Applications of the Spherical Multiple-Cell Grid in Ocean Surface Wave Models

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Abstract: The spherical multiple-cell (SMC) grid is an unstructured latitude-longitude (lat-lon) grid, supporting flexible domain shapes and multi-resolution in desired areas. It retains the quadrilateral cells as in the standard lat-lon grid so that the simple finite difference schemes could be used. Sub-time-steps are applied on different cell sizes to speed up propagation calculations with a choice of 2nd or 3rd order advection schemes. Grid cells are merged at high latitudes to relax the CFL restriction and a fixed reference direction is used to define wave spectra in the polar region so that the SMC grid can be extended to the full globe, including the Arctic. The multi-resolution refinement is particularly useful to resolve small islands and coastline details, which are important in ocean surface wave propagations but could be too expensive in standard lat-lon grid models. The SMC grid has been implemented in the WAVEWATCH III community wave model and used by a few wave modelling groups. A SMC 3-6-12-25 km multi-resolution global wave forecasting model has been used operationally in the UK Met Office since October 2016, leading to great reduction of model errors in comparison with our old 35km model and other global wave forecasting systems in the world. This presentation will introduce some of the applications and validation works with the latest WW3 V5.16 public release, including the Met Office 3-6-12-25 km global and North Atlantic ensemble wave forecasting models, and a SMC 50-km global wave model for the Met Office coupled climate modelling system. The SMC grid can also be applied on a rotated lat-lon grid for high latitude regions and this will be illustrated with a UK regional model at 1.5-3 km resolutions and an Arctic model at 1/16-1/8-1/4 degree spatial resolutions. Other applications outside the UK Met Office will be outlined as well, including the 12-25 km Arctic and 50-100 km global wave climate models in Environment Canada, the 0.5-1-2 km Great Lakes model in NCEP/NOAA, the 4-8-16 km Gulf of Mexico model used in Swinburne University of Technology, Australia, and a 3-6-12-25 km Mediterranean Sea wave model for collaborations with UPC, Spain and CNR-ISMAR, Italia. The National Marine Environment Forecasting Centre of China is also preparing to switch their west Pacific wave forecasting model to a multi-resolution SMC grid model.

1. Introduction

The ice edge in the Arctic is retreating at unexpected speeds in recent years and reached as high as 86^o N in summer 2007, opening new shipping routes cross the Arctic and calling ocean surface wave models to extend at high latitudes. The major problem to extend a latitude-longitude (lat-lon) grid wave model at high latitudes is the diminishing longitude grid-length towards the Pole, which exerts a severe restriction on time steps of finite-difference schemes (advection and diffusion in particular). Another problem is the increased curvature of the parallels at high latitudes. The rapid change of the local east direction renders the scalar assumption invalid for any vector component defined relative to the local east direction. The spherical multiple-cell (SMC) grid has been developed to tackle the polar problems (Li 2011). The SMC grid has the flexibility to remove all land points out of the wave propagation schemes and requires minimal changes to the lat-lon grid finite-difference schemes because the lat-lon "rectangular" cells are retained.

The SMC grid relaxes the Courant-Friedrichs-Lewy (CFL) restriction of the Eulerian advection time-step by merging longitudinal cells towards the Poles as in the reduced grid (Rasch 1994). Round polar cells are introduced to remove the polar singularity of the spherical coordinate system. Vector component propagation errors caused by the scalar assumption at high latitude is removed by replacing the local east with a fixed reference direction, for instance, the map-east as viewed in a polar stereographic projection, to define the wave spectral components in the Arctic.

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Besides, unresolved small islands incur errors in global ocean surface wave models as they are important sinks of the ocean surface wave energy (Tolman 2003). Missed island groups in coarse resolution global models lead to a persistent under-prediction of wave energy blocking. Although the far field errors can be alleviated with sub-grid obstructions, using high resolution cells is still the most appropriate approach for accurate swell prediction close to islands (Chawla and Tolman 2008). One feature of the unstructured SMC grid is that it can also handle multiple resolutions within the same model, so that small islands and coastlines are resolved at high resolutions while the vast open oceans are kept at an affordable resolution. This is a very appealing option for operational models as increasing resolution throughout the full model domain to resolve small islands is not economical. The multi-resolution feature of the SMC grid also allows regional models to be merged into one global model, another desirable advance in future operational wave modelling.

The SMC grid has been implemented in the WAVEWATCH III® wave model (WW3, Tolman 1991; Tolman et al 2002, WW3DG 2016) and updated in the last V5.16 public release. Scalar advection schemes on the SMC grid are described in Li (2011) and ocean surface wave propagation on a SMC grid is presented in Li (2012). Details of the map-east method for the Arctic part in the SMC grid are given in Li (2016). Here we summarise the essentials of building a SMC grid model from the WW3 model and its applications in operational wave forecasting and research models.

2. The SMC grid and numerical schemes

A global SMC grid is shown in Fig.1. For clarity, only the Arctic region is shown here. The highest resolution of the SMC grid (for size-1) cell is set to be $\Delta\lambda$ = 360º/(1024*4) = 0.3515625º/4 and $\Delta\varphi$ = 180º/(768*4) = 0.234375º/4 such that the latitudinal grid length is about 6 km. The SMC grid uses only the sea points or cells and refines model resolution (by two levels) to these 6 km cells around islands and coastlines. Away from the coast a two level increase in cell sizes results in a global 3-level (6-12-25 km) SMC grid on ocean surface. This SMC grid will be referred to as the SMC6-25 grid. Cells are merged longitudinally at high latitudes following the same rules in Li (2011) to relax the CFL restriction. A unique 5-element integer array is assigned to each cell to hold its SW corner *x*-, *y*-indices (i, j), cell *x*-, *y*-sizes (Δi, Δj), and water depth (h), as illustrated in Fig.2. The *x*- and y-indices are measured in size-1 cell increment so the cell centre latitude and longitude can be worked out with

$$\varphi_j = \varphi_0 + (j + 0.5 * \Delta j)\Delta\varphi; \qquad \lambda_i = \lambda_0 + (i + 0.5 * \Delta i)\Delta\lambda \tag{1}$$

where λ_0 and φ_0 are the origin of the cell *x*- and *y*-indices. For the SMC6-25 grid, the origin of the grid indices is set at zero-meridian on the Equator so both λ_0 and φ_0 are zero. The mapping rule (1) is exactly the same as that for the lat-lon grid cells except for that the SMC grid cells are not arranged in spatial sequence (hence is called an unstructured grid) and their sizes may change by a multiple of 2 (size-1, size-2, size-4, …). The depth h is also rounded to an integer so the whole cell array can be declared as an integer array. The cells are listed as a 1-D array and sorted by their *y*-size for use of sub-time steps on refined cells. Please note that the sorting is on the *y*-size not the *x*-size because the cell *x*-size may change on the same resolution level due to the longitudinal merging at high latitudes. The cell *y*-size will be in ascending order in the sorted cell array list and the number of cells for each resolution level (of a given *y*-size) is listed on the first line of the cell array file after the total cell number. This cell number counts will be used for declaring the cell array variable and setting the sub-loops for propagation schemes.

It should be emphasized that the cell size has to be increased no more than one level for any neighbouring cells, that is, around a size-1 cell the neighbouring cells can be either size-1 or size-2. Similarly, size-2 cells can be linked to cells of the same size-2 or either one level down (size-1) or one level up (size-4). This one level size change rule ensures resolution varies gradually and simplifies the face flux formulation. Putting cells of more than one level difference in sizes side by side would jeopardise the present face array generating program.

Once the cell arrays are compiled in the sorted order, cell face arrays can be generated with an extra FORTRAN program. Cell faces are named by its normal velocity components as *u*- or *v*-faces. A 7-element integer array is pre-calculated for each face to store its face position, size and its upstream-central-downstream (UCD) cell indices. An extra *y*-size integer is added for the υ-face array for sorting purposes. Face sizes are chosen to be the minimum size between the two neighbouring cells. For a cell face neighbouring two cells of one level below, the face is divided into two faces of the lowered level size. This minimised face size ensures one face links two cells only. The face arrays are also sorted by its *y*-size so that the multi-resolution advection/diffusion loops can be divided into multi-step sub-loops. The total face number and sub-level face numbers are listed on the first line of the face array file for propagation and mapping purposes. The face arrays are used to calculate the advection-diffusion and the depth gradient.

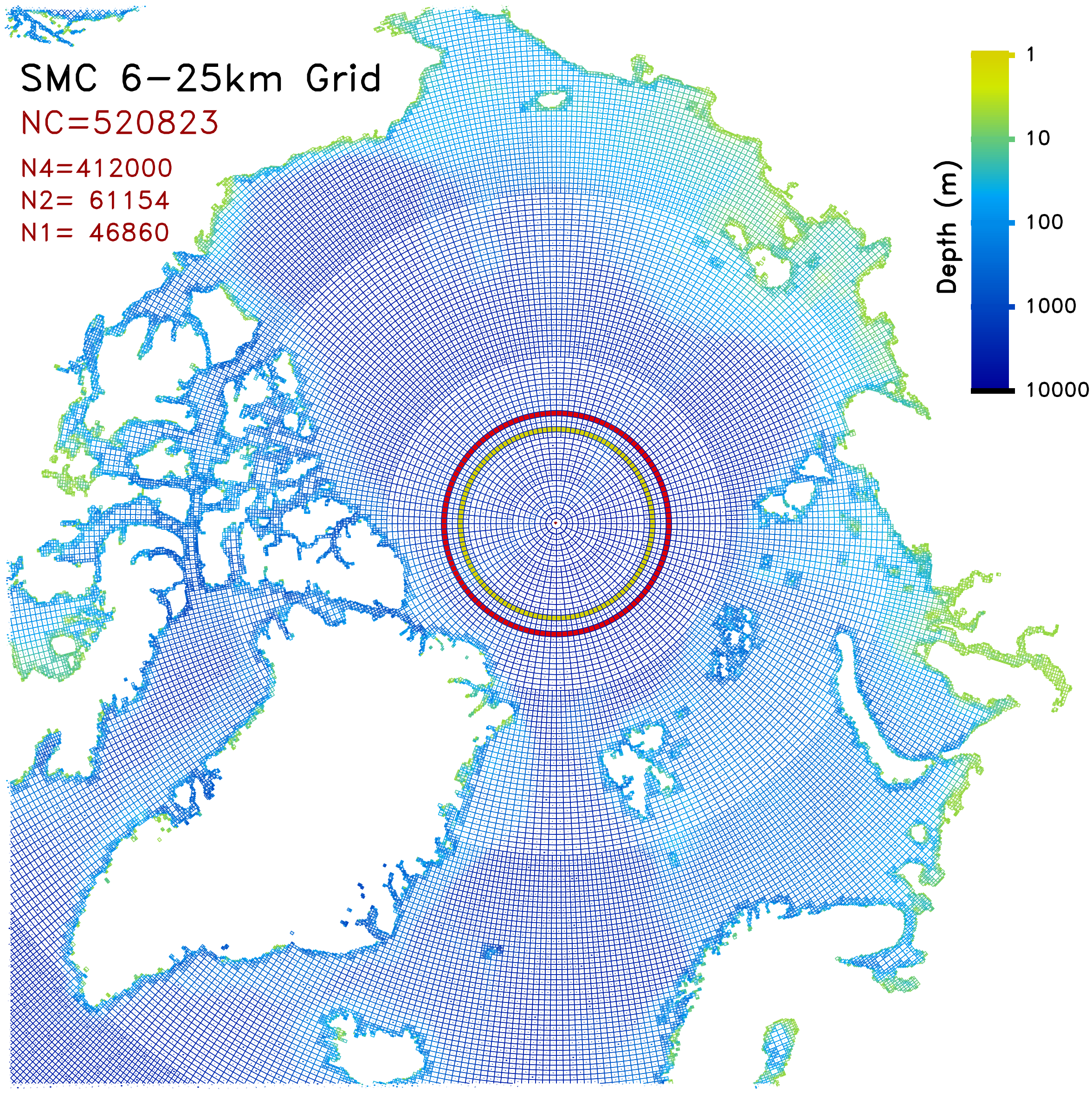


Fig.1. The Arctic part of the SMC6-25 grid.

For the multi-resolution SMC6-25 grid, the face and cell loops are sorted into 3 sub-loops according to their *y*-sizes, thanks to the unstructured nature of the SMC grid. Advection-diffusion terms for the refined 6- and 12-km cells are calculated at ¼ and ½ of the base level time step, that is, the 6-km flux and cell loops are done twice before the 12-km flux and cell loops are calculated once. The base level flux and cell loops are only calculated at each base level time step. A temporary net-flux variable is used to accumulate fluxes between different levels and is reset to zero once it is used for its cell update. The simple loop–regrouping technique for multi-resolution SMC grid allows a smooth transfer from a single resolution SMC grid to a multi-resolution grid with optimised efficiency.

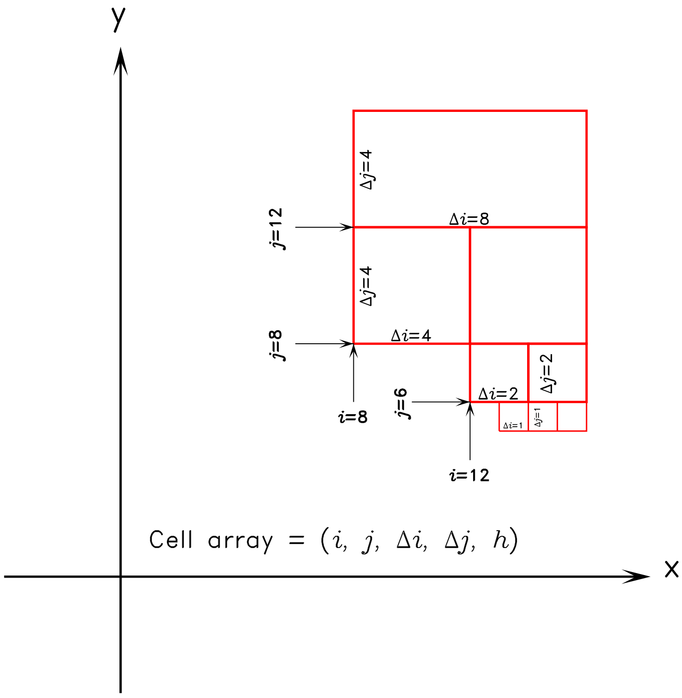


Fig.2. Illustration of SMC grid cell arrays.

Another feature of the SMC grid is the unification of boundary conditions with internal flux evaluations. Cell faces at coastlines are assumed to be bounded by two consecutive empty cells when the face arrays are generated. Thus, any wave energy transported into these empty cells will disappear, and no wave energy will be injected out of these zero cells into any sea cells. This convenient setup conforms to the zero wave energy boundary condition at land points used by ocean surface wave models and allows all the boundary cell faces to be treated in the same way as internal faces in one face loop. In addition, the periodic boundary condition for a global model is automatically included by the unstructured grid. So short boundary loops are avoided in the SMC grid propagation schemes and the full face and cell loops are streamlined for vectorization and parallelization.

An additional benefit of using two consecutive zero-boundary cells beyond the coastline is the complete blocking of wave energy by single-point islands. On a conventional lat-lon grid, wave energy can 'leak' through a single-point island due to the interpolation with neighbouring sea points in transport schemes which use a 5-point stencil like the UNO2 scheme (Li 2008), which is an improved version of the MINMOD scheme (Roe 1985). In the SMC grid, any single-point island is extended with two zero cells beyond its boundary face. As a result, wave energy cannot pass through such 'expanded island'. Nevertheless, the sub-grid obstruction scheme from the original WAVEWATCH III model is kept in order to account for islands unresolved by the highest resolution cells. The sub-grid obstruction scheme follows the approach of Hardy et al (2000) with some modifications (Tolman 2003).

For the updated WW3 V5.16 model, an optional 3rd order advection UNO3 scheme (Li 2008) is added so users may replace the default UNO2 scheme with the UNO3 scheme by defining a namelist (PSMC) variable UNO3 = .TRUE. in the ww3_grid.inp file. The UNO3 scheme is equivalent to the Ultimate Quickest (UQ) scheme (Leonard 1991, Leonard et al 1996) used for regular lat-lon grids but it replaces the UQ scheme's flux limiters with the UNO2 scheme. As the UNO3 scheme is less diffusive than the UNO2 scheme, an additional 1-2-1 weighted smoothing term is also available by defining the PSMC namelist variable AVERG = .TRUE. in the ww3_grid.inp file. The default diffusion term is still in effect for the UNO3 scheme and users may adjust this term with the swell age parameter as in the default UNO2 scheme case. In practise, we would advise the use of the default UNO2 scheme, as the UNO3 option does not bring much improvement to wave model performance but adds about approximately 30% extra computation load. This is because the strong smoothing for controlling the garden sprinkler effect will degrade any high order advection scheme to an equivalent

1st order scheme. This is also true to regular grid models, where the 2nd order scheme (UNO option) allows sufficient freedom to choose directional bins and set diffusion parameters. The results are expected to be similar to using the UQ option, whilst saving computing time.

In the SMC grid wave model, a rotation scheme is substituted for the advection-like scheme to estimate the refraction term so that the CFL limit can be avoided. The rotation scheme is similar to a re-mapping advection scheme and is unconditionally stable. Although the rotation scheme does not have any limit on the refraction increment, the refraction angle should not pass beyond the depth gradient line and this physical limiter on the total refraction angle is included in the rotation scheme. A user-defined maximum refraction angle is also built for easy control of the refraction effect and it also creates room for merging the refraction with other directional changing terms, such as the refraction by ambient current and the great circle turning (GCT) term. The rotation subroutine shifts each directional component by the combined angle and partitions its energy into the two directional bins which the rotated one strides across after the rotation. This simple rotation subroutine not only removes the time step restriction on the refraction angle but also adds an implicit directional diffusion because its implicit diffusivity is equivalent to that of the first order upstream scheme. This additional smoothing in the transverse direction is desirable for wave models to mitigate the GSE.

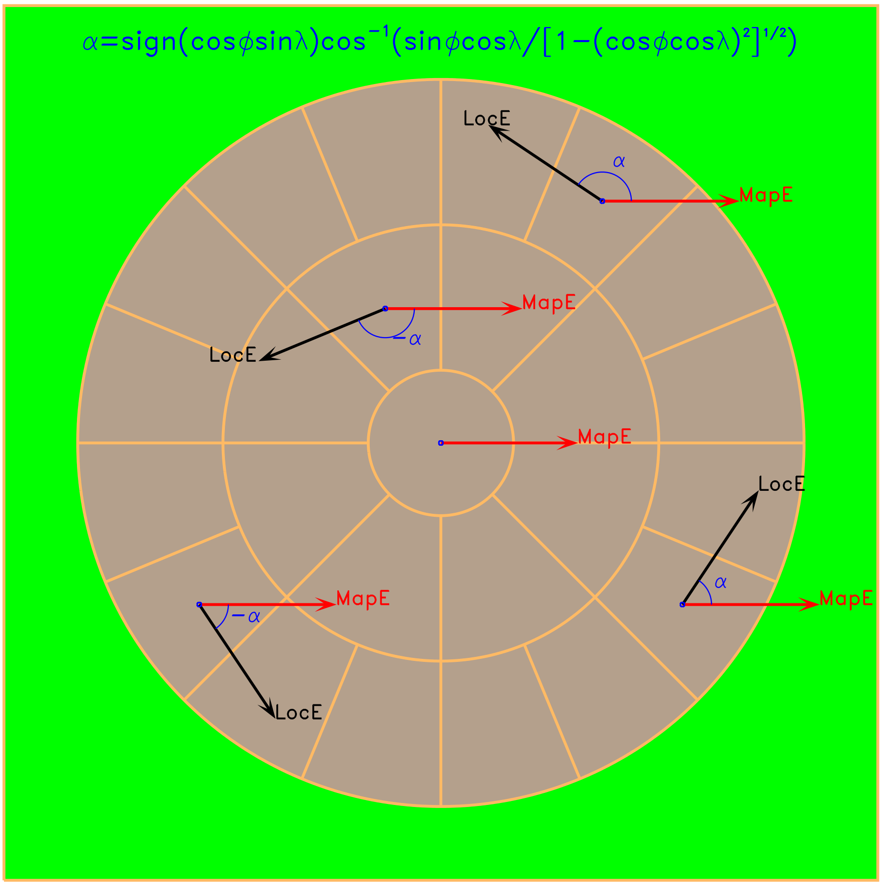


Fig.3. Relationship of local east and map-east reference direction systems.

The current induced spectral shift is calculated with an advection-like UNO2 scheme in *k*-space because the spectral shift is usually small enough to meet the CFL condition. The term is calculated at the advection sub-time step for all cell spectra and is subject to a user defined CFL limiter. There is a possible interaction between the current spectral shift and source terms, which may result in erroneous high waves in high current speed areas when running high resolution models. This interaction is not fully understood at present and the most effective way to remove the high waves has been to switch off the current induced spectral shift, or massively reduce the k-shift limit (e.g. to a value of 0.01) if such high wave pockets appear. This issue has been found for regular grid models as well, so is not thought to be specific to the SMC model propagation scheme. Notice that the current induced refraction rotation and extra propagation are still in effect even the spectral shift is switched off.

This map-east system used to define vector components in the Arctic part can be conveniently approximated with a rotated grid with its rotated pole on the Equator. The standard polar region becomes part of the 'tropic region' in the rotated grid so the longitudinal direction of the rotated grid can be substituted for the map-east direction. As the angle (α in Fig.3) from map-east to the local east direction changes with longitude, there are no fixed corresponding components between the two systems. For this reason, wave spectra defined by those two different systems could not be mixed and they are separated in the SMC grid into two parts, and linked up by two-way boundary conditions. In the SMC grid shown in Fig.1, the reference direction change is set between the 3rd (at about 83º) and 4th (at 86.4º) size-changing parallels (see definition in Li 2011), where the local east direction changes less than 3º over one cell length as there are 128 cells in one row. The Arctic part and the rest (will be referred to as the *global part*) are linked together through 4 over-lapping rows. Because of the unstructured nature of the SMC grid, the Arctic cells are appended behind the global part in the single cell list so the two parts can be conveniently separated by sub-loops. The boundaries beyond both parts are treated as imaginary coastlines so the overlapping rows are treated in the same way as inner cells. This allows the propagation over that whole globe to be calculated together in one single loop.

Wind velocity and other direction related source terms have to be converted into the map-east system within the Arctic part. This conversion is automatically performed inside the WW3 model when the ARC option is selected so users could still used a regular grid wind (and current velocity if any) input file for the whole globe. However, output wind and other directional related variables for the Arctic part are left in the map-east system at present. We are providing an update to convert the Arctic part directional variables back into the regular local east system for output but this update has not merged into the WW3 trunk yet.

The great circle turning (GCT) term has to be modified in the Arctic part to use the rotated grid latitude, which is close to zero in the Arctic part because the rotated Equator passes the North Pole. If the Arctic part is kept small around the polar region (like above 85ºN in the SMC625 grid), this GCT term becomes negligible. The refraction term retains the formulation in the Arctic part except for that the gradients of water depth and current component along the wave direction have to be rotated to the map-east system. As the Arctic Ocean above 85ºN is considered deep for wind waves, the refraction is also negligible in the small Arctic part.

The spatial gradient of a given field variable on a SMC grid or at a sea point may be required for other terms, such as wave scattering or bottom friction. The SMC grid module contains a convenient subroutine, `SMCGradn(HCel, GrHx, GrHy)`, which calculates the spatial gradients, `GrHx(1:NSEA), GrHy(1:NSEA)`, of the given field `HCel(-9:NSEA)`. Note that the input field is extended from the normal range (`1:NSEA`) to (`-9:NSEA`) for the SMC grid. The `HCel(-9:0)` should be assigned the boundary values (on land points) used for this given field. For water depth as an example, set `HCel(-9:0)=0`. The spatial gradient will be in the map-east direction system for the Arctic part while in the local east system for the global part.

3. Input/output files for SMC option

The regular lat/lon grid WW3 wave model requires input files for wind forcing, sea-ice coverage, water depth, sub-grid obstruction and land-sea masking. For the SMC grid option these regular grid input files are no longer required in the latest public release V5.16 and they are replaced with sea-point only files. The water depth is stored in the last column of the cell array file in unit of meter. If the accuracy at a metre is not enough, users may use other units (such as cm) and convert the depth integer back to units of metres inside the ww3_grid program. Using integer to represent water depth in the cell array is to ensure the cell array is completely an integer array. A corresponding regular grid at the base level resolution of the SMC grid is used to set up the WW3 model for the SMC grid option. For example, a 25km regular lat-lon global grid is used for setting up the input files for the SMC6-25 grid. When compiling the WW3 executables, the spherical regular lat/lon grid option should be selected and the propagation options (PR3 UQ as default) should be replaced by the SMC option (and ARC option if Arctic part is included). Most of the WW3 model will work the same way as on the regular grid except for the propagation (advection, diffusion, refraction, and GCT), which will be calculated with the newly added SMC module. Extra input files for the SMC grid are required in the ww3_grid program. These extra input files include the SMC grid cell and face arrays and the modified cell-only sub-grid obstruction file. If input boundary condition is required for a regional model on a SMC grid, an extra boundary cell file is also required for the ww3_grid program. If the Arctic part is included, they require extra cell and face arrays for the Arctic part as well. These Arctic part cell and face arrays will be merged with the global part cell and face arrays inside the ww3_grid program so that propagation on the whole global grid is calculated within one loop.

In the ww3_grid.inp file a new namelist PSMC is added for input of some SMC grid related parameters. The PSMC namelist for the SMC6-25 grid has the following parameters

```
  &PSMC DTIME=39600.0, RFMAXD=36.0, LvSMC=3, JEQT=1344  /
```

where DTIME is the swell age (in unit of s) used to define the smoothing (diffusion) coefficient, RFMAXD is the maximum refraction angle in degree, LvSMC defines the resolution levels of the SMC grid and JEQT is a shifting integer for the j-indices to match the SMC grid cells with the regular grid. The swell age has the same meaning as in the regular grid WW3 model (Booij and Holthuijsen 1987) and it has to be adjusted according to the base-level grid length and the advection time step (diffusion is calculated at the same time step as advection) to ensure the maximum Fourier number is less than 1 or usually set to be 0.5. A guide rule for the maximum swell age T_s is given by

$$T_s \le \frac{6}{\Delta t_a}\left(\frac{\Delta x_0}{c_{gm}\Delta\theta}\right)^2 \qquad (2)$$

in which Δt_a is the advection time step, Δx_0 is the base level grid length on the Equator, $\Delta\theta$ is the directional bin width (in radian) and c_{gm} is the maximum group speed in the model spectral range (usually at the lowest frequency end). If the swell age is set too large, the diffusion term will become unstable and eventually bring the model to a crash. It would be convenient if the swell age was reduced automatically inside the model when users accidentally set it too large. This automatic adjustment, however, has not been set up yet.

The maximum refraction angle is the user-defined limiter for the refraction rotation scheme. It is recommended to be less than 60 degrees to avoid caustic-like focus of spectral energy. Setting it equal to one directional bin width will bring the refraction rotation scheme to be equivalent to the regular grid refraction term.

The LvSMC integer represents the number of levels of the multi-resolution SMC grid. The above quoted line is for the SMC6-25 grid, which has 3 levels: the 6km, 12km and the base level 25km. The JEQT integer (1344) is a shifting number for conversion of the SMC grid cell *y*-indices to the regular grid *j* indices. It is equal to the distance from the regular grid *y*-origin to the SMC y-index origin (on the Equator for SMC6-25 grid and hence JEQT) and divided by the SMC size-1 unit length. Because the regular 25km grid y-indices starts from southern boundary of the model domain while the SMC6-25 cell y-indices are referred from the Equator, the SMC6-25 cell *j* indices have to be shifted by this number for mapping with the regular grid *y*-indices. There is a similar namelist integer parameter ISHFT for the x-index conversion to the regular grid *i*-indices. As the SMC6-25 cell *x*-indices share the same origin as the regular grid (at zero meridian) this parameter is not defined in the namelist but takes the default value 0. Also note that, the SMC6-25 cell indices are in units of the smallest cell (size-1) while the 25km regular grid is at the base level (size-4). So SMC6-25 cell indices are 4 times larger than the regular grid ones.

If the model needs boundary conditions, users should set another PSMC namelist integer NBISMC, which is the number of boundary cells. A non-zero number will invoke extra lines to read the boundary cell list from an extra input file and setup boundary condition interpolation arrays for the model. If not defined in the namelist, it will take the default value 0 and tell the ww3_grid program to skip those lines as no boundary conditions are required. Note that SMC grid boundary conditions are set up by the boundary cell list (the sequential number of the boundary cells in the full cell array list) instead of using masks. To generate boundary conditions for other models, however, the SMC grid uses the normal longitude and latitude settings as a regular lat-lon grid model. This is also true for generating boundary conditions for a SMC grid model. Users need to convert the cell list into corresponding list of lat-lon pairs for the mother model to generate boundary condition files for it. The lat-lon pair list does not need to be in the same order as the boundary cell list.

The extra SMC grid input files for the ww3_grid program are read in a similar way as the regular grid input files. The following lines are added after the regular grid depth file line (which line is still kept to read the minimum depth parameter though the regular grid depth file is not actually read for a SMC grid):

```
  32  1  1  '(....)'  'S6125MCels.dat'
  33  1  1  '(....)'  'S6125ISide.dat'
  34  1  1  '(....)'  'S6125JSide.dat'
  31  1  1  '(....)'  'S6125Obstr.dat'
```

The regular grid sub-grid obstruction file read line has been moved after the cell and face array files because the smc cell only obstruction file requires the cell array for mapping the obstruction ratio.

If boundary conditions are required for a regional SMC grid with NBISMC = n is nonzero in the PSMC namelist, the ww3_grid program will then read the boundary cell sequential numbers from a file specified by

```
  35  1  1  '(....)'  'S6125Bundy.dat'
```

If no boundary condition is required (NBISMC = 0), the above line in ww3_grid.inp should be commented off with a first column $ sign.

If the Arctic part is included (compiled with SMC ARC options), additional lines for Arctic cell and face arrays should follow

$ Extra cell and face arrays for Arctic part. JGLi16Jan2014

```
  36  1  1  '(....)'  'SMC625BArc.dat'
  37  1  1  '(....)'  'S625AISide.dat'
  38  1  1  '(....)'  'S625AJSide.dat'
```

If the Arctic part is not required, these lines should be commented off as well. The input line for the regular grid masks file is kept though the masks file is not read for SMC grid. The masks are defined inside ww3_grid for the SMC grid model using the cell array (and boundary file if any).

```
  39  1  1  '(....)'  'NAME'  'S625Masks.dat'
```

Once these input files are put in place, the mod_def.ww3 file is generated with the ww3_grid program as for the regular grid model. The SMC grid WW3 model can then be run the same way as a regular grid model. See the WW3 manual for running the ww3_grid and the ww3_shell program.

Output from the SMC grid WW3 model can be saved as the regular base-level lat-lon grid ones or in the raw ww3 format output. The former regular grid output will lose the refined resolutions. The raw ww3 format saves output at all sea points, i.e., all the SMC cell points, and requires re-mapping with the cell array for visualisation. In WW3 V5.16, the ww3_outf program could be used to generate cell-only text output files by the type-4 option. Because the output cell sequence is exactly the same as the SMC grid input cell array, there is no need to save the cell mapping info in the output files and simply re-use the input cell array for visualization. There are some example IDL programs under the sub-directory `smc_docs/SMCG_TKs/` in the WW3 V5.16 package to illustrate how the text output files could be mapped into a global view frame. Revisions to the ww3_ounf package, enabling generation of seapoint only or regularly gridded netCDF outputs have been applied for use with the Met Office operational wave models (based on WW3 V4.18) and are planned to be added to a public release version in the near future.

4. Applications of the SMC grid in the Met Office

4.1. *Global 3-6-12-25 km wave forecasting model*

Since October 2016 the Met Office has updated its global wave forecast model to a 3-6-12-25 km 4 level resolution SMC grid (SMC36125), replacing the old 35 km global multi-grid model (G35). The new global model has a base resolution of about 25 km in open oceans and refined to 12 and 6 km close to most coastlines (Li and Saulter 2014). Fig.4 zooms in the Arctic and European region of the SMC36125 global model and it has a refined area in the European waters at 12 km resolution and down to 6 and 3 km near coastlines. This refined area is intended to replace our European 8 km nested model. The total number of sea points in SMC36125 reaches 592,570 in comparison with the G35 sea points of approximately 290,000, or more than doubled. Spectral resolution is also increased from 24x25 in G35 to 30x36 in SMC36125 or 80% increase, plus source terms update from ST3 to ST4, which is alone a 10% computing cost increase. Considering the massive spatial and spectral resolution enhancement, the computation cost increase is moderate (2 times more than that of G35).

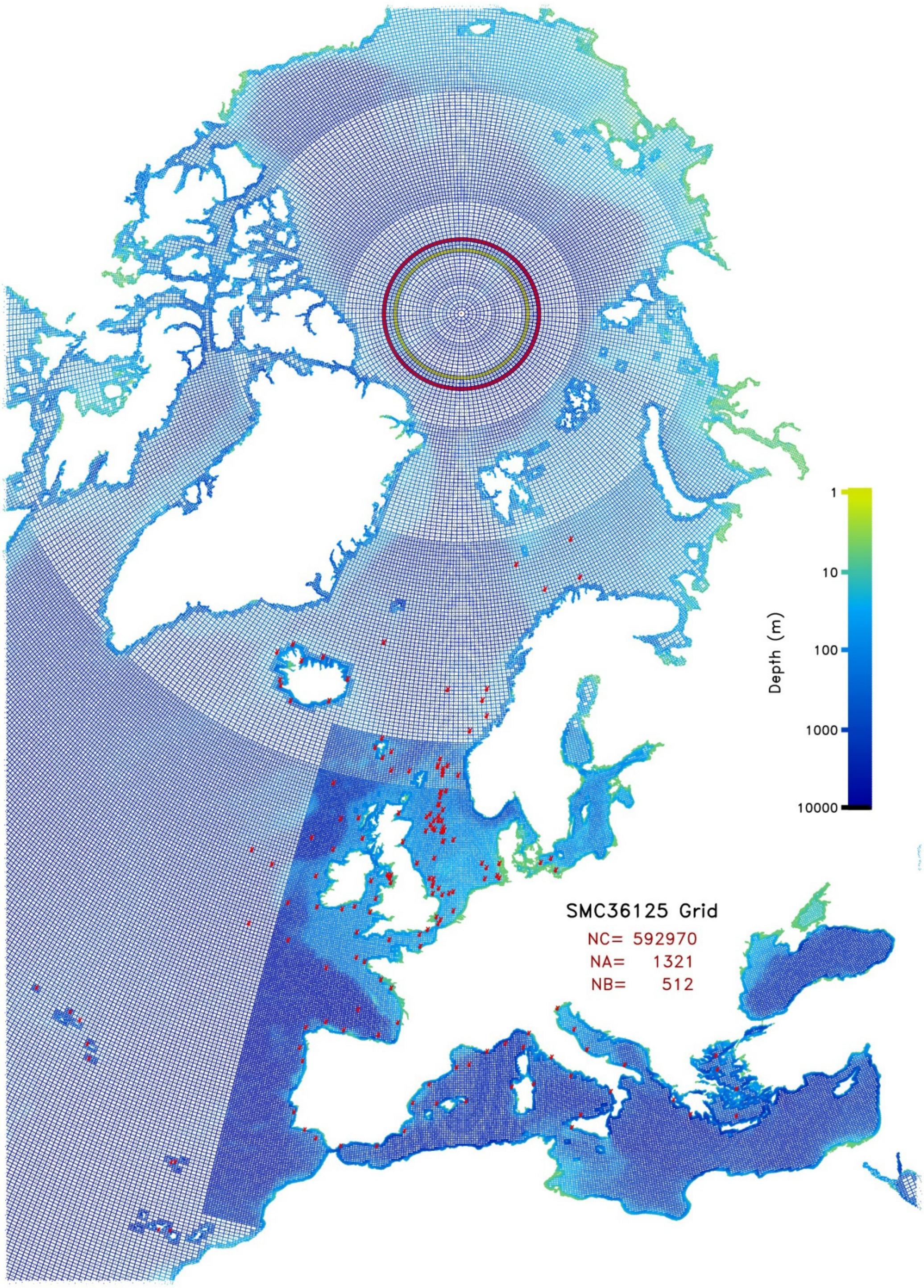
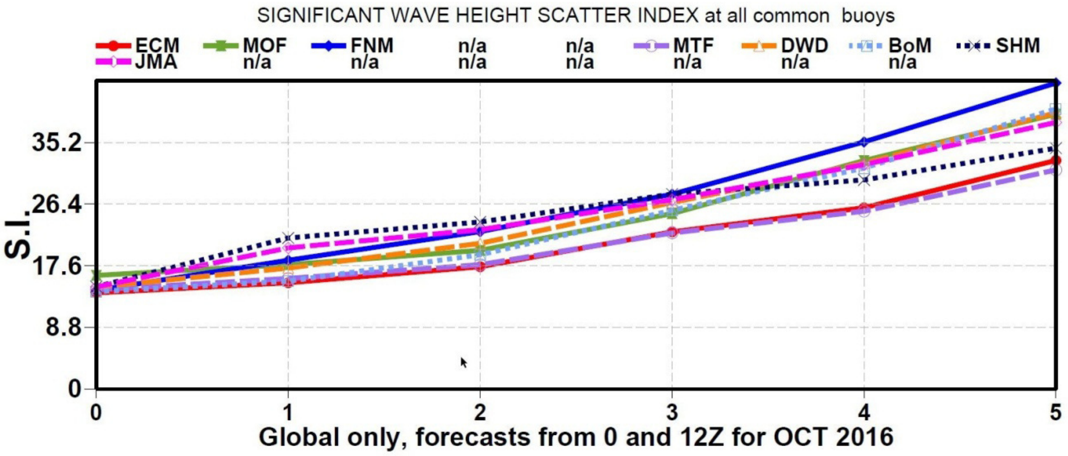


Fig.4. The Arctic and European region of the Met Office global 3-6-12-25 km SMC grid.

The model performance is significantly improved after this update as indicated by the JCOMM buoy data inter-comparison (Bidlot et al 2007). Fig.5 shows the scatter indicator (SI) plots before and after our global model update, in comparison with other global wave forecasting models. The Met Office forecast (MOF) model is indicated by the green line in Fig.5 and the October 2016 plot (top panel in Fig.5) used our old global wave model data while the November 2016 plot (lower panel in Fig.5) were draw with our updated SMC36125 model data. The improvement is significant considering the slight increase in computational cost.

4.2. *North Atlantic ensemble model*

The second SMC grid wave model used in the Met Office wave forecasting system is the north Atlantic wave ensemble model, which uses a grid similar to SMC36125 but removes the European refined area and all ocean surface except for the north Atlantic main water body (see Fig.6). To minimise boundary input, the selected north Atlantic domain follows coastlines all the way except for a few short crossings, such as the over the Gibraltar Strait, Baltic Sea, Hudson Bay, and particularly over the Southern Atlantic Ocean along the Tropic of Capricorn. To minimise the computing cost, the base resolution 25 km cells are used as much as possible and the highest resolution 3 km cells are used only near European coastlines. A deterministic 2D wave spectral boundary condition is provided by our SMC36125 global model, only for the southern crossing along the Tropic of Capricorn. The ensemble model is driven by a set of 18-member perturbed wind forcing and run 4x18 times a day to provide ensemble forecasts for UK coastal areas.



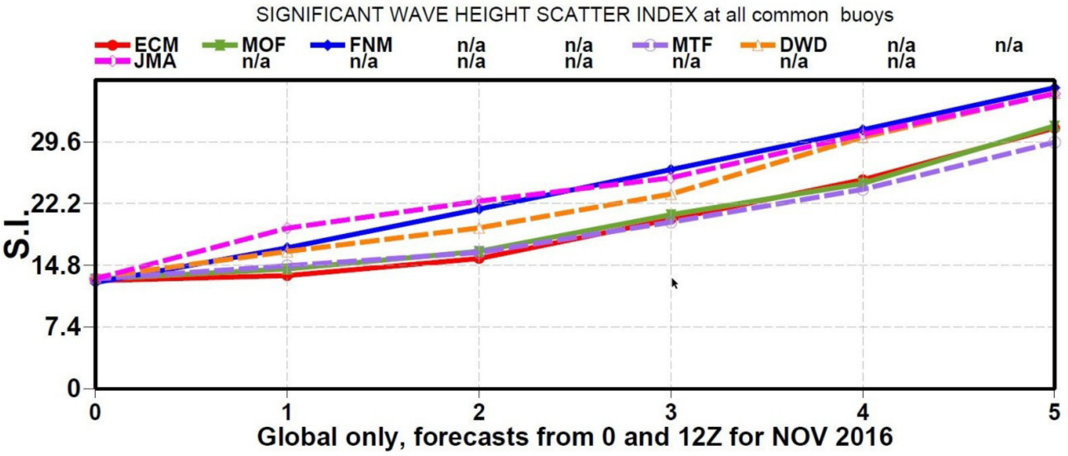


Fig.5. Scatter indicator plots of the ECMWF inter-comparison of 5-day wave model forecasts against all available buoy data for October and November 2016. The Met Office model performance is indicated by the forest green line (MOF).

4.3 *UK 1.5-3 km rotated SMC grid model*

The UK 1.5-3km SMC grid model uses a rotated grid (rotated north pole at 177.50ºE 37.50ºN) with a uniform increment of 0.0135º in each dimension for the 1.5 km refined cells and a base resolution of 0.0270º, or about 3 km, for the rest (see Fig.7). The refinement criterion is based on both proximity to the coast and water depth. The 1.5 km cells are used for all locations where averaged depths are less than 40m. The grid covers a region from approximately 45°N, 20°W to 63°N, 12°E and is derived from a 1.5 km ocean model, which generates the input surface current for the wave model. This is a model in preparation to replace the present UK 4 km regional model. Validation work has been done and the model performs well except for a small but stubborn problem with the current induced refraction. Somehow, the current gradient induced wave-number shift may trigger an unknown mechanism in the non-linear wave interaction terms (both DIA or WRT options), which generate extra wave energy and lead to spurious high waves around the French island (Isle of Ushant) as shown in Fig.8. This is mitigated by switching off, or massively limiting, k-shift effects.

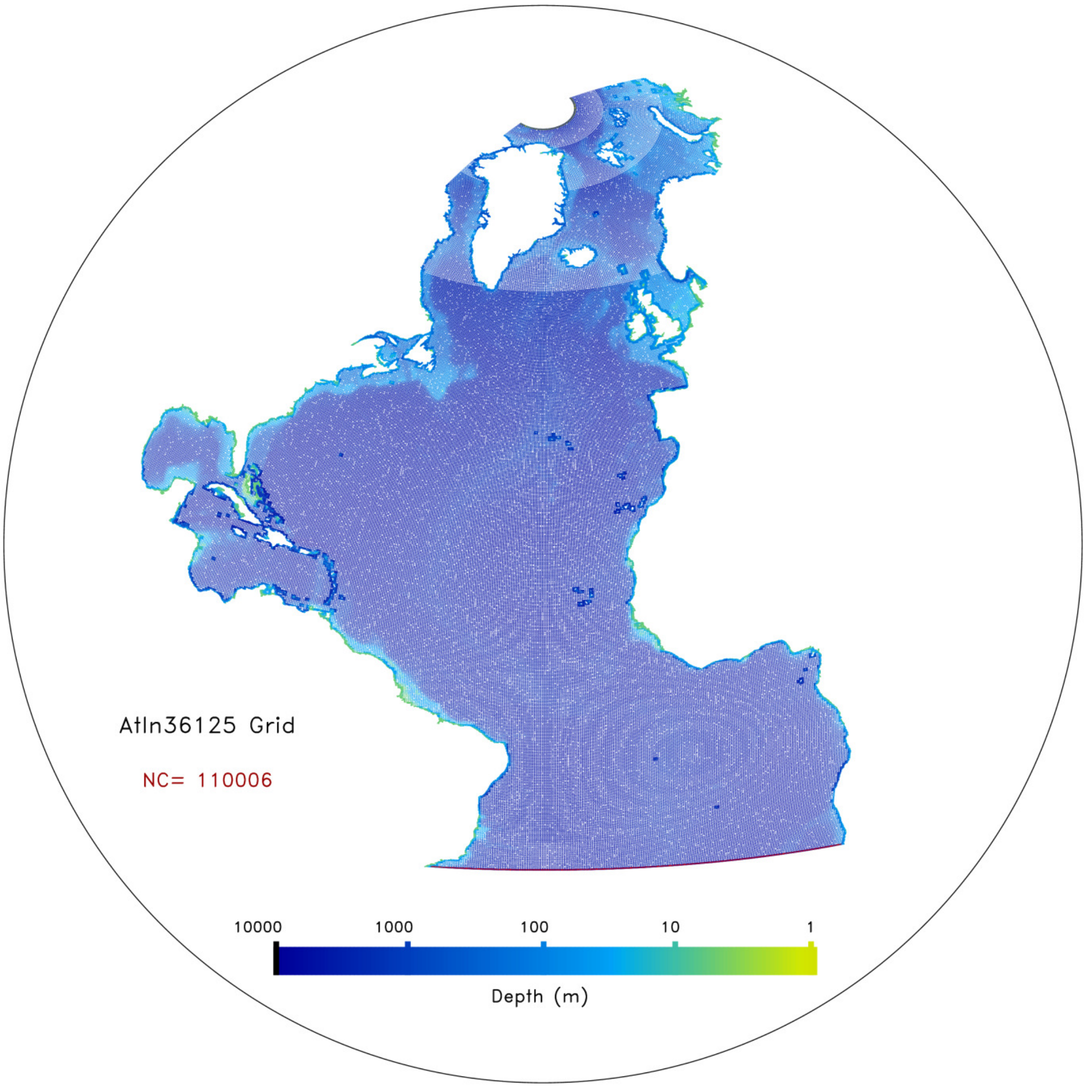


Fig. 6. The Atlantic SMC36125 grid ensemble model.

4.4 *Global SMC 50 km grid wave model for coupled system*

A 50 km SMC grid global wave model including the whole Arctic is used for wave climate studies in the Met Office coupled system (including atmospheric, ocean, ice and wave models). The relaxed CFL restriction and full Arctic inclusion in the SMC50km grid allow an efficient wave component in the coupled system to study various climate scenarios, including full open ice-free Arctic case. This SMC50km grid is used as a sample grid in the WW3 model public release and users may try to generate it from files provided in the WW3 V5.16 model package, particular in smc_docs/SMC_TKs/. The following steps may be used to generate the SMC50km grid and run the wave model WW3 V5.16, plus some sample visualization.

1) To generate SMC50km cell arrays, run IDL program `Glob50SMCels.pro`, which will read the bathymetry data `G50kmBathy.dat` and produce the global cell array file, `G50SMCelsA.dat`, including the Arctic.

2) Decide how large the Arctic part you like and divide the `G50SMCelsA.dat` into the global and Arctic parts. We chose all cells above ~ 84ºN or j >= 179. Note that there are 4-overlapping rows between the global and Arctic part so the global part actually contains all cells upto j = 182.

3) Sort the global part cells with Linux script `countcells.` The Arctic part is already in the right order so there is no need to sort it but the first count line has to be entered manually by counting the

total number of cells in the Arctic part and the boundary cells for the first two and next two overlapping rows. In our example file `G50SMCBAr.dat` the first count line is

489 128 128

You may now visualize the SMC50km cells with the IDL program `g50smcgrids.pro`, which has some tuneable parameters to choose your view point and area. The projection will be saved in 3 data files, which can be used later for wave output visualization from the same view point as the grid.

4) Generate face arrays with `G50SGlSide.f90` and `G50SAcSide.f90` for the global and Arctic part, respectively. They will read the sorted cell arrays as input.

5) Sort the face arrays (output from `G50SGlSide.f90` and `G50SAcSide.f90`) with the Linux script `countijsd`.

6) Generate the subgrid obstruction ratio with `Glob50SMCObstr.pro`, which will read the subgrid obstruction data (on full grid), `G50kmObstr.dat`, and produce a SMC cell only sub-grid obstruction file `G50GObstr.dat`, plus a text information file for setting up the regular grid input files (wind, ice, etc if any).

7) Prepare the model input files. Here are the list of files used in inp/

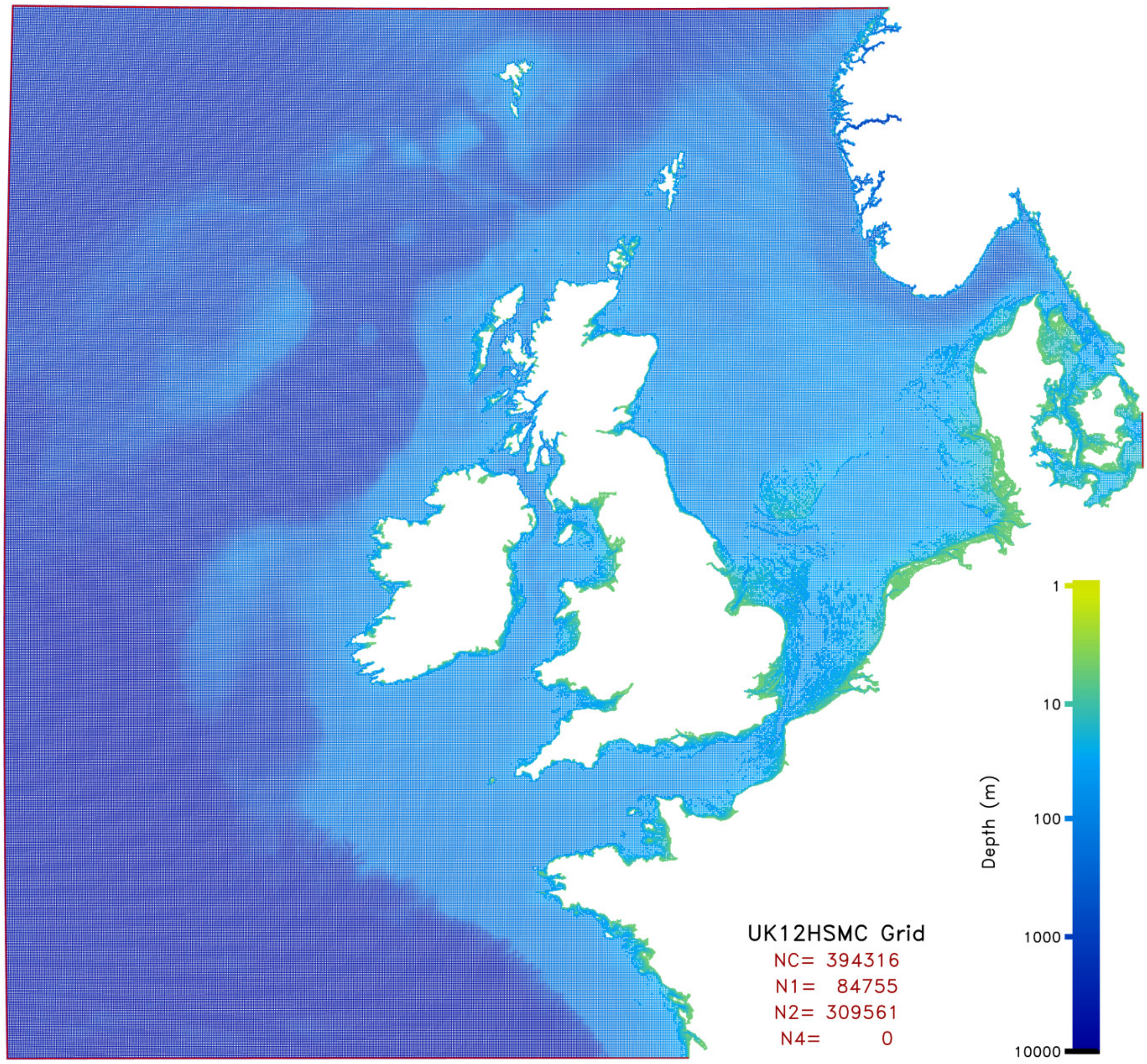


Fig.7. The rotated SMC 1.5-3 km UK grid.

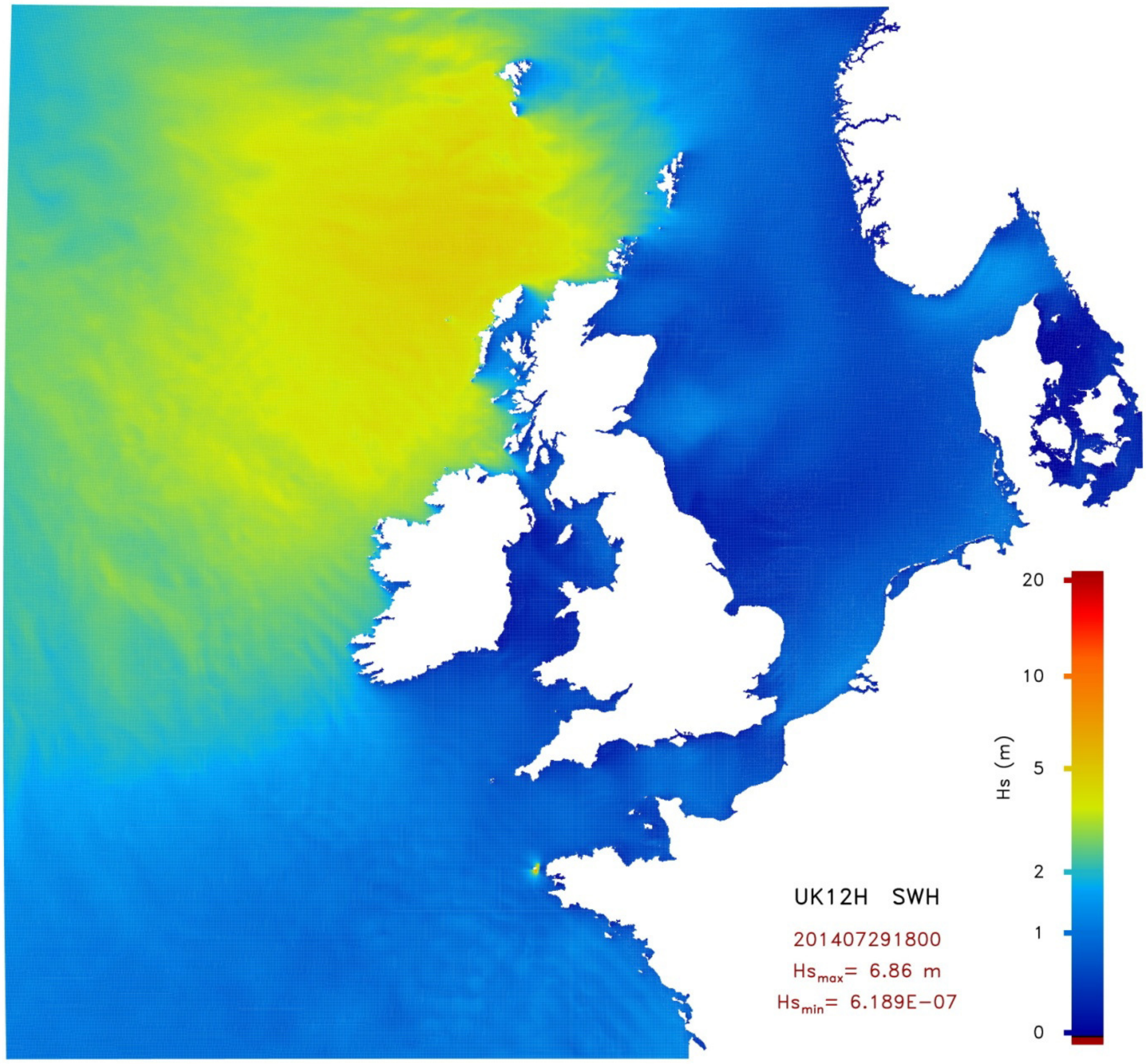


Fig.8. SWH from the rotated SMC 1.5-3 km UK wave model with k-shifts activated.

```
  23932 Nov 18  2013 G50AISide.dat
  29276 Nov 18  2013 G50AJSide.dat
5356170 Oct 20  2014 G50GISide.dat
5792134 Oct 20  2014 G50GJSide.dat
 537907 Oct 20  2014 G50GObstr.dat
  13223 Oct 20  2014 G50SMCBAr.dat
2904635 Oct 20  2014 G50SMCels.dat
  26530 Apr 13 15:50 ww3_grid.inp
   3146 Apr  5 10:55 ww3_outf.inp.template
   7097 Oct  4  2013 ww3_outp.inp.template
   8770 Apr  5 13:50 ww3_shel.inp.template
   3401 Dec 20  2012 ww3_strt.inp
```

Note the `*.inp.template` files require substituting the model start/end times to generate the final `*.inp` files.

8) Prepare the forcing wind, ice and current files if any. These forcing files remain the same as on a regular lat-lon grid. The corresponding regular grid settings are available from the `inp/ww3_grid.inp`.

9) Compile the WW3 model with the given switch file `switch` for series executables or for parallel ones by swapping '`SHRD`' with '`DIST MPI`' in the switch file.

10) Run the `ww3_grid` program to generate the `inp/mod_def.ww3` file

11) Run the `ww3_shel` with all input files to generate wave model output.

12) Post-processing from `out_grd.ww3`, depending on which format you like. We use the full text output converted by `ww3_outf (type 4)` and then visualized with the idl program `smc_docs/SMCG_TKs/g50smcswhglb.pro`, which will use the same projection as saved in step (3b) for the SMC50km cell visualization.

5. Applications of the SMC grid outside the Met Office

5.1 *The Great Lakes SMC0512 grid*

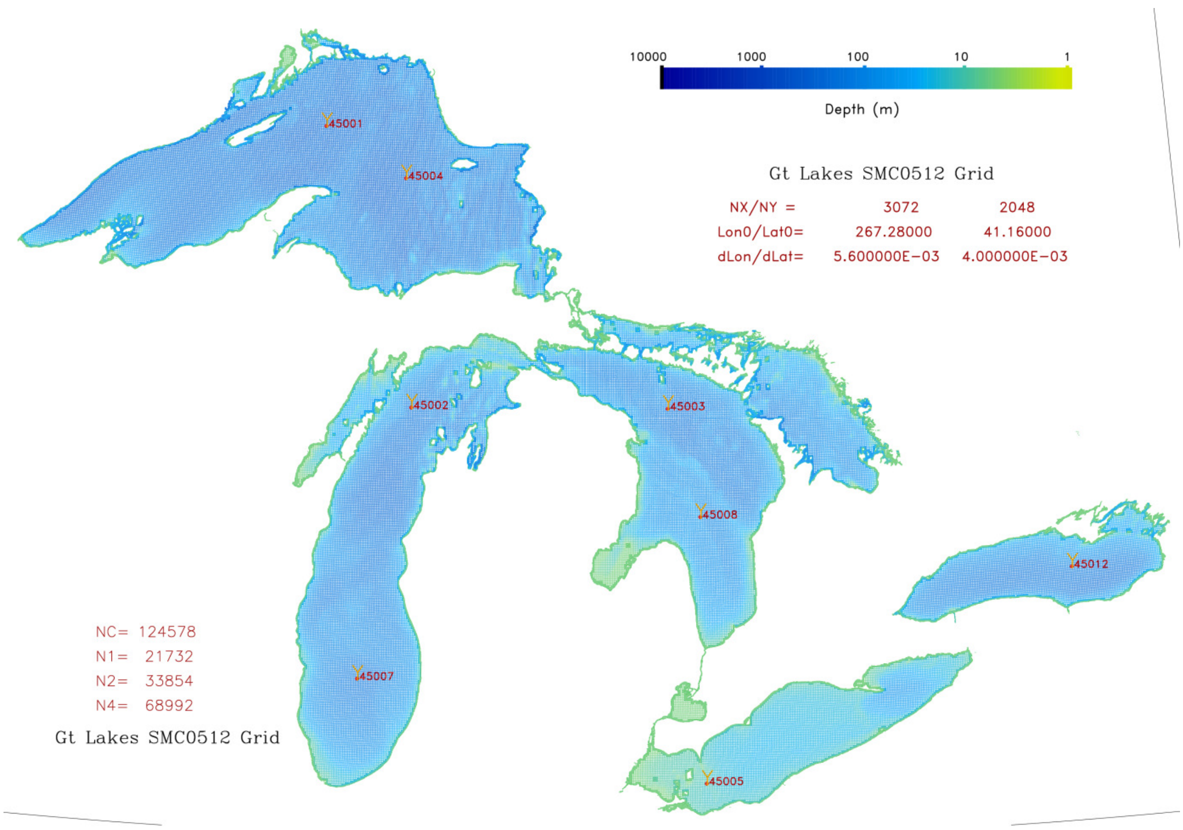


Fig.9. The SMC 0.5 – 1 – 2 km grid for the Great Lakes.

This is a regional model to cover the Great Lakes at variable resolution from 0.5 km to 2 km. As the Great Lakes are isolated from the oceans, this model domain does not need any boundary conditions. Also note the SW corner of the corresponding regular grid at 2 km resolution is chosen as the SMC grid index reference point so the index shifting numbers for the SMC0512 grid becomes zero. The model is designed for a research project in NCEP/NOAA to compare the different grids in the WW3 model, including the regular lat-lon grid, the curvilinear grid, the triangle cell unstructured grid, and the SMC grid. The project is stranded for some reasons and has not finished yet.

5.2. *The Arctic wave climate system*

This is an application in Environment Canada for wave climate study over the Arctic. It consists of an Arctic regional model at 12-25 km resolution and a global 50-100 km SMC grid to generate boundary conditions for the Arctic regional model. Only the regional Arctic model 12-25 km SMC grid is shown here in Fig.10.

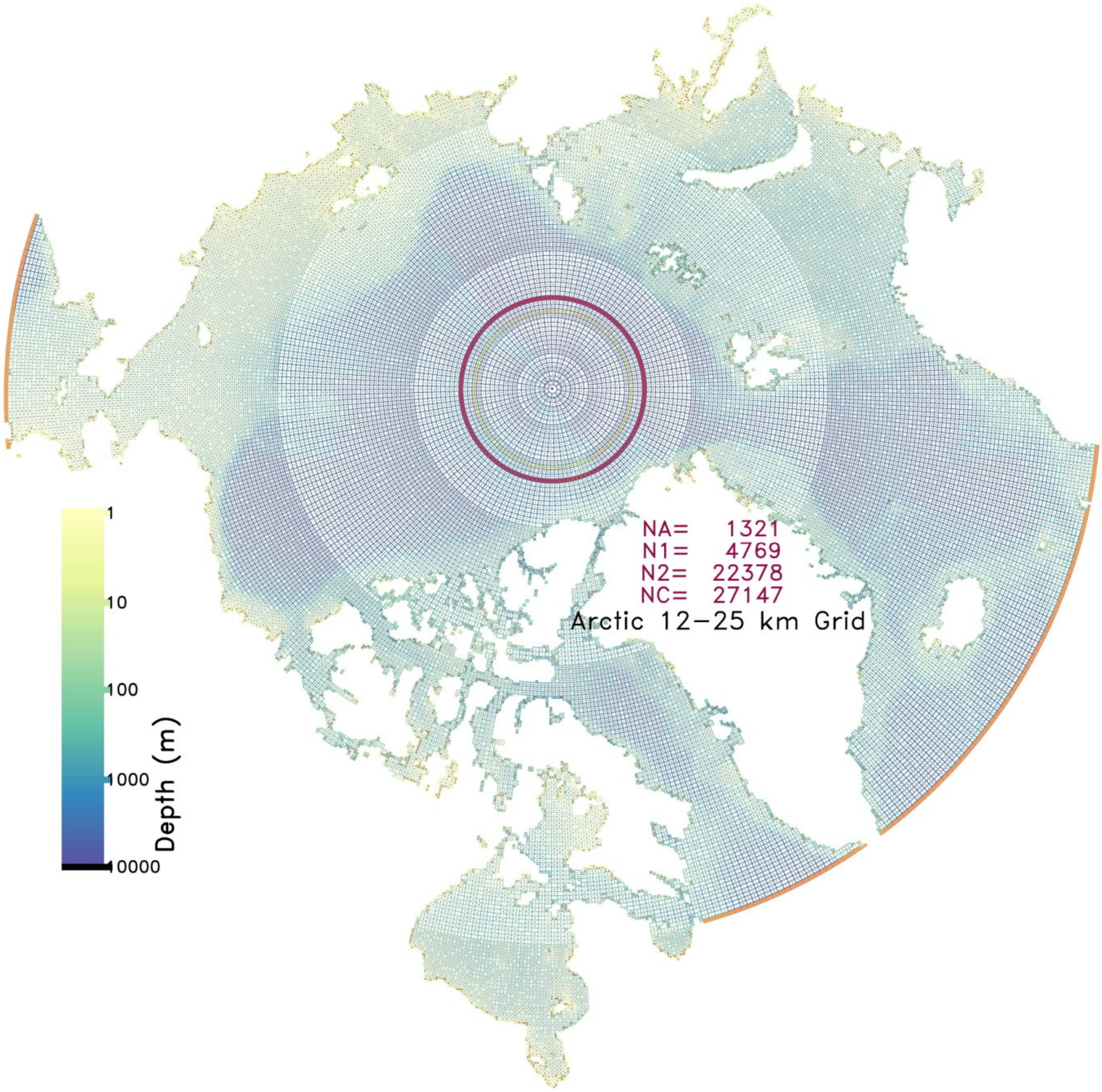


Fig.10. The EC Arctic wave climate model. Courtesy of Dr Mercè Casas-Prat (EC).

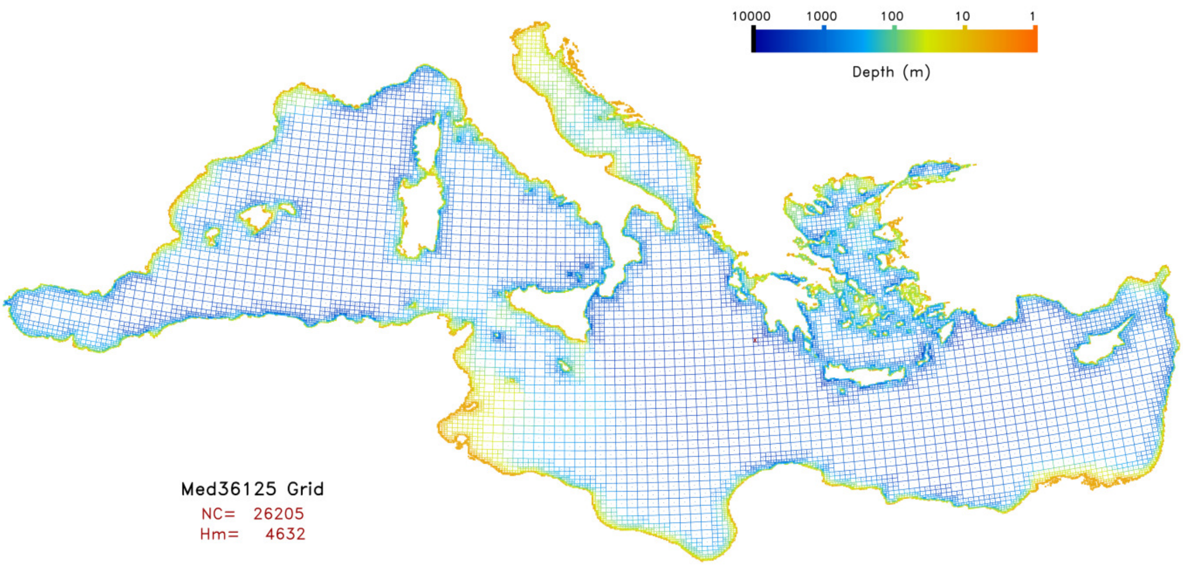


Fig.11. The SMC36125 Mediterranean grid.

5.3 *The SMC36125 Mediterranean model*

This is a regional cut out from our SMC36125 north Atlantic ensemble model grid, covering the Mediterranean Sea only (see Fig.11). Boundary conditions for the short crossing at the Gibraltar Strait can be ignored if the area of interest is away from it. Two European partners, the Spanish UPC and the Italia CNR-ISMAR, have shown an interest in using this model for their regional wave environmental studies.

5.4. *The Gulf and Mexico and Caribbean Sea 12-25 km model*

This Gulf of Mexico and Caribbean Sea 12-25 km SMC grid (see Fig.12) wave model was created for an Australian project to study hurricane wind effects on wave generation. Unfortunately, the model was abandoned at the last minute due to a current induced refraction problem in the WW3 V4.18 SMC grid module. This problem has since been found to be a mixture of a code bug and interactions between the current gradient induced wave number shift and the non-linear wave interaction schemes (both DIA and WRT). A temporary remedy to this problem is either to suspend the current gradient part in the wave number shift term or switch of the wave number shift term completely. It will not cause any noticeable change in the wave output fields except for removing those spurious high waves at strong current gradient area.

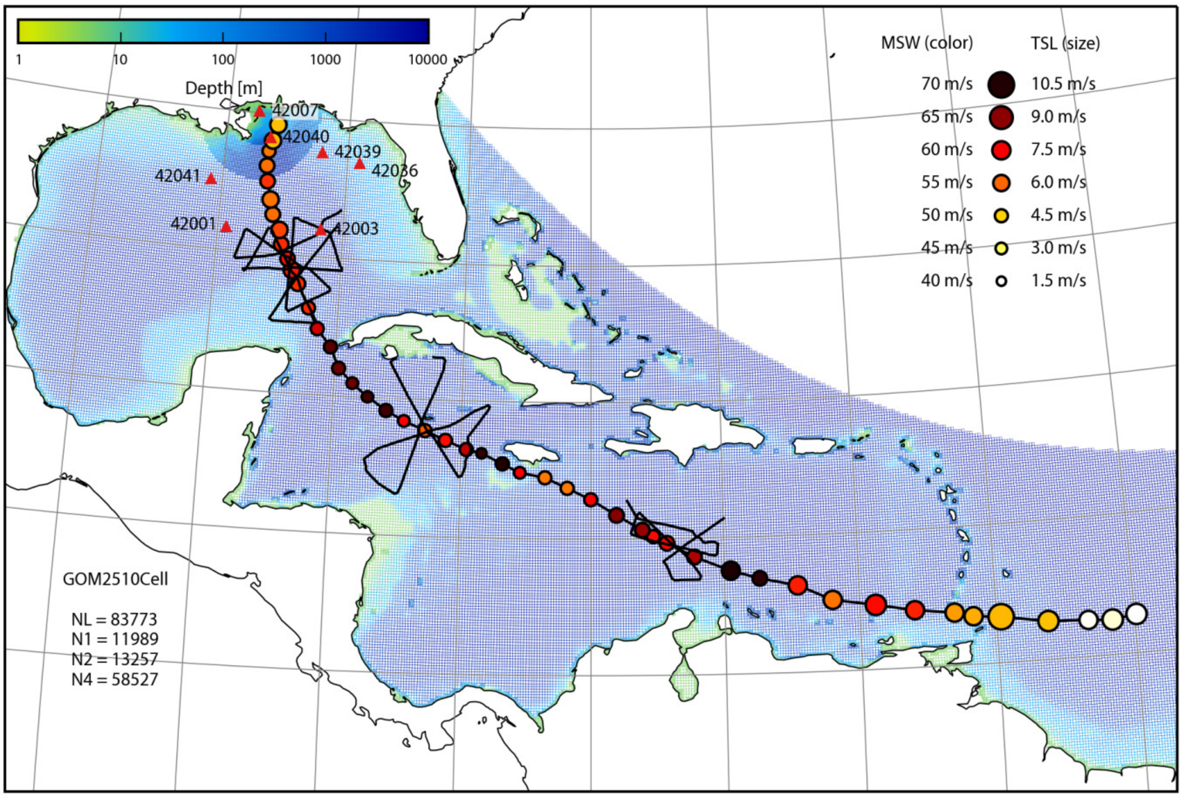


Fig.12. The Gulf of Mexico and Caribbean Sea 12-25 km SMC grid. Courtesy of Dr Qingxiang Liu (UM, Australia).

5.5. *The west Pacific regional wave model*

The National Marine Environment Forecast Centre (NMEFC) of China is planning to replace their west Pacific wave forecast model with a SMC grid model, similar to that shown in Fig.13. It is still in its development stage and has not been finalised yet. A research institute (ITRI) in Hsinchu, Taiwan is also interested in a similar system to replace their west Pacific nested grid ensemble model (Fig.14).

5.6. *The rotated SMC grid Arctic model*

This model is proposed by us to replace the Environment Canada Arctic regional model. It uses a rotated SMC grid (the rotated north pole at 135ºE and 10ºN) with uniform size-1 increment at dlon = dlat = 0.0625º and 3 resolution levels at about 6-12-25 km (Fig.15).

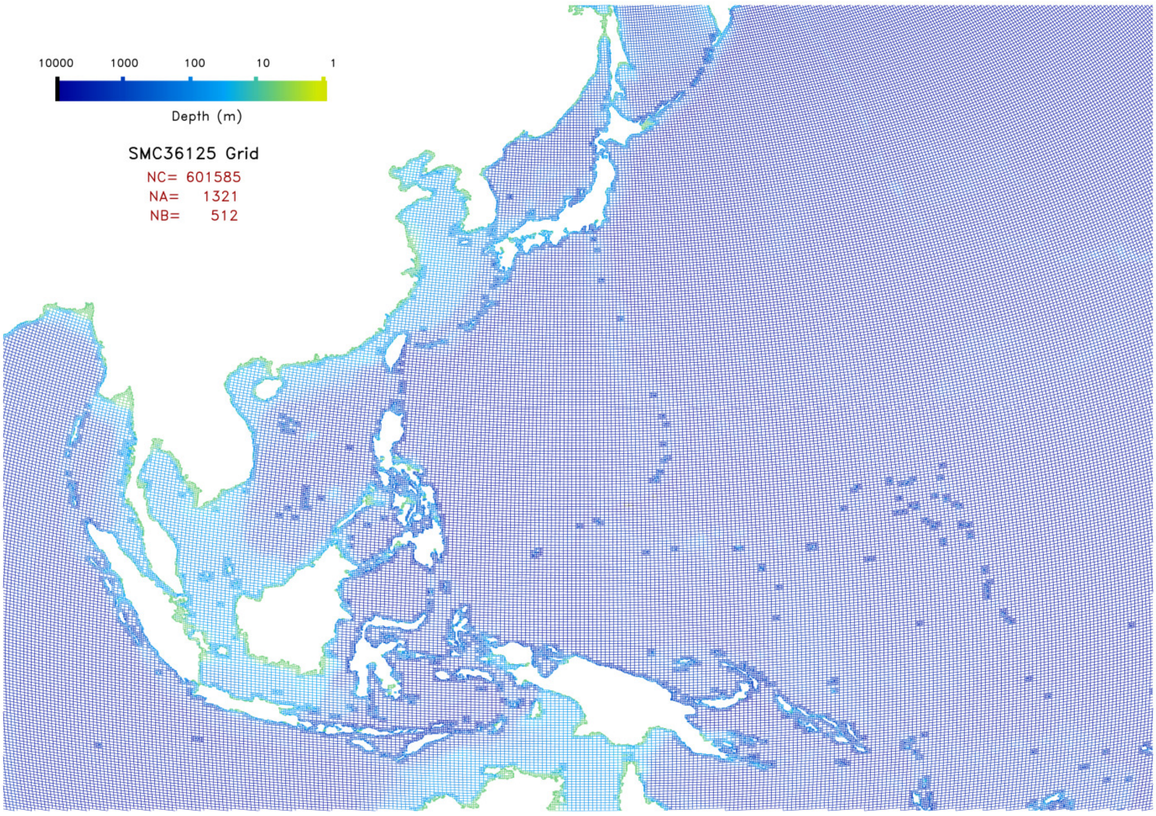


Fig.13. The west Pacific region of the global SMC36125 grid.

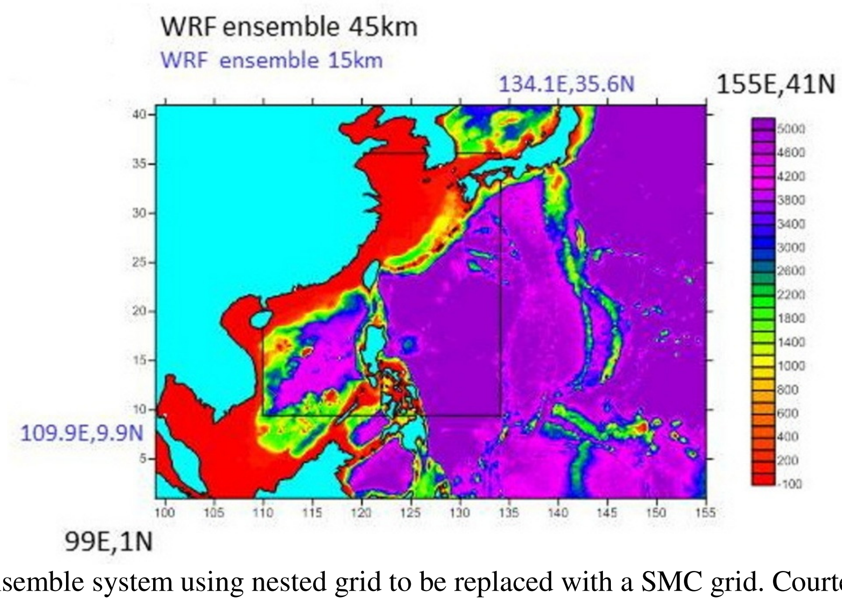


Fig.14. An ensemble system using nested grid to be replaced with a SMC grid. Courtesy of Dr Hengwen Chang (ITRI, Hsinchu, Taiwan).

One advantage to use a rotate grid is that there is no need for the map-east system in the high latitude region as it has changed Arctic into an equivalent Equatorial area. Another advantage of the rotated grid is the evenly spaced mesh within the model domain. The model has been compared with our operational global SMC36125 model and they agree well. A sample SWH output in an ice-free case from this rotated SMC grid Arctic model is shown in Fig.16. The model was driven by our global 17 km wind and 2D spectral boundary conditions for the red cells in Fig.15 are provided by our SMC36125 global wave model. Note that the forcing wind has to be converted into the rotated grid orientation before feeding into the model. The 2D spectral boundary conditions are, however, kept in the standard lat-lon grid orientation for the convenience of generation them by standard lat-lon model. The boundary 2D spectra will be converted to the rotated grid orientation inside the WW3 model.

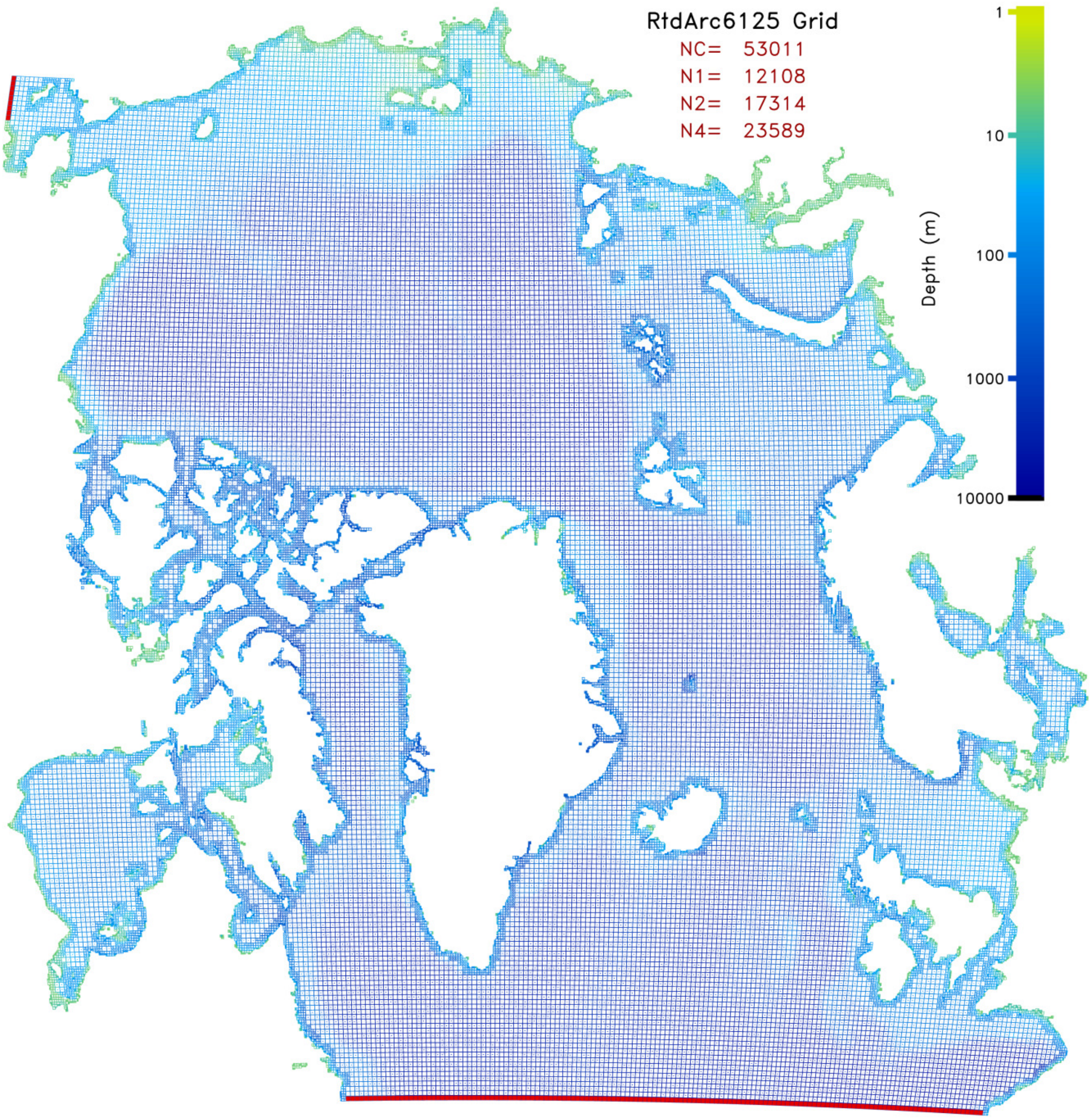


Fig.15. The rotated SMC grid Arctic model at 6-12-25 km resolution.

6. Summary and conclusions

The SMC grid is briefly reviewed and existing applications of this grid in the UK Met Office and outside it are summarised. This unstructured, multi-resolution latitude-longitude grid, which incorporates a practical solution to the polar problems, has shown its efficiency and convenience in various operational and research wave models. The Met Office global wave model update in October 2016 to a 3-6-12-25 km SMC grid is a successful demonstration of how the grid could help improve model performance without majorly increasing computation demands. The SMC grid supports flexible domain shapes and refined resolution in desired areas, making it suitable for both global and regional models. It retains the quadrilateral cells as in the standard lat-lon grid so that the efficient finite difference schemes could be used. Sub-time-steps are applied on different cell sizes to speed up propagation calculations, with a choice of 2nd or 3rd order advection schemes. Grid cells are merged at high latitudes to relax the CFL restriction and a fixed reference direction is used to define wave spectra in the polar region so that the SMC grid can be extended to the full globe, including the Arctic. The multi-resolution refinement is particularly useful to resolve small islands and coastline details, which are important in ocean surface wave propagation but could be too expensive in standard lat-lon grid models.

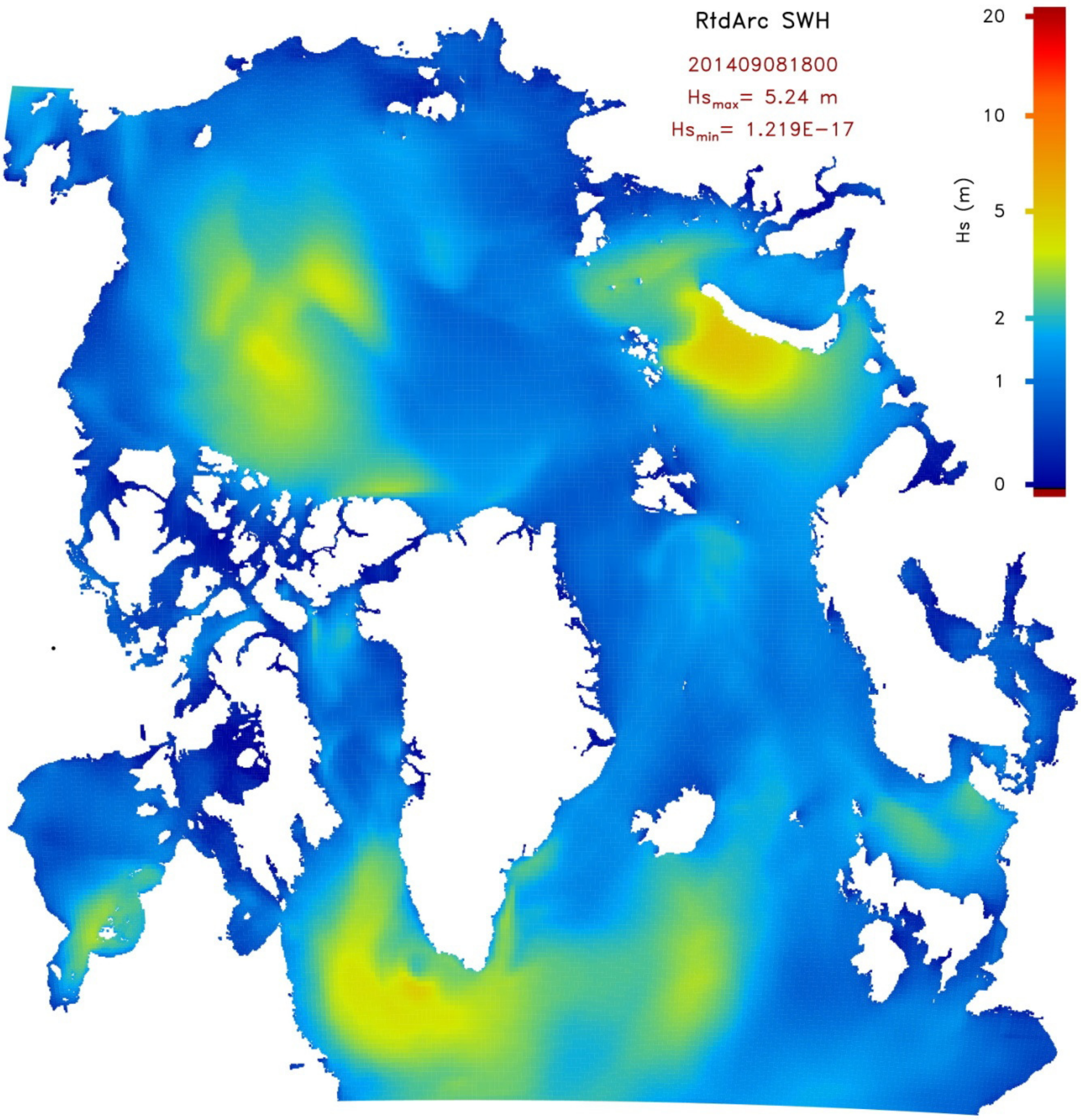


Fig.16. Simulated SWH field from the rotated Arctic SMC6125 grid model in an ice-free case.

The SMC grid has been implemented in the WAVEWATCH III community wave model since V4.18 and updated in the latest V5.16 with some bug-fixing and added new features. This grid has caught the attention of a few international users and collaborators, thanks to the wide spread user network of the WAVEWATCH III model. Four operational and research SMC grid models are in use in the Met Office now, including the global SMC36125 operational wave forecast wave model, the north Atlantic ensemble model, the global 50 km SMC grid wave model in coupled climate system and the rotated SMC grid UK 1.5-3 km regional model in preparation stage. Worldwide outside the Met Office, six SMC grid regional and global models have been used or are in preparation. They are the Great Lakes 0.5-1-2 km SMC grid model for a NCEP/NOAA research project, the Environment Canada Arctic climate system (consisting of the 12-25 km Arctic SMC grid model and the 50-100 km global SMC grid to generate boundary conditions), the Gulf of Mexico and Caribbean Sea 4-8-16 km SMC grid model for hurricane studies in an Australia project, the Mediterranean Sea SMC36125 grid model for our European partners (UPC, Spain and CNR-ISMAR, Italia), and west Pacific multi-resolution regional SMC grid models, which are in preparation for NMEFC, China and ITRI, Taiwan. The combination of rotated lat-lon grid and the SMC grid has created a more suitable platform for high latitude regional models than the SMC grid on the standard lat-lon grid. A rotated SMC grid Arctic model has been demonstrated to show its evenly-spaced mesh in the whole Arctic region and the minimal changes required to run the model. It is expected that more users will find applications of this grid in their wave environmental studies.

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