Incorporating Breaking Wave Predictions in Spectral Wave Forecast Models

Russel Morison and Michael Banner
School of Mathematics, The University of New South Wales,
Sydney 2052, Australia

Motivation

- **wave breaking at sea** - widespread air-sea interfacial process with very significant geophysical and maritime importance
- present spectral wave forecast models **do not provide explicit forecasts** of breaking wave properties.
- recent advances in understanding wave breaking have made it possible to redress this deficiency
- to describe a novel methodology that adds to standard spectral wave model output - **accurate forecasts** of

(i) the **spectral density of breaking crest length per unit area** and

(ii) the associated **breaking strength**

We did this initially for the **dominant wind waves** and have now extended it across the **full spectrum**

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Modeling Wave Breaking Spectrally

Radiative transfer equation (deep water, no currents)

The radiative transfer equation for describing the evolution of the wave height spectrum *F(k)* is given by:

$$\frac{\partial F}{\partial t} + c_g \cdot \nabla F = S_{in} + S_{nl} + S_{diss}$$

where

- $F=F(k,\theta)$ is the directional wave spectrum
- c_g is the group velocity
- $S_{tot} = S_{in} + S_{nl} + S_{ds}$ is the total source term.
- S_{in} is the ***atmospheric input*** *spectral source term*
- S_{nl} is the ***nonlinear spectral transfer*** *source term representing nonlinear wave-wave interactions within the spectrum*
- S_{ds} is the spectral ***dissipation rate*** *here taken as due primarily to wave breaking*

Saturation Threshold-based Dissipation Rate S_{ds}

• based on treating spectral bands as nonlinear wave groups. Uses a low power of the spectral saturation ratio (~steepness ratio) to simulate observed threshold behaviour *[extension of Alves & Banner (JPO, 2003)]*

$$S_{ds}(k,\theta) = [C_1 * D * (\tilde{\sigma} - \tilde{\sigma}_T)/\tilde{\sigma}_T)^{a_1} + C_2 * D * E_{tot} * k_p{}^2\ ](\sigma/\sigma_m)^{a_2}\ \omega F(k,\theta)$$

'local S_{ds}' **'non-local S_{ds}'**

This formulation uses

• normalized azimuthally-integrated saturation: $k^4 F(\mathbf{k})/\theta(k) = (2\pi)^4 f^5 F(f) / 2g^2$

• measured threshold of the normalized spectral saturation (Banner et al., JPO, 2002) with a_1=2

• tail exponent a_2 = 4 to match dissipation to input behavior in the spectral tail

• nonlocal dissipation rate component

• coefficient D for the local S_{ds}: non-dimensional and linear in the wind speed to match to the wind input term.

• C_1 and C_2 constants

Modified Jansen Wind Input

$$S_{in}(k,\theta) = \varepsilon\,\beta(k,\theta)\,\omega\,(u_*^{red}(k)\cos\theta / c)^2 * E(k,\theta)$$

$$S_{in}(k,\theta) = \varepsilon\,\beta(k,\theta)\,\omega\,(u_*^{red}(k) / c)^2 * E(k,\theta)\cos\theta$$

$$\beta(k,\theta) = J_2\,\mu(\ln(\mu))^4 / \kappa^2$$

where J_2=1.2 (Janssen (1991) .

$$\beta(k,\theta) = 0$$ for $$\mu > 1$$

$$\mu(k,\theta) = (u_*/c)^2\,(g z_0/u_*^2)\exp(J_1\kappa/(u_*\cos\theta/c)^2)$$

$$z_0 = \frac{0.01 u_*^2}{g} / \sqrt{1 - C_0(\tau_w/\tau)}$$

$$u_*^{red}(k_n) = \sqrt{[\tau_{tot} - J_0\sum_{i=1}^{n}(\tau_w(i) + \tau_{bw}(i))]/\rho_{air}}$$

Brief description of the Methodology

$\Lambda(\mathbf{c})$ is the spectral density of breaking wave crest length per unit area

$\Pi(\mathbf{c})$ is the spectral density of the total wave crest length per unit area

The breaking probability $P_{br}(\mathbf{c})$ for wave scales **c** is defined as:

$$\Pr_{br}(\mathbf{c}) = \frac{\int \mathbf{c}\,\Lambda\,d\mathbf{c}}{\int \mathbf{c}\,\Pi\,d\mathbf{c}}$$

$$\sim \frac{\text{passage rate of breaking wave crests}}{\text{passage rate of wave crests}}$$

Λ(c): spectral density of *breaking wave crest length* per unit area with velocities in the range (c, c+dc) (Phillips,1985)

$$b\frac{\rho}{g}c^5\Lambda(c)dc$$

wave energy dissipation rate at scale c

$$b\frac{\rho}{g}c^4\Lambda(c)dc$$

momentum flux from waves of scale c to currents

The sea state threshold variable used for breaking probability was the normalised spectral saturation

$$\tilde{\sigma}(k) = \sigma(k) / < \theta(k) >$$

where $\sigma(k)$ is the azimuth-integrated spectral saturation given by

$$\sigma(k) = k^4 \Phi(k)$$

$$= (2\pi)^4 f^5 G(f)/2g^2$$

and $<\theta(k)>$ is the mean spectral spreading width given by

$$< \theta(k) > = \int_{-\pi}^{\pi} (\theta - \bar{\theta})\, F(k,\theta)\, k\, d\theta \;/ \int_{-\pi}^{\pi} F(k,\theta)\, k\, d\theta$$

where $\bar{\theta}$ is the mean wave direction, and $\Phi(k)$, $G(f)$ and $F(k,\theta)$ are, respectively, the spectra of wave height as a function of scalar wavenumber, frequency and vector wavenumber.

$$\Pr_{br}(\tilde{\sigma}) = H(\tilde{\sigma} - \tilde{\sigma}_T) * \alpha_{br} * (\tilde{\sigma} - \tilde{\sigma}_T)^{0.5}$$

$$b_{br}(\tilde{\sigma}) = H(\tilde{\sigma} - \tilde{\sigma}_T) * c_{br} * (\tilde{\sigma}^{0.5} - \tilde{\sigma}_T{}^{0.5})^{1.0}$$

$$S_{ds}^{loc}(c)\,dc = b(c)\,g^{-2} c^5 \Lambda(c)\,dc$$

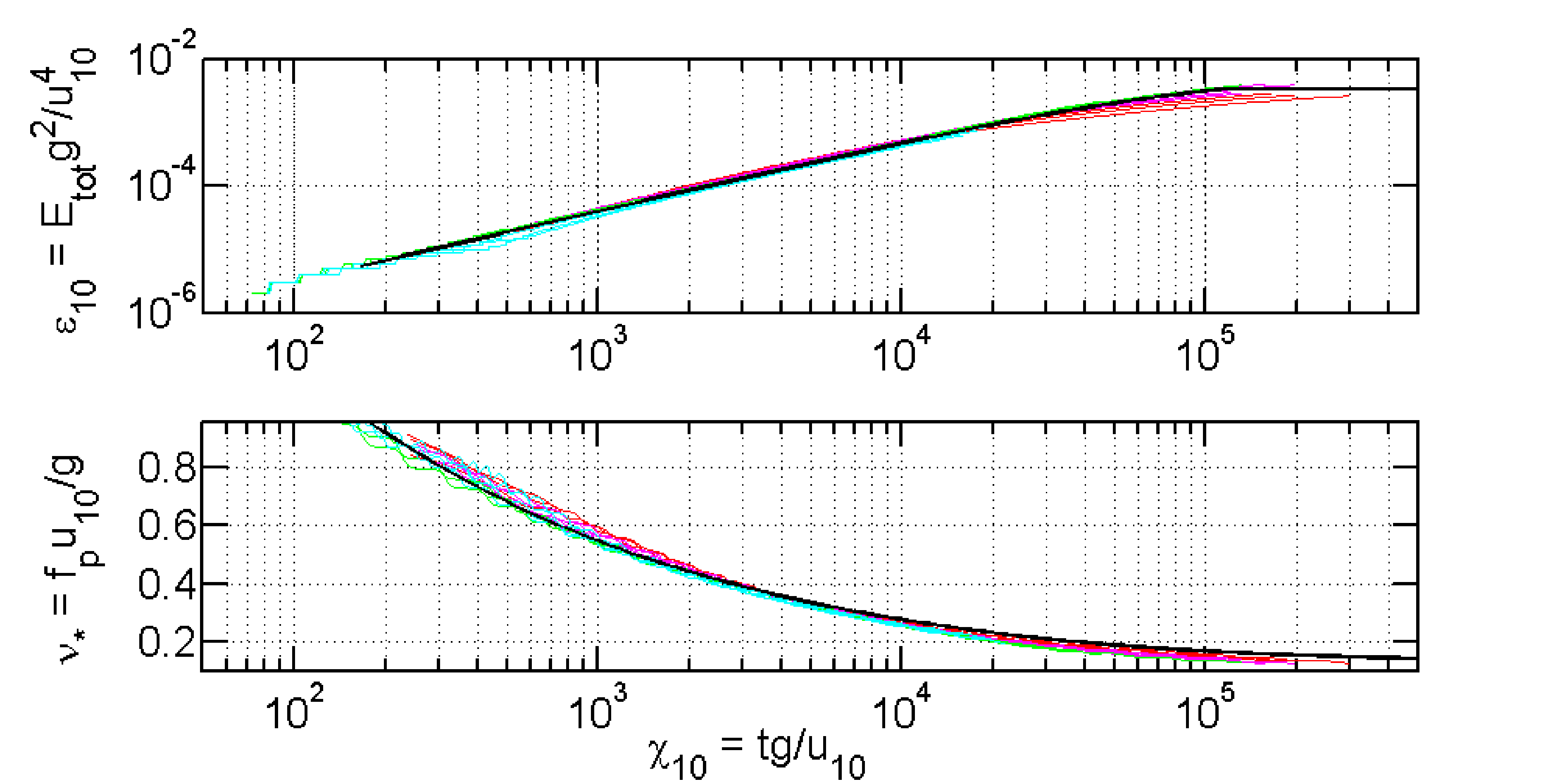
$$\Lambda(c) = S_{ds}^{loc}(c) * g^2 / (b(c) * c^5)$$

$$B_{eff} = \int S_{ds}(c)dc \,/ g^2 \int c^5 * b(c) * \Lambda(c)dc$$

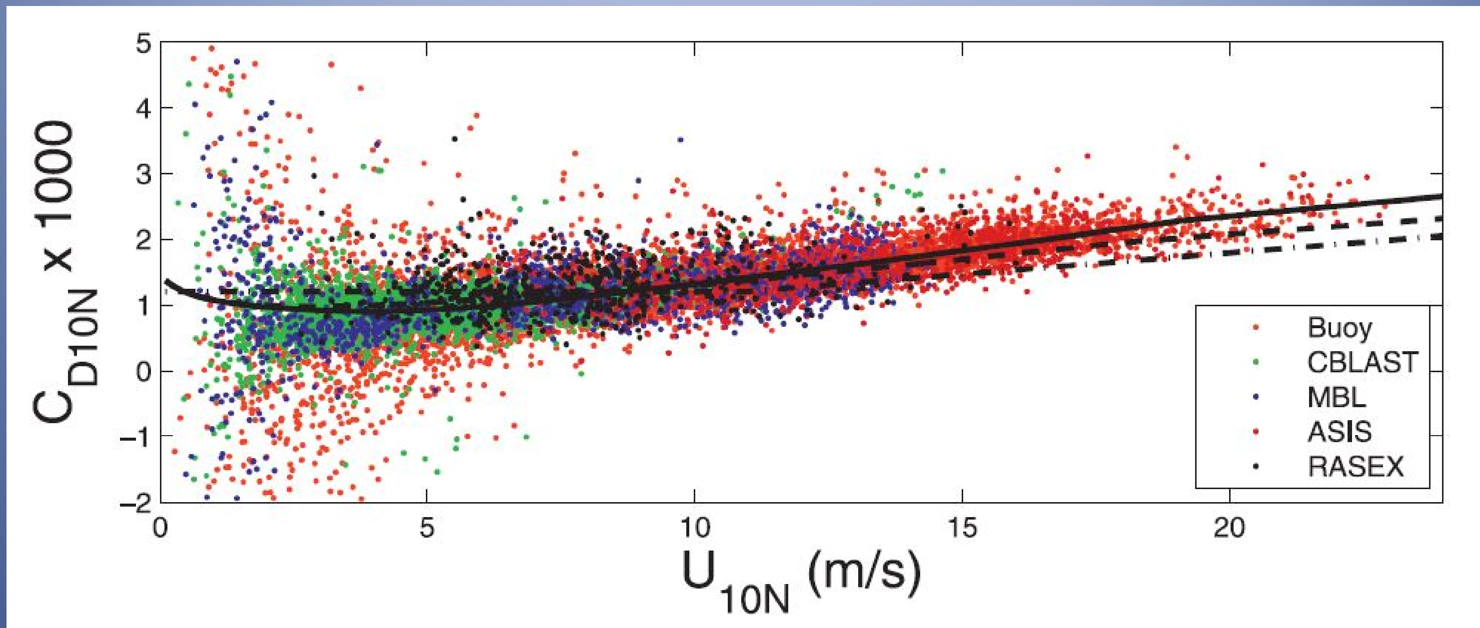
$$B_{eff}^{loc} = \int S_{ds}^{loc}(c)dc \,/ g^2 \int c^5 * b(c) * \Lambda(c)dc$$

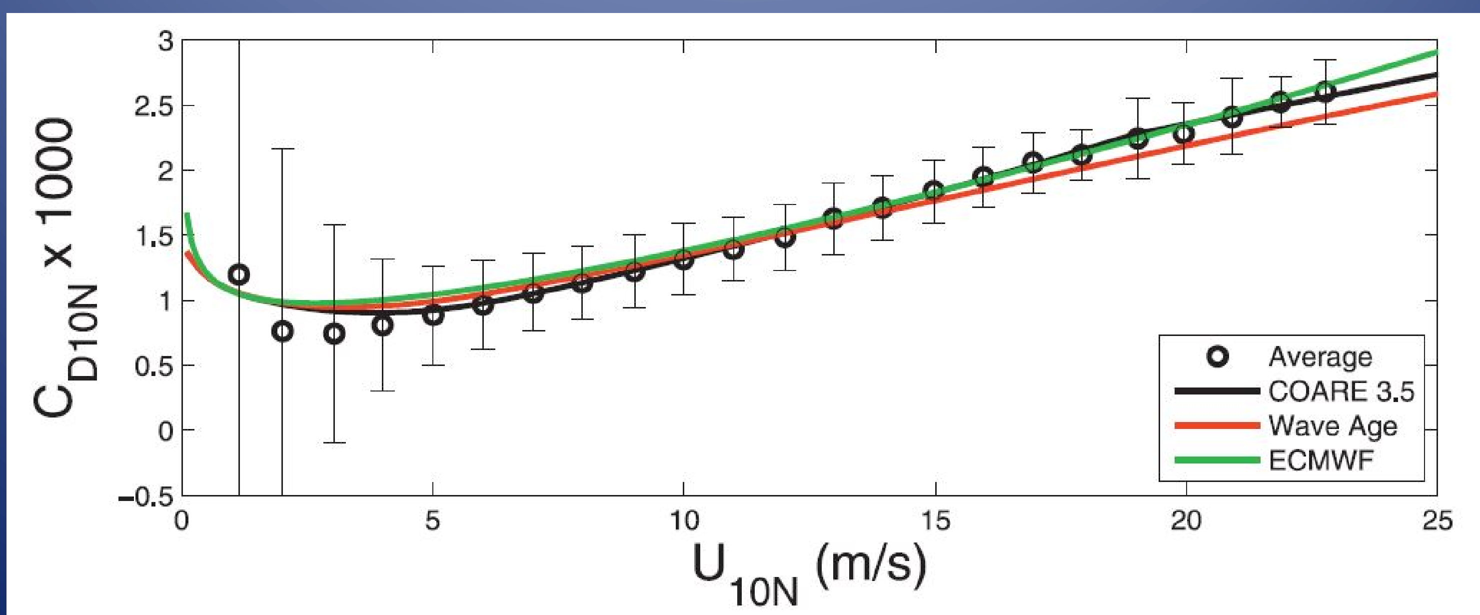
$$\Pr_{br}(c) = \int_{0.7C}^{1.3C} c\Lambda(c)dc \,/ \int_{0.7C}^{1.3C} c\Pi(c)dc$$

Non-Dimensional Evolution

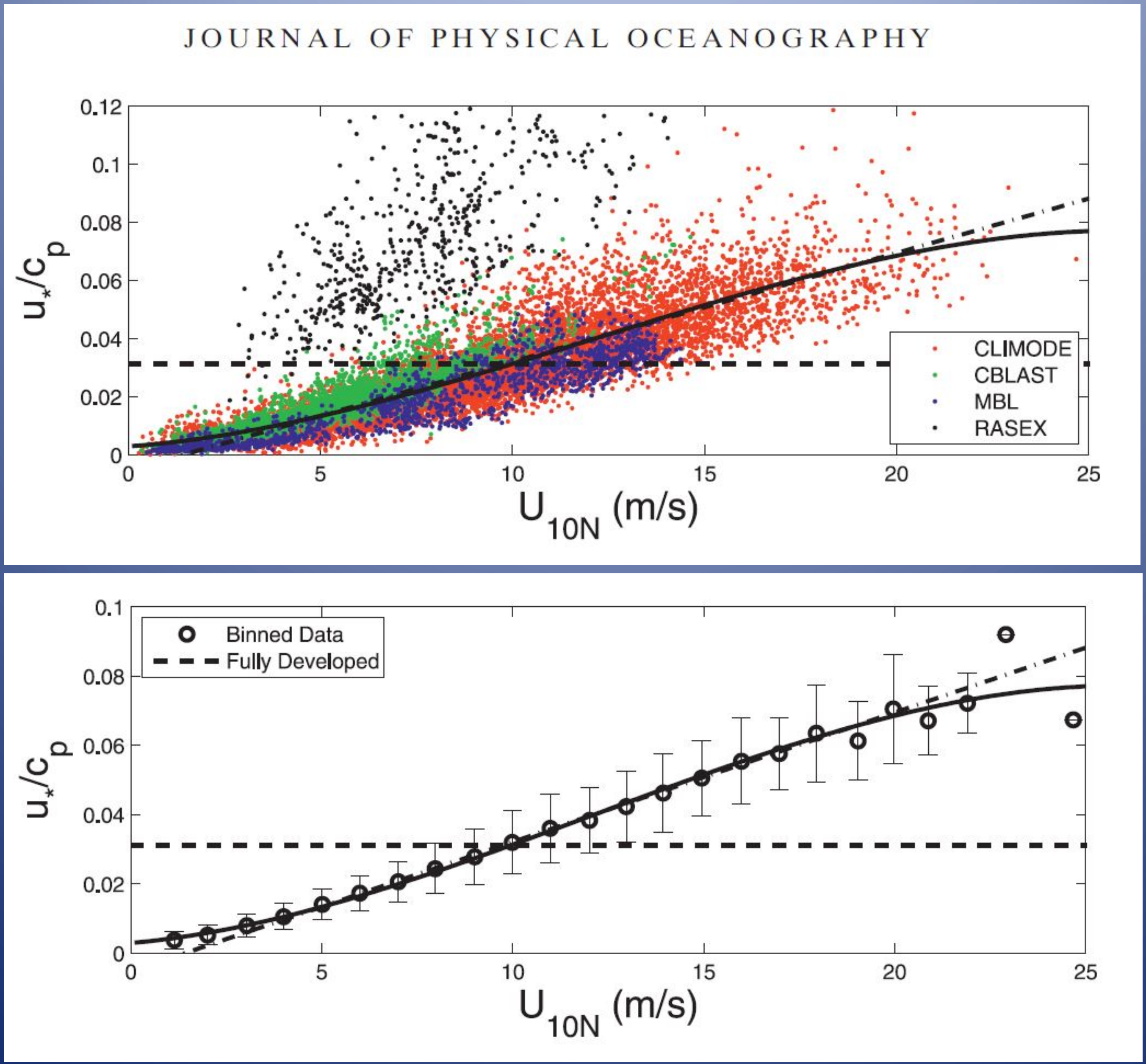


Edson et al, 2013. Drag Coefficient Observational Data.

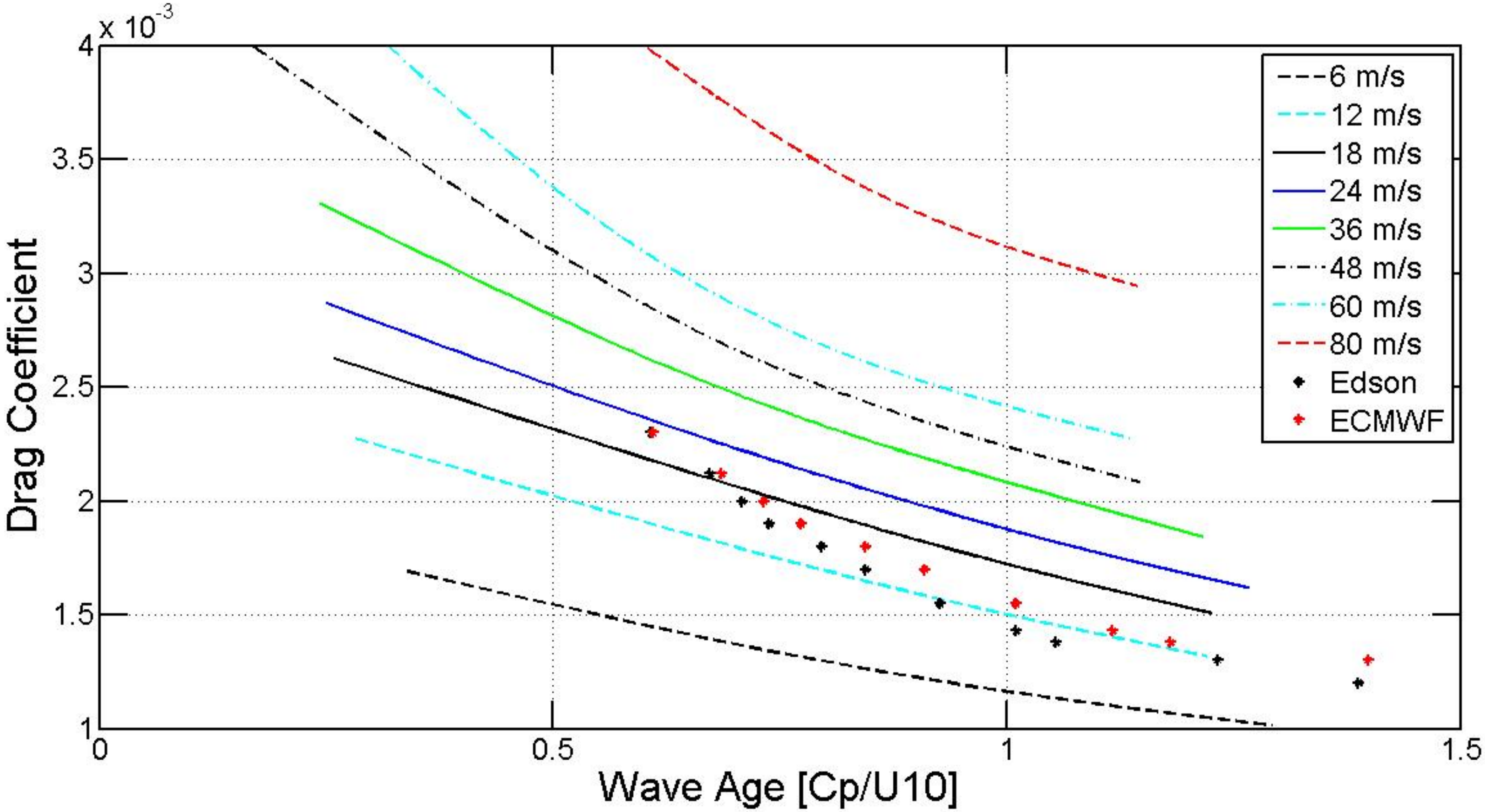




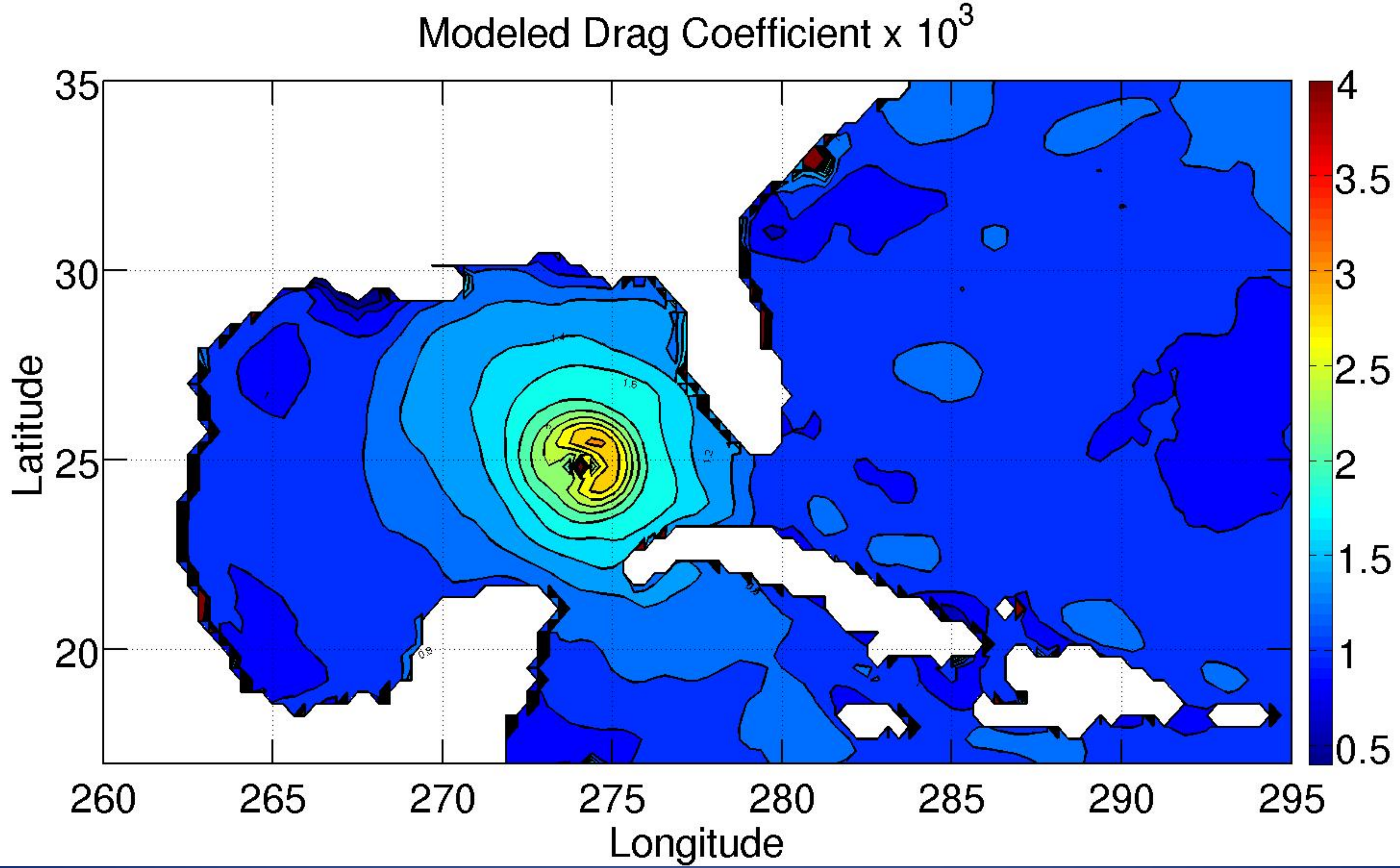
Edson et al, 2013. Observational Data: Wave Age .vs. U10



Model Forecast : Drag Coefficient forecast for Fetch Limited for multiple Wind Speeds 6 m/s to 80 m/s



Modeled Drag Coefficient: Hurr. Katrina



$$\Pr_{br}(\tilde{\sigma}) = H(\tilde{\sigma} - \tilde{\sigma}_T) * \alpha_{br} * (\tilde{\sigma} - \tilde{\sigma}_T)^{0.5}$$

$$S_{ds}^{loc}(c)\, dc = b(c)\, g^{-2} c^5 \Lambda(c)\, dc$$

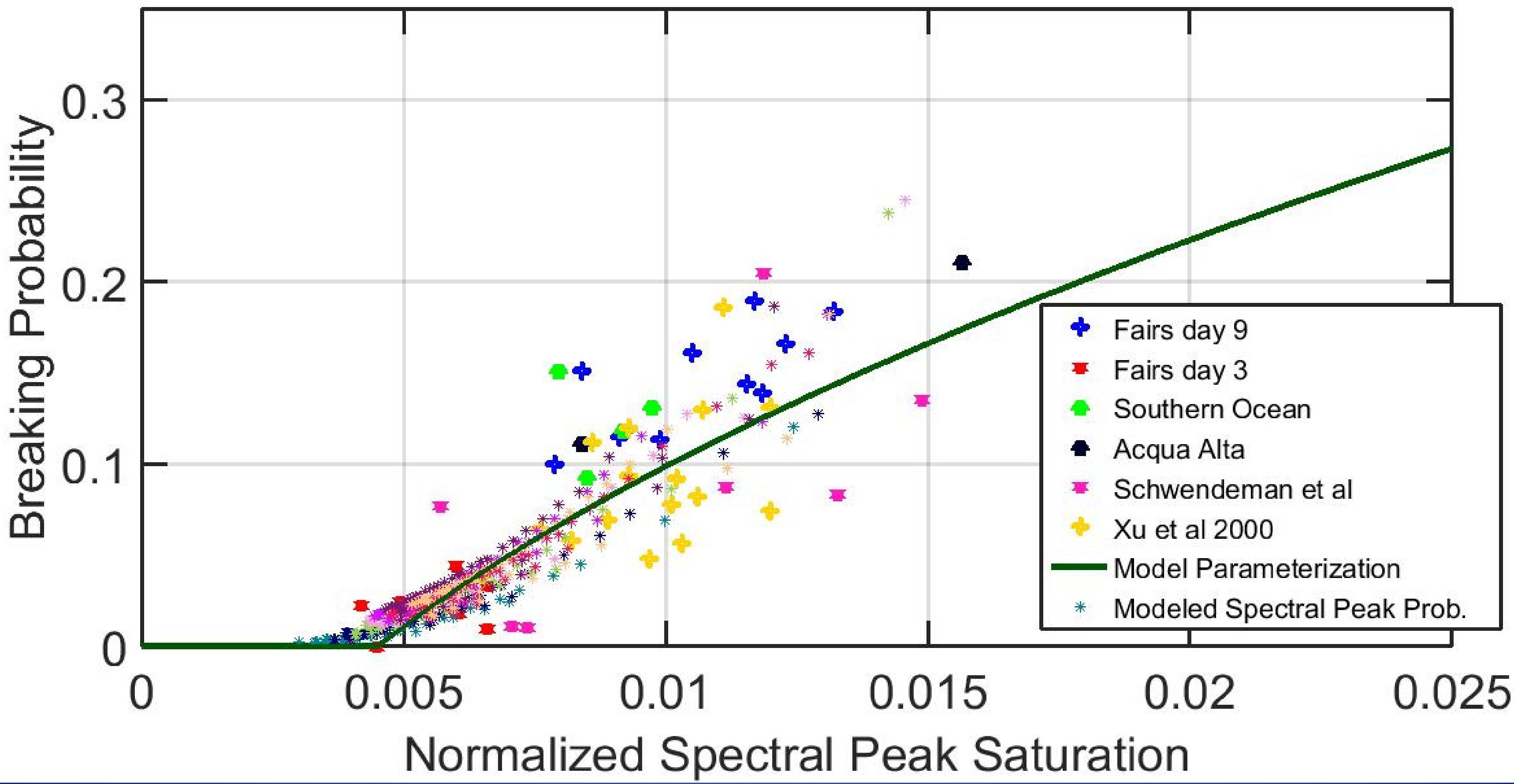
$$b_{br}(\tilde{\sigma}) = H(\tilde{\sigma} - \tilde{\sigma}_T) * c_{br} * (\tilde{\sigma}^{0.5} - \tilde{\sigma}_T{}^{0.5})^{1.0}$$

$$\Lambda(c) = S_{ds}^{loc}(c) * g^2 / (b(c) * c^5)$$

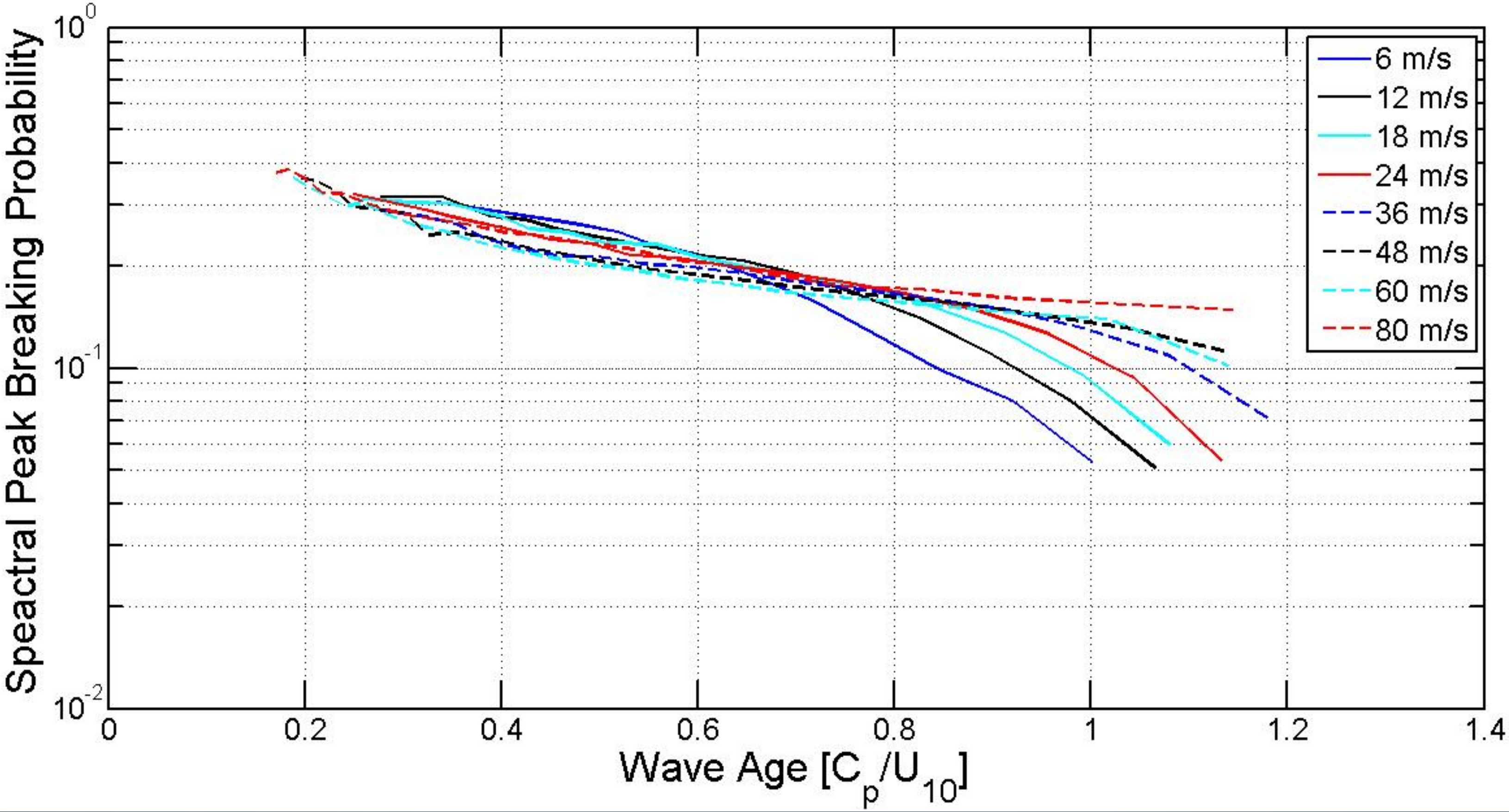
$$\Pr_{br}(c) = \int_{0.7C}^{1.3C} c\Lambda(c)dc \Big/ \int_{0.7C}^{1.3C} c\Pi(c)dc$$

Breaking Probability .vs. Normalized Saturation.

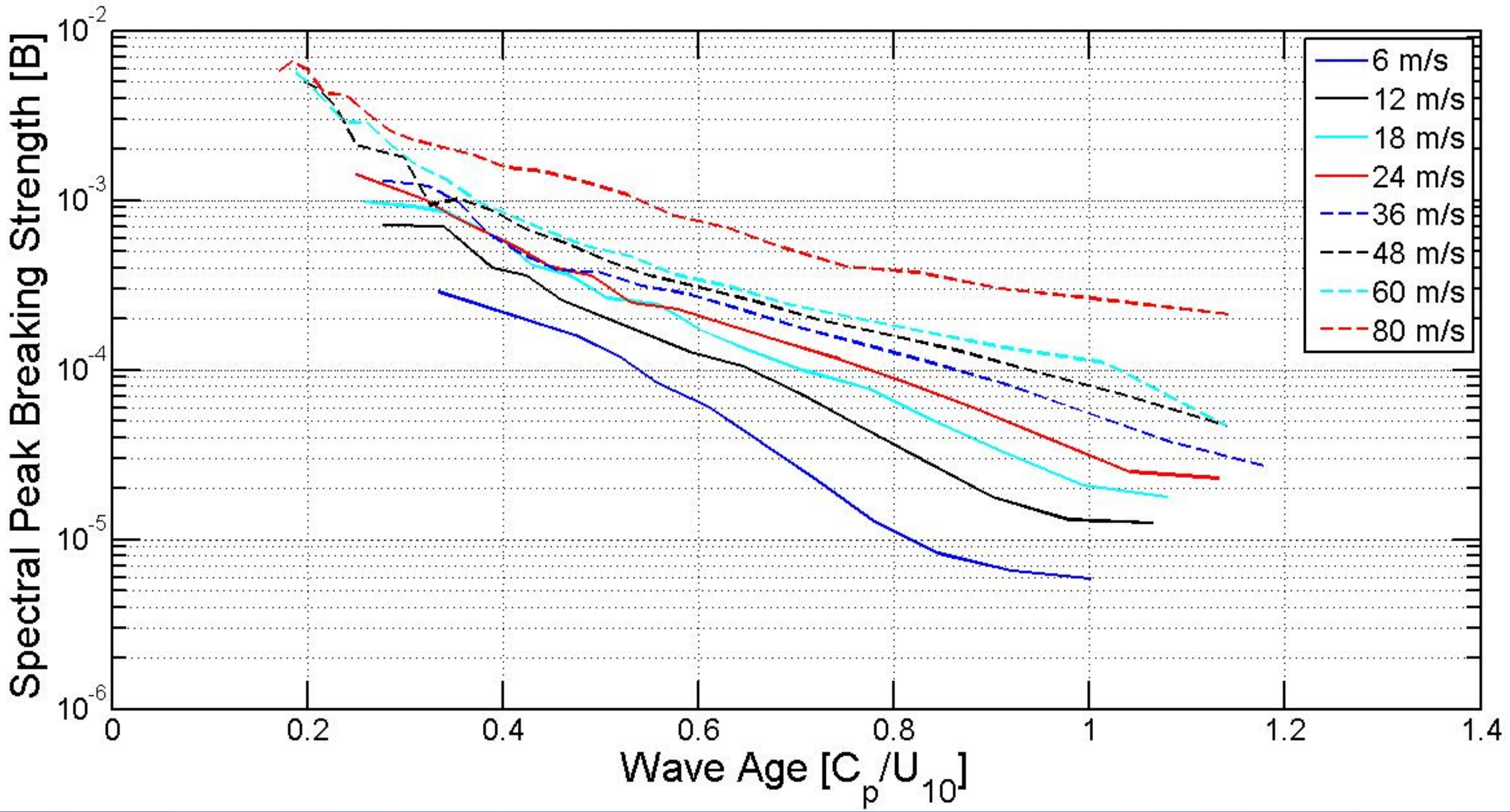
Observed Data; Model Parameterization; from c*Λ(c) dc



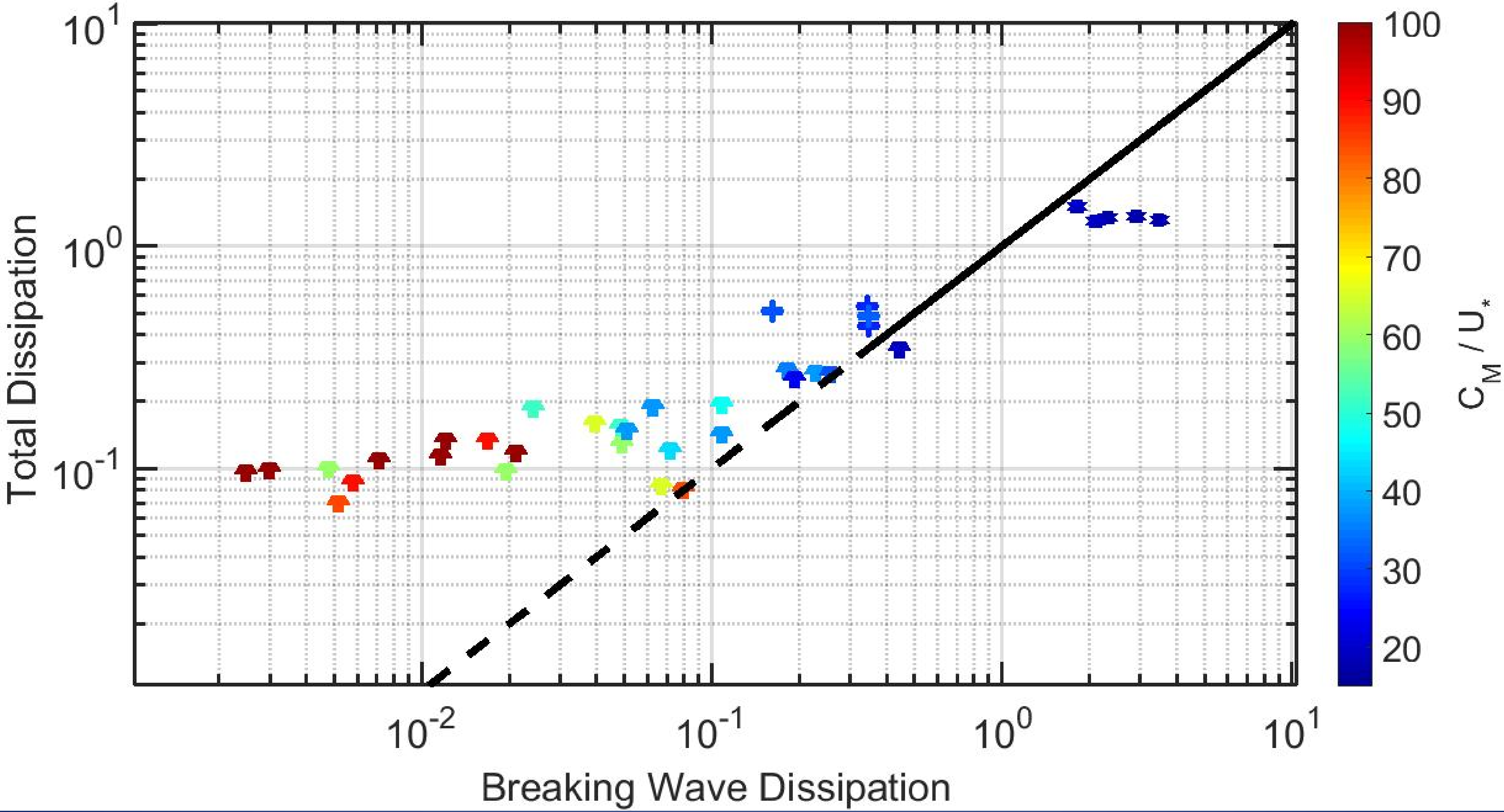
Model Forecast: Breaking Probability .vs. Wave Age for multiple wind speeds 6 m/s to 80 m/s.



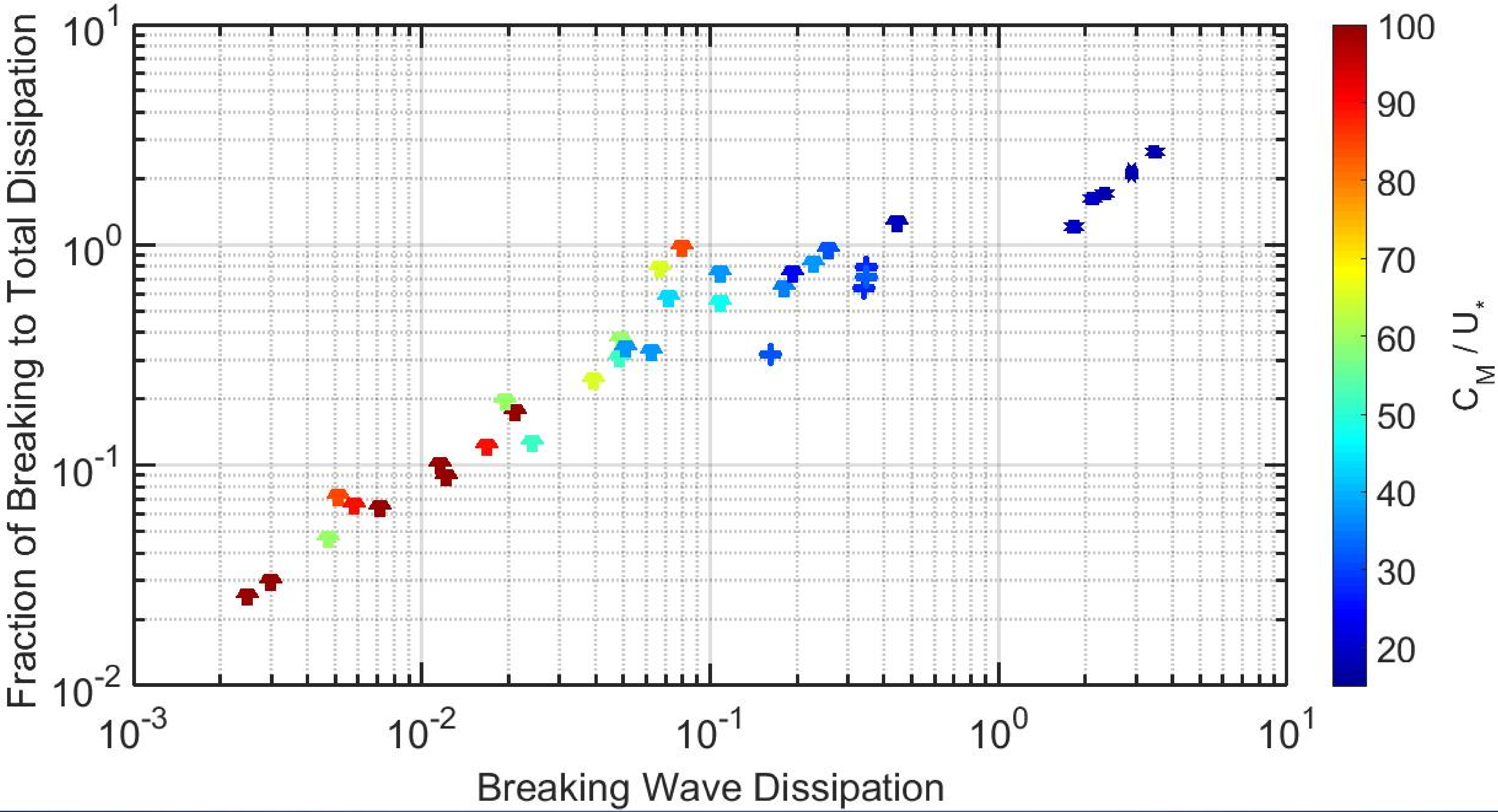
Model Forecast: Breaking Strength .vs. Wave Age
For multiple wind speeds 6 ms/ to 80 m/s



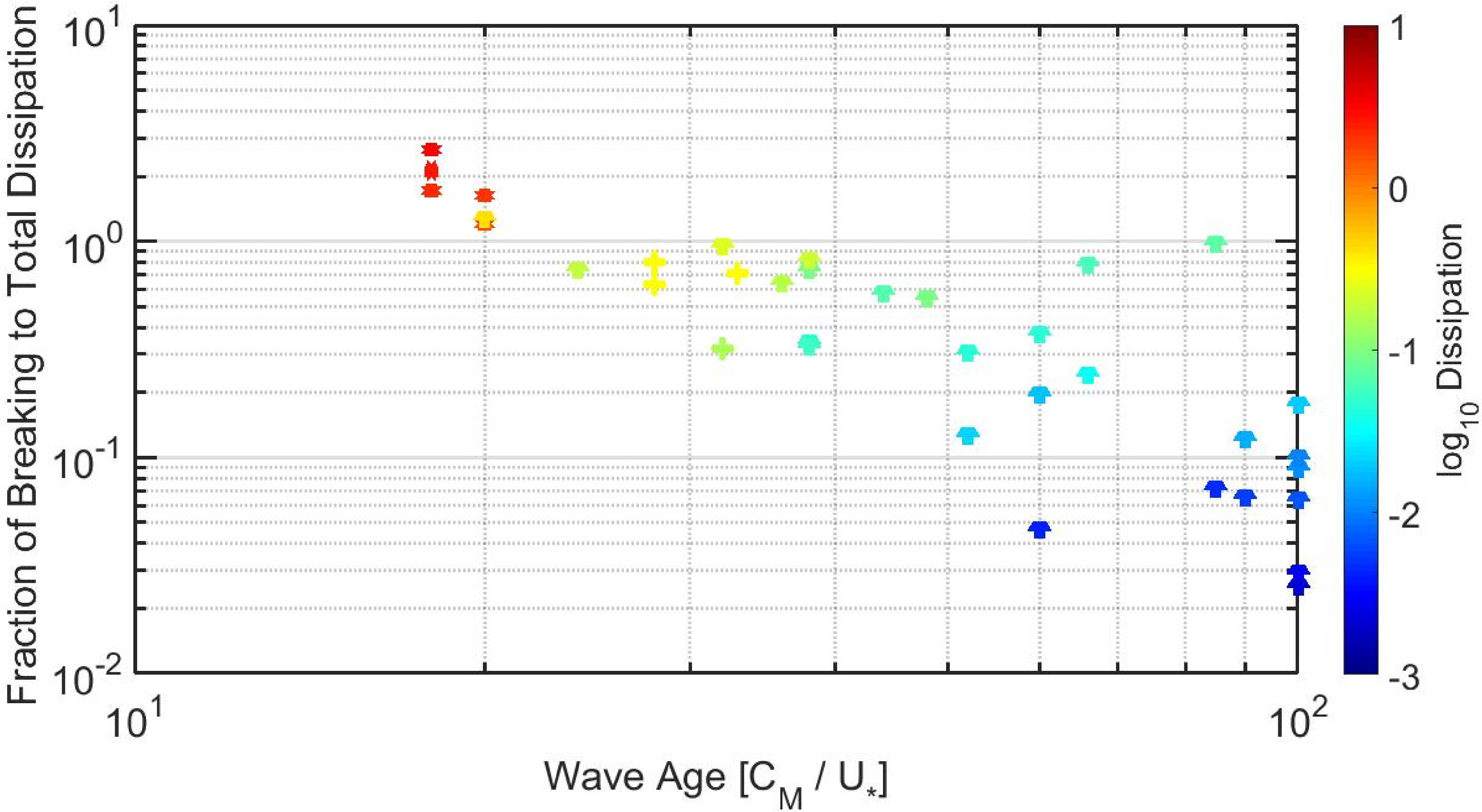
Breaking Dissipation vs Total Dissipation (Sutherland & Melville 2015)



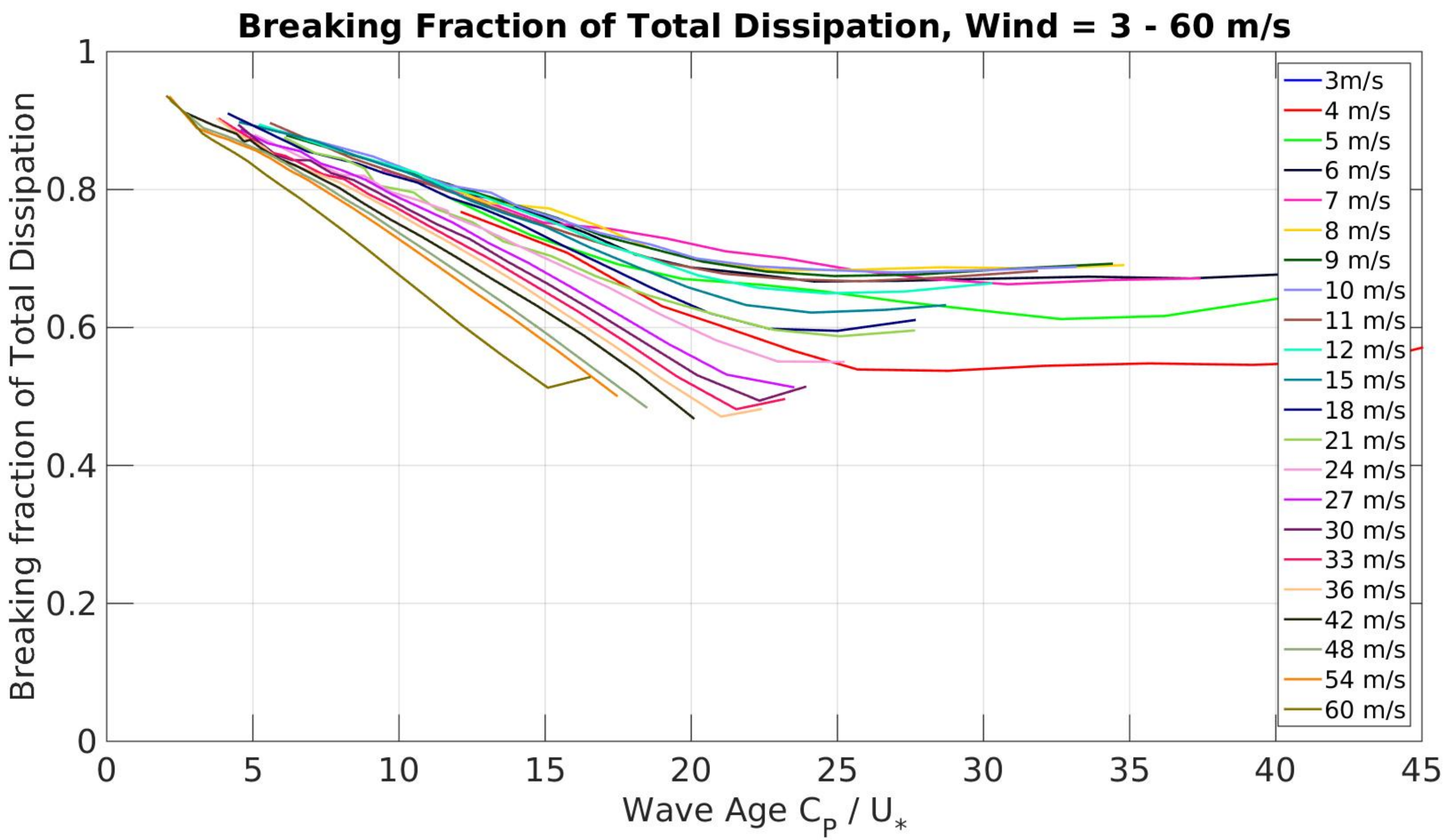
Breaking Dissipation fraction Total Dissipation (Suth. & Mel. 2015)



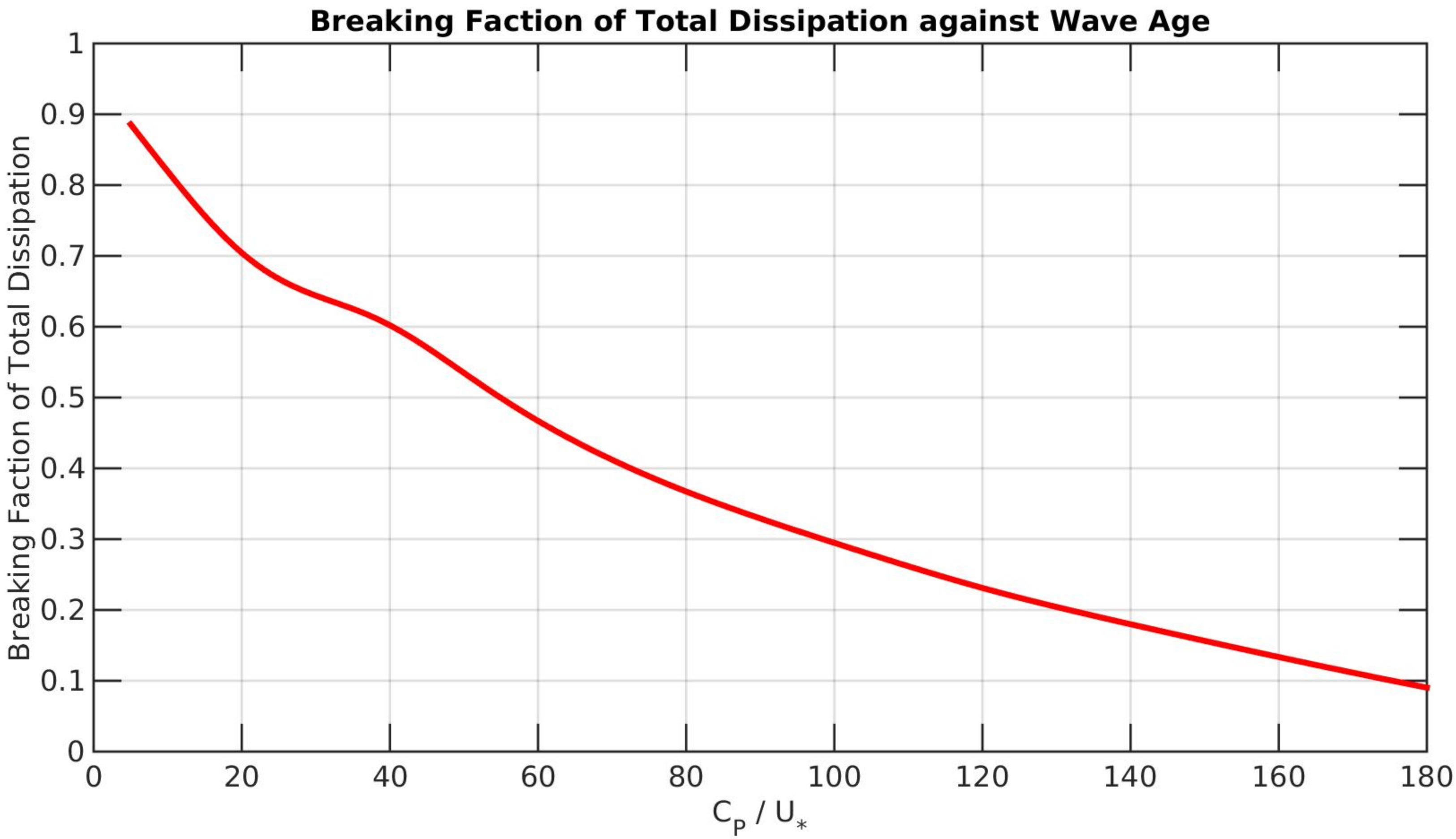
Breaking Dissipation fraction Total Dissipation (Suth. & Mel. 2015)



Breaking Dissipation as a fraction of Total Dissipation 3 to 60 m/s

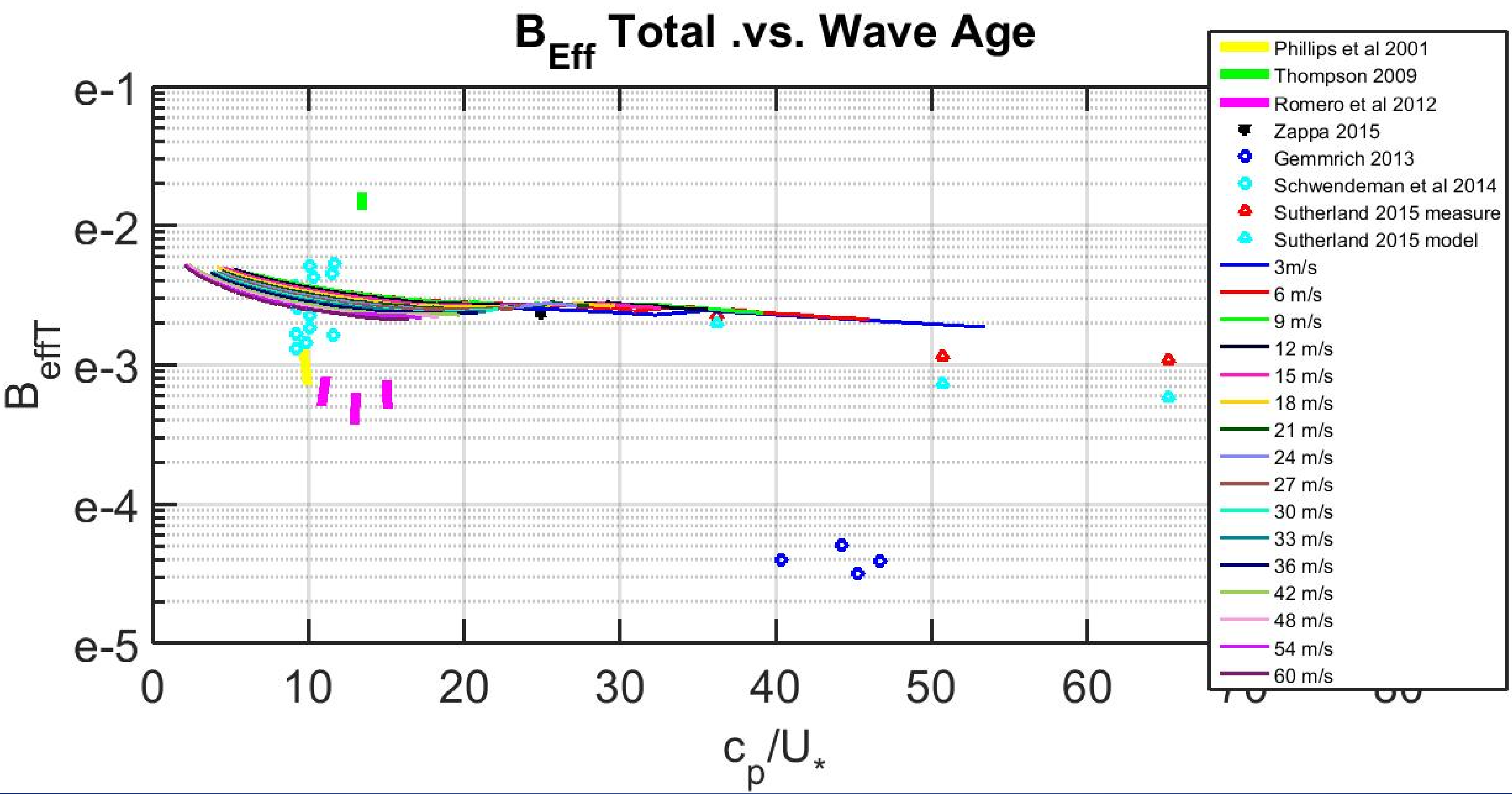


Breaking Dissipation as a fraction of Total Dissipation for Decaying Seas



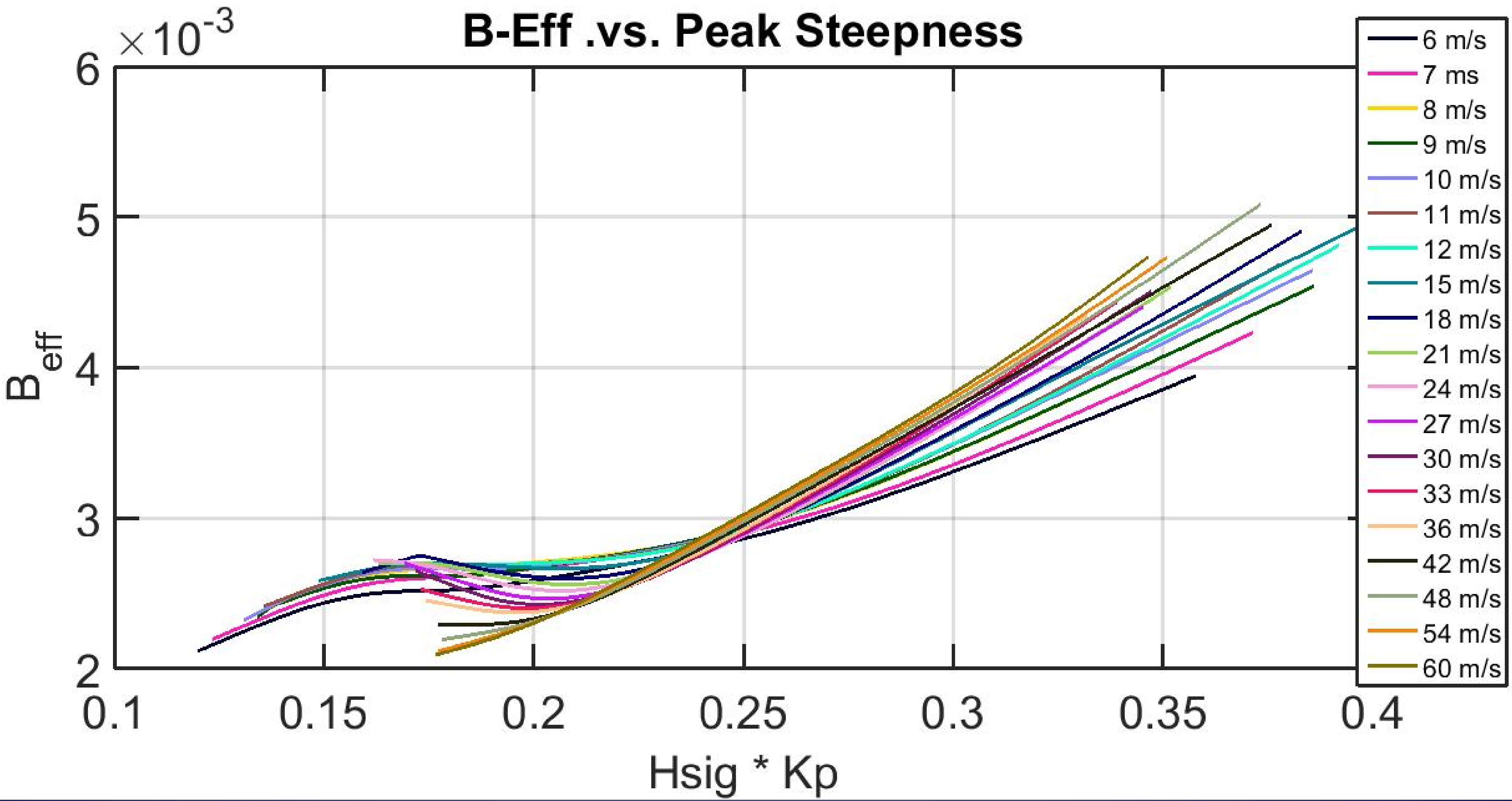
Effective Breaking Strength versus Wave Age (Total Dissipation)

$$B_{eff} = \int S_{ds}(c)dc \,/ g^2 \int c^5 * \Lambda(c) dc$$



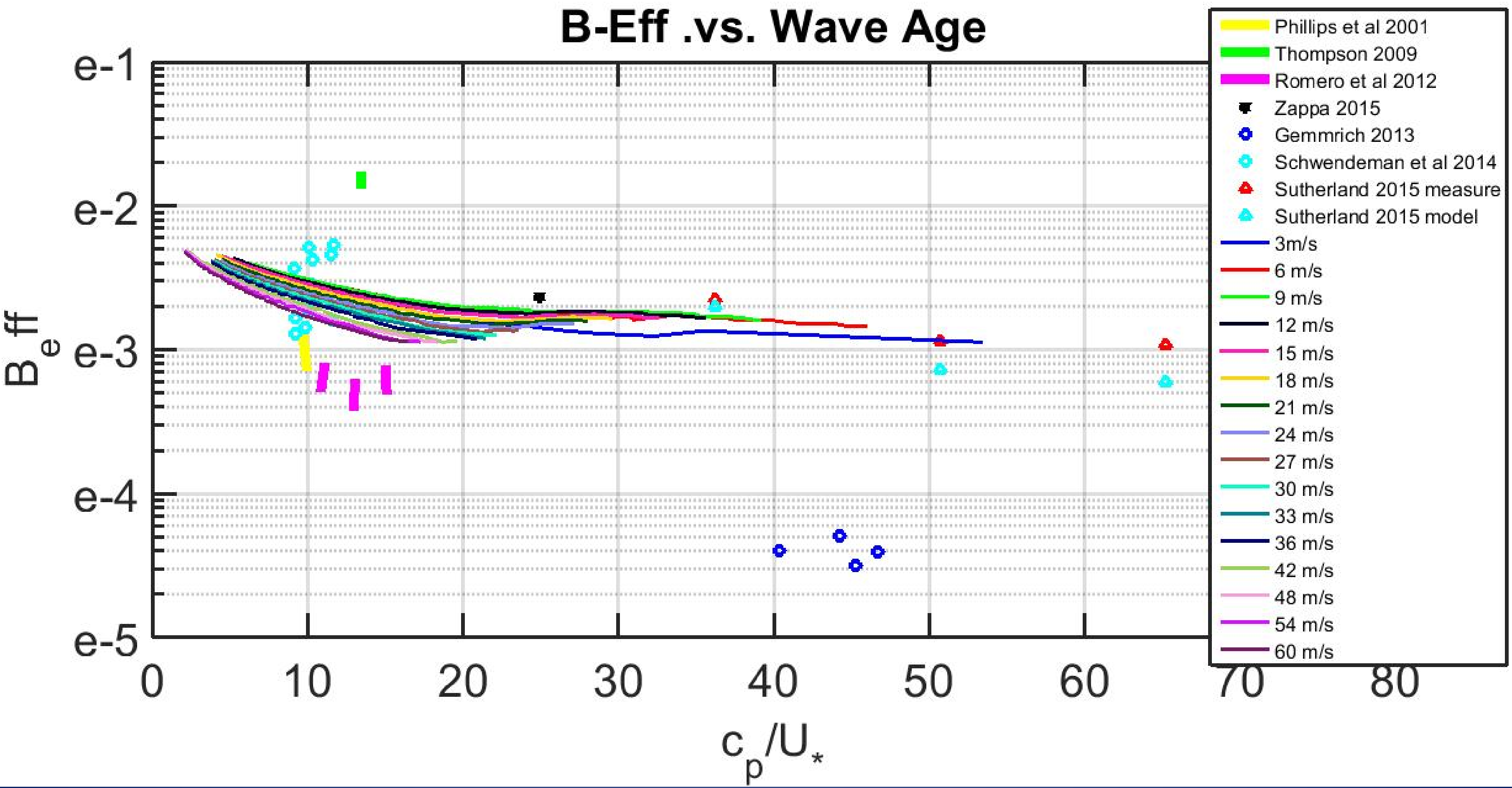
Effective Breaking Strength versus Peak Steepness (Total Dissipation)

$$B_{eff} = \int S_{ds}(c)dc \,/ g^2 \int c^5 * \Lambda(c) dc$$

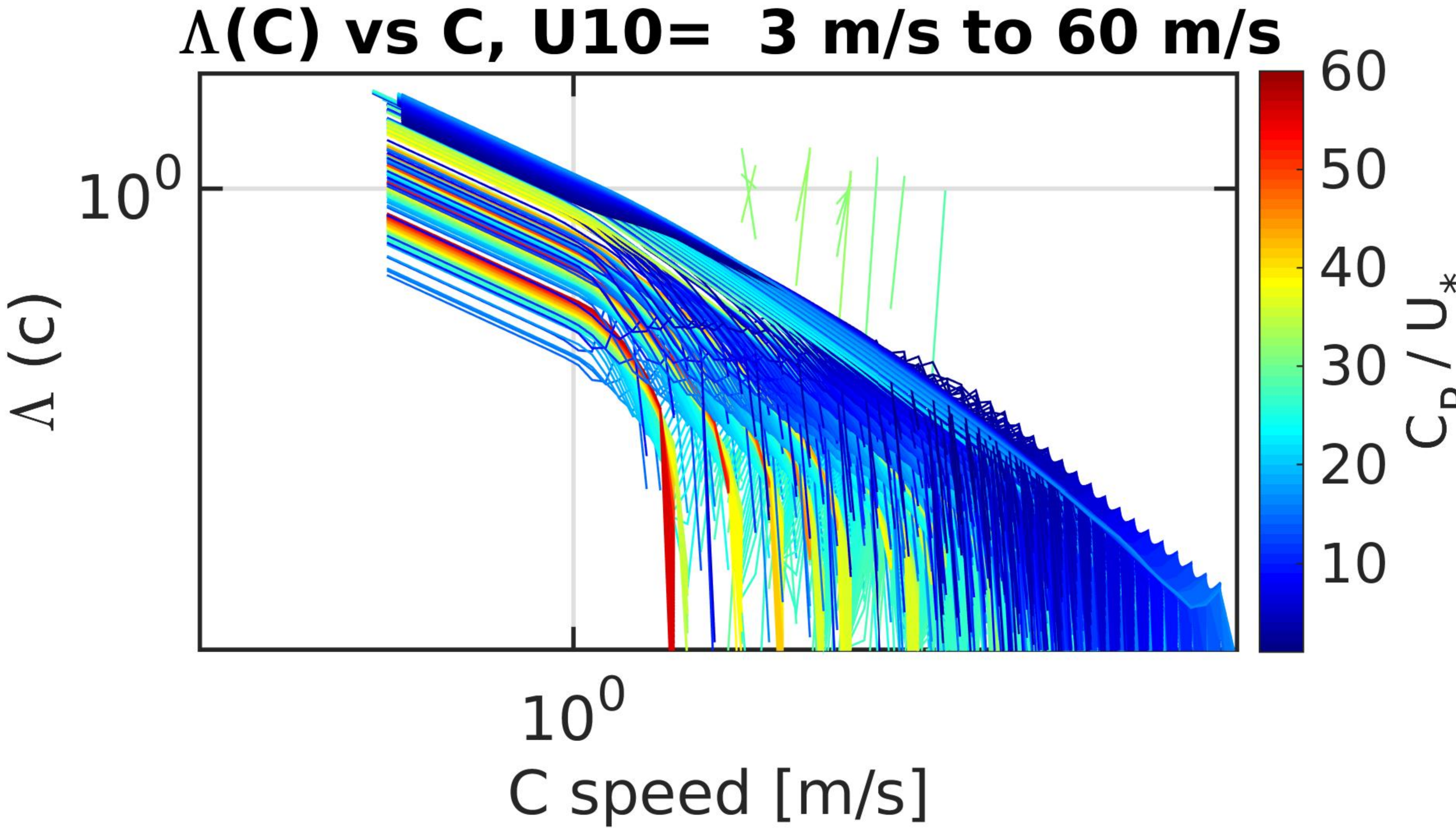


Effective Breaking Strength versus Wave Age (Breaking Dissipation)

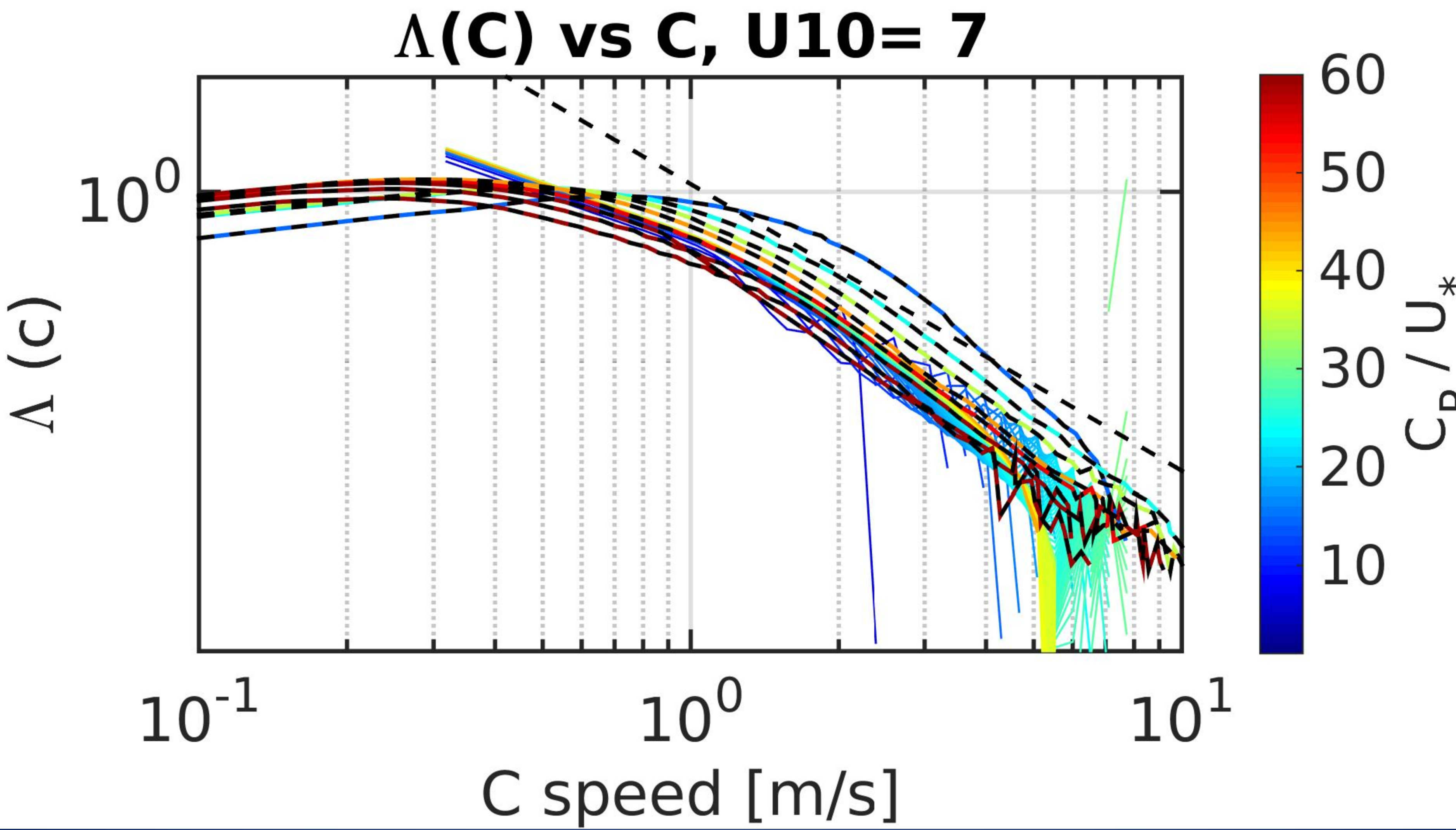
$$B_{\mathrm{eff}}^{\mathrm{loc}} = \int S_{\mathrm{ds}}^{loc}(\mathrm{c})\mathrm{dc} \,/\mathrm{g}^2 \int \mathrm{c}^5 * \Lambda(c)dc$$



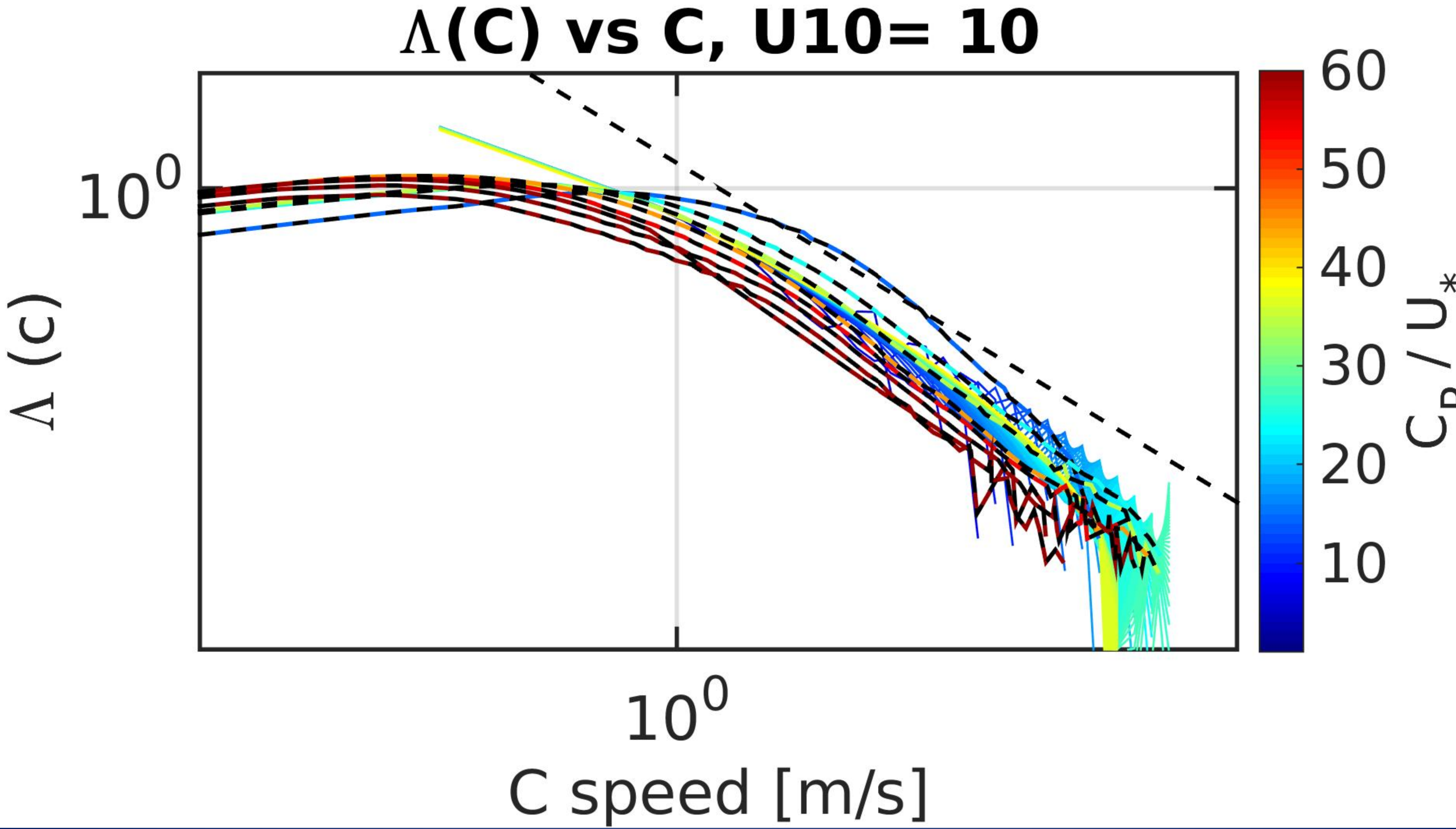
Model Forecast: Lambda(c) .vs. Wave Age.
For multiple wind speeds 3 to 60 ms.



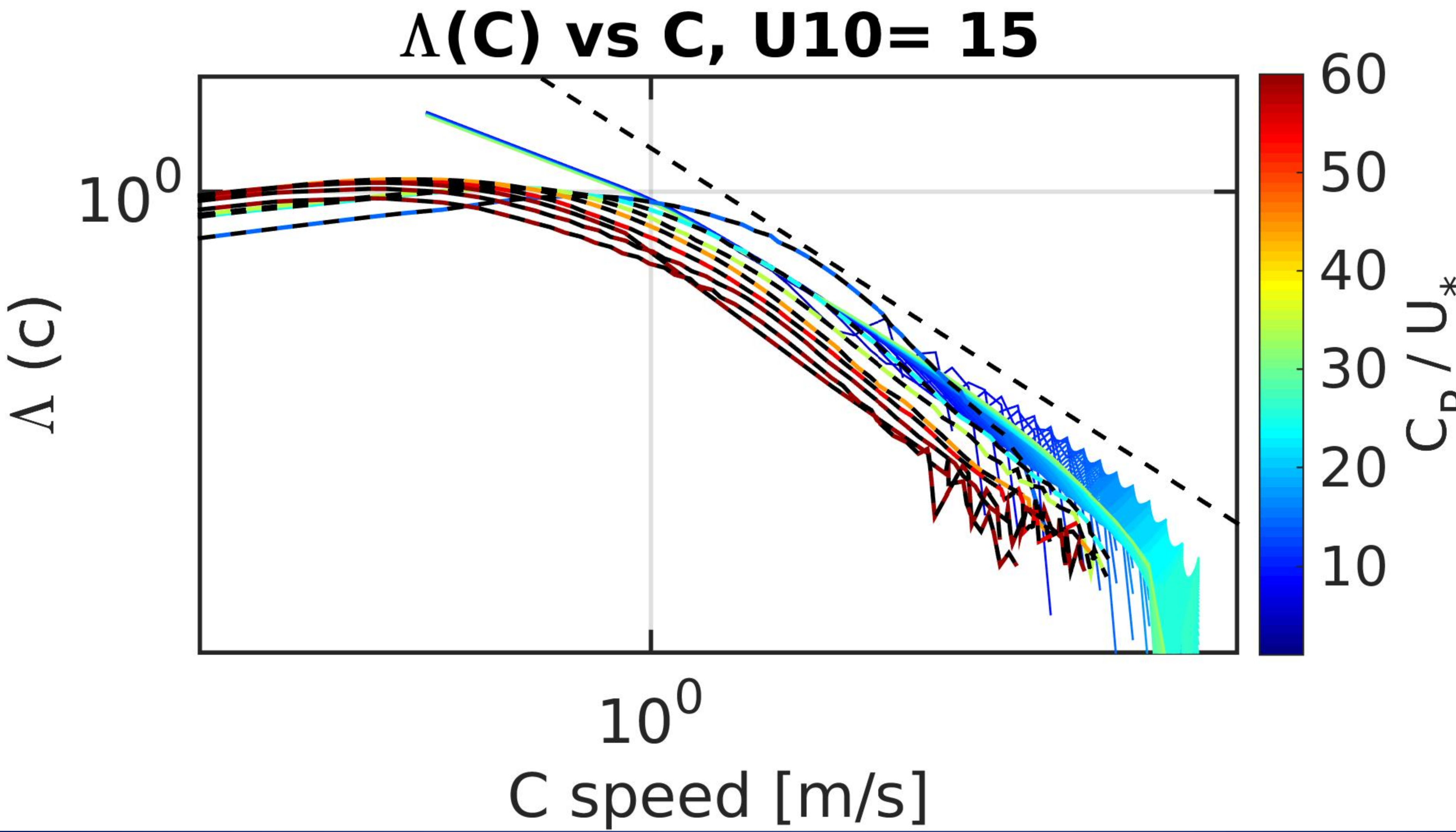
Model Forecast: Lambda(c) .vs. Wave Age. For U10= 6 m/s. Dashed lines from observed Λ(c) by Sutherland & Melville 2015



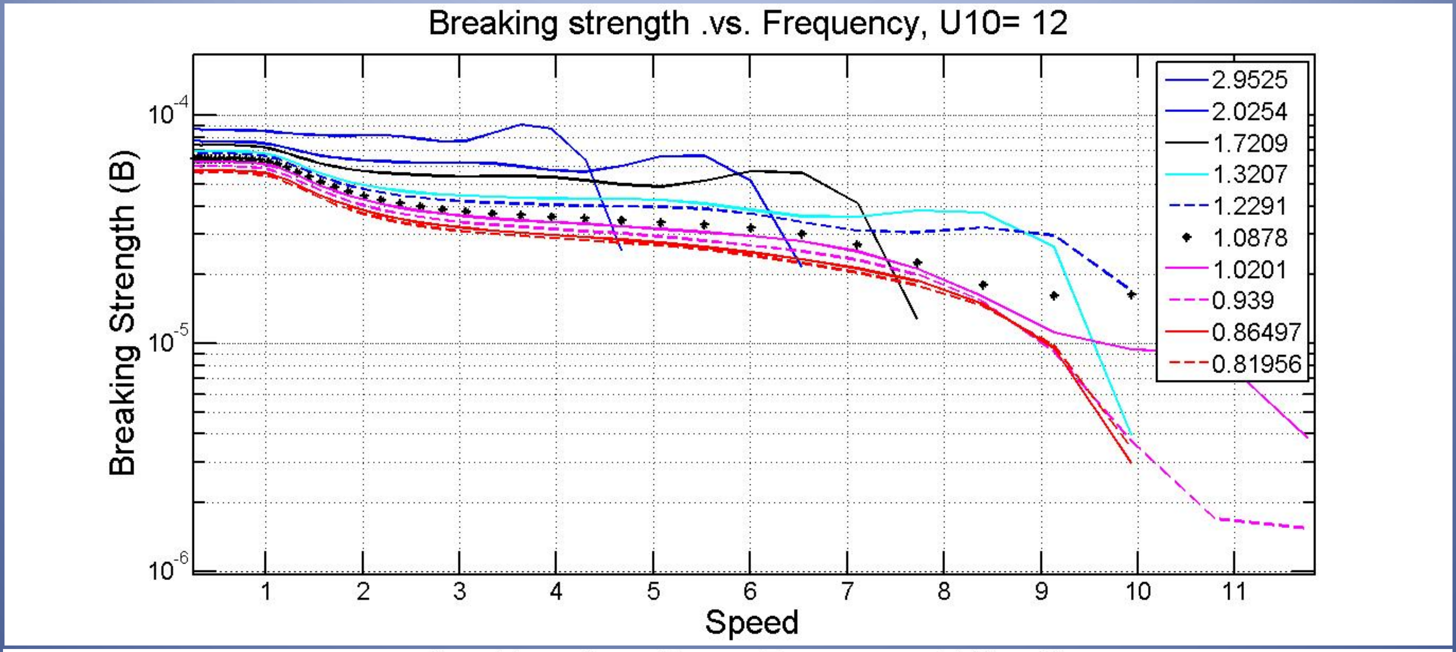
Model Forecast: Lambda(c) .vs. Wave Age.
For U10= 10 m/s. Dashed lines from observed Λ(c) by Sutherland & Melville 2015

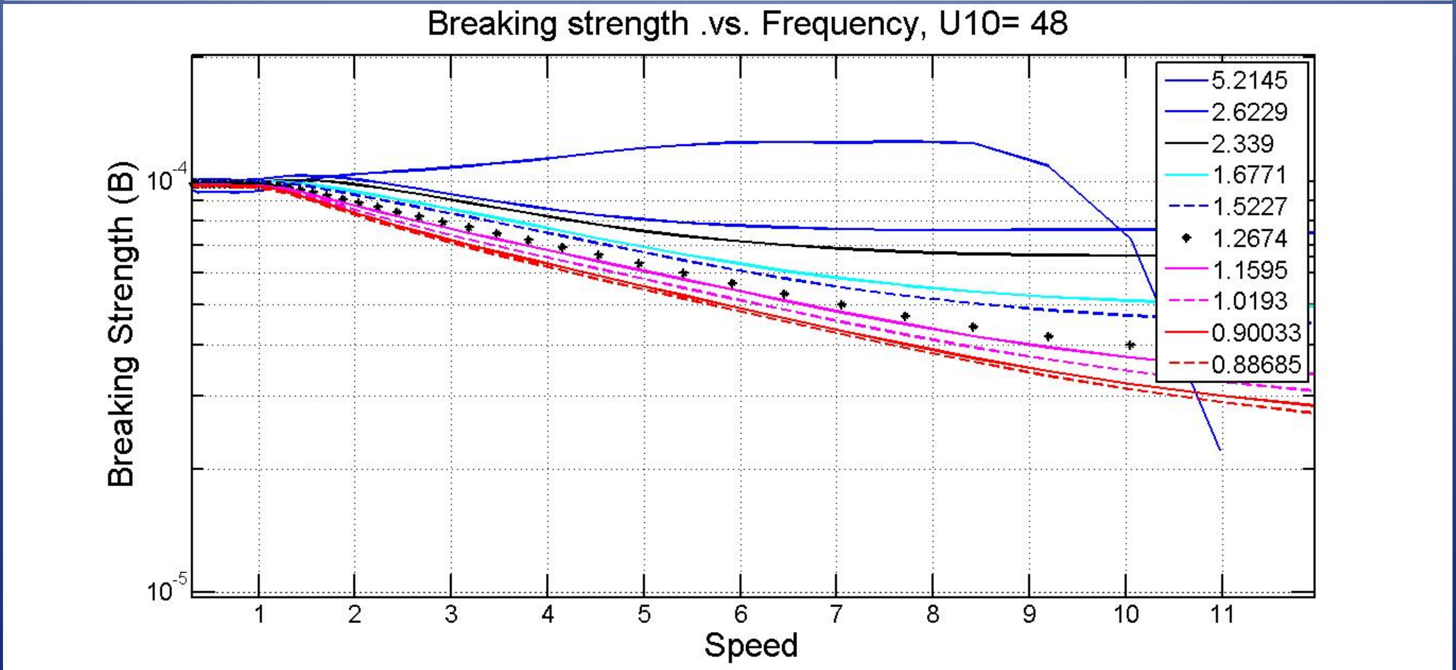


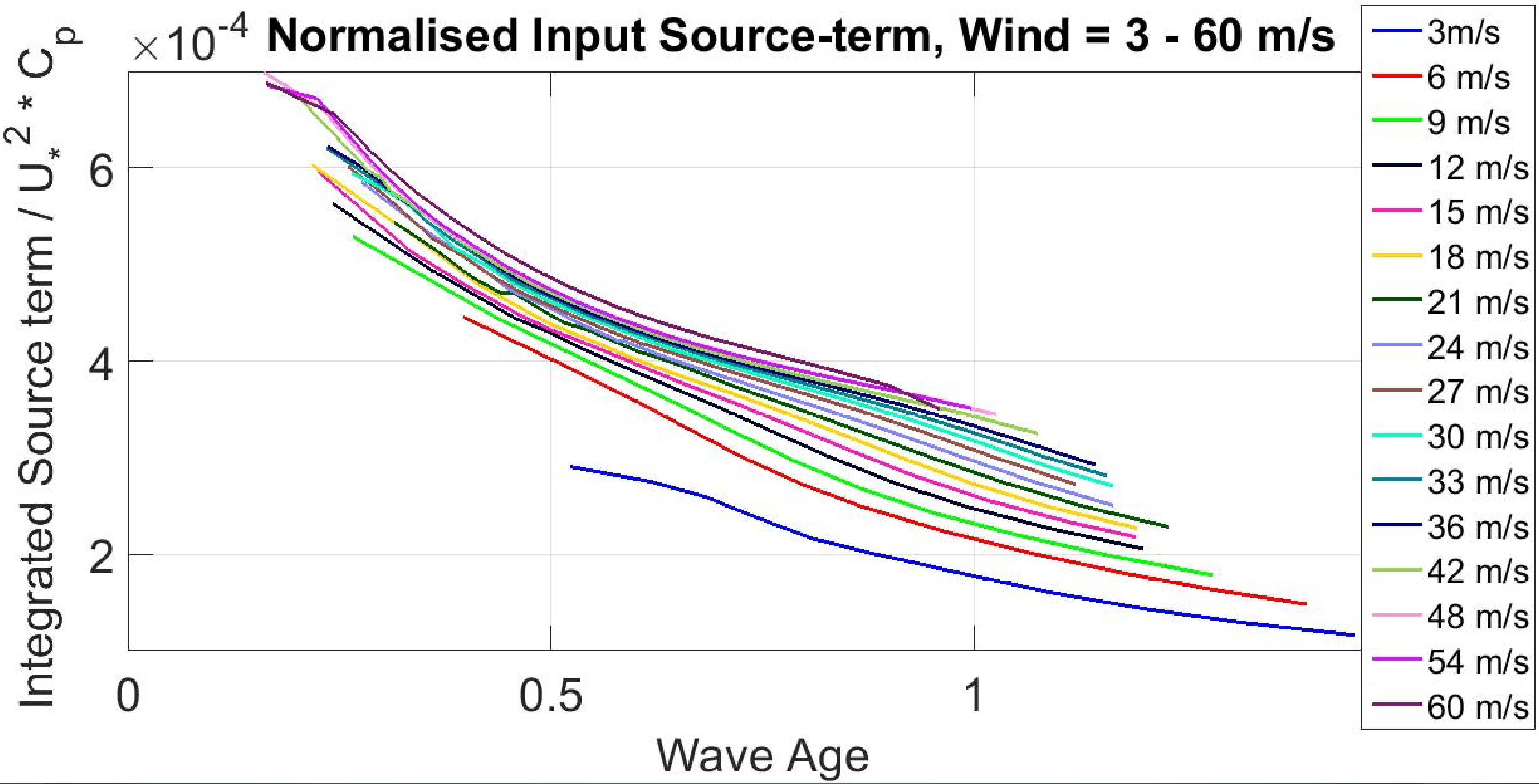
Model Forecast: Lambda(c) .vs. Wave Age. For U10= 15 m/s. Dashed lines from observed Λ(c) by Sutherland & Melville 2015

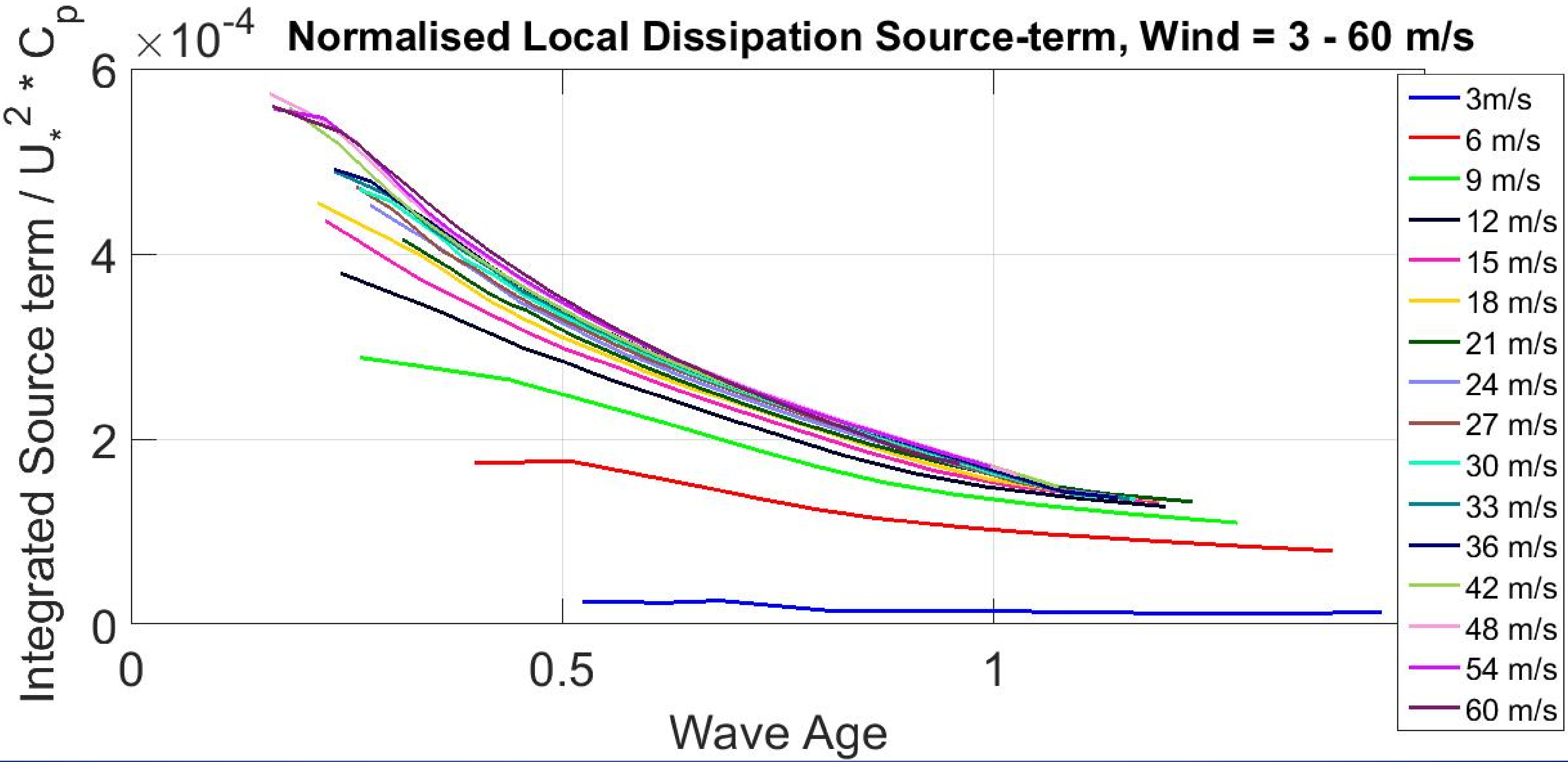


Model Forecast: Breaking Strength [b] .vs. Speed for 12m/s & 48 m/s.

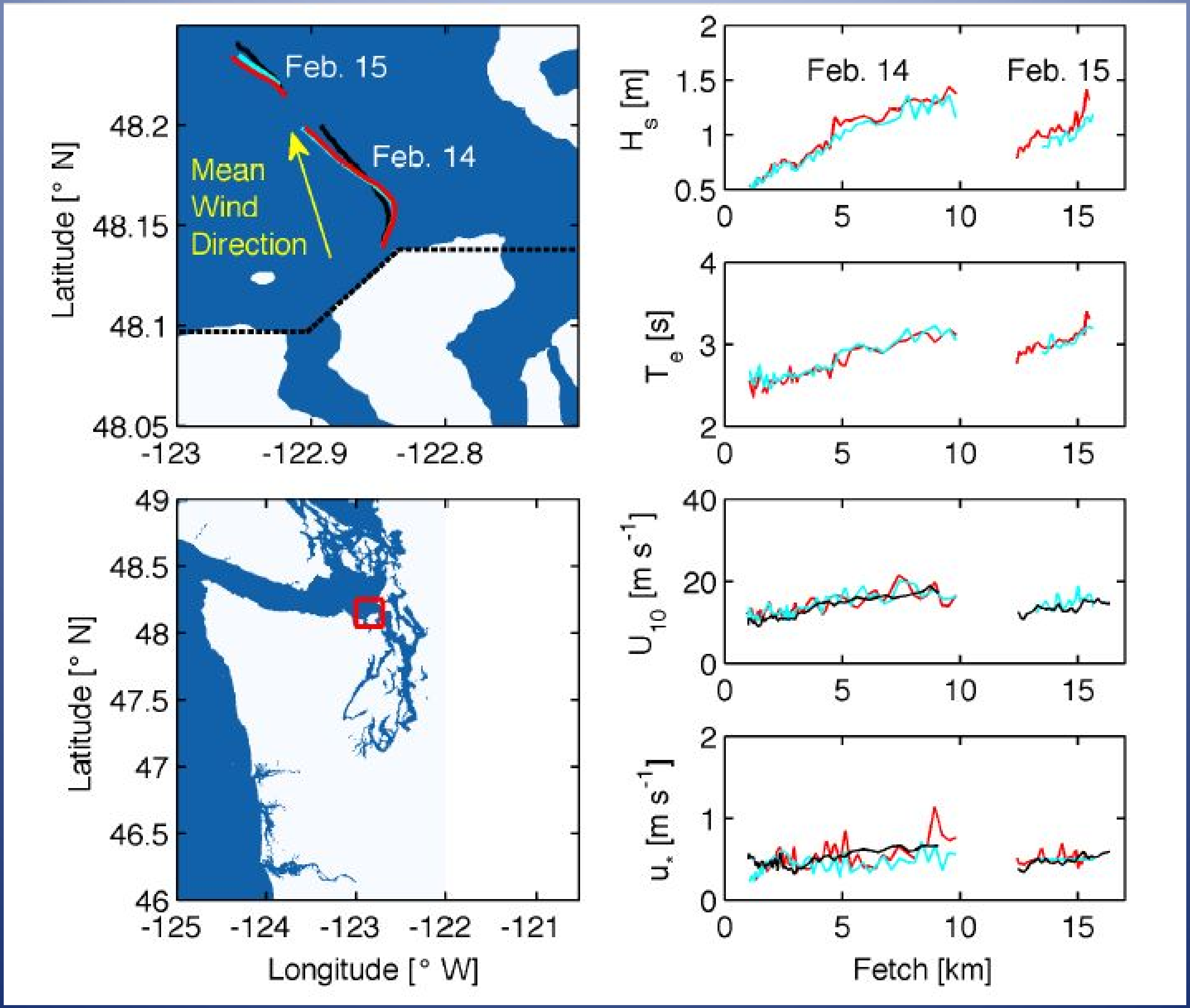


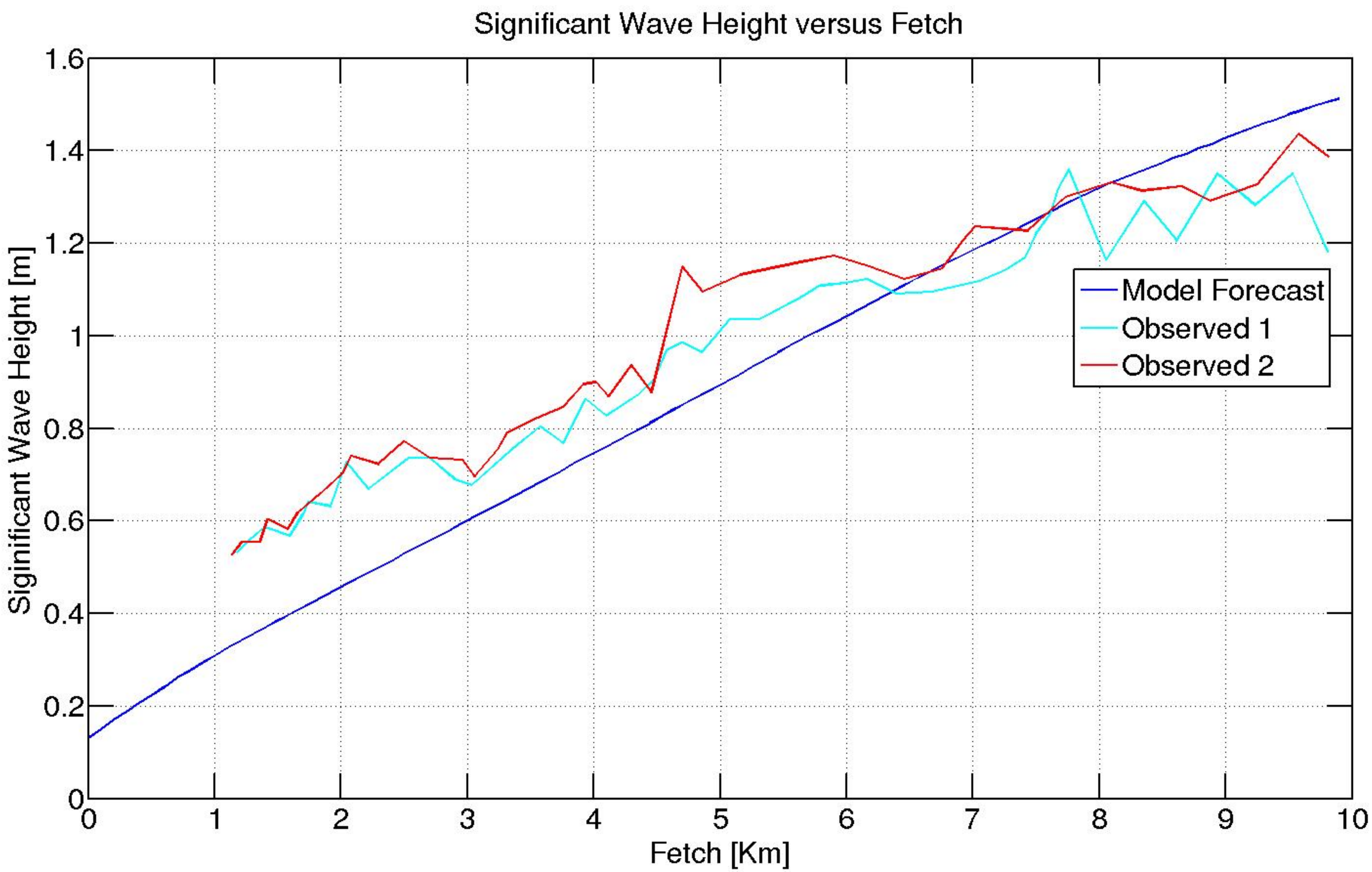


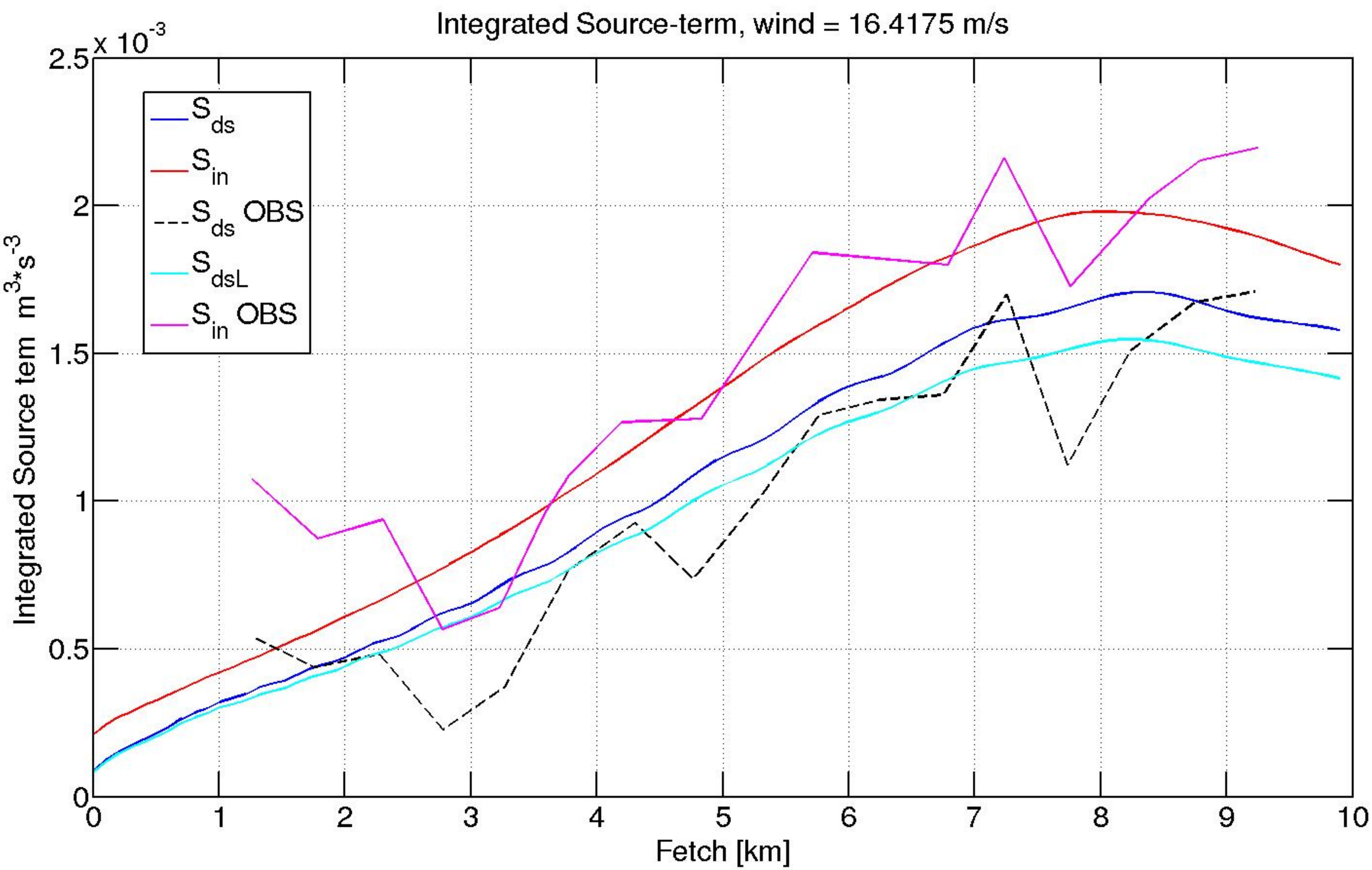

Normalised Input Source-term, Wind = 3 - 60 m/s
×10^-4
Integrated Source term / U*^2 * Cp
6
4
2
0
0.5
1
Wave Age
3m/s
6 m/s
9 m/s
12 m/s
15 m/s
18 m/s
21 m/s
24 m/s
27 m/s
30 m/s
33 m/s
36 m/s
42 m/s
48 m/s
54 m/s
60 m/s


Normalised Local Dissipation Source-term, Wind = 3 - 60 m/s
Integrated Source term / U*^2 * Cp
×10^-4
6
4
2
0
0
0.5
1
Wave Age
3m/s
6 m/s
9 m/s
12 m/s
15 m/s
18 m/s
21 m/s
24 m/s
27 m/s
30 m/s
33 m/s
36 m/s
42 m/s
48 m/s
54 m/s
60 m/s

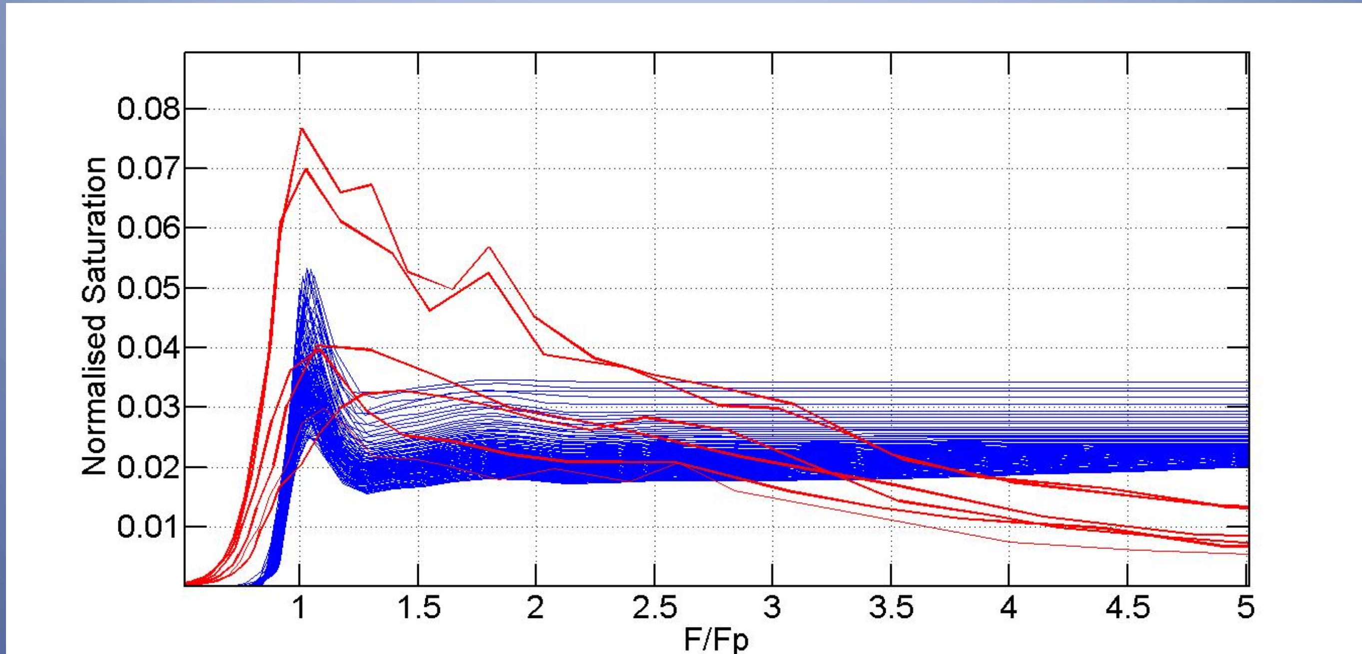
Strait of Juan de Fuca: ***Experiment Conditions***



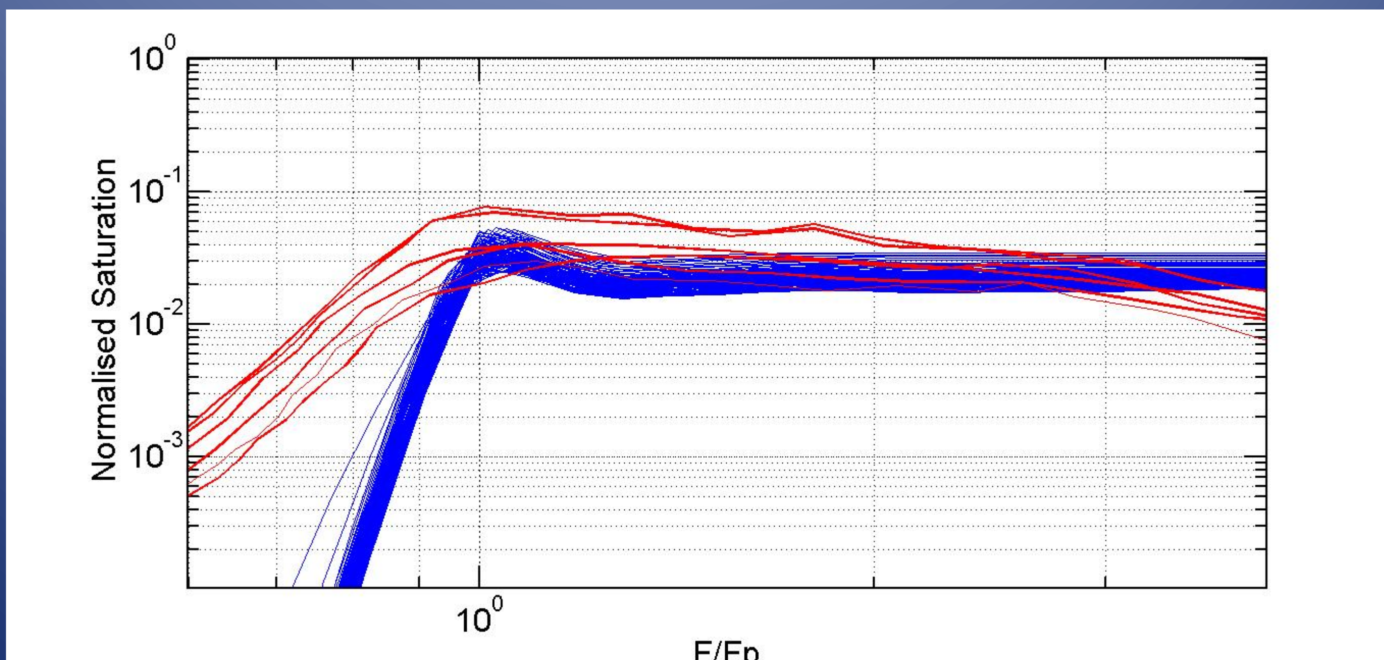

Significant Wave Height versus Fetch
Siginificant Wave Height [m]
Fetch [Km]
Model Forecast
Observed 1
Observed 2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
1
2
3
4
5
6
7
8
9
10



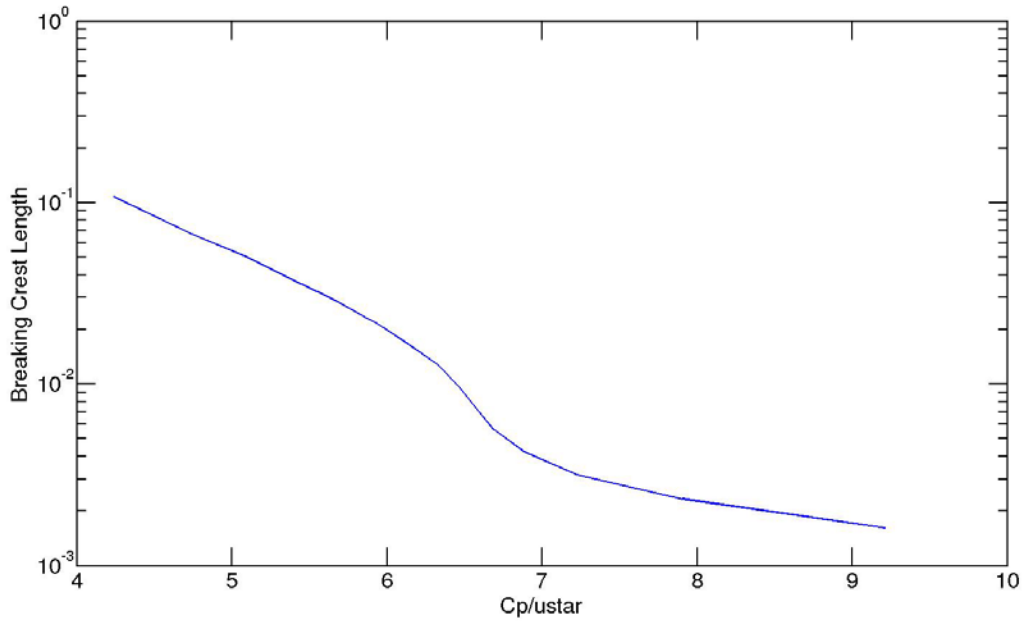
Integrated Source-term, wind = 16.4175 m/s
x 10^-3
S_{ds}
S_{in}
S_{ds} OBS
S_{dsL}
S_{in} OBS
Integrated Source tem $m^{3*}s^{-3}$
Fetch [km]
0
0.5
1
1.5
2
2.5
0
1
2
3
4
5
6
7
8
9
10



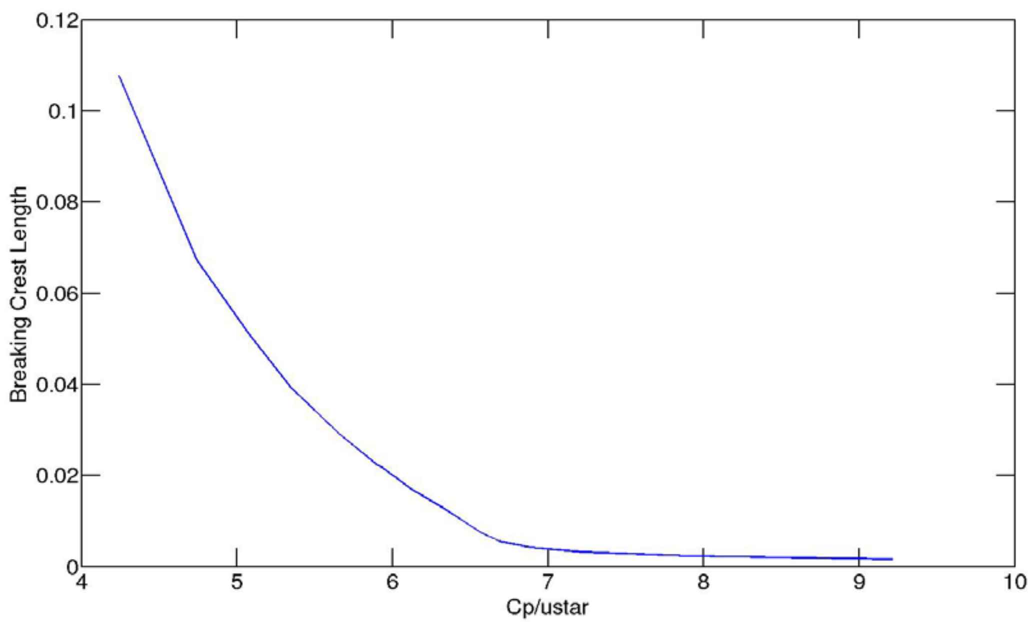
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Normalised Saturation
1
1.5
2
2.5
3
3.5
4
4.5
5
F/Fp



10^{0}
10^{-1}
10^{-2}
10^{-3}
Normalised Saturation
10^{0}
F/Fp



10^0
10^{-1}
10^{-2}
10^{-3}
Breaking Crest Length
4
5
6
7
8
9
10
Cp/ustar



0.12
0.1
0.08
0.06
0.04
0.02
0
Breaking Crest Length
4
5
6
7
8
9
10
Cp/ustar

Concluding Remarks

- our framework provides predictions of wave breaking properties (breaking probability, breaking crest length spectral density per unit area and breaking strength) using standard spectral wave models.

- it provides accurate predictions for the limited breaking data available in developing and mature wind seas

- further validation against data will be made as suitable new data sets become available.

- it has been added to existing spectral wave forecasting models. Upgrading the form of the DIA is desirable.