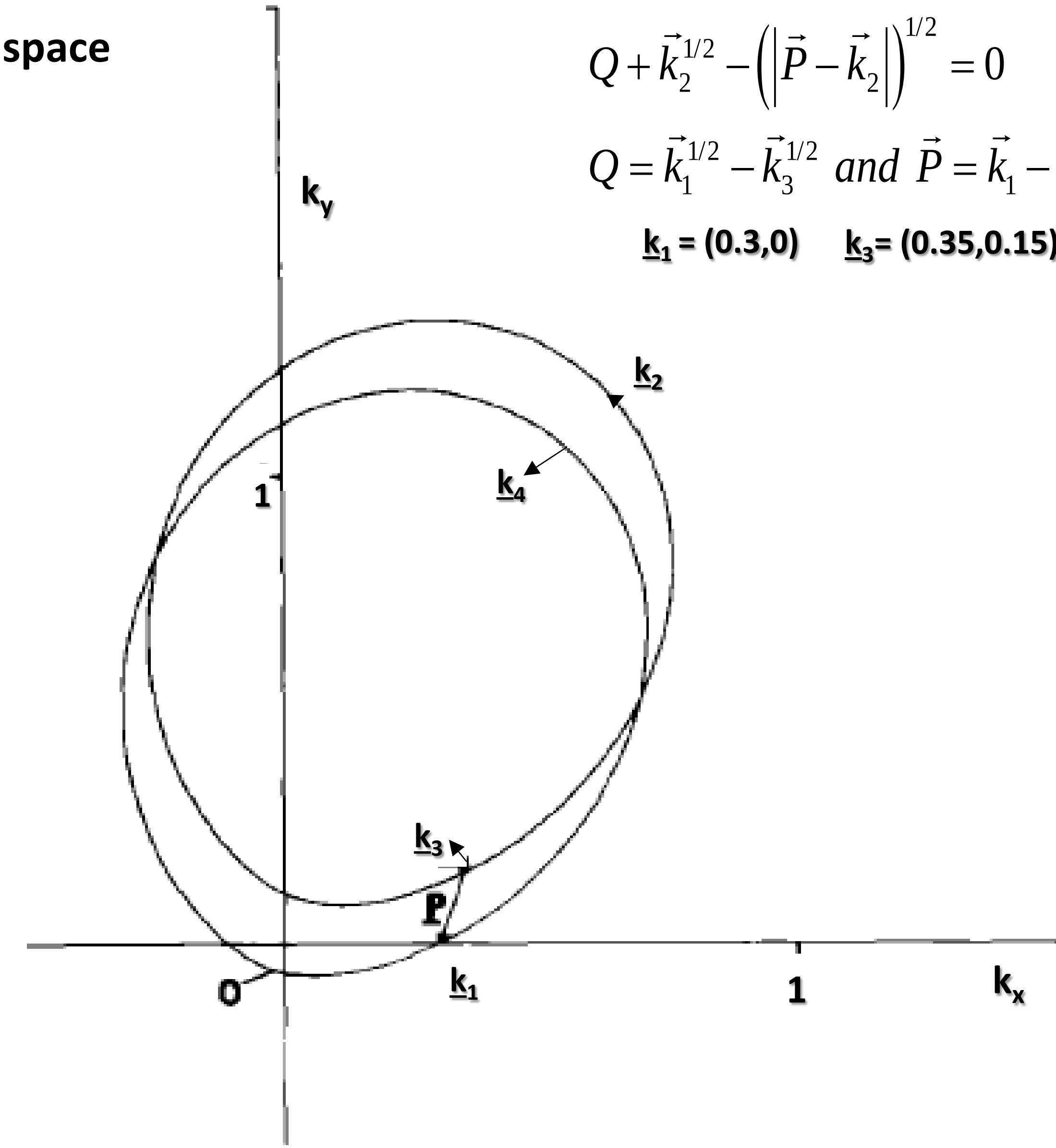
Full Boltzmann Integration (FBI)

Wave number space

$$Q + \vec{k}_2^{1/2} - \left(\left|\vec{P} - \vec{k}_2\right|\right)^{1/2} = 0$$

$$Q = \vec{k}_1^{1/2} - \vec{k}_3^{1/2} \; and \; \vec{P} = \vec{k}_1 - \vec{k}_3$$

$\underline{k}_1$ = (0.3,0) **$\underline{k}_3$= (0.35,0.15)**



(Webb 1978)

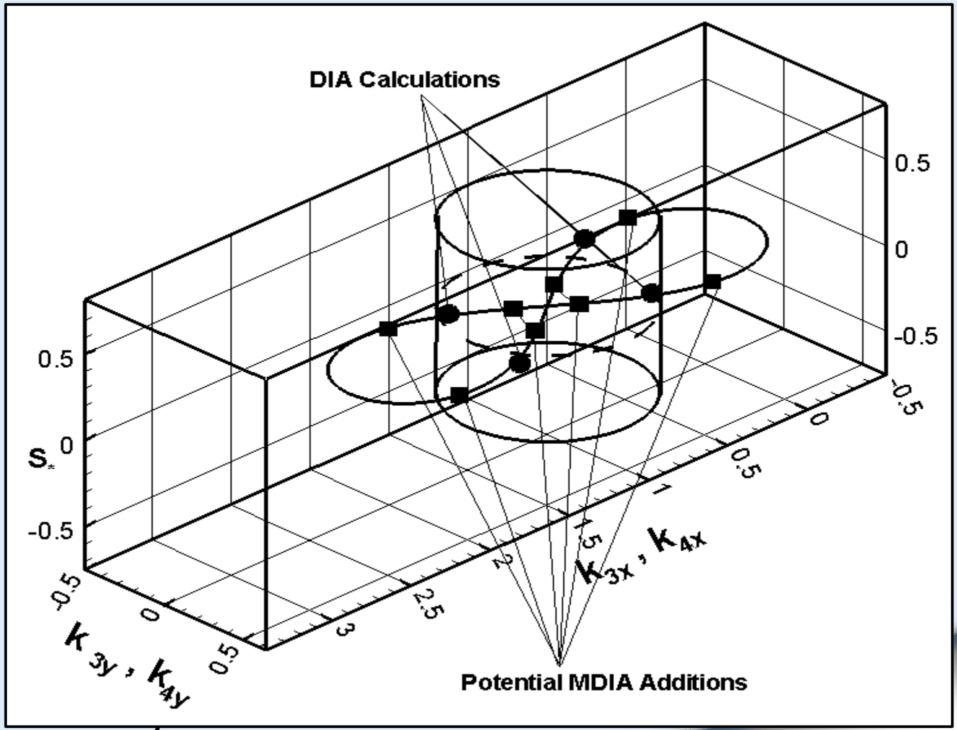
Resonant Volume

- Vertical lines in this shape represent the loci and the Discrete Interaction Approximation (DIA) points are shown for a single k_1.

	$\vec{k}_1$	$\vec{k}_3$
FBI Points:	**Same**	**(30 Locus x 7 frequency x 10 angle)**
DIA Points:	**Same**	**4 pre-selected points**

- This figure shows the interaction volume for each k_1 considered (Typically 20-50 frequencies and 18-36 angles).

- It also shows the potential DIA additions which includes $\delta(k_1-k_2)$ and reduces the dimension of the integral.



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Spectral Evolution following a Perturbation

- Wave-wave interactions restore the spectrum to a self-similar form following a perturbation in the overall source term balance **(Hasselmann et al 1976; Young and Van Vladder 1993)**.

- To test this, an f^{-4} based spectrum was used here and in the remaining simulations **(Long and Resio 2007)**. Results are shown by normalized integrated energy spectrum compensated by,

$$\hat{E}(f) = \frac{E(f)f^{4}}{\beta} \qquad \beta = \alpha_4 ug(2\pi)^{-3}$$

- α_4 is a dimensionless equilibrium range constant **(Resio et al. 2004)**. The f^{-4} equilibrium zone switches to a parametric f^{-5} form at an $f_{ti} = n \times f/f_p$:

$$\beta E(f,\theta) f^{-4} = \alpha_5 E(f,\theta) f^{-5}$$

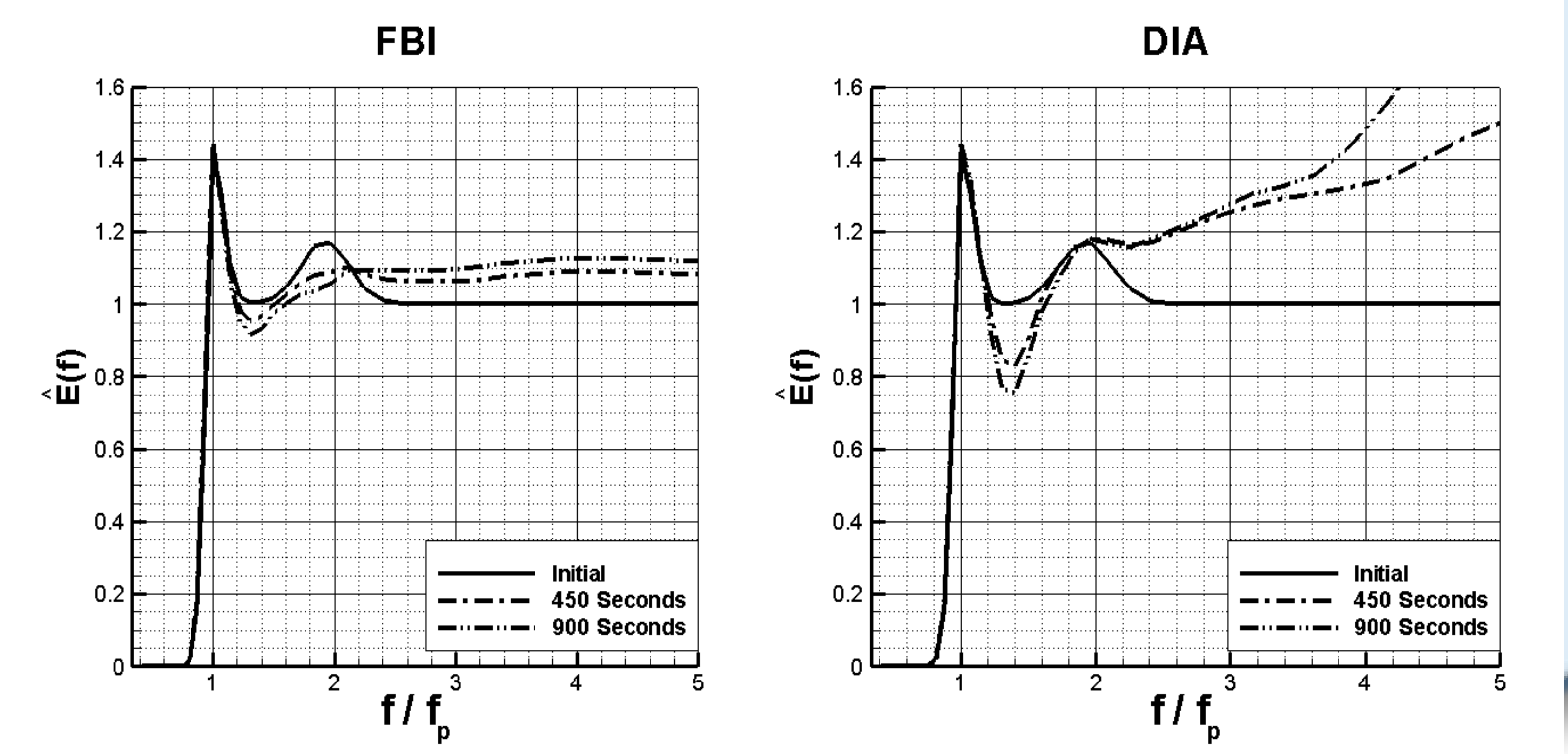
Spectral Evolution following a Perturbation

- f_p of 0.2 Hz, γ_r of 1.41 and f_{ti} = 8 to ensure stability. 1 and 10 s time steps.

Υ_r	4.32	3.09	2.04	1.23	0.62
Υ	7	5	3.3	2	1

γ_r = Relative peakedness.
γ = JONSWAP peakedness.

- The perturbed energy diffuses within the equilibrium range and increases the equilibrium range constant temporarily.



Source Term Balance in Equilibrium Range

- **Kitaigorodskii (1983)** stated that the balance of energy sources in the equilibrium range could be met only if the sum of the wind input (S_{in}) and dissipation due to wave breaking (S_{ds}) were infinitesimally small following **Zakharov and Filonenko (1967)**, had shown that the condition of zero flux divergence for resonant nonlinear fluxes would naturally support this spectral behavior.

$$S_{in} + S_{ds} \approx 0$$

Source Term Balance in Equilibrium Range

- **Phillips (1985)** argued that the previous equation oversimplified the energy balance in the equilibrium range and suggested a more general form,

$$S_{in} + S_{ds} + S_{nl} \approx 0$$

- This form implies that the shape of the spectrum will not deviate substantially for combinations of wind input and wave breaking that do not sum to zero within the equilibrium range.

Source Term Balance in Equilibrium Range

- Total wind induced momentum introduced into the eq. region ($1.5 < f/f_p < 4$):

$$M_i = \lambda_i \frac{\rho_a}{\rho_w} C_d U_{10}{}^2 g^{-1}$$

- λ is the percentage of wind induced momentum rate that varies between +0.05 to -0.05. Constant U_{10} and direction,

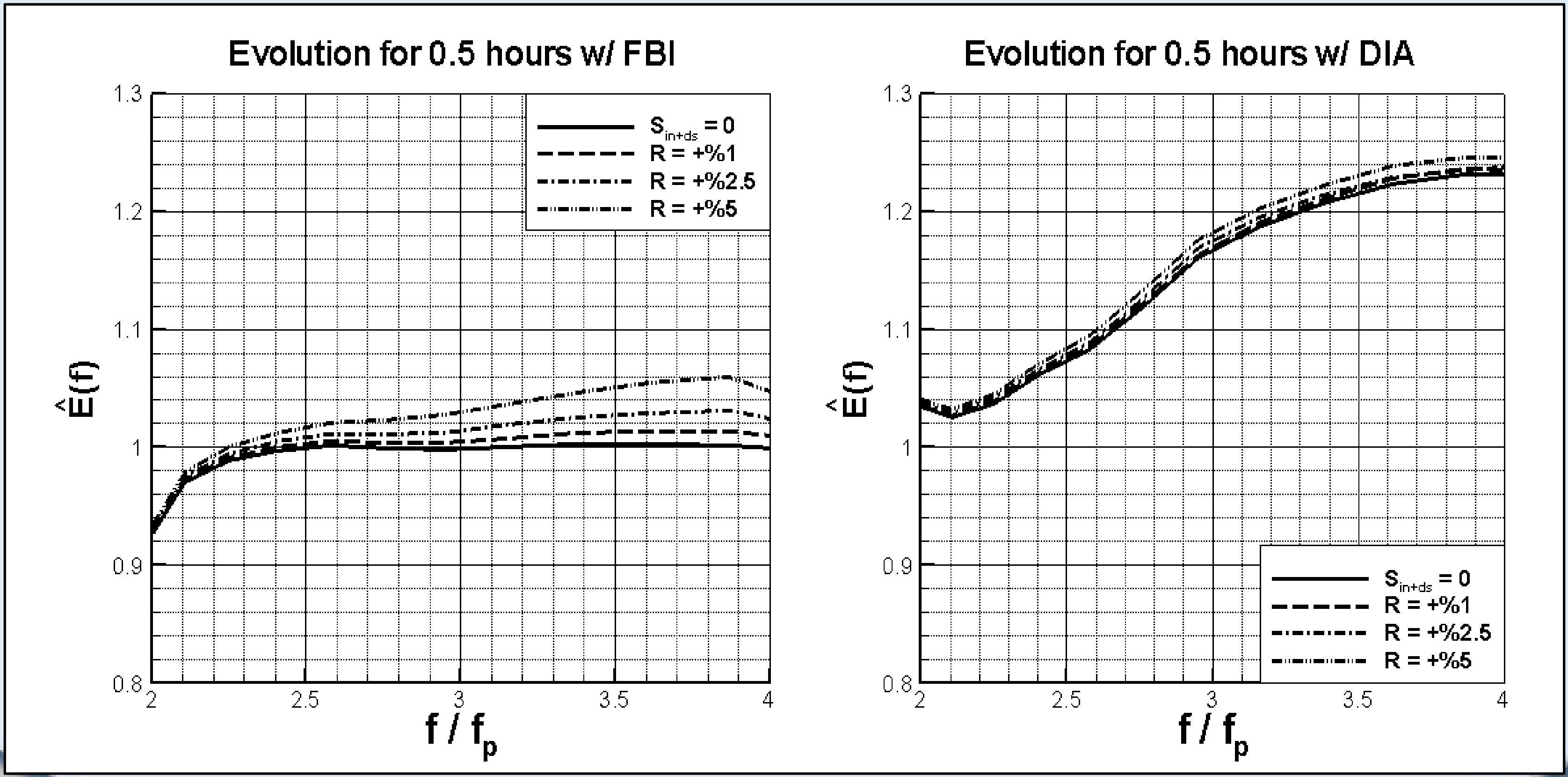
$$S_{(in+ds)_i}(f,\theta) = q_i D(\theta) c(f)$$

$$q_i = M_i (\Delta \mathbf{k})^{-1}$$

- γ_r of 1.4, 10 ms^{-1} wind speed and 0.2 Hz peak frequency over half an hour. f_{ti} is 11 to ensure that there was no feedback into the equilibrium range.

Source Term Balance in Equilibrium Range

- $S_{in}>S_{ds}$ results. Deviations from the f^{-4} shape in equilibrium range due to the positive residual momentum distorting the energy fluxes.



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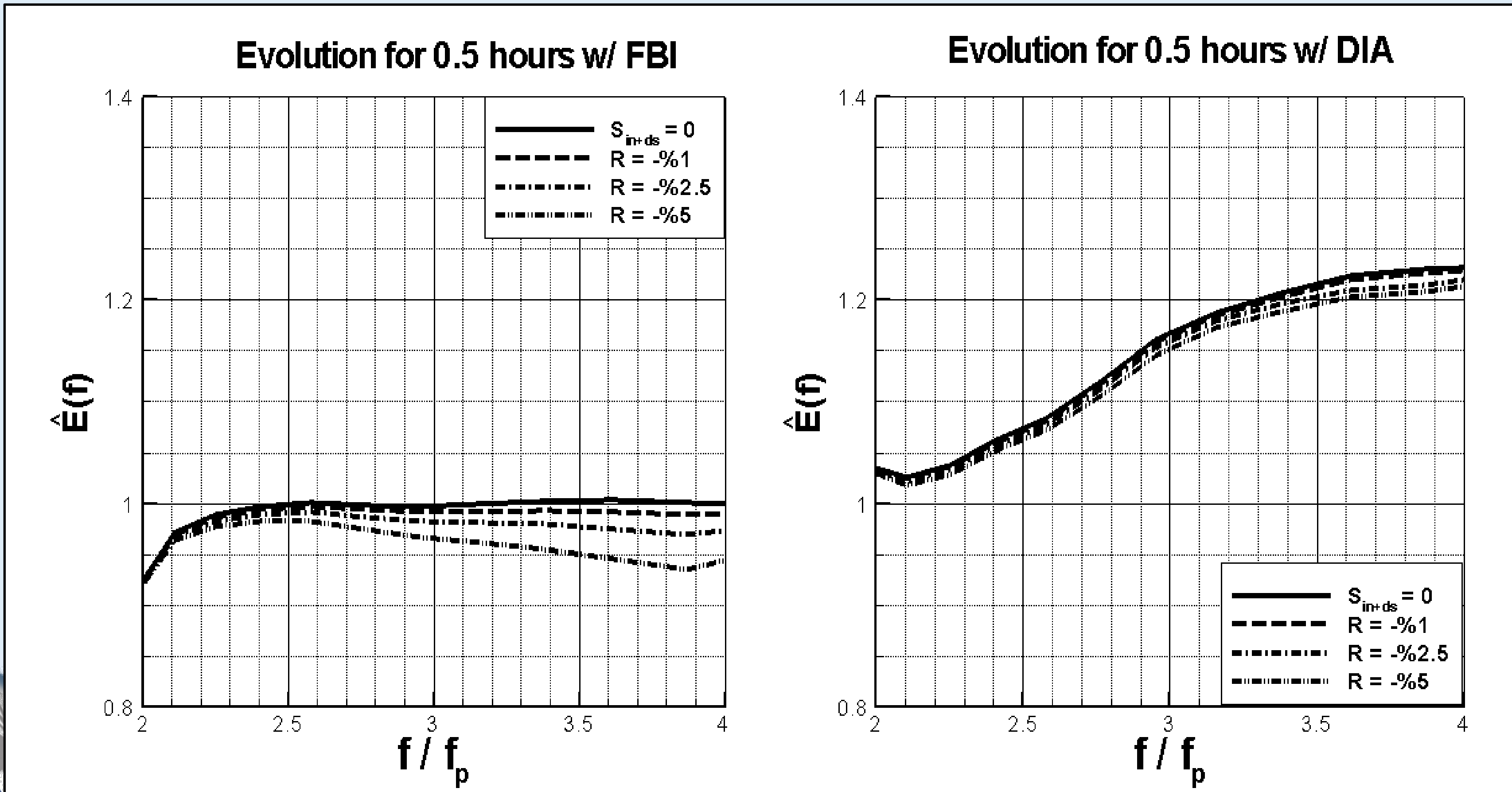
Source Term Balance in Equilibrium Range

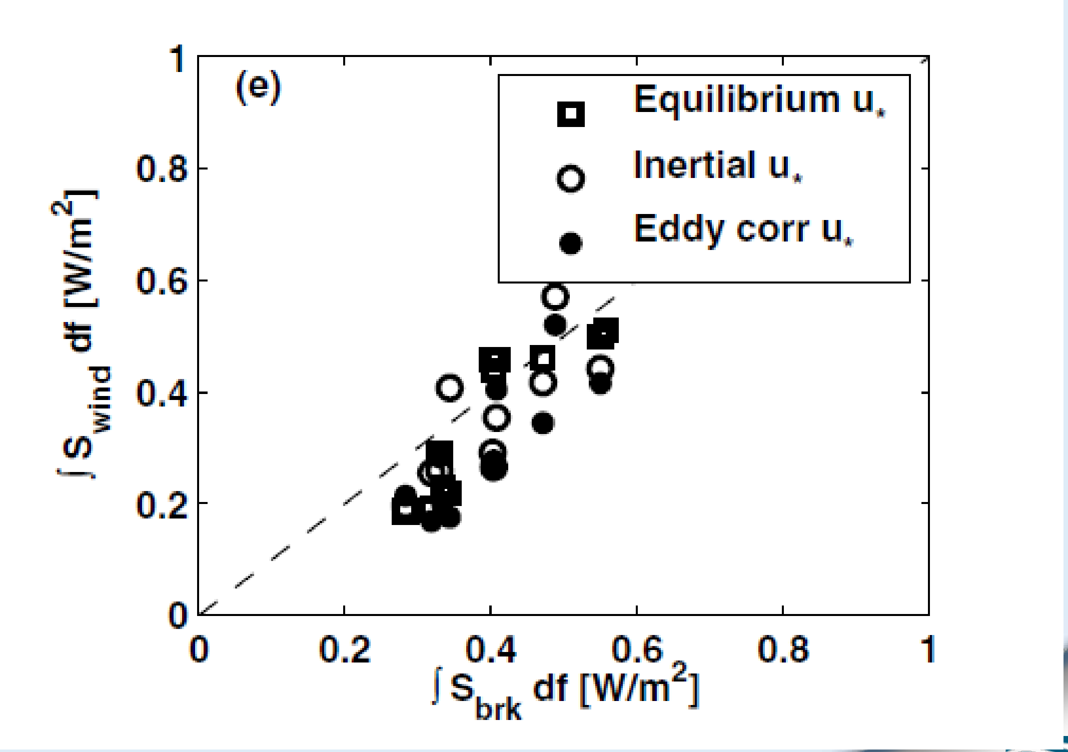
- S_{ds}>S_{in} results. Similar deviations are observed, S_{nl} redistributes instabilities in the energy flux balance and creates an increasing spectral density at high frequencies.



Source Term Balance in Equilibrium Range

- Inequalities between the input and the dissipation term affect the shape throughout the evolution of the spectra.

- Terms in the theoretical equilibrium balance of wind input and breaking dissipation from shipboard and drifter measurements at ocean station PAPA. **(Thomson et al 2013)**



Evolution of Swell Spectrum

- It was noticed that the swell heights were overestimated in wave models.

- **Collard et al. (2009)** and **Young et al. (2013)** assumed that nonlinear interactions can be neglected in swell dissipation. **Ardhuin et al. (2010)** developed a quasi-empirical method to include swell decay within operational models.

- **Badulin and Zakharov (2016)** showed that there is a net energy loss in swell propagation associated with the nonlinear interactions.

Evolution of Swell Spectrum

- A quasi-steady circular storm is considered, where waves fully develop at its circumference and have no wind/dissipation effects beyond this limit. Also zero spatial gradients, similar to **Badulin and Zakharov (2016)**, so that:

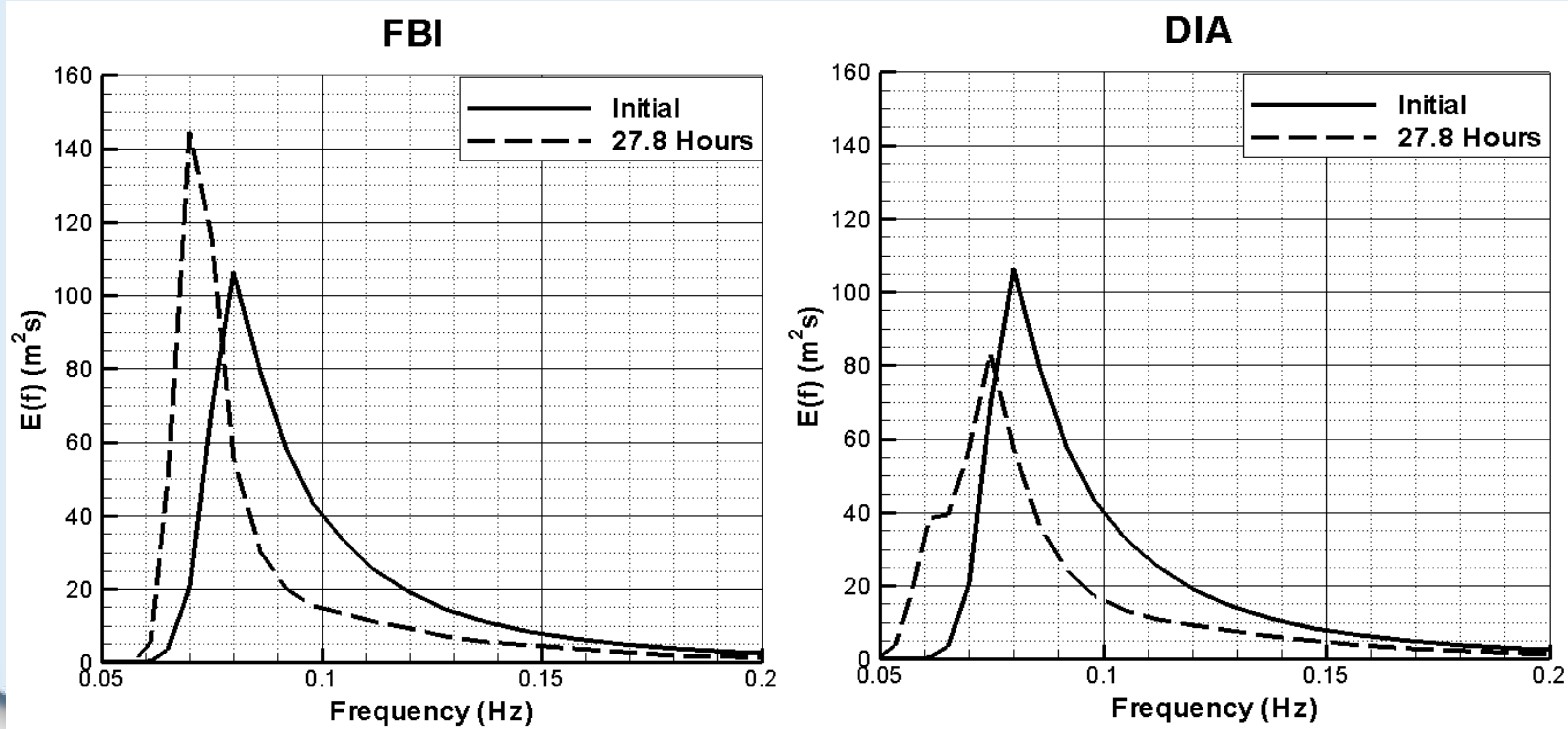
$$E(f,\theta)_R = \frac{R_0}{R} E(f,\theta)_0$$

- R_0 is the radius of the storm whereas the R is the distance from the storm.

- Purpose: To observe the influence of the nonlinear interactions.

Evolution of Swell Spectrum

- Pierson-Moskowitz spectrum with a peak frequency of 0.08 Hz simulated for 27.8 hours.
- The swell spectrum gets narrower and the peak frequency shifts toward lower frequencies.



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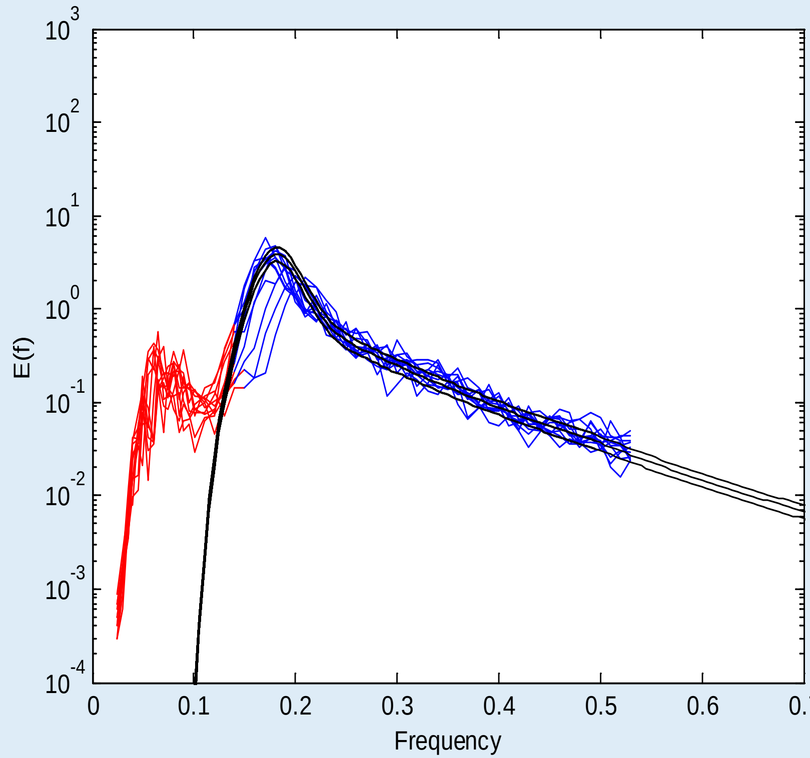
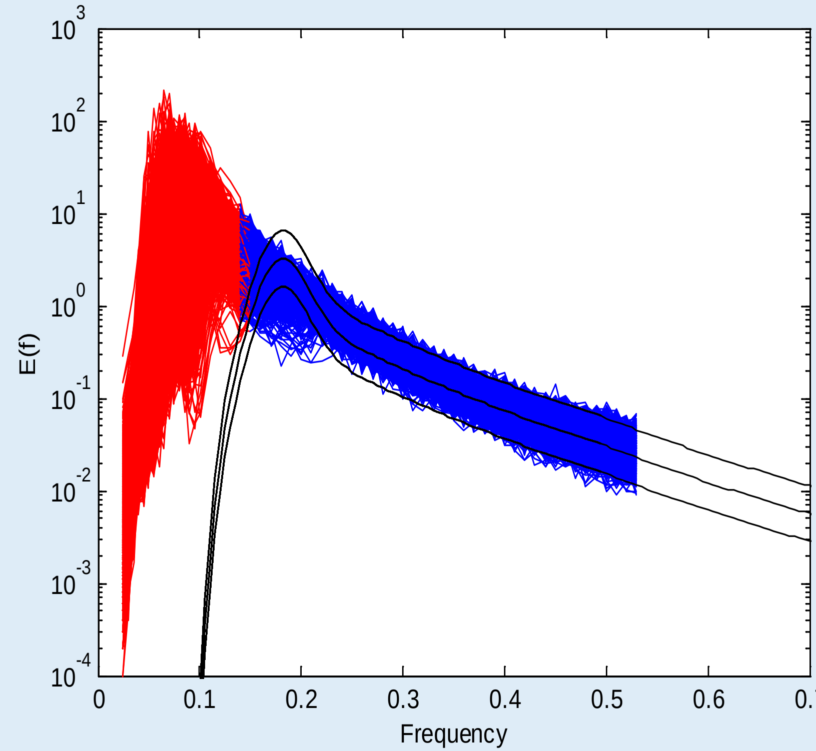
Evolution of Swell Spectrum

Sea



Swell>50%



Vincent (2016) shows spectra from ocean station PAPA, the energy levels are different However the tail still keeps the f^{-4} shape for swell dominated spectra.

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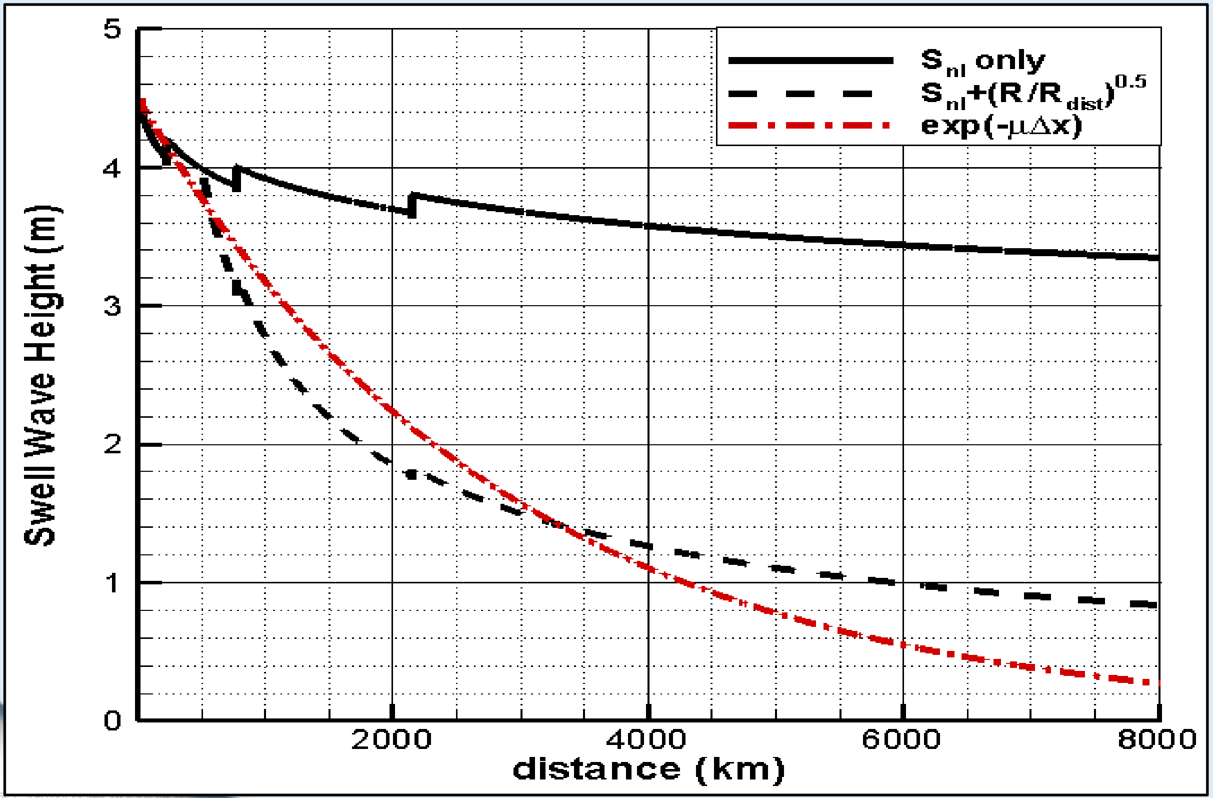
Swell Attenuation

- Estimated decay rate found by swell observations (Snodgrass et al. 1966; Collard et al. 2009) and used by Young et al. (2013) to validate his empirical decay rate:
- $$H_{swl} = H_o \exp(-\mu \mathbf{c_g} \Delta t)$$ μ=3.5x10^{-7}



FBI rate:

$$H_s \cong 4\sqrt{E}$$

$$S_{nl} + \left(\frac{R_{storm}}{R_{total}}\right)^{0.5}$$

Swell Attenuation

- After long propagation distances, the arrival time for peak swell conditions could be significantly earlier:

  -The propagation distance is 1067 km (663 miles) with S_{nl} for the test,
  -It would be 975 km (605 miles) without S_{nl},
  -The arrival time for swell would be approximately several hours earlier.

- If these interactions are not properly modeled by the DIA, an empirical source term might be required to simulate the spectral source term. **(Zieger et al 2015).** b_1 is an empirical coefficient.

$$S_{swl}(k,\theta) = -\frac{2}{3} b_1 \omega \sqrt{B_n} N(k,\theta)$$

Conclusions

- Returning to an equilibrium form following a perturbation is only possible with accurate detailed-balance exchanges. The DIA was unable to reproduce due to,

  - Insufficient degrees of freedom in the DIA to replicate the full integral form,
  - Inappropriate form for dimensional closure of the DIA which was based on an f^{-5} rather than an f^{-4} spectral form.

- Even if the source terms are posed with the same f^{-4} form, they force the equilibrium to deviate from the "constant flux" form in the equilibrium range if they do not sum to zero.

- The inability of 3G models to predict increased energy levels and period associated with the spectral peak could severely impact accurate predictions of swell arrival time and the shape and magnitude of these spectra.

- **We need to move forward**

4^{th} Generation Paradigm

1) Better S_{nl} representation to capture spectral shape evolution.

2) All the source terms need to be represented in their appropriate degrees of freedom.

3) Exploring evolutionary solutions for the different parts of the spectrum to tackle the time constraint.

4) Optimistically this approach would yield a stable model (with no limiters) that has faster execution times.

Questions?

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