Modelling storm surges with multi-scale finite element methods

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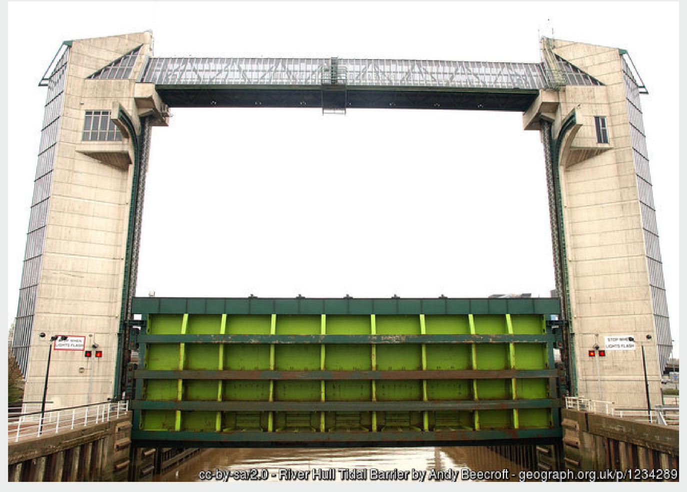
Background

- The December 2013 storm surge in the North Sea came within 40 cm of overtopping Hull's Tidal Surge Barrier [1], and was underestimated by the current UK operational surge forecasting model which uses a 12 km resolution structured mesh.
- This project aims to set up a storm surge model using finite element methods, and apply adjoint technology to perform calibration, sensitivity analysis and data assimilation, and to complement existing ensemble techniques for uncertainty quantification.



The Model

- This project uses the Thetis coastal ocean model, which is based on the Firedrake finite element code generation framework [2].
- The use of finite elements facilitates the use of multi-scale unstructured meshes, which allow high resolution to be used sparingly and efficiently, without requiring nested meshes.
- The tidal model set up here uses the two-dimensional non-linear shallow water equations, with a fully implicit wetting-drying scheme [3].

THETIS

Firedrake

Tidal Model Setup

- High-resolution bathymetric data from Digimap [4] (1 arcsecond, approx 30 m), including inter-tidal regions.

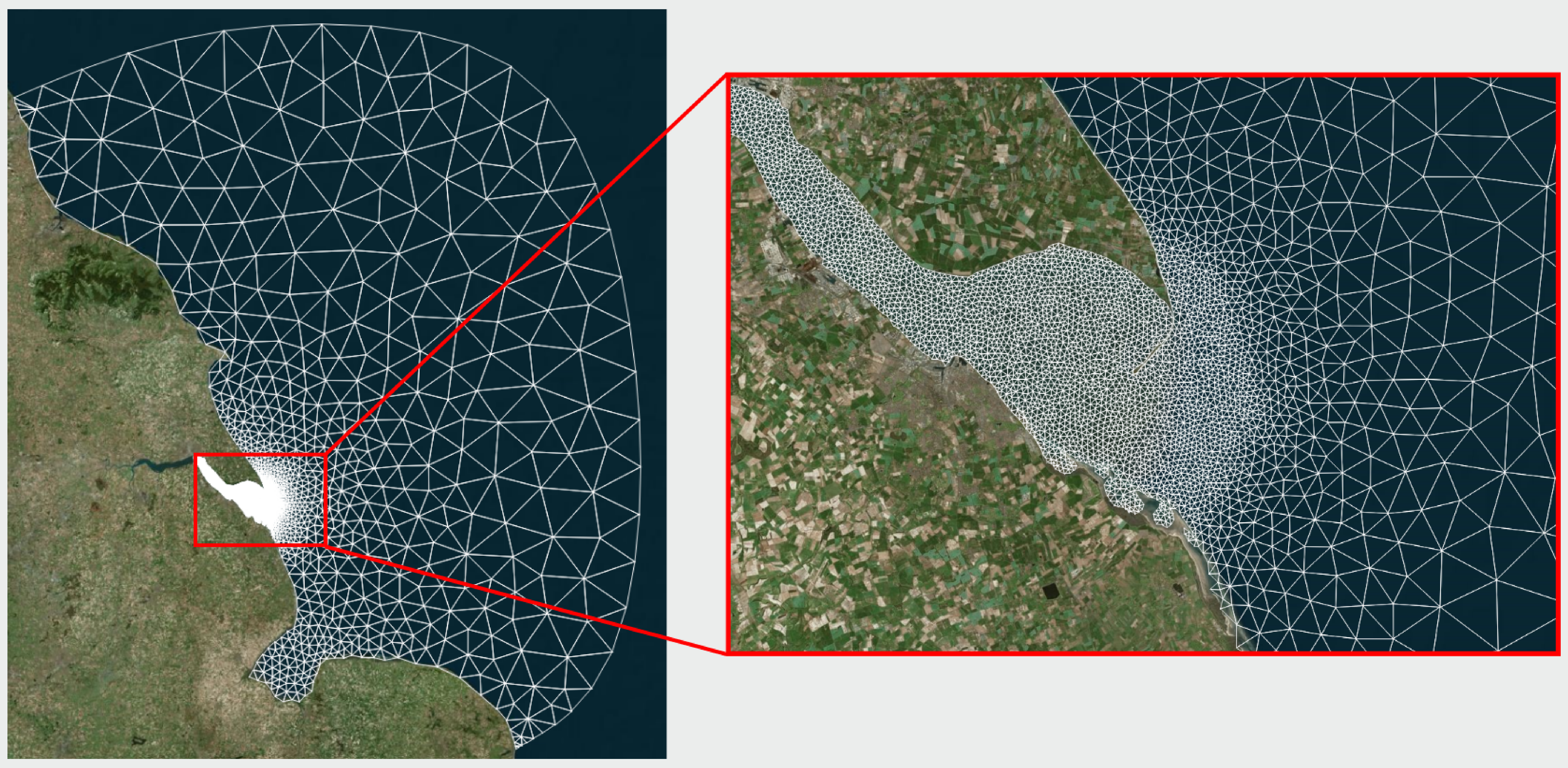


Fig. 1: Mesh for Humber Estuary simulations, overlayed on satellite image.

- Tidal forcing from TPXO dataset, using 8 constituents, applied at ocean boundary.
- Model includes intertidal region and wetting-drying scheme, to model coastal boundary.

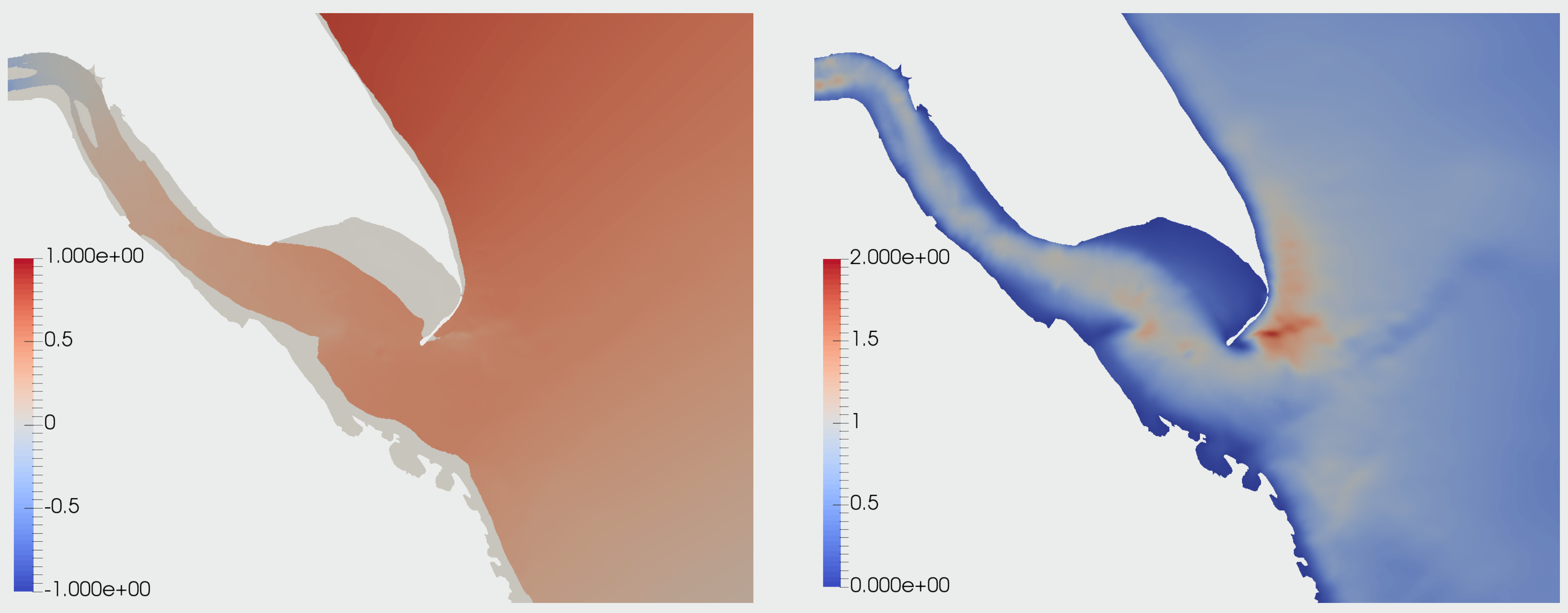


Fig. 2: Elevation and velocity snapshots from tidal simulation. Grey indicates dry regions.

References

[1] Steve Wragg. Hull City Council Flood Investigation Report: December 2013 City Centre Tidal Surge Flood Event. 2014. [2] Florian Rathgeber et al. Firedrake: automating the finite element method by composing abstractions. ACM Transactions on Mathematical Software, abs/1501.0, 2015. [3] Tuomas Kärnä et al. A fully implicit wetting-drying method for DG-FEM shallow water models, with an application to the Scheldt Estuary. Computer Methods in Applied Mechanics and Engineering, 200:509–524, 2011. [4] Edina Digimap. Gridded Bathymetry [ASC], Scale 1:50,000, Updated August 2013, SeaZone Solutions Ltd., UK. Using: EDINA Marine Digimap Service, Downloaded: April 2017. [5] P E Farrell et al. Automated derivation of the adjoint of high-level transient finite element programs. SIAM Journal on Scientific Computing, 35(4):369–393, 2013.

Model Calibration

- Calibration is undertaken using adjoint methods; gradient of a user-defined functional with respect to input parameters is computed by Firedrake (via Dolfin-Adjoint [5]) and used in a gradient-based optimisation algorithm.
- Here, the optimal Manning bottom friction parameter field is determined by minimising the misfit between the model and tide gauge data at Immingham, inside the Humber Estuary.
- Figures below, of preliminary results, show sensitivity of tidal gauge misfit to Manning parameter at the start of the calibration process, and final bottom friction field after minimisation of squared tide gauge misfit over a simulated time of 8 days.

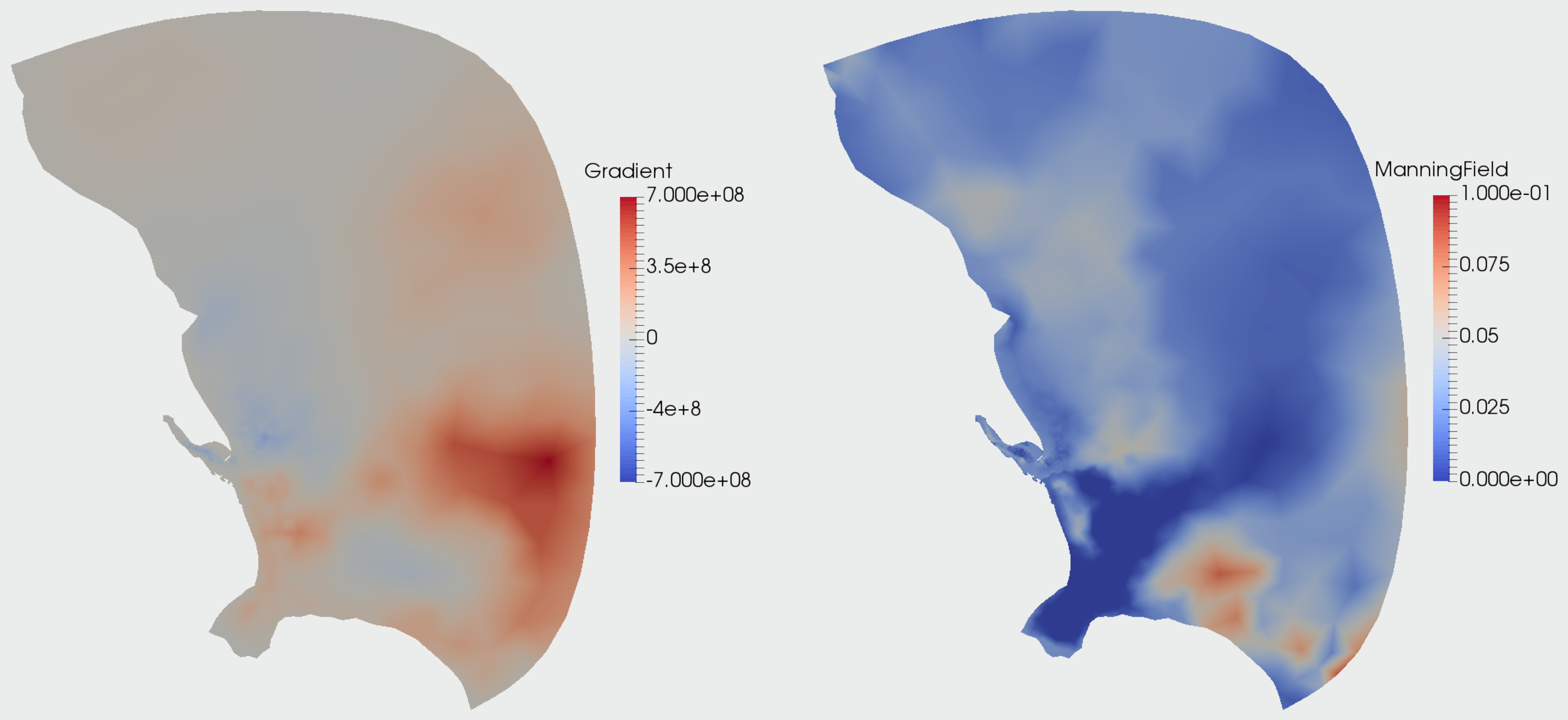


Fig. 3: Initial sensitivity field and optimised friction field after 14 iterations.

- The impact of the optimisation can be directly observed in the tide gauge time series, as well as in harmonic analysis. The benefit of the optimisation lasts well beyond the 8-day optimisation period.

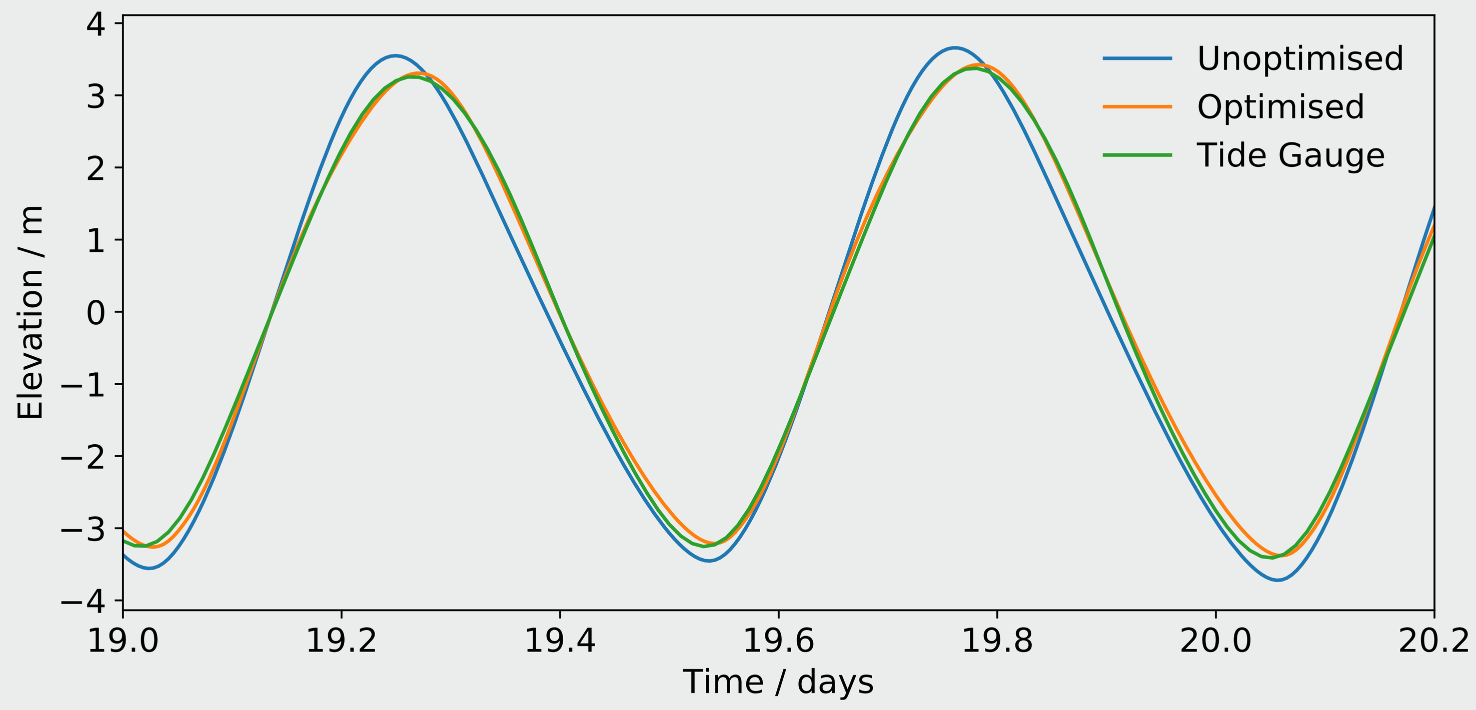


Fig. 4: Tide gauge timeseries at Immingham.

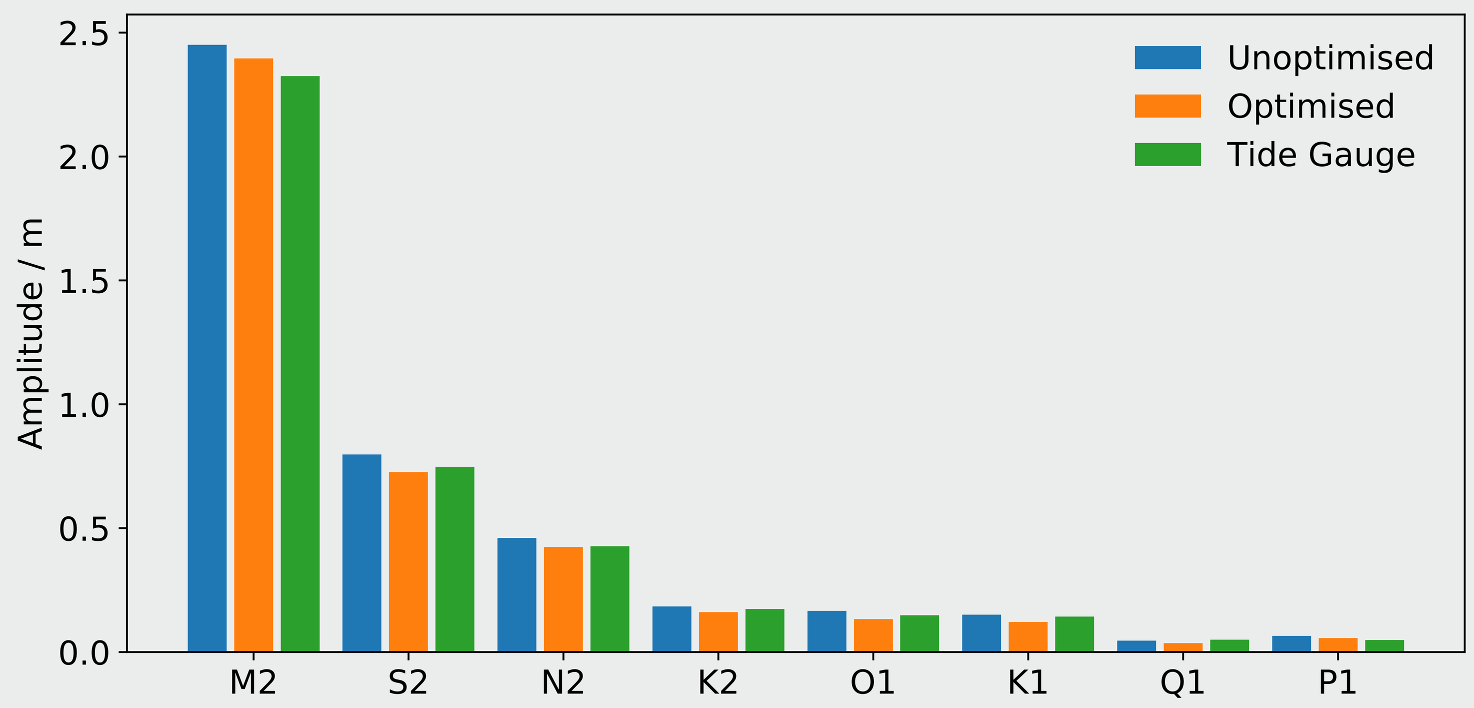


Fig. 5: Tide gauge harmonic analysis at Immingham, based on 30-day simulation.

Summary & Future Work

- Tidal model of the Humber Estuary has been set up and calibrated with respect to bottom friction coefficient, using adjoint methods and gradient-based optimisation.
- Next step is the addition of storm surge forcing to the Humber model, for hindcasting the December 2013 event.
- Adjoint-calculated sensitivities with respect to various input parameters and source terms will give useful insight into the development and propagation of the surge, and could be used in forecast mode to complement ensemble methods.

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