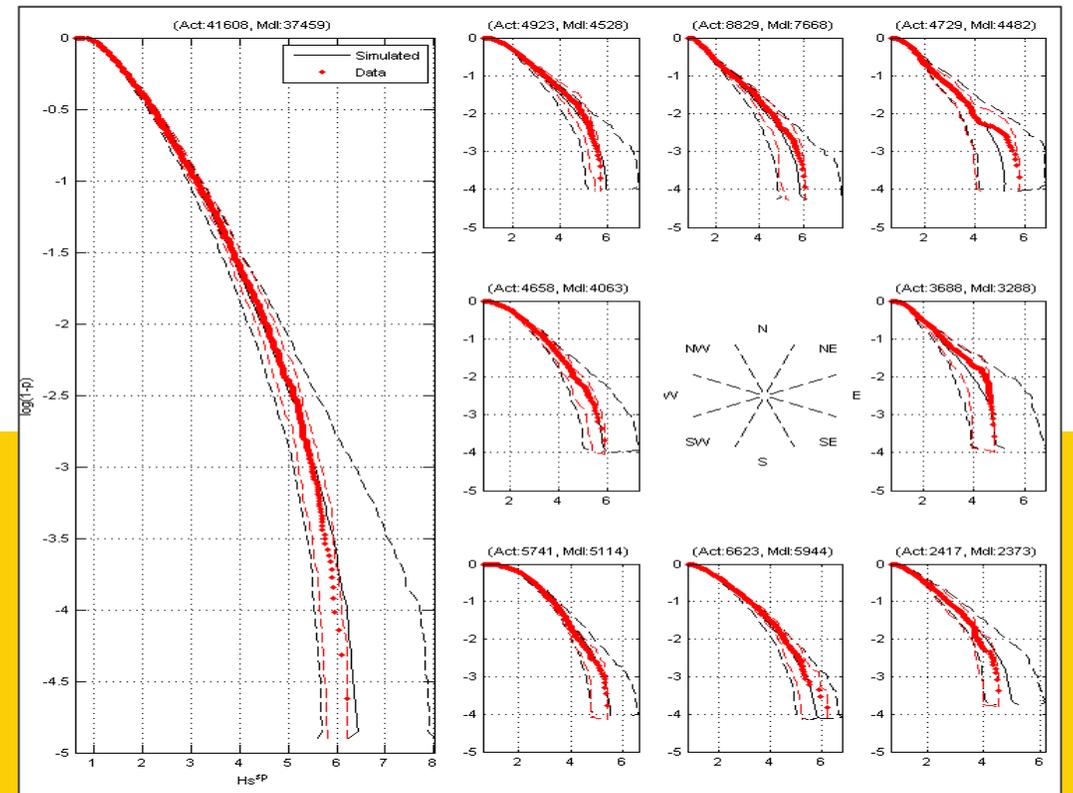




# Derivation of total extreme water levels using covariate extreme value analysis

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# Agenda

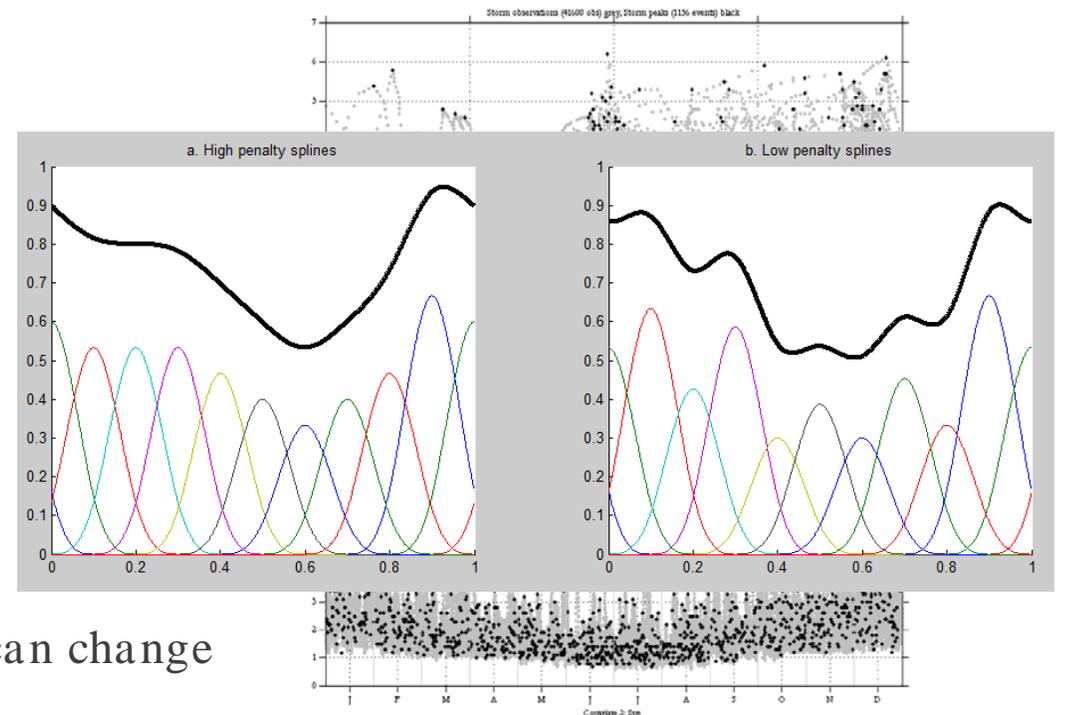
- Motivation
- Overview of Covariate Extreme Value (CEVA) approach for storm peaks
- Storm profiles
  - Seastates
  - Surges
- Inclusion of tides
- Simulations within storms
- Return period simulations
- Summary

# Motivation

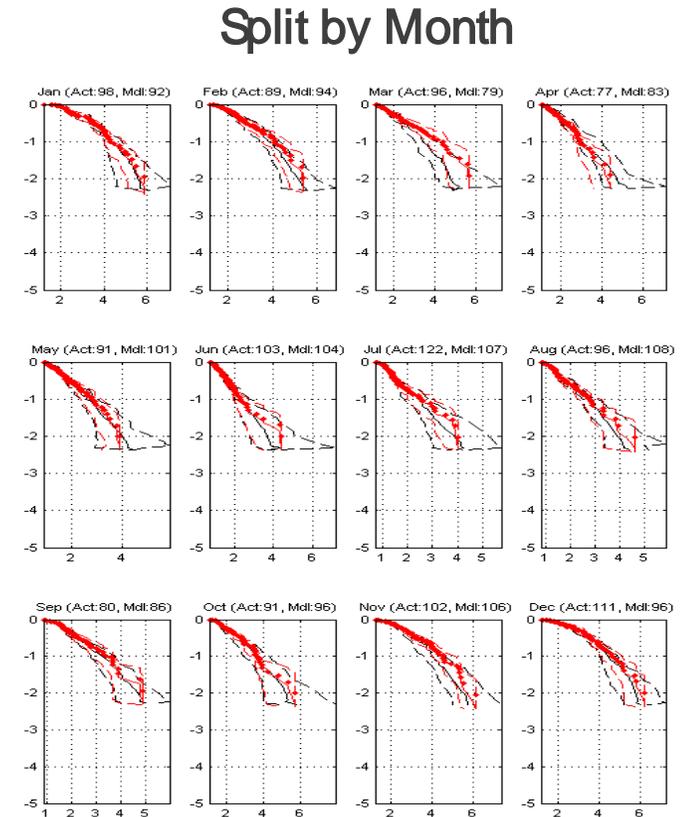
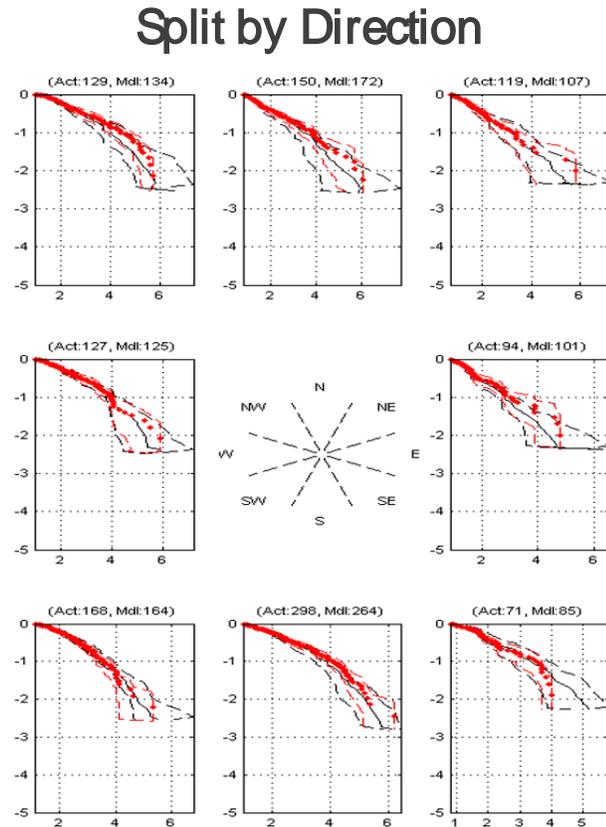
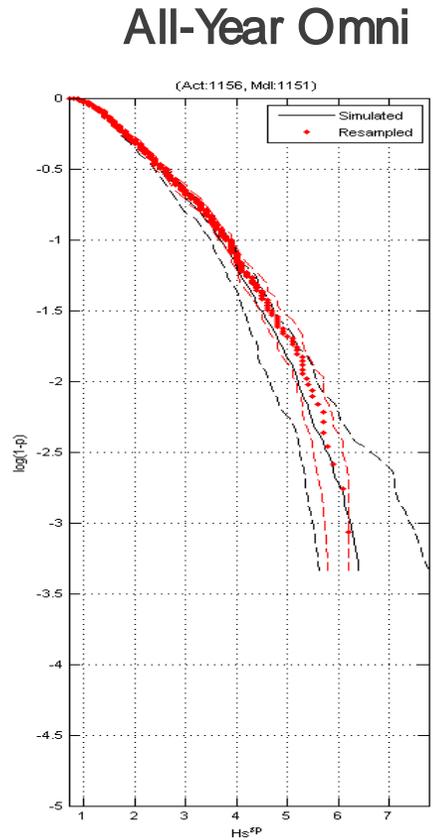
- Wave impact on decks of offshore platforms has a big impact on return period of structural collapse
- Extent of perceived impact is increasing due to:
  - Increased crest heights due to better understanding:
    - spatial effect across platform area,
    - non-linearities,
  - maybe climate change effects ...
  - Platform subsidence
- For non-evacuated structures can mean platform evacuation in severe storms or even shut down completely
- In the past, simplistic approaches/ assumptions have been used to derive the total water levels in extreme storms.
- Trying to improve the approach ....

# Overview of CEVA for storm peak $H_s$

- Define an array of seasonal-directional bins – typically 32  $Drc$  ( $11.25^\circ$ ) x 24  $Ssn$  (~2 weeks)
- Select a storm threshold as a *quantile* varying across all bins – captures calmer directions/ seasons
- Save from each storm:
  - storm peak
  - storm profile (more of this later)
- For all  $Drc$ - $Ssn$  bins
  - Select an EVA threshold based on a quantile,  $\mu$
  - Fit the storm occurrence rate,  $\lambda$ , as a Poisson process
  - Fit GP shape,  $\sigma$ , and scale,  $\gamma$ , parameters
- Variation of all parameters is by B-splines
- A smoothing parameter defines how quickly their magnitudes can change
- Perform a Monte Carlo analysis for storm peaks for RP
- Identify the largest  $H_s$  within each storm and assign to a  $Drc$ - $Ssn$  bin



# Comparison of modelled and observed storm peaks



■ Observed: data - red dots, 95% UI red dashed lines

■ Model: median – solid black line, 95% UI black dashed lines

# Definition of total water level

Max total water level,  $TWL =$

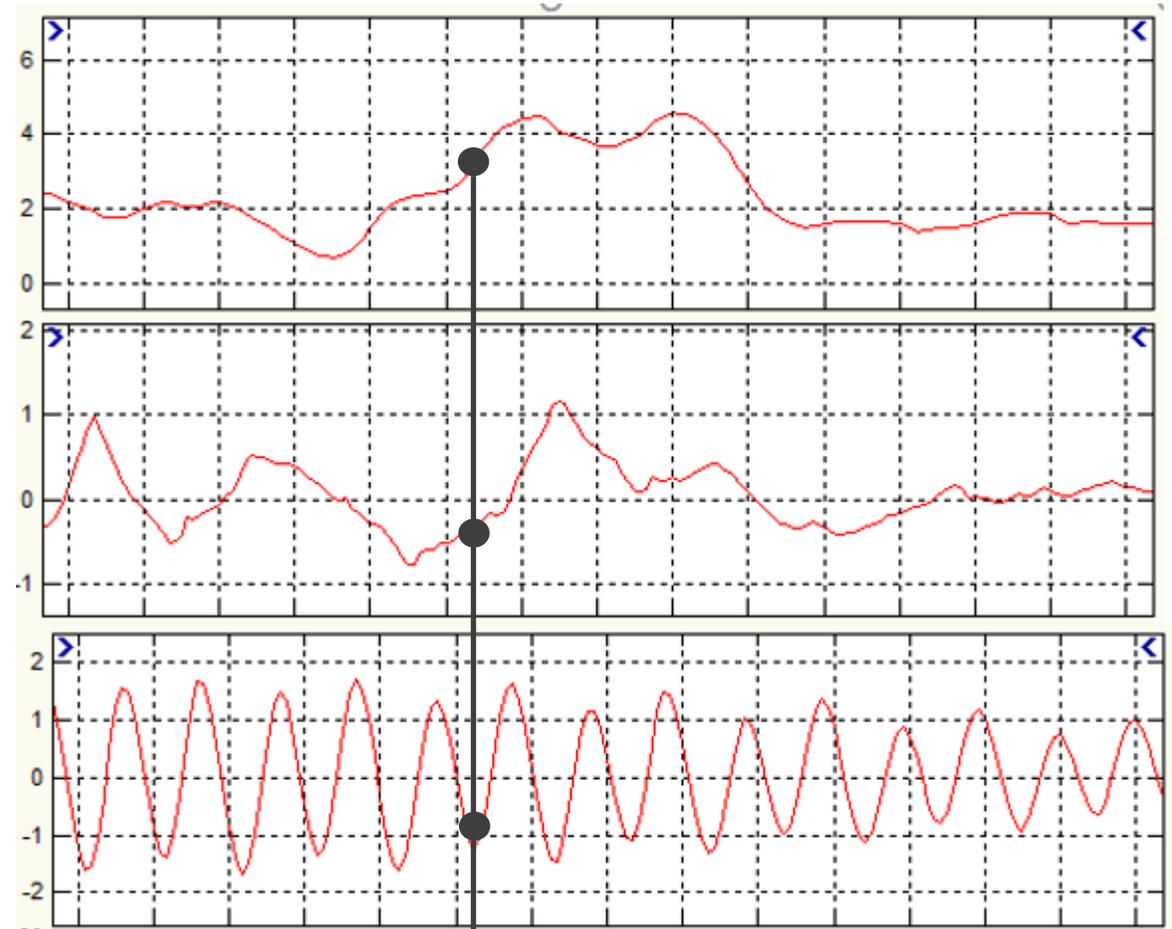
Instantaneous max wave crest

+

Hourly mean surge

+

Hourly mean tidal level



# Storm profiles

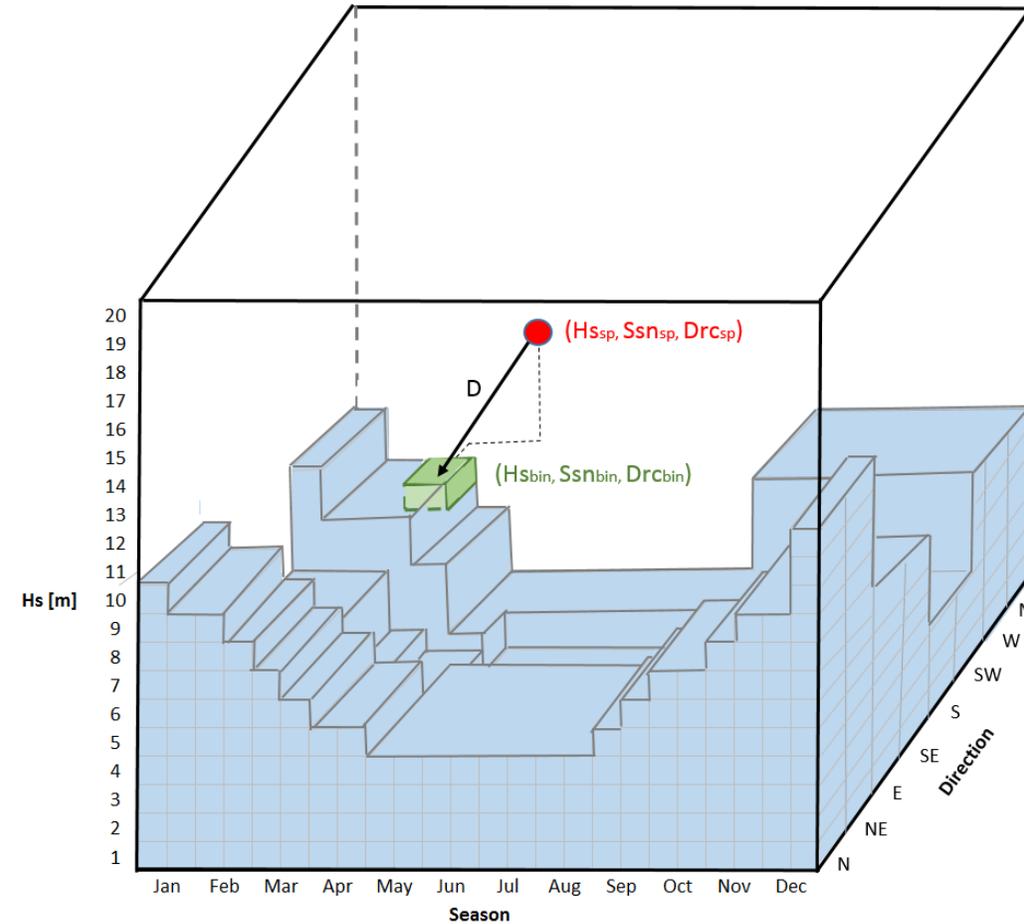
It is not sufficient to just base the analysis on storm peaks because ....

- Storms last longer than one sea-state so ...
  - ... largest individual crests may occur during smaller sea states ...
  - ... in a direction sector *different* to storm peak.
  
- The *TWL* may occur at a time of
  - ... large tide away from peak *Hs*...
  - ... or at a time of large surge.
  
- So, need to define realistic storm profiles to associate with each storm peak:
  - Want to reflect the variability in characteristics with season, direction and severity of storms

# Selection of storm profiles for a simulated storm peak

- Assume that observed storm shapes are representative.
- Put observed “archetype” storm profiles into bins based on their peak  $H_s$  and assoc  $Drc$ ,  $Ssn$
- Find distance,  $D$ , from any simulated  $H_{s_{sp}}$ ,  $Ssn_{sp}$ ,  $Drc_{sp}$  to each bin

$$D = \sqrt{\frac{(H_{s_{sp}} - H_{s_{bin}})^2}{\alpha_{H_s}} + \frac{(Ssn_{sp} - Ssn_{bin})^2}{\alpha_{Ssn}} + \frac{(Drc_{sp} - Drc_{bin})^2}{\alpha_{Dir}}}$$





# Storm profiles - surge

## Storm archetype

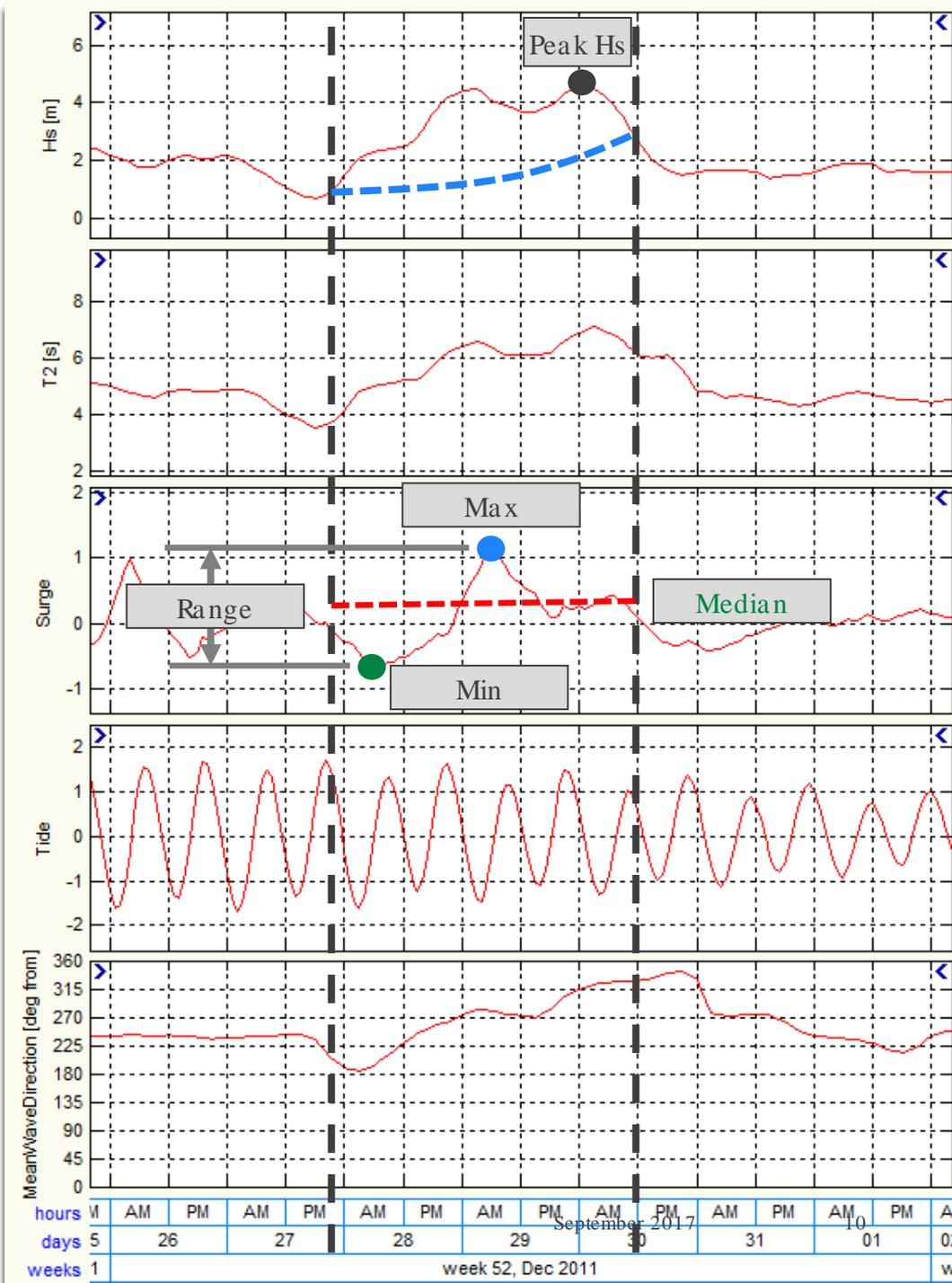
--- Hs quantile varying by direction

- - - Start and end of storm based on  $H_s$

● ● Surge characteristic values

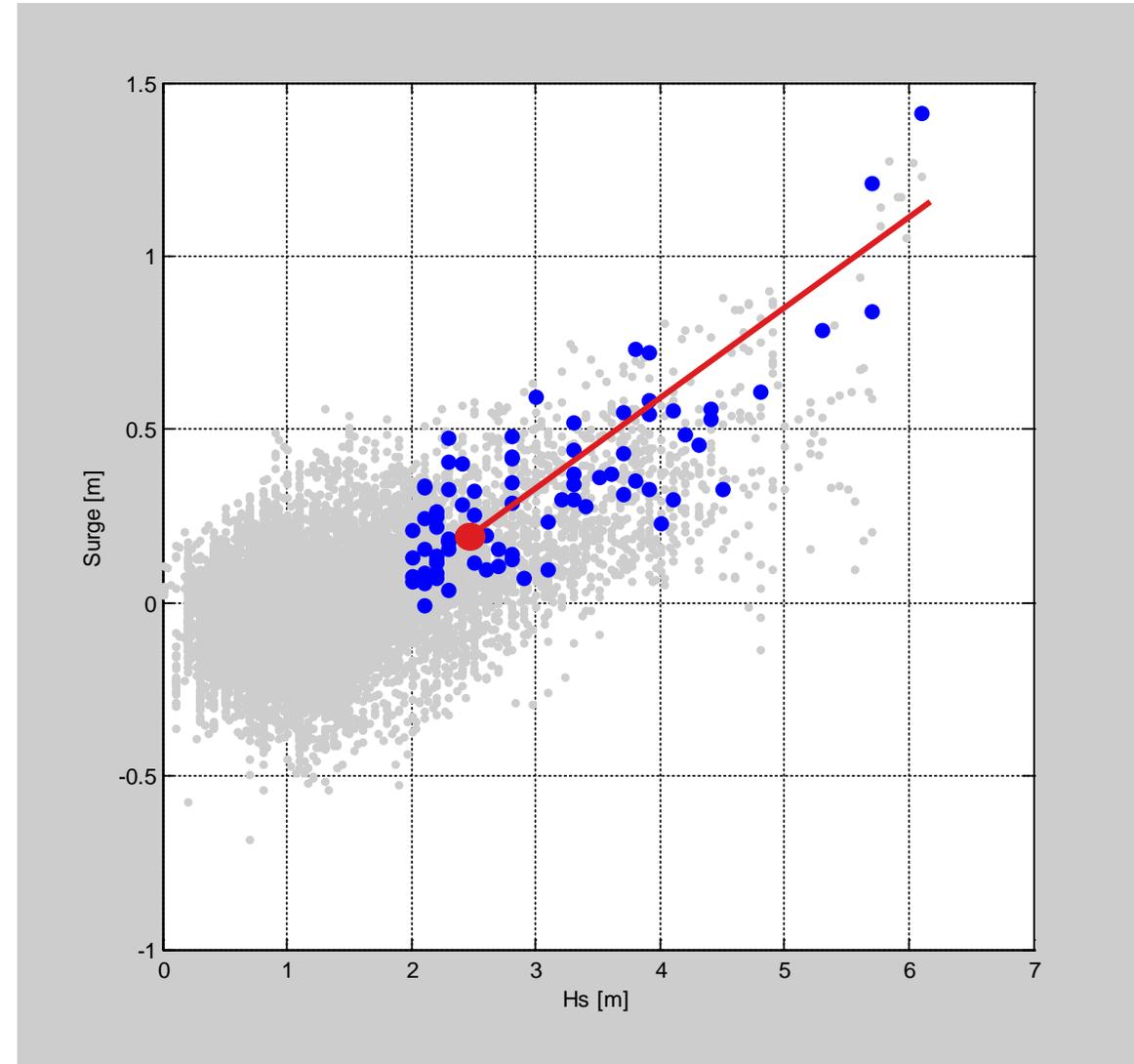
- - - Median surge

- Surge and wave profiles are selected as a pair from same storm
- Modify storm surge profile based on relationships between peak storm  $H_s$  and either:
  - Surge max, min and median, or
  - Surge median and range



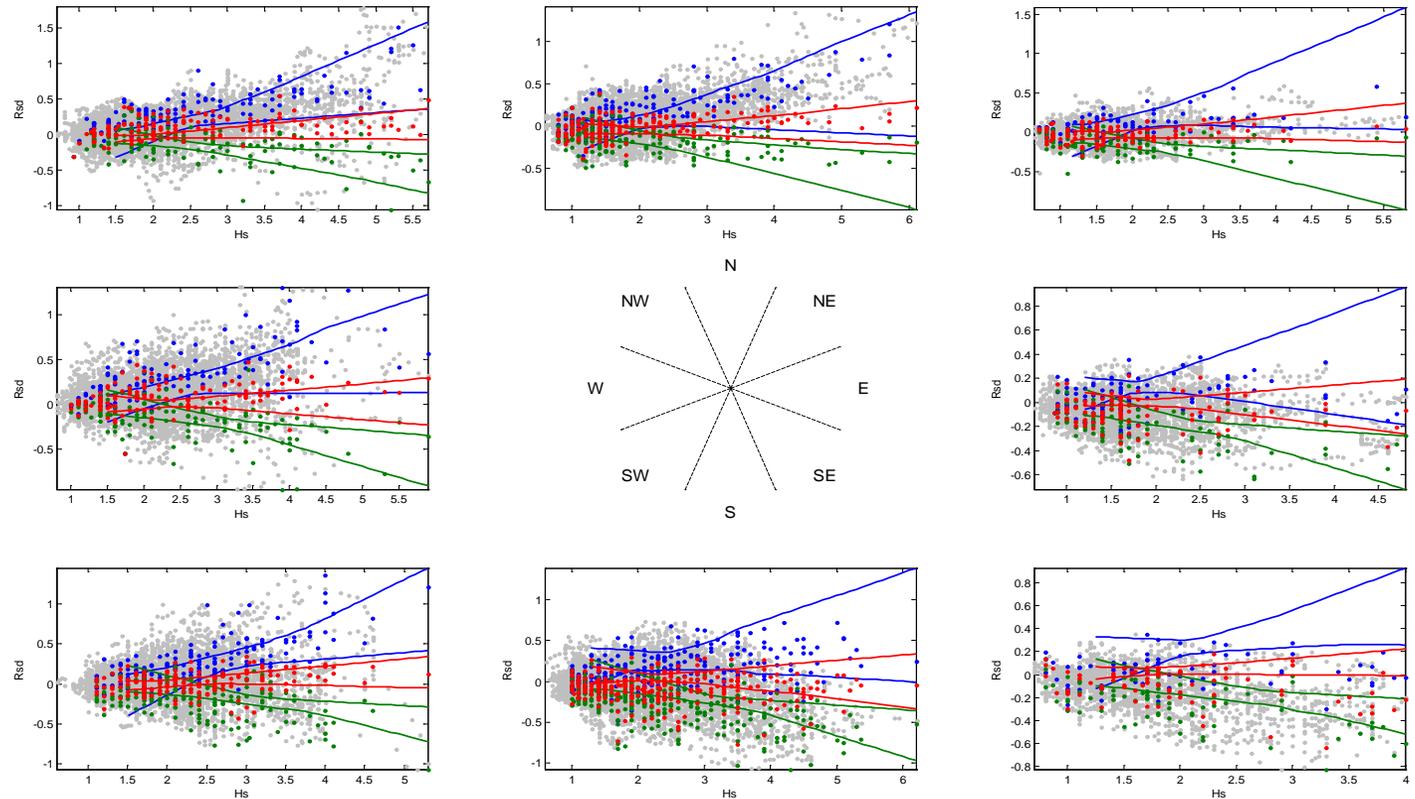
## Example – max storm surge versus $H_s$

- Base on:
  - Max storm  $H_s$
  - Max storm surge
- Define an  $H_s$  “lock point” as a quantile
- Derive median value of max surge for  $H_s$  lock point
- Fit a slope based on linear regression
- Fit to all Drc-Ssn bins in one go using splines to capture variability across array



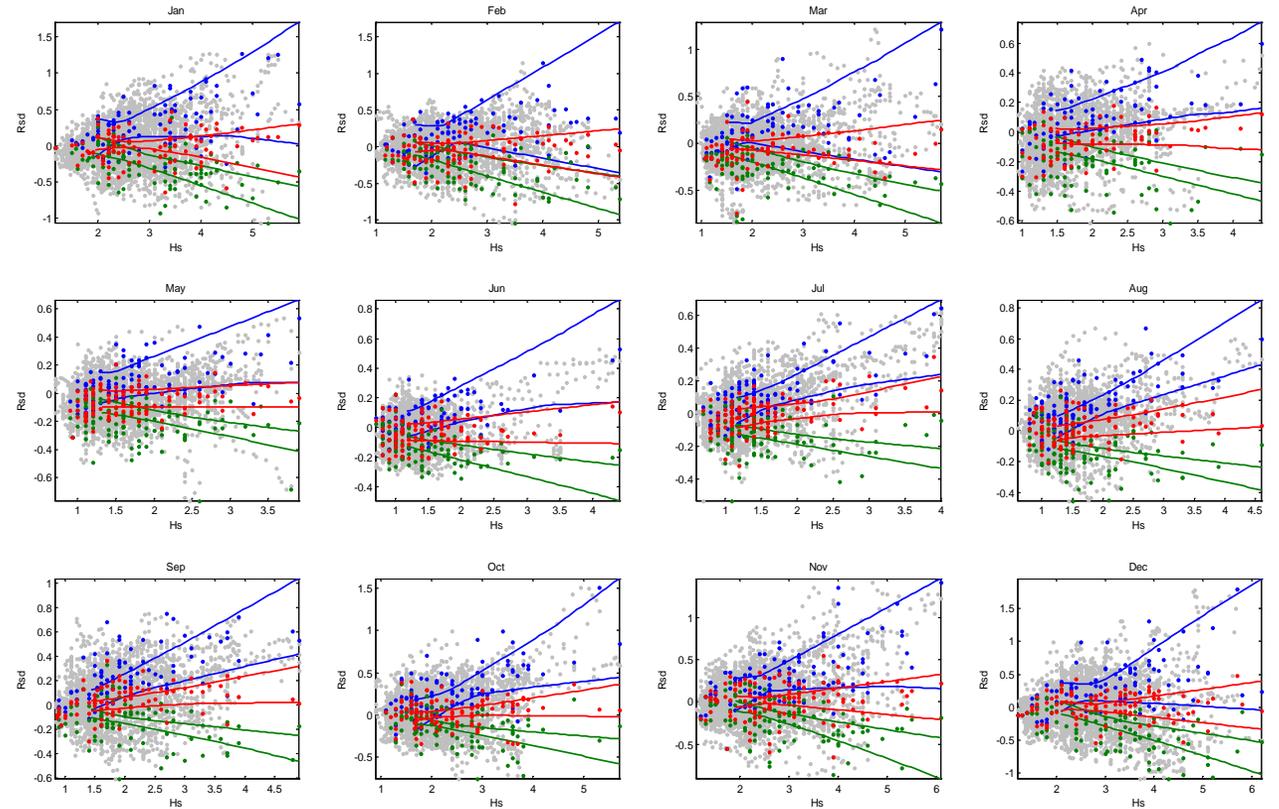
# Storm surge versus Hs – max, min, median by direction

- Storm maximum
- Storm median
- Storm minimum



■ Each plot shows range of fits from 4 *Drc* x 24 *Ssn* bins in each octant

# Storm surge versus Hs – max, min, median by month

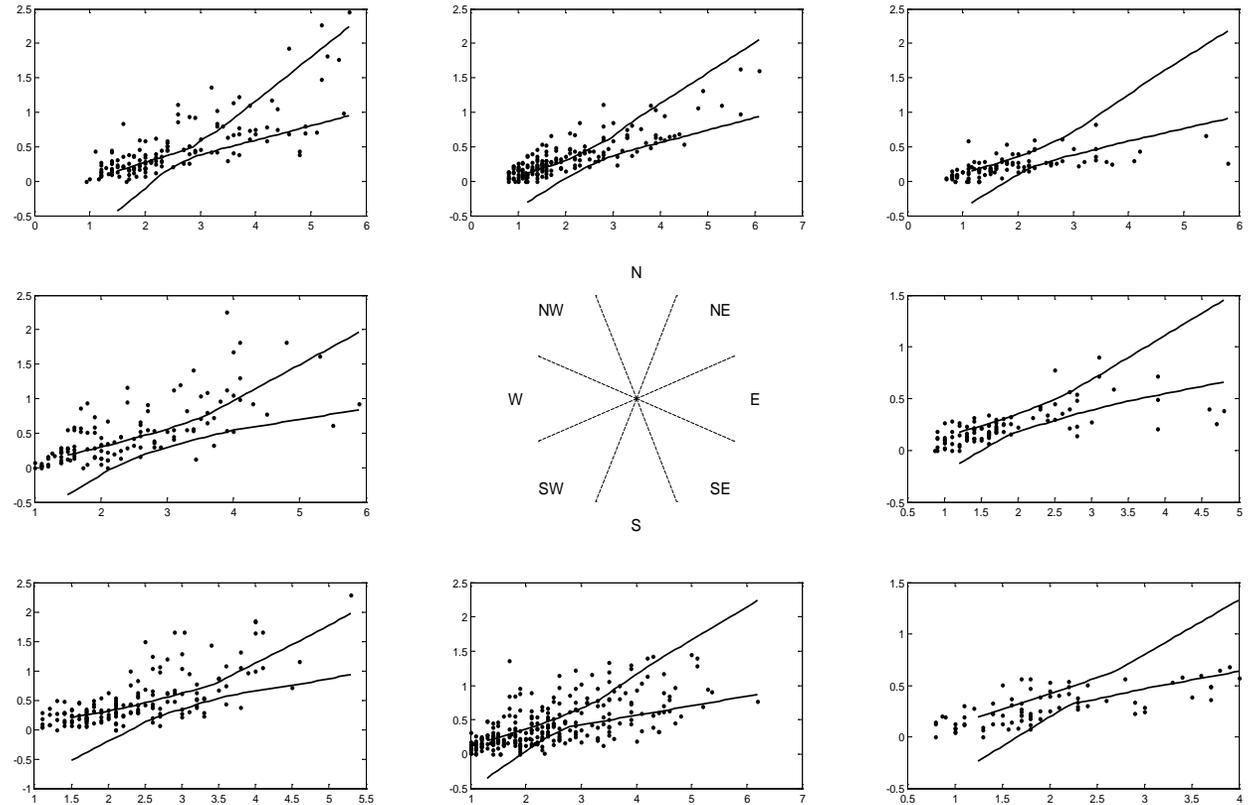


- Storm maximum
- Storm median
- Storm minimum

■ Each plot shows range of fits from 32 *Drc* x 2 *Ssn* bins in each month

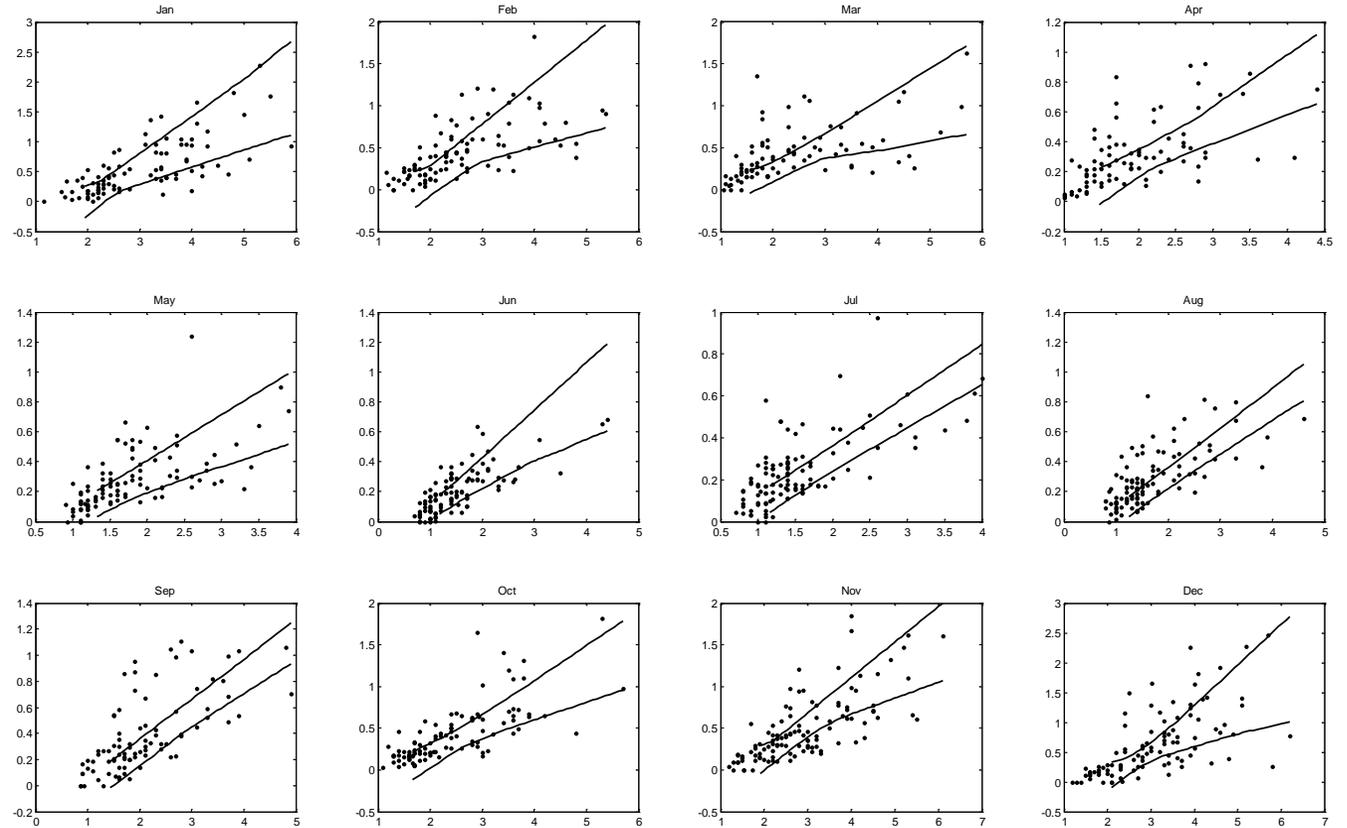
# Storm surge versus Hs – range by direction

● Storm range



■ Each plot shows range of fits from 4 *Drc* x 24 *Ssn* bins in each octant

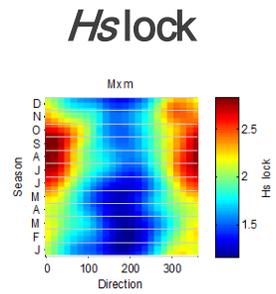
# Storm surge versus Hs – range by month



● Storm range

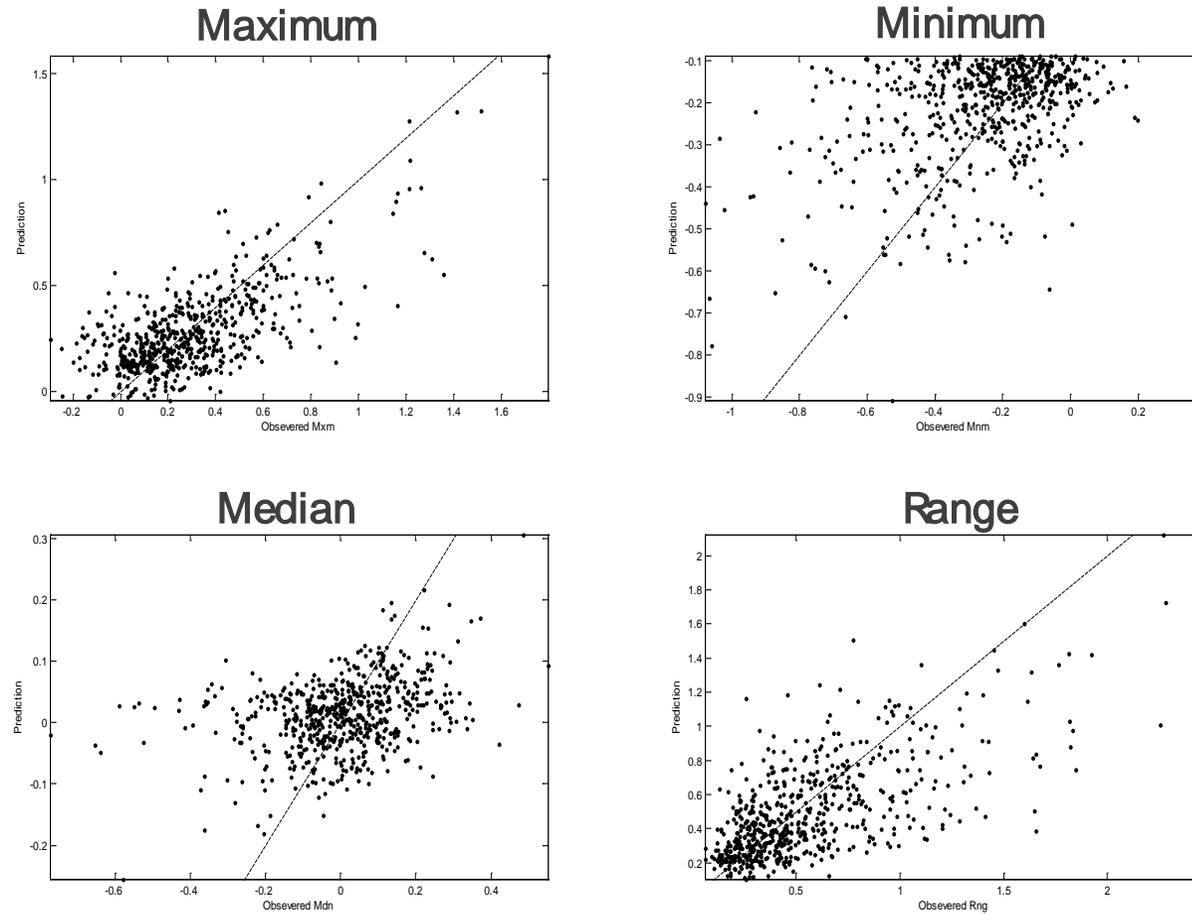
■ Each plot shows range of fits from 32 *Drc* x 2 *Ssn* bins in each month

# Storm surge versus Hs summary



Slope	Surge lock	
<p>Mxm</p> <p>Hs slope: 0, 0.1, 0.2, 0.3, 0.4, 0.5</p>	<p>Mxm</p> <p>Sig Lock: 0, 0.1, 0.2, 0.3</p>	Maximum
<p>Minm</p> <p>Hs slope: -0.25, -0.2, -0.15, -0.1, -0.05</p>	<p>Minm</p> <p>Sig Lock: -0.14, -0.13, -0.12, -0.11, -0.1, -0.09, -0.08</p>	Minimum
<p>Mdn</p> <p>Hs slope: -0.1, -0.05, 0, 0.05, 0.1</p>	<p>Mdn</p> <p>Sig Lock: -0.08, -0.06, -0.04, -0.02, 0, 0.02, 0.04</p>	Median
<p>Rng</p> <p>Hs slope: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7</p>	<p>Rng</p> <p>Sig Lock: 0.1, 0.2, 0.3, 0.4</p>	Range

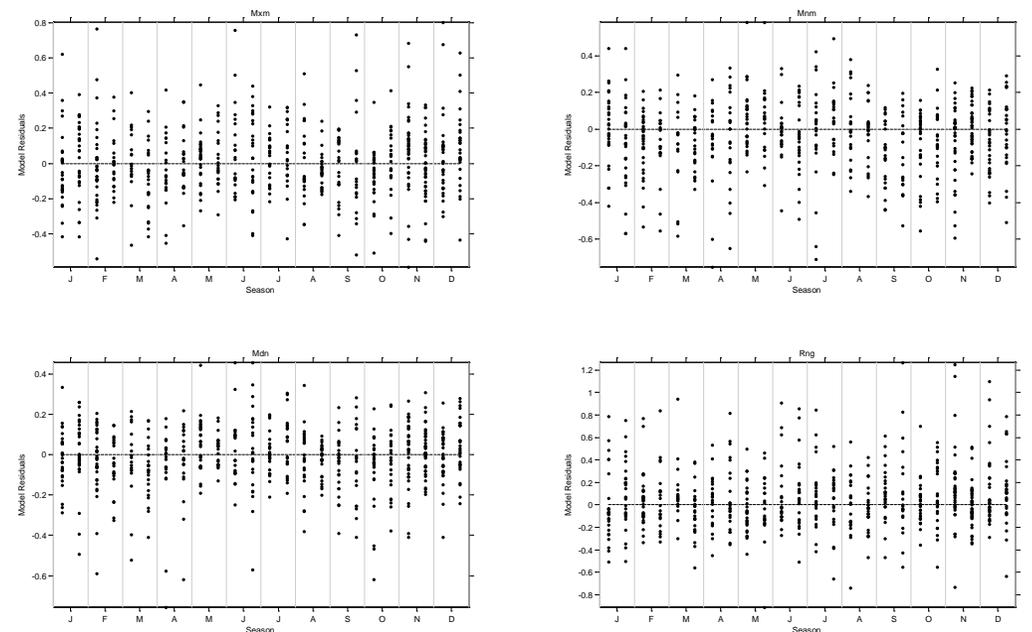
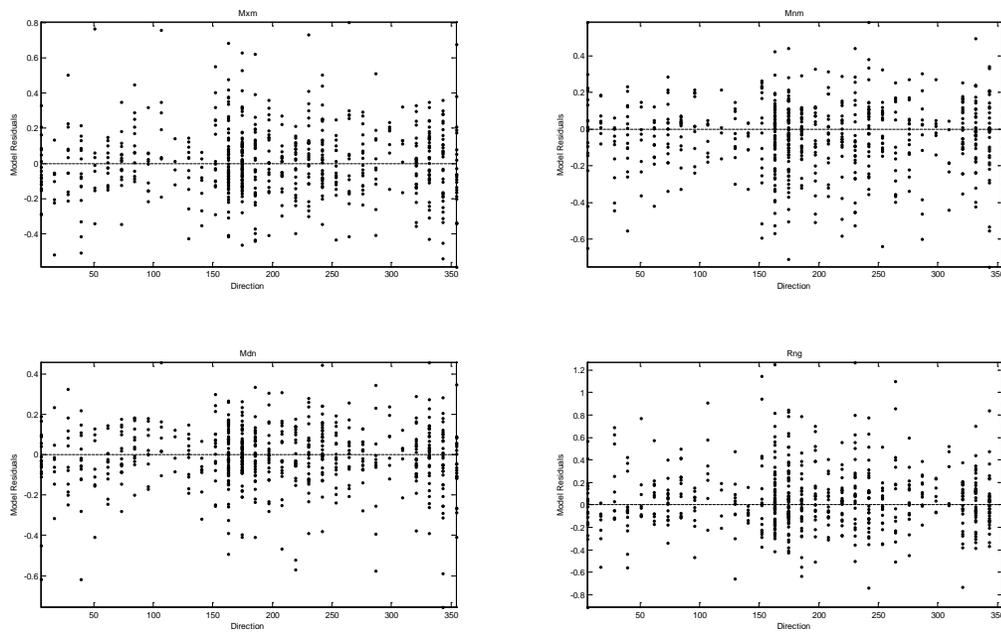
# Comparison of observed and predicted storm surge



# Residuals between observed and predicted storm surge

## By Direction

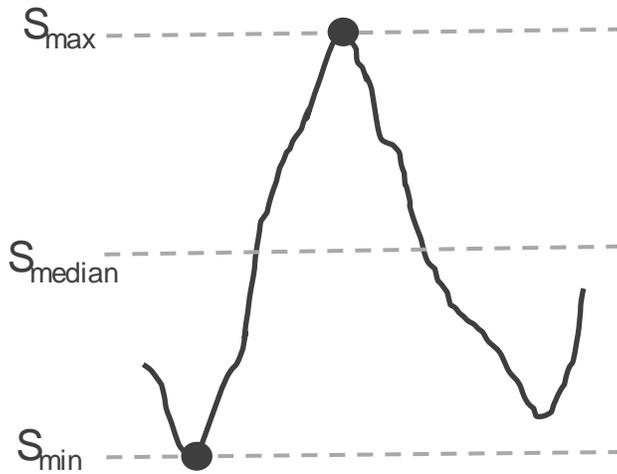
## By Season



■ Residuals are sampled during Monte Carlo analysis

# Storm surge scaling – based on min, max, median

Archetype storm



Scaled storm profile

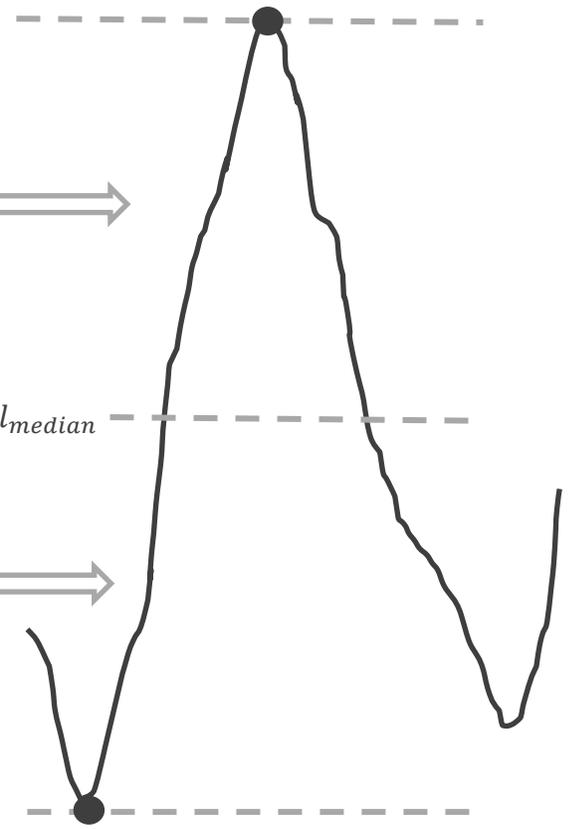
$$S'_{max} = f(\mathbf{H}S_{sp}, HS_{lock}, surgeLock_{max}, slope_{max}) + RandomResidual_{max}$$

$$scaling_{>median} = \frac{S'_{max} - S'_{median}}{S_{max} - S_{median}} \Rightarrow$$

$$S'_{median} = f(\mathbf{H}S_{sp}, HS_{lock}, surgeLock_{median}, slope_{median}) + RandomResidual_{median}$$

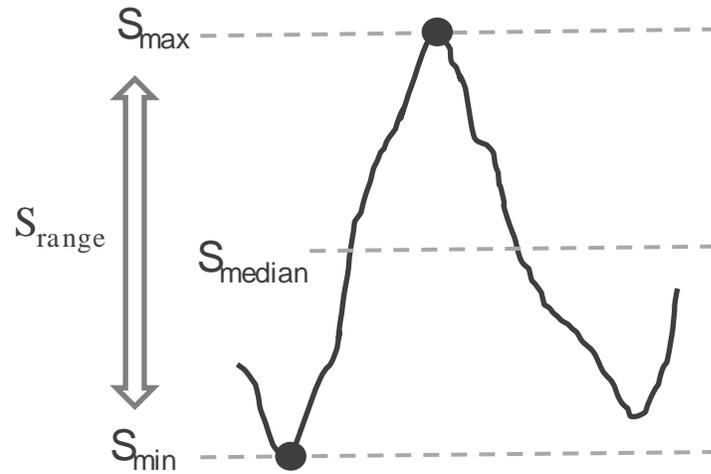
$$scaling_{<median} = \frac{S'_{min} - S'_{median}}{S_{min} - S_{median}} \Rightarrow$$

$$S'_{min} = f(\mathbf{H}S_{sp}, HS_{lock}, surgeLock_{min}, slope_{min}) + RandomResidual_{min}$$



# Storm surge scaling – based on range and median

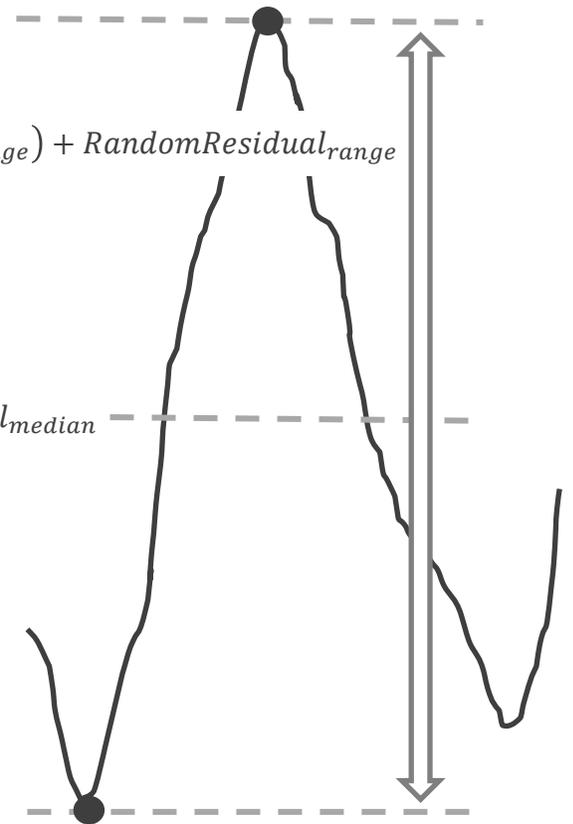
Archetype storm



Scaled storm profile

$$S'_{min} = f(\mathbf{H}S_{sp}, HS_{lock}, surgeLock_{range}, slope_{range}) + RandomResidual_{range}$$

$$S'_{median} = f(\mathbf{H}S_{sp}, HS_{lock}, surgeLock_{median}, slope_{median}) + RandomResidual_{median}$$



# Storm surge modelling performance

## ■ Based on:

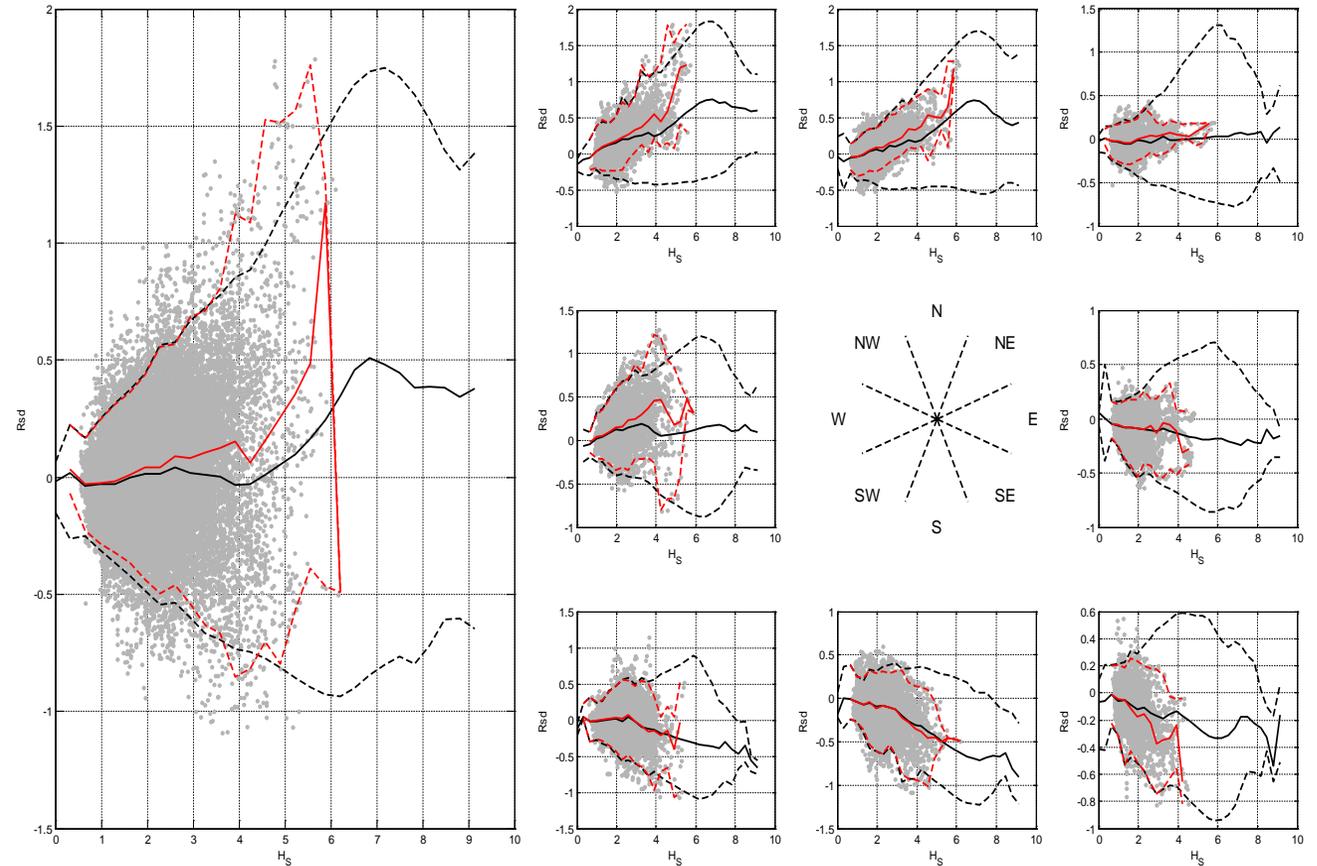
- Max, min, median
- Observed data points (grey)

## ■ Comparison of:

- observed (red)
- modelled (black)

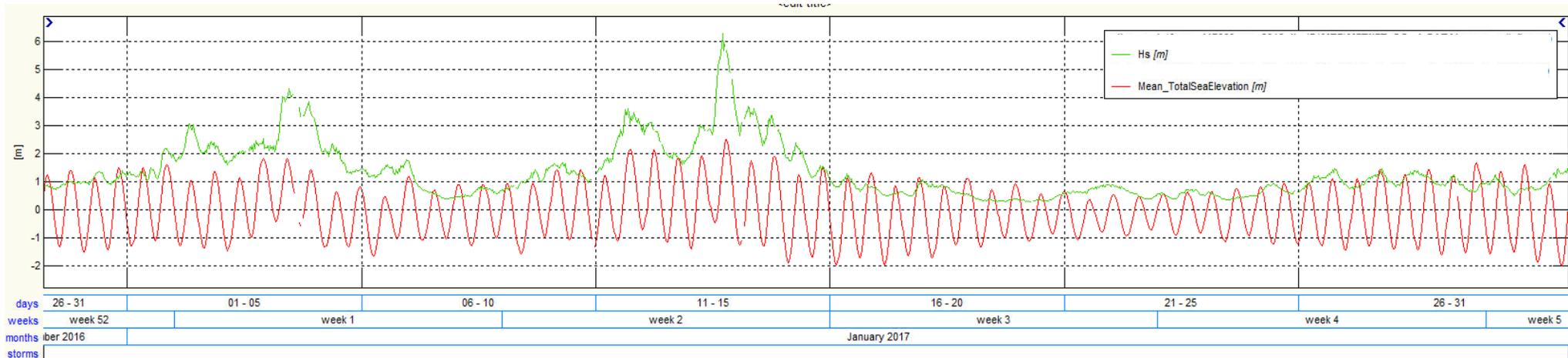
## ■ Medians (solid)

## ■ 95% UI (dashed)



# Inclusion of tides

- Sample tide
  - In “deep water” by just a random selection
  - In shallow water, tide is sampled from the same storm as the waves, to preserve interactions



## Simulation within storms

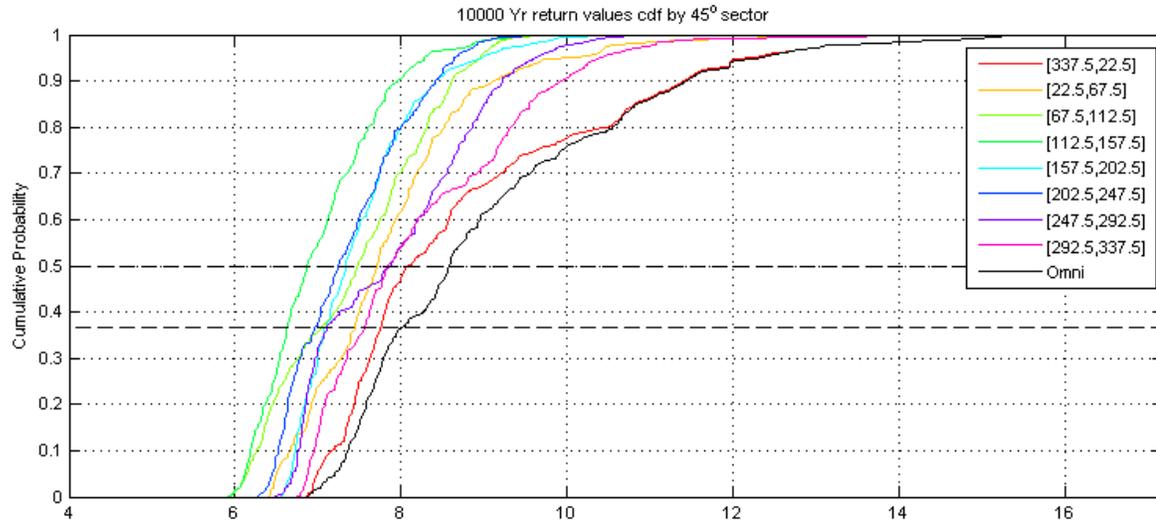
- $Hs_{sp}$  simulated with an associated  $Drc_{sp}$  and  $Ssn_{sp}$  using varying Poisson and GP distributions
- Randomly-selected archetype storm from the “closest” bin
- Re-scale:
  - entire storm profile  $Hs$  to match the peak to  $Hs_{sp}$ ,  $Drc_{sp}$  and  $Ssn_{sp}$
  - wave periods scaled to maintain wave steepnesses
  - surge history using the selected method
- Add tide (or re-using tide, if a shallow-water location)
- Calculate total water depth for every sea state as *tide + surge + water depth*
- Randomly sample  $C_{max}$  from each sea state using Forristall distribution based on total water depth
- Calculate the maximum  $TWL$  from every sea-state within the storm as:  $C_{max} + surge + tide$
- For each storm, max value of  $TWL$  is saved for every  $Drc$ - $Ssn$  bin impacted by storm

# Return period simulation

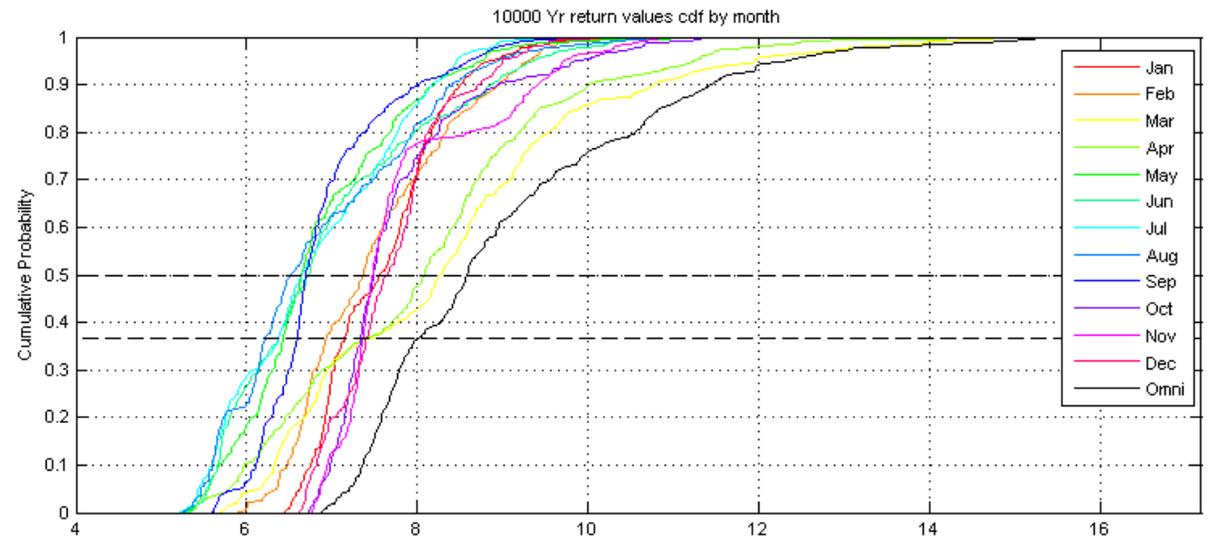
Use Monte Carlo approach to simulate all storms in a RP of interest multiple times:

- Fit Poisson and GP models to  $B$  bootstraps of the original data
- Make  $R$  realisations of the full return period of interest for each bootstrap
- Produces  $B \times R$  simulations
- Save largest values  $TWL$  for every  $Drc$ - $Ssn$  bin
- Produce CDFs for the maxima for RP of interest in each  $Drc$  and  $Ssn$  bin
- RV for each bin is taken as the most-probable value  $\sim \exp(-1)$
- Other percentiles, can also be captured, e.g. 95% UI

# 10,000-year return values ranges



- Based on ranked storm maxima from all simulations
- The more simulations, the smoother the lines
- This example based on 300 simulations
- The spread reflects uncertainties ....



# Uncertainty

- The natural randomness (aleatory uncertainty)
  - Sampling storm peaks multiple times from the fitted model
  - Sampling storm profiles
  - Sampling wave height and crests
  - Re-scaling surge histories from the residuals of the fitted distribution
  - (Sampling tide randomly)
- Uncertainty in model based on finite sample and modelling assumptions (epistemic uncertainty)
  - EVA threshold
  - Bootstrapping the original data multiple times
- Aleatory uncertainty dominates the confidence interval range within CEVA.

# Results

## Wave crest directional return values

Crest	Return Period [years]					
	1	10	50	100	1000	10000
N	4.7	6.6	7.7	8.1	9.5	10.6
NE	3.8	5.8	6.9	7.3	8.5	9.8
E	3.6	5.4	6.5	6.8	8.2	9.2
SE	3.9	5.5	6.3	6.7	7.7	8.7
S	5.0	6.5	7.3	7.6	8.7	9.5
SW	4.9	6.3	7.1	7.5	8.4	9.4
W	4.9	6.6	7.4	7.8	8.9	9.9
NW	4.9	6.8	7.7	8.0	9.1	10.3
Omni	6.2	7.6	8.4	8.8	10.0	11.0

## TEWL directional return values

TEWL	Return Period [years]					
	1	10	50	100	1000	10000
N	5.7	7.7	8.9	9.3	10.7	11.9
NE	4.7	6.7	7.9	8.3	9.6	10.8
E	4.4	6.1	7.2	7.5	8.8	10.1
SE	4.5	6.1	6.9	7.3	8.4	9.4
S	5.5	7.0	7.8	8.1	9.1	10.1
SW	5.6	7.0	7.8	8.2	9.1	10.1
W	5.9	7.5	8.5	8.8	10.0	11.1
NW	6.1	8.0	9.0	9.5	10.7	11.7
Omni	7.0	8.7	9.6	9.9	11.3	12.4

## SWL directional return values

Implied SWL	Return Period [years]					
	1	10	50	100	1000	10000
N	0.9	1.1	1.2	1.2	1.1	1.3
NE	0.9	0.9	1.0	1.0	1.1	1.0
E	0.8	0.7	0.7	0.7	0.7	0.9
SE	0.6	0.6	0.6	0.6	0.7	0.7
S	0.5	0.5	0.5	0.6	0.5	0.5
SW	0.7	0.6	0.7	0.7	0.7	0.8
W	1.0	0.9	1.1	1.0	1.1	1.2
NW	1.2	1.2	1.3	1.4	1.6	1.4
Omni	0.8	1.1	1.2	1.1	1.3	1.4

## Wave crest monthly return values

Crest	Return Period [years]					
	1	10	50	100	1000	10000
Jan	4.8	6.6	7.5	7.9	9.0	9.9
Feb	4.2	6.0	6.9	7.3	8.4	9.4
Mar	3.7	5.4	6.6	7.0	8.5	9.7
Apr	3.4	5.2	6.3	6.8	8.1	9.7
May	2.9	4.6	5.7	6.2	7.2	8.3
Jun	2.6	4.4	5.4	5.8	7.1	8.1
Jul	2.7	4.3	5.5	5.9	7.2	8.1
Aug	3.0	4.7	5.8	6.2	7.3	8.2
Sep	3.4	5.3	6.3	6.6	7.7	8.6
Oct	4.1	6.1	7.1	7.5	8.8	10.0
Nov	4.6	6.6	7.6	7.9	9.1	10.0
Dec	5.1	6.8	7.6	8.0	9.1	10.1
All Year	6.2	7.6	8.4	8.8	10.0	11.0

## TEWL monthly return values

TEWL	Return Period [years]					
	1	10	50	100	1000	10000
Jan	5.6	7.4	8.4	8.7	10.0	10.8
Feb	4.9	6.7	7.9	8.3	9.4	10.4
Mar	4.6	6.3	7.5	7.9	9.2	10.6
Apr	4.4	6.0	7.1	7.6	9.0	10.4
May	3.7	5.4	6.6	7.0	8.0	9.1
Jun	3.6	5.2	6.3	6.8	8.1	9.1
Jul	3.7	5.4	6.6	7.0	8.3	9.3
Aug	4.1	5.8	6.9	7.2	8.4	9.4
Sep	4.4	6.6	7.4	7.8	9.0	9.9
Oct	5.2	7.3	8.4	8.9	10.2	11.3
Nov	5.5	7.7	8.8	9.2	10.5	11.5
Dec	5.8	7.7	8.6	9.0	10.2	11.3
All Year	7.0	8.7	9.6	9.9	11.3	12.4

## SWL monthly return values

Implied SWL	Return Period [years]					
	1	10	50	100	1000	10000
Jan	0.9	0.8	0.9	0.8	1.0	0.9
Feb	0.8	0.8	1.0	1.0	1.0	1.0
Mar	0.9	0.9	0.9	0.9	0.8	0.9
Apr	0.9	0.8	0.8	0.7	1.0	0.7
May	0.9	0.8	0.9	0.8	0.8	0.8
Jun	1.0	0.8	0.9	1.0	1.0	1.0
Jul	1.0	1.0	1.1	1.1	1.1	1.1
Aug	1.1	1.1	1.1	1.1	1.1	1.2
Sep	1.0	1.2	1.1	1.2	1.2	1.3
Oct	1.1	1.3	1.3	1.4	1.3	1.3
Nov	0.9	1.1	1.3	1.3	1.4	1.5
Dec	0.7	0.9	1.0	1.1	1.1	1.2
All Year	0.8	1.1	1.2	1.1	1.3	1.4

## Summary

- Described a process to derive TEWL using the CEVA approach.
- Storm peak modelling based on spline-smoothed: thresholds, Poisson rates, GP parameters.
- Wave storm profiles are sampled and scaled to match peaks of simulated storms.
- Associated storm surge profiles re-scaled using linear regressions and residuals.
- Addition of random or associated tides, depending on water depth.
- Monte Carlo simulation is used based on multiple bootstraps and realisations
  
- Still under development:
  - Better scale storm characteristics to very large storms, e.g. storm length
  - Use of Heffernan & Tawn conditional extremes approach to extrapolate surge characteristics to higher  $H_s$
  - Increasing use of Numerical Integration (rather than Monte Carlo approach)
  - Incorporation of more recent wave crest probability distributions