Estimation of directional spreading within surface water waves

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Introduction

- Crest height statistics important for design of marine structures;
- Evidence of amplifications beyond 2nd-order from the field; (Christou & Ewans, 2012)
- Directionality acts to reduce these non-linear amplifications;
- Limited amount of data from the field in steep sea-states;
- Laboratory simulations incorporating non-linearity and directionality;
- Role of directionality in steep sea-states and the formation of large wave events.

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Key points

1. Generation of directionally spread sea-states in a laboratory environment;

2. Estimation of the degree of directional spreading within a sea-state and large individual waves;

3. Changes in the directional spreading with increasing steepness.

Experimental investigation

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Water depth: $d = 1.25$ m

Measurements: η and u, v, w.

20 seeds x 1024 s

JONSWAP spectra, $\gamma = 2.5$

$H_s = [0.10, 0.15, 0.20]$ m

$T_p = 1.6$ s, $\sigma_\theta = 15^o$

For the frequency spectrum, $S_{\eta\eta}(\omega)$, the directional spectrum is:

$$F(\omega, \theta) = S_{\eta\eta}(\omega) D(\omega, \theta) \, , \tag{1}$$

where $D(\omega, \theta)$ is the directional spreading function (DSF).

In this study: Gaussian DSF - frequency independent

$$D(\omega, \theta) = \frac{A}{\sigma_\theta \sqrt{2\pi}} e^{\left[-\frac{(\theta-\theta_m)^2}{2\sigma_\theta^2}\right]} \tag{2}$$

with σ_θ the standard deviation, θ_m the mean wave direction and A a scaling parameter.

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Methods of directional wave generation: DSM

Three methods of generation:

- Double Summation Method (DSM):
  - Each frequency (*i=1:N*) has M wave components at different directions
  - Total MN components
  - Issues with ergodicity, cancellation of wave components with same frequency but different directions

$$\eta(x, y, t) = \sum_{i=1}^{N} \sum_{j=1}^{M} A_{ij} \cos[\omega_i t - k(x \cos\theta_j + y\, sin\theta_j) + \epsilon_{ij}] , \quad (3)$$

where

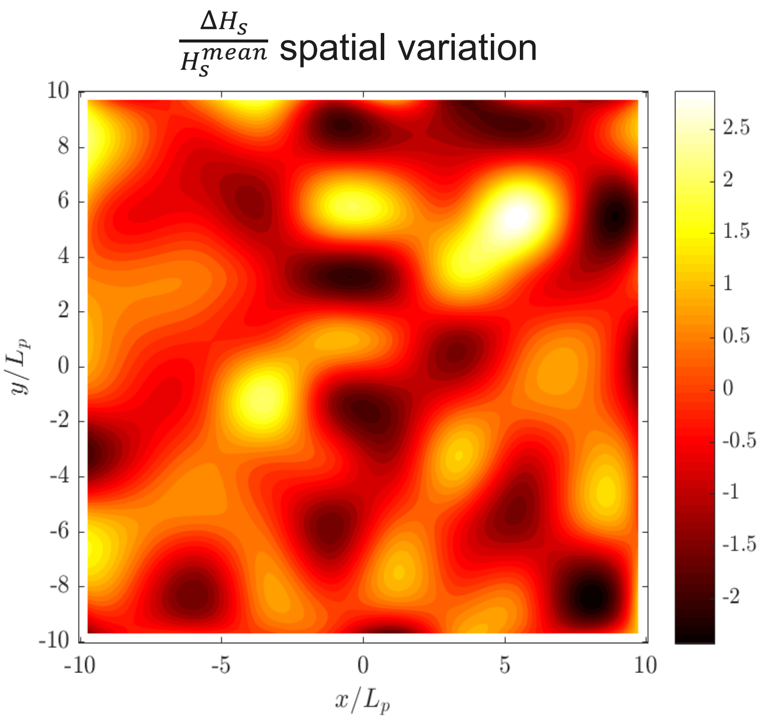
$$\omega_i = i(2\pi\Delta f),\ \theta_j = j\, \Delta\theta \text{ and } A_{ij} = \sqrt{2\, S_{\eta\eta}(\omega, \theta)\Delta\omega\Delta\theta}$$

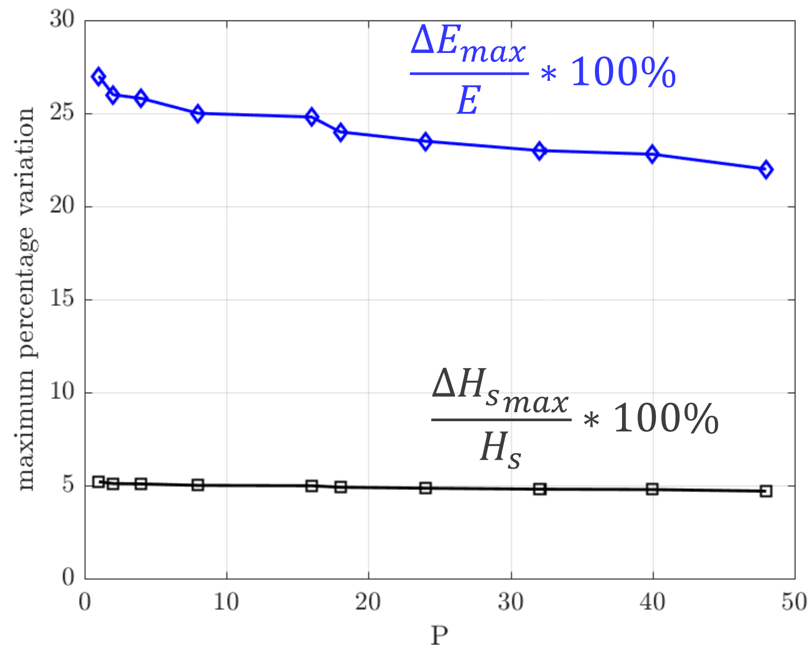
Methods of directional wave generation: DSM

Numerical Simulations with LRWT:

- Repeat time = 1024 s (3 hrs field equivalent) $\Delta f = 1/1024$ Hz
- Variations in H_s~5% → unwanted in model testing
- Increase frequency discretization $\frac{1}{P\Delta f}$, $\mathrm{P} = 2,4,8 \ldots 48$

→ practical problems with wave-makers





Methods of directional wave generation: SSM,RDM

- Single Summation Method (SSM):
  - Widely applied
  - Division in frequency bands,
  - Each frequency – 1 direction
  - No problems with ergodicity
  - Sensitive to the discretization of the frequency spectrum

$\eta(x,y,t) = \sum_{i=1}^{N} A_i \cos[\omega_i t - k(x \cos\theta_i + y \, sin\theta_i) + \epsilon_i]$ (4) where

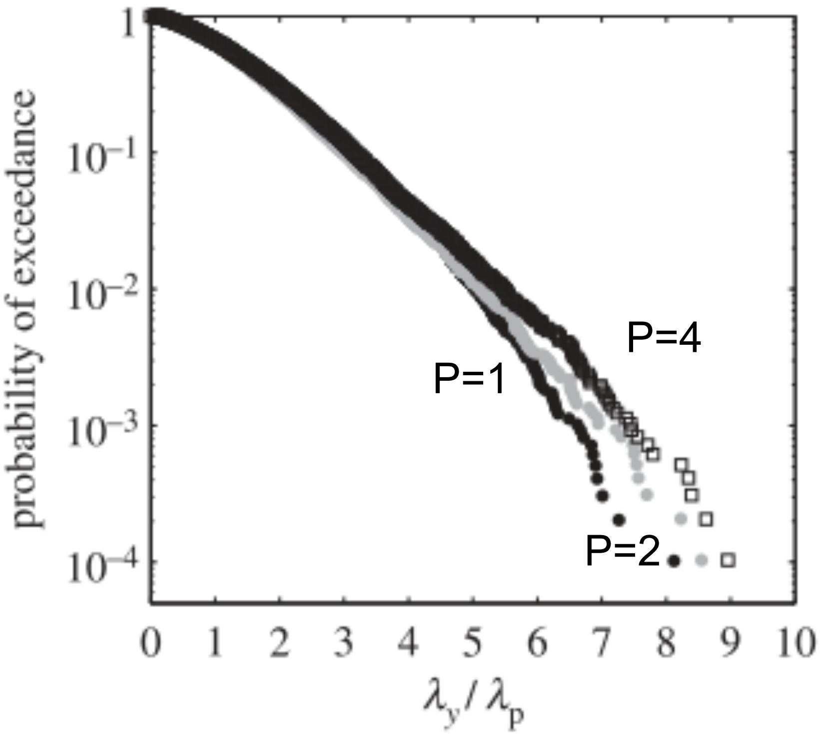
$\omega_i = \frac{i \, \Delta\Omega}{M}$, where $\Delta\Omega$ is the width of the band.

- Random Directional method (RDM)
  - Same as above, single summation
  - Each frequency – 1 direction across the whole spectrum
  - Directions chosen randomly from a weighting function
  - Less sensitive to discretization

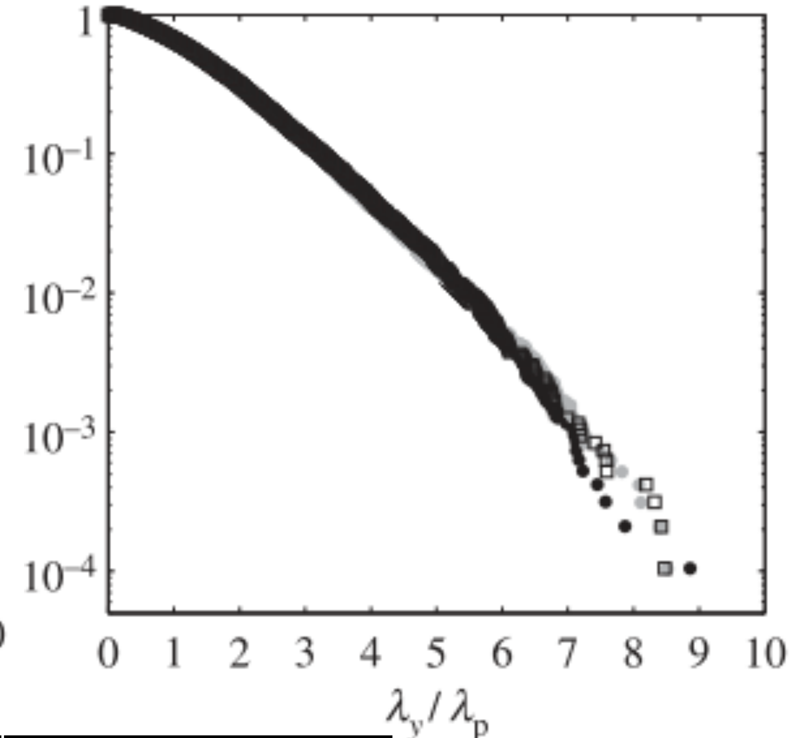
Methods of directional wave generation: SSM,RDM

Normalised crest lengths

SSM



RDM



P	Δf (Hz)	no. directional components	no. frequency bands
1	1/1024	25	64
2	1/2048	30	108
4	1/4096	40	161

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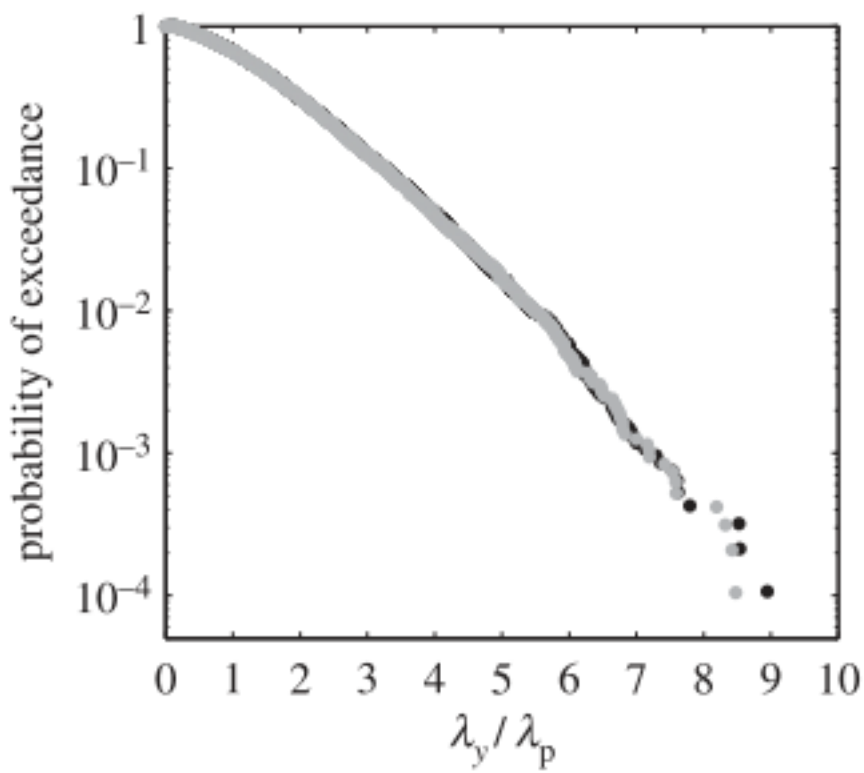
Methods of directional wave generation: SSM,RDM

Same results for finer discretization

SSM: $\Delta f = 1/8192$ RDM: $\Delta f = 1/4096$

SSM/RDM



All methods give same results for:

$$\frac{1}{\Delta f} \rightarrow 0$$

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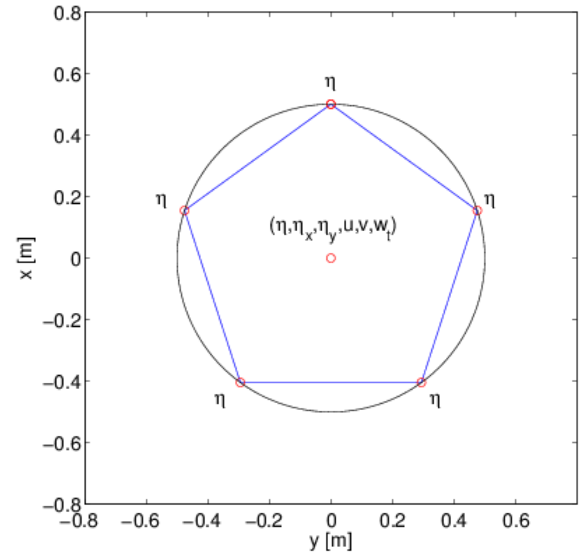
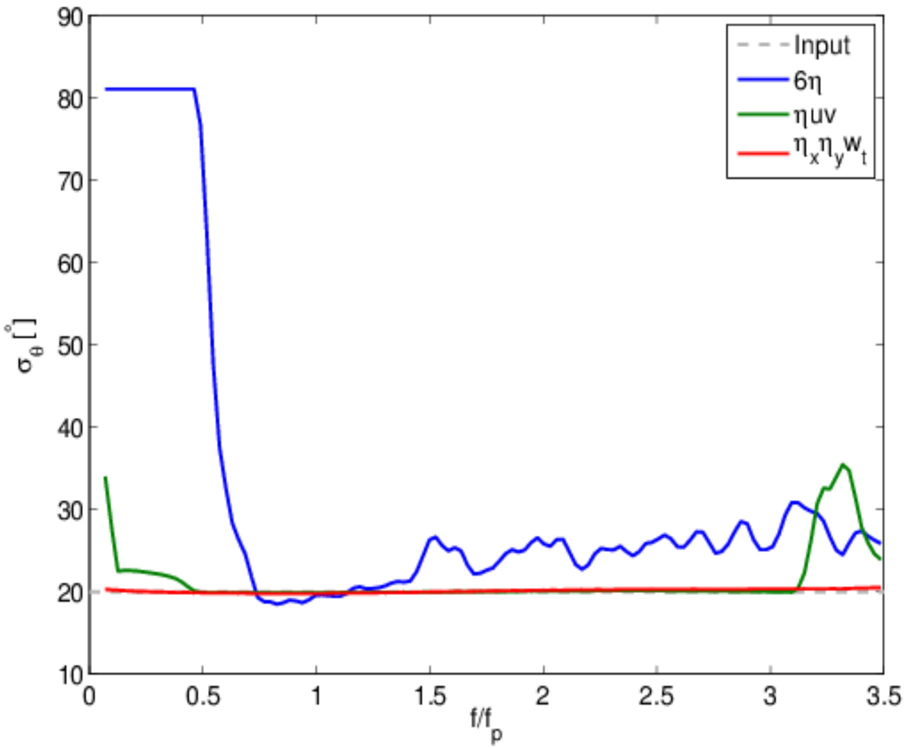
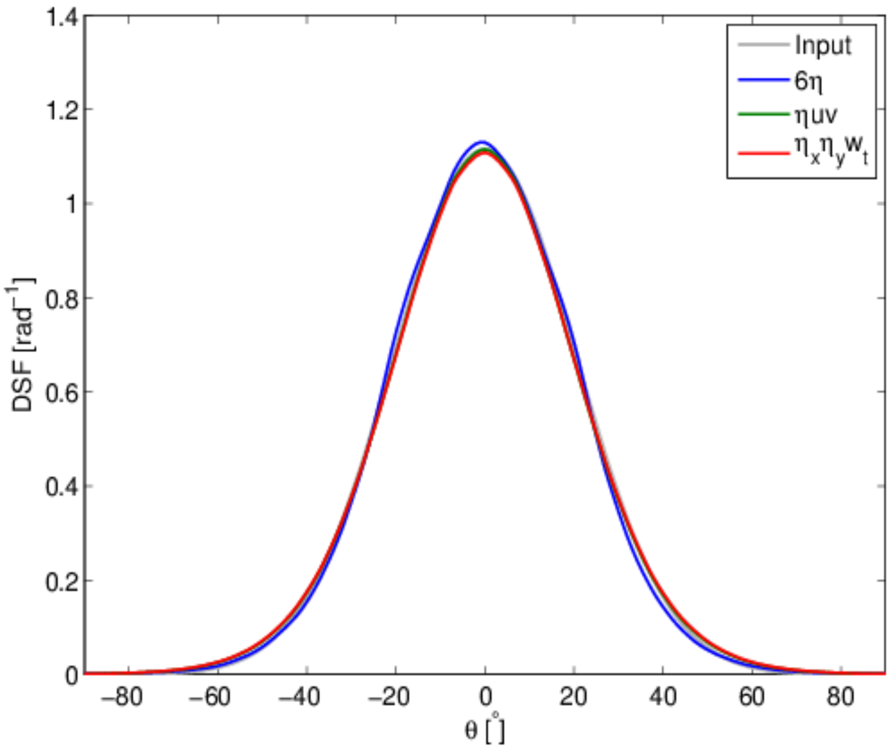
Directional analysis: input data

- σ_θ=20°, 20 seeds x 1024 s
- calculations based upon the EMEP
- comparisons between various input data





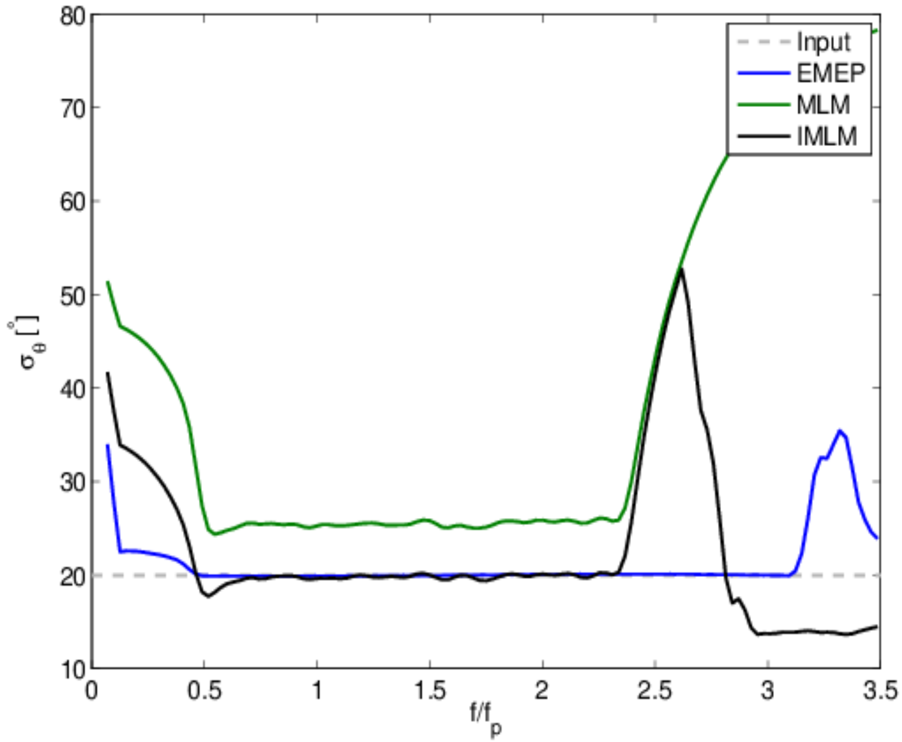
σ_θ vs. f/f_p



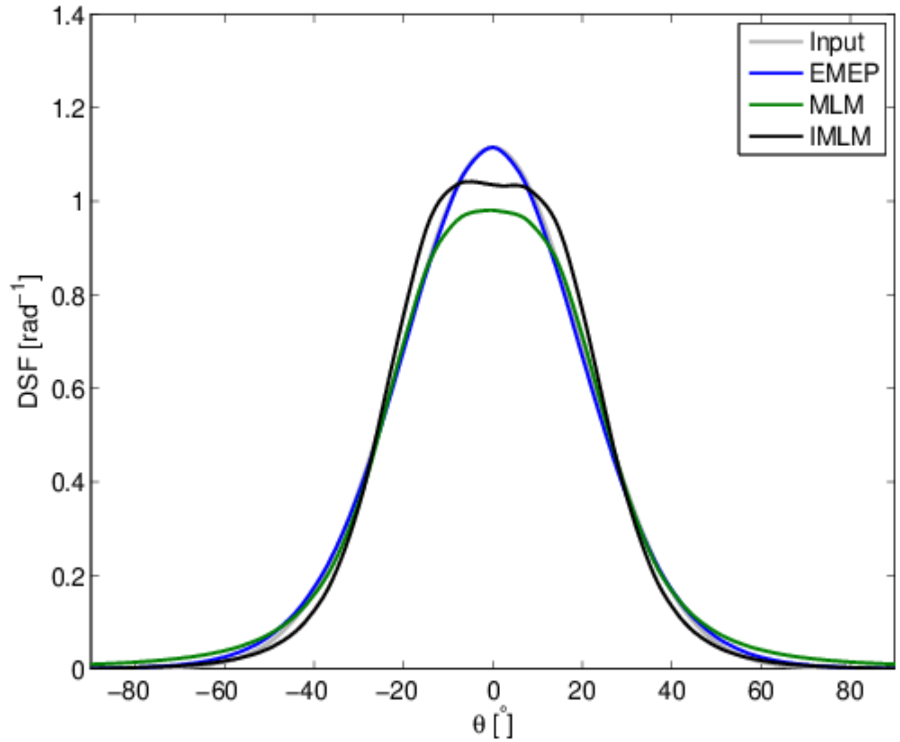
Directional spreading function, DSF

Directional analysis: methods

- σ_θ=20°, 20 seeds x 1024 s
- calculations based upon *η*,*u*,*v*
- comparisons between various analysis methods



σ_θ vs. f/f_p

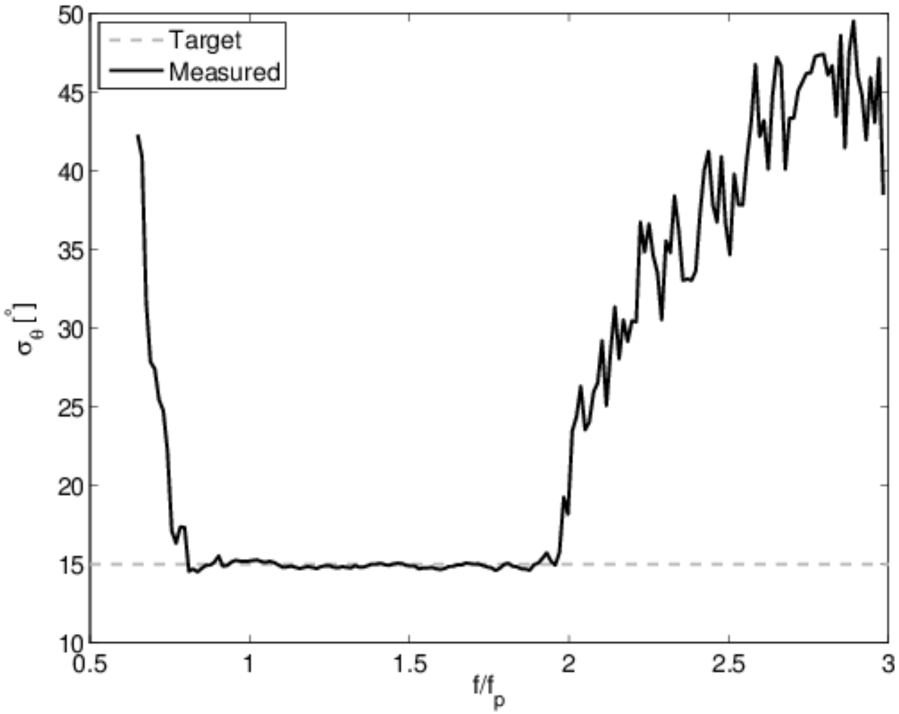


Directional spreading function, DSF

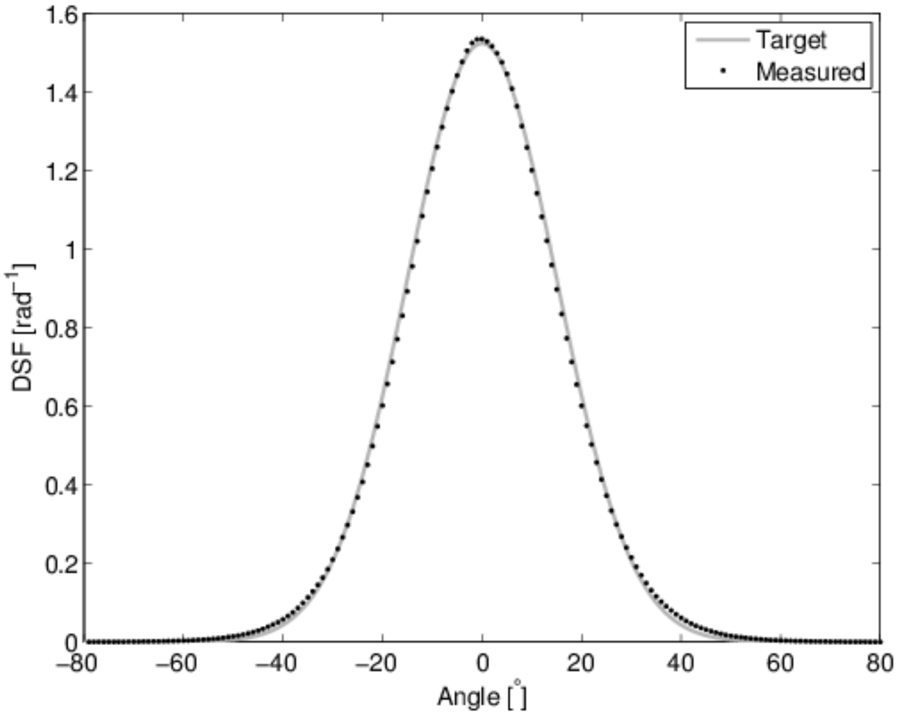
Directional analysis: sea-state

Comparisons to laboratory data (H_s=10m, ½$H_s k_p$=0.081)

- σ_θ=15°
- input data: η,u,v
- calculated using the EMEP
- sea state generated using RDM



σ_θ vs. f/f_p



Directional spreading function, DSF

Directional analysis: sea-state

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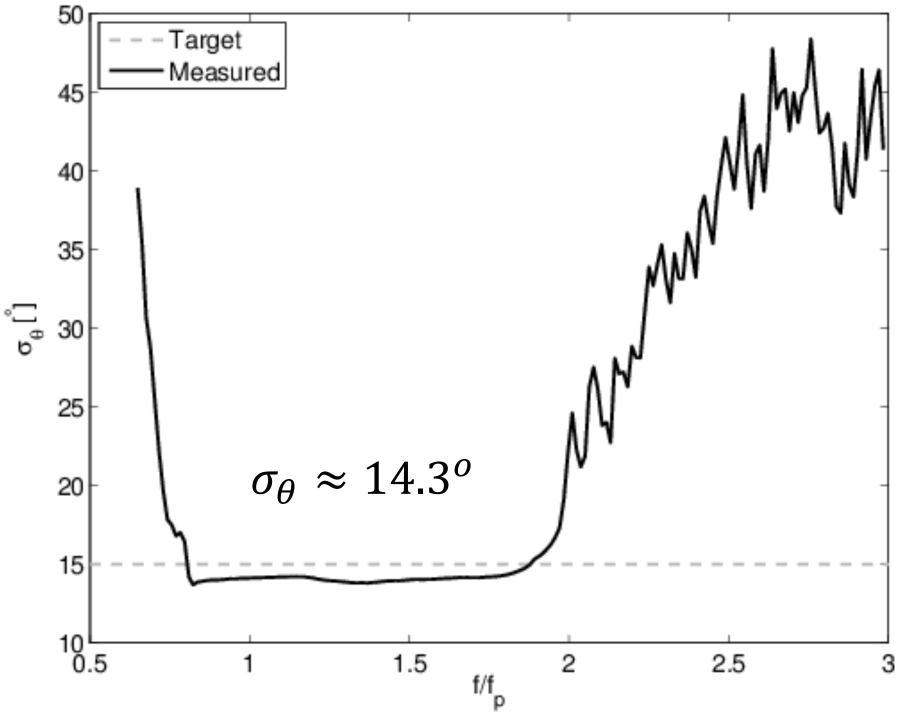
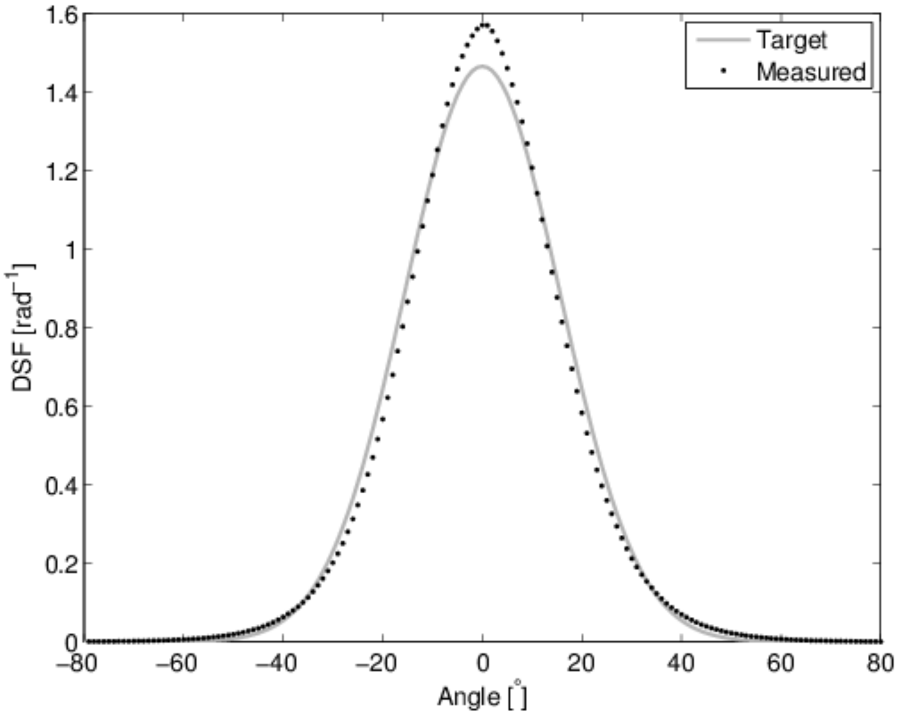
Comparisons to laboratory data (H_s=15.0m, ½H_sk_p=0.122)

- σ_θ=15°
- input data: η,u,v
- calculated using the EMEP
- sea state generated using RDM



σ_θ vs. f/f_p

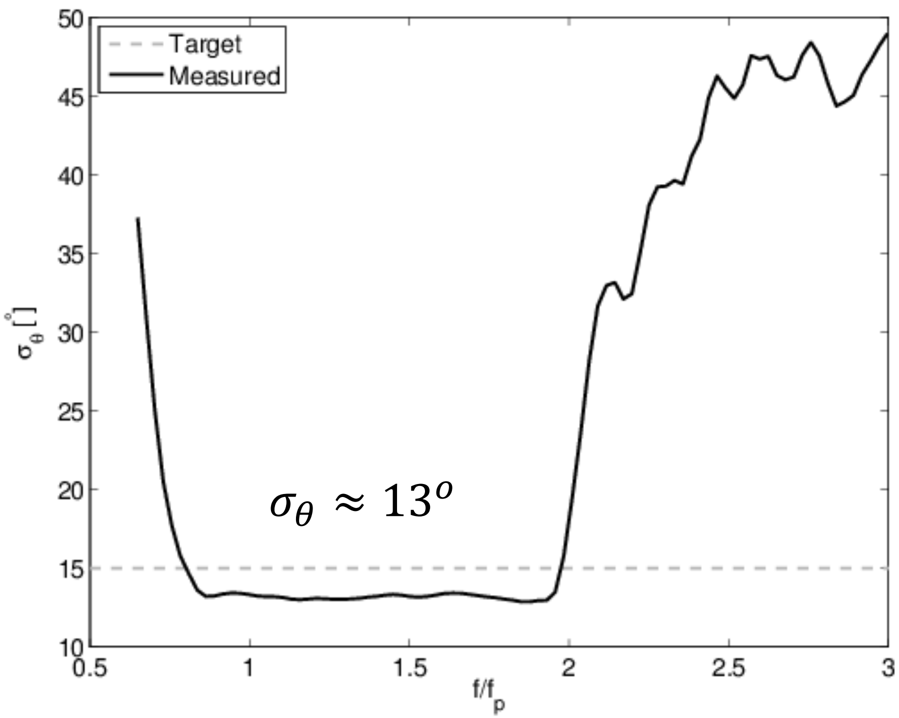


Directional spreading function, DSF

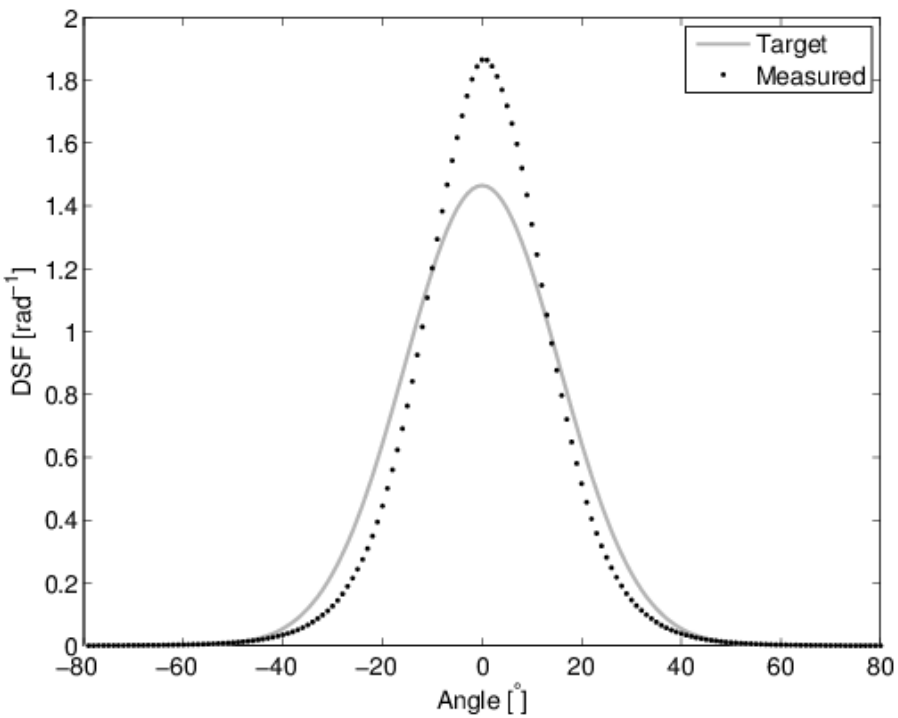
Directional analysis: sea-state

Comparisons to laboratory data (H_s=20.0m, $\frac{1}{2}H_s k_p$=0.163)

- σ_θ=15°
- input data: η, u, v
- calculated using the EMEP
- sea state generated using RDM



σ_θ vs. f/f_p



Directional spreading function, DSF

Directional analysis: alternative method

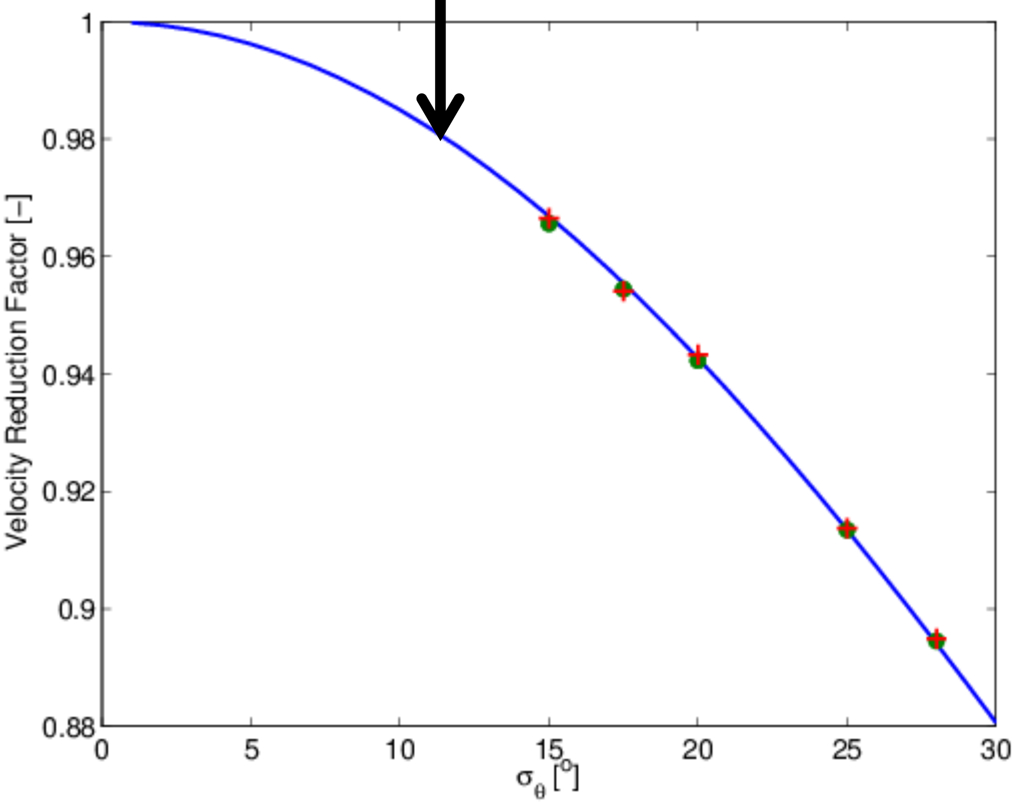
Based upon the velocity reduction factor (VRF)

$$\text{VRF}=\left[\frac{1}{2}+\frac{1}{2}exp\left(-2\sigma_\theta^2\right)\right]^{\frac{1}{2}} \quad (5)$$

(Tucker & Pitt, 2001)

or

$$\text{VRF}=\frac{(rms\ u_x\ in\ directional\ sea)}{(rms\ u_x\ in\ uni{-}directional\ sea)}$$

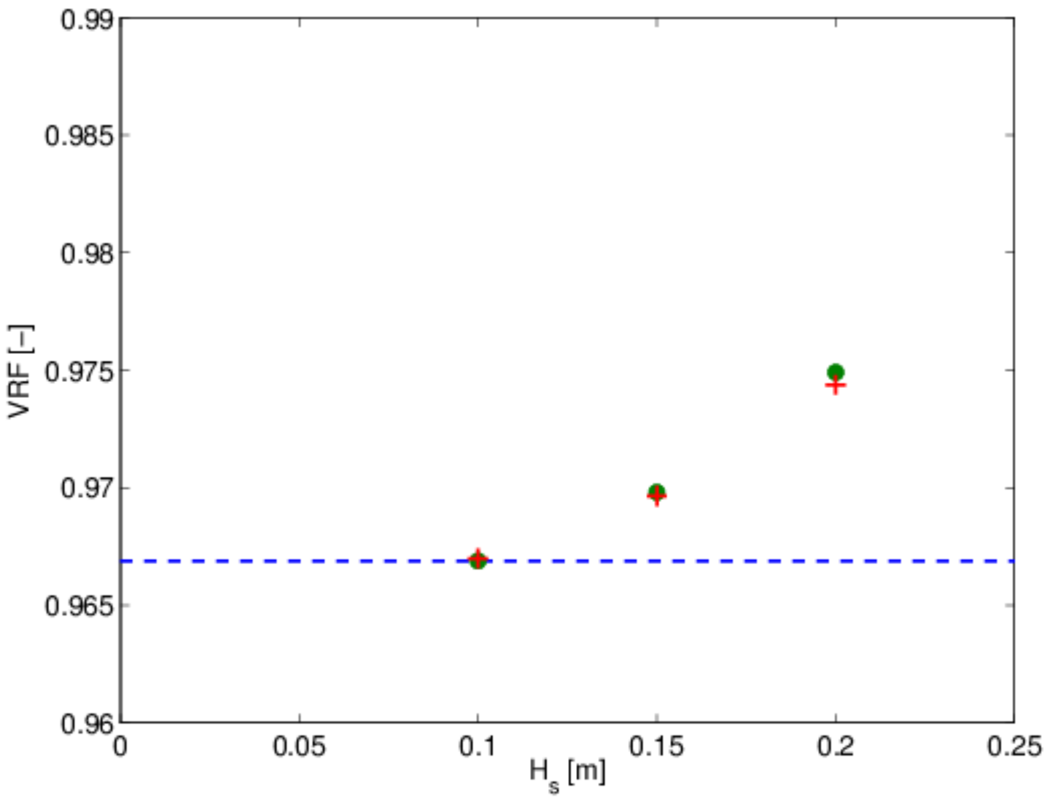


Based upon:

- ● the analysis of an entire wave record
- + the average of a wave-by-wave analysis

Directional analysis: alternative method

- comparisons to laboratory data
- VRF averaged over 20 x 3-hour seeds for each sea state
- changes with H_s

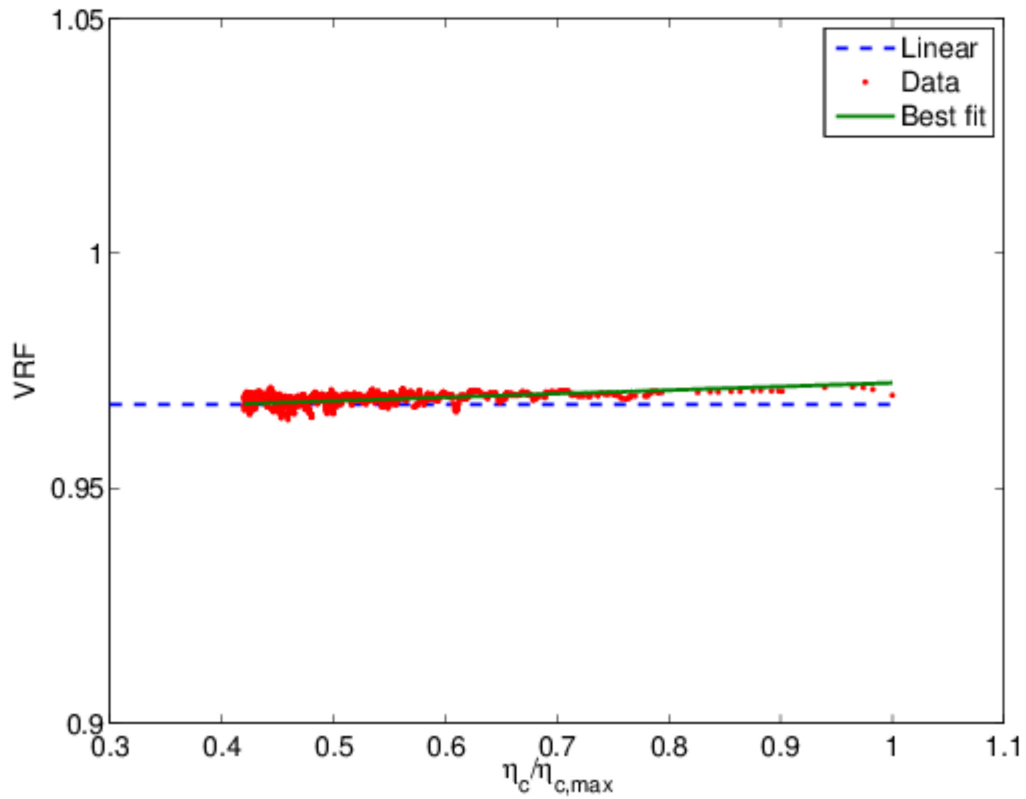


- - - linear input σ_θ
● velocity ratio
+ earlier EMEP σ_θ

Steeper sea-states are more uni-directional

Velocity reduction factor (VRF)

- Comparisons to laboratory data (H_s=10m, σ_θ=15°, ½$H_s k_p$=0.081)
- VRF calculated for individual waves
- Plotted in terms of the normalised crest elevation, η_c/η_{cmax}



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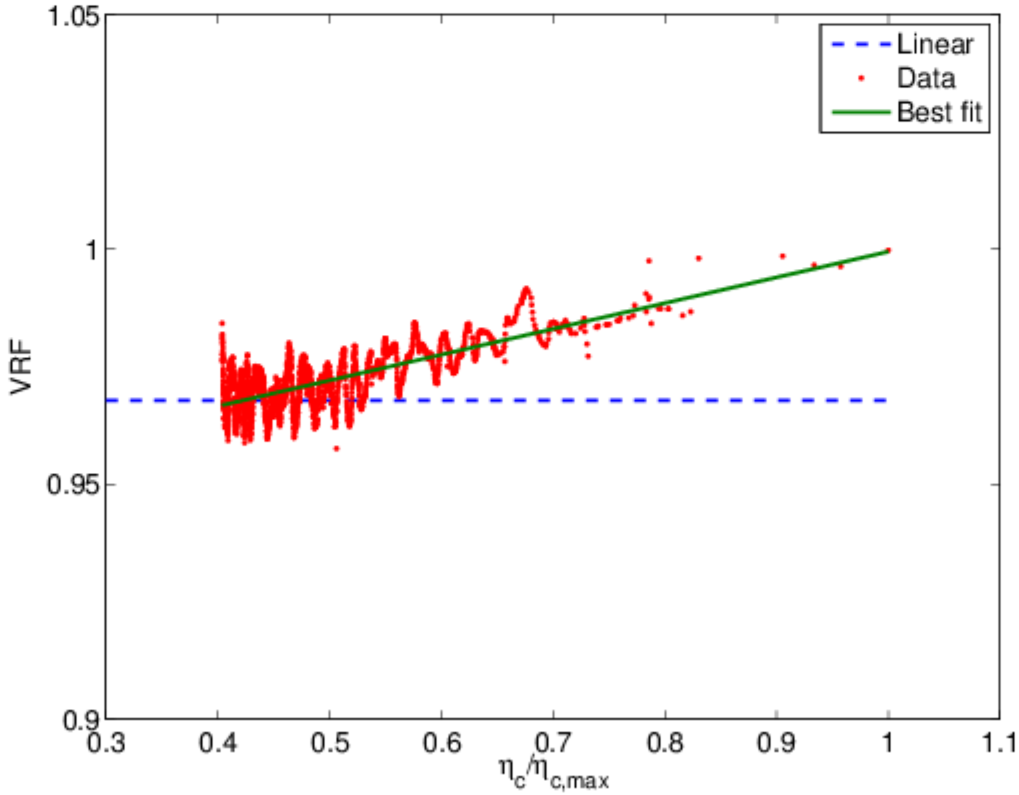

Velocity reduction factor (VRF)

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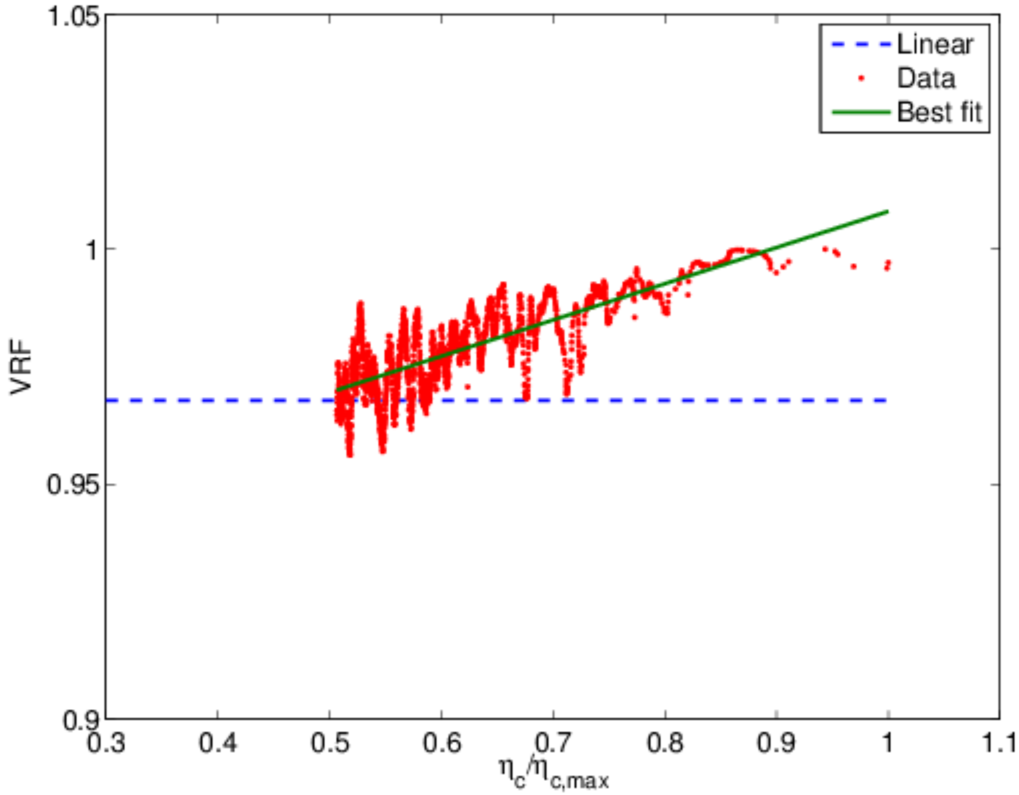

Concluding remarks

- Preferred method of directional wave generation: RDM
  - Computationally efficient
  - Ergodic
  - Easy to implement
- Directional spreading decreases in steeper sea-states
- Large individual waves are less directionally spread
- Results agree with experimental and numerical studies (e.g. Johannessen & Swan, 2001 & 2003; Adcock, et al. 2012 & 2015)
- Extension to intermediate and shallow water depths through LoWiSh JIP (currently restricted)
- Part of results available in Proc. Royal Society:

"A laboratory study of nonlinear changes in the directionality of extreme seas" (2017), M.Latheef, C.Swan, J.Spinneken

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Thank you for your attention!

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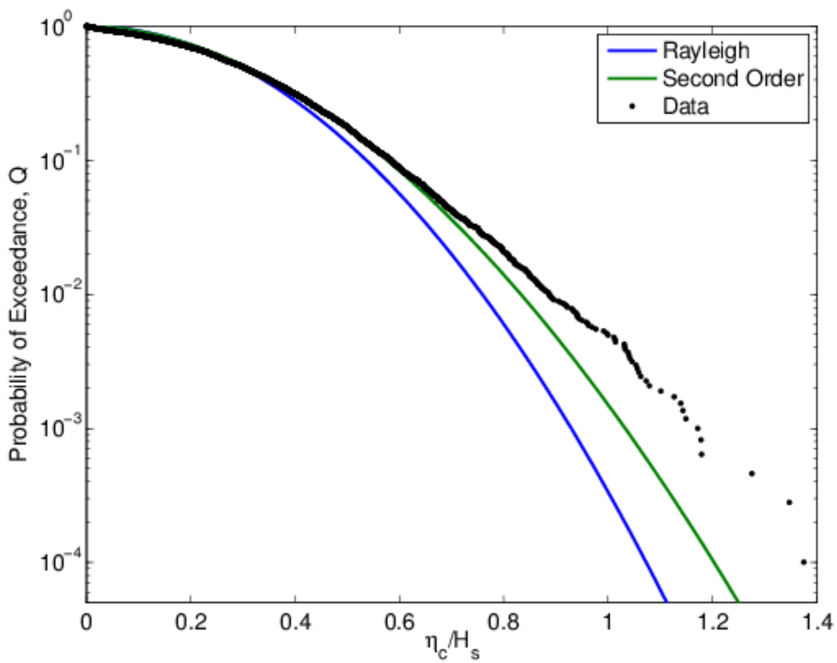
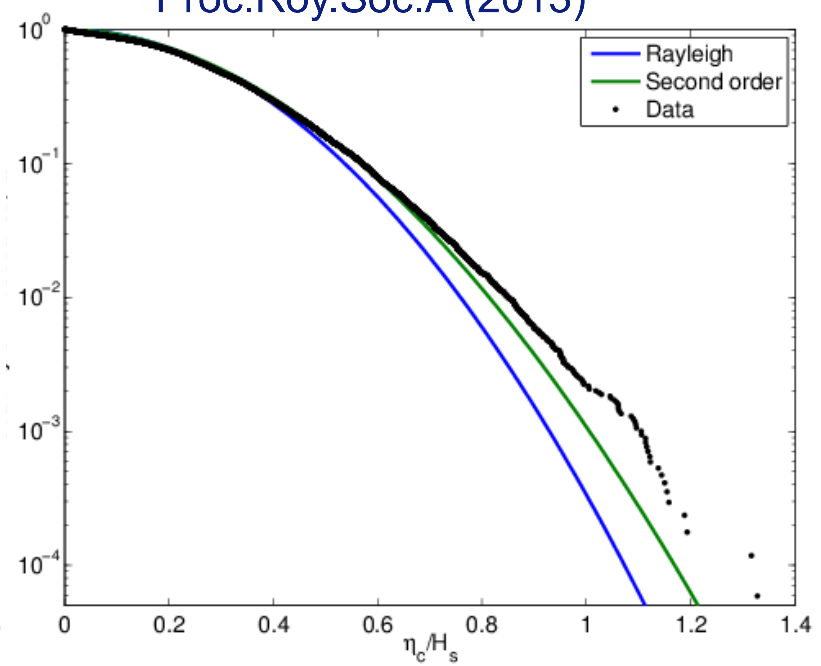
Short-term distribution of crest heights

- Effects beyond $O(a^2k^2)$
- Both in field data (North Sea) and laboratory data (ICL)

Field data (H_s>12 m)
Christou & Ewans (2012)
CREST JIP

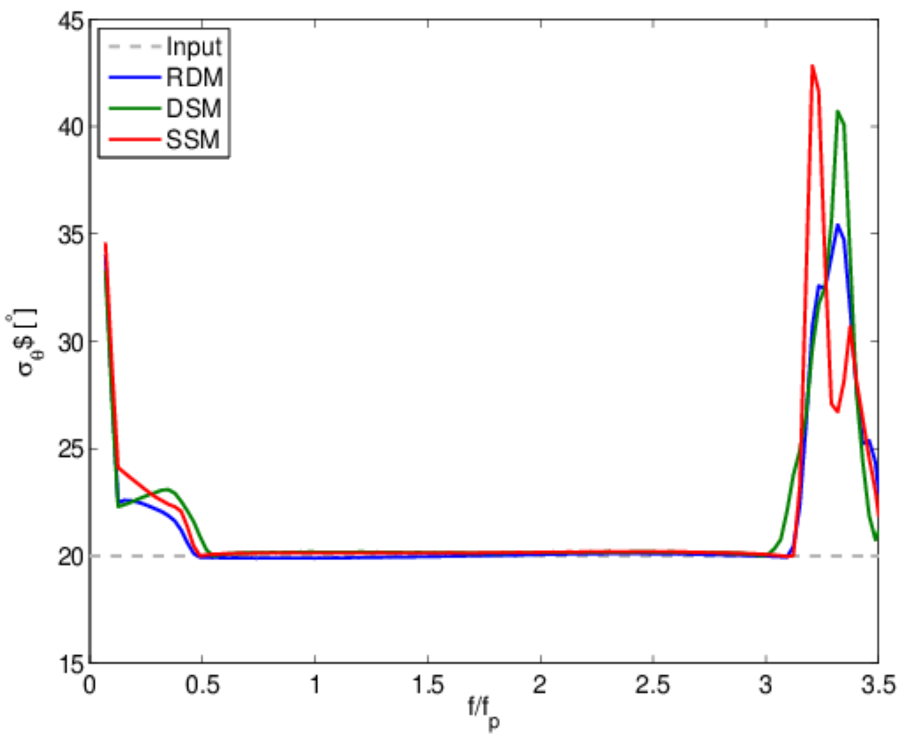


Experimental data (H_s=12.5 m, T_p=16 s, σ_θ=15°)
Latheef & Swan
Proc.Roy.Soc.A (2013)

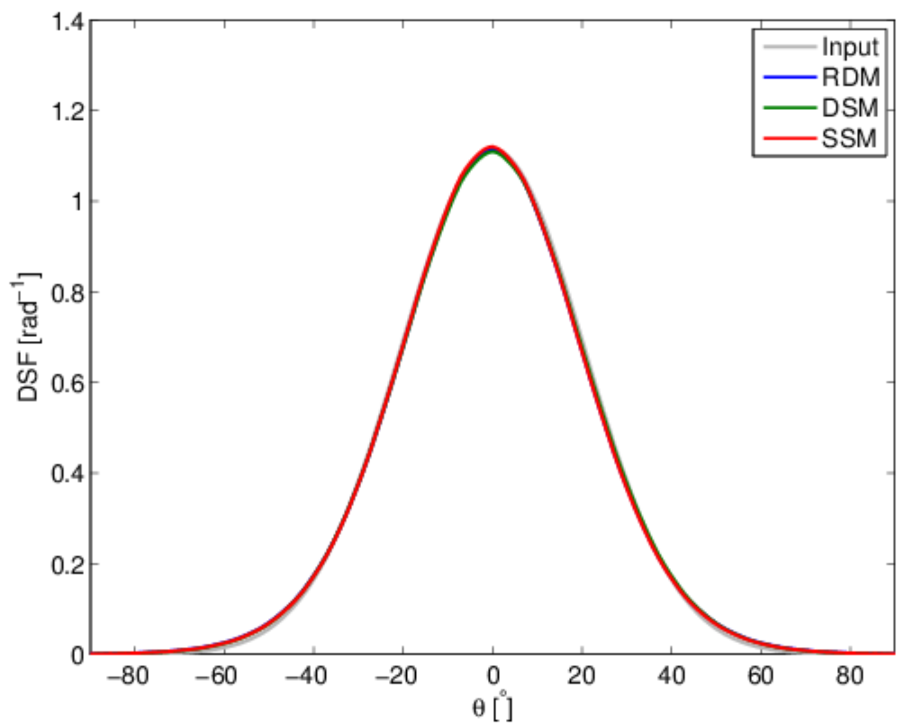


Directional analysis: Generation method

- σ_θ=20°
- calculations based upon the EMEP
- comparisons between different methods of directional simulation



σ_θ vs. f/f_p



Directional spreading function, DSF

Directional spectrum

Given the frequency spectrum, $S_{\eta\eta}(\omega)$, the directional spectrum is:

$$F(\omega,\theta) = S_{\eta\eta}(\omega)D(\omega,\theta)\ ,$$

where $D(\omega,\theta)$ is the directional spreading function (DSF).

In terms of Fourrier series:

$$D(\omega,\theta) = \frac{1}{\pi}\left\{\frac{1}{2} + \sum_{n=1}^{\infty} A_n(\omega)\cos n\theta + B_n(\omega)\sin n\theta\right\}, \qquad (1)$$

where $A_n(\omega) = \int_{-\pi}^{\pi} D(\omega,\theta)\cos n\,\theta$ and $B_n(\omega) = \int_{-\pi}^{\pi} D(\omega,\theta)\sin n\,\theta$

In this study: Gaussian DSF - frequency independent

$$D(\omega,\theta) = \frac{A}{\sigma_{\theta\sqrt{2\pi}}}\, e^{\left[-\frac{(\theta-\theta_m)^2}{2\sigma_\theta^2}\right]} \qquad (2)$$

with σ_θ the standard deviation and θ_m the mean wave direction

RMS spreading: $\sigma_\theta = \sigma_1(\omega) = \sqrt{2[1-\sqrt{A_1^2(\omega)+B_1^2(\omega)}]}$

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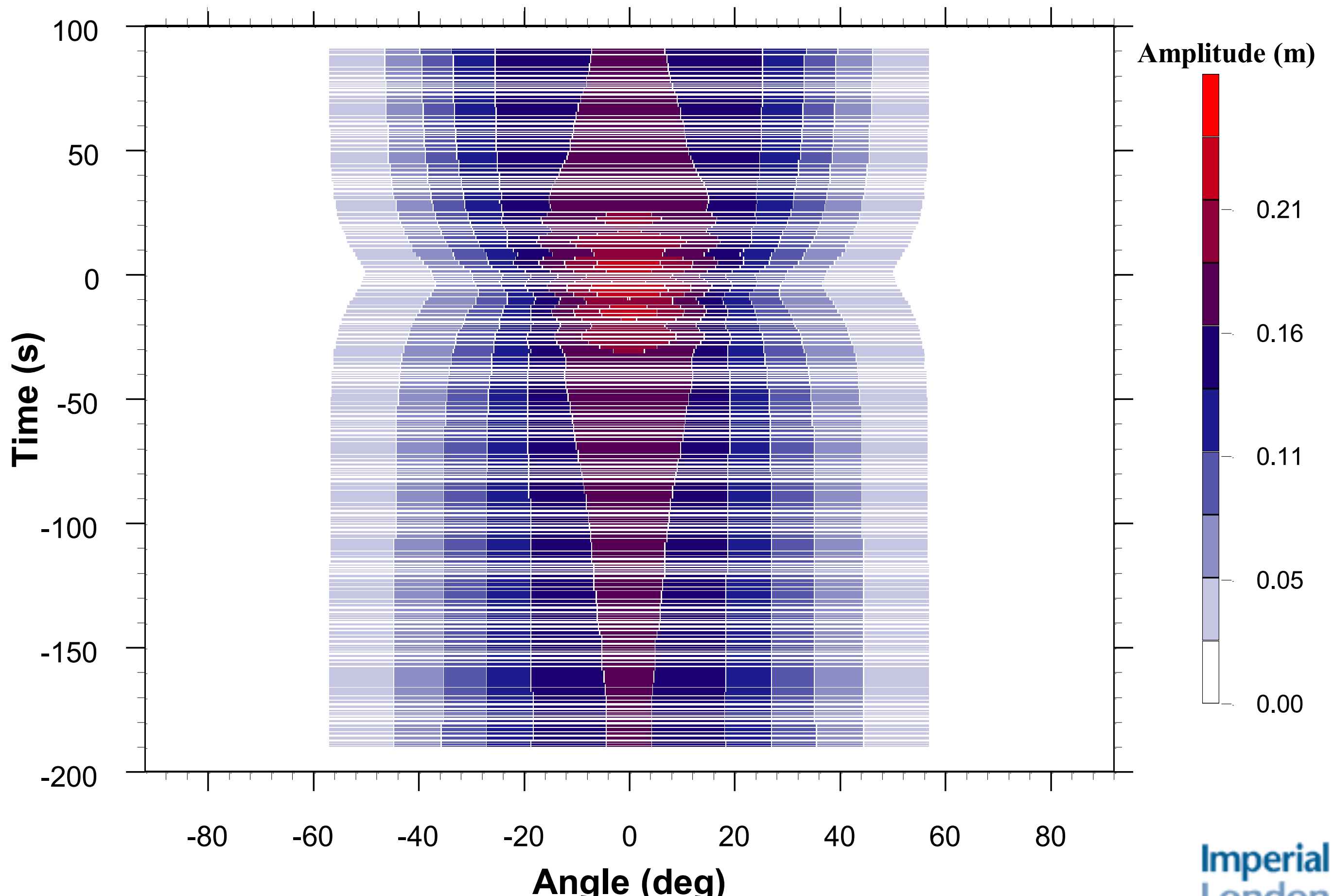
Earlier work:

- Numerical calculations of focused waves (spectral model - BST)
- Local reduction in directional spreading
- Supporting laboratory data (Johannessen & Swan, 2001 & 2003)



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