TSA
- the Two-Scale Approximation in WW3

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What are the objectives?

1. Replace DIA – Discrete Interaction Approximation – in WW3
2. Implement TSA into WW3 using an efficient accurate new formulation for quadruplet (nonlinear) wave-wave interactions
3. Tests including: waves - swell, turning winds, shallow water…
4. Tests for real North Atlantic storms

Wave generation and growth…

a balance equation …

$$\frac{\partial E(f,\theta)}{\partial t} = -\vec{c}_g \cdot \vec{\nabla} E(f,\theta) + \sum_k S_k(k,\theta)$$

where source terms are:

- $\vec{c}_g$ = group velocity
- S_{in} = wind input
- S_{ds} = wave dissipation
- S_{nl} = nonlinear transfer due to wave-wave interactions

$S_{nl} \Rightarrow$ full Boltzmann Integral - FBI

For internal transfer of wave action (or energy) in the spectrum at n_1 *(e.g. at* $\mathbf{k}_1$) *via* wave-wave interactions by $\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4$) - Hasselmann (1962), Zakharov (1966)

$$\frac{\partial n_1}{\partial t} \equiv S_{nl} = \int\int T(\mathbf{k}_1, \mathbf{k}_3)\, d\mathbf{k}_3$$

where

$$T(\mathbf{k}_1, \mathbf{k}_3) = 2\oint [n_1 n_3(n_4 - n_2) + n_2 n_4(n_3 - n_1)] C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)\theta(|\mathbf{k}_1 - \mathbf{k}_4| - |\mathbf{k}_1 - \mathbf{k}_3|)|\partial W/\partial n|^{-1}\, ds,$$

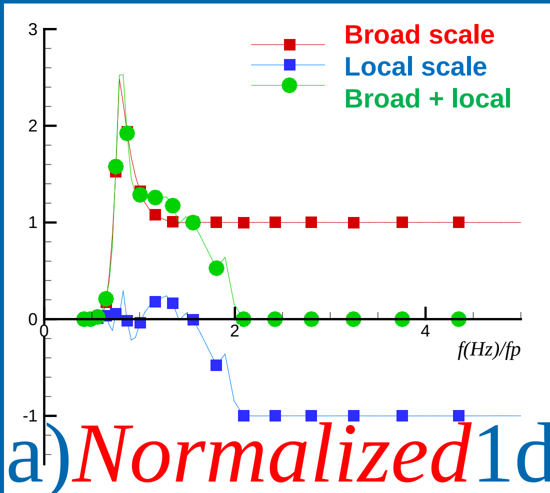
TSA – Two-Scale Approximation

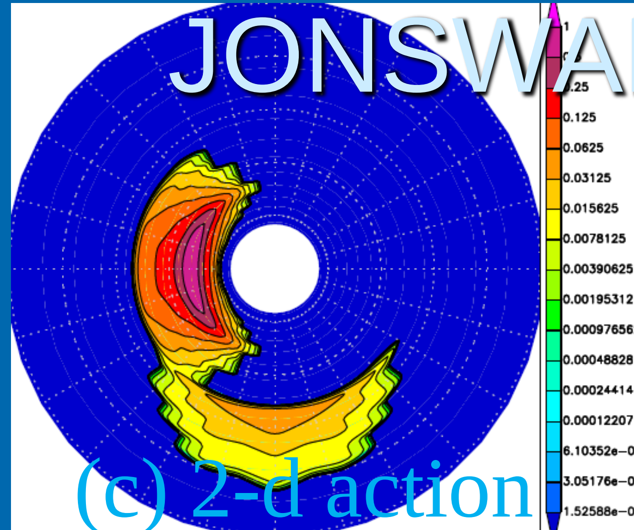
$$n_i = n_{i\,[\text{broad-scale}]} + n_{i\,[\text{local-scale}]}\ ;\ i = 1,2,3,4$$

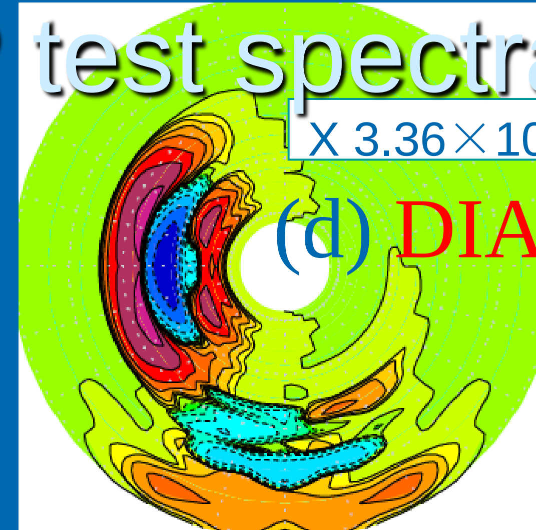
Neglect $n_{2\,[\text{local-scale}]}$ and $n_{4\,[\text{local-scale}]}$

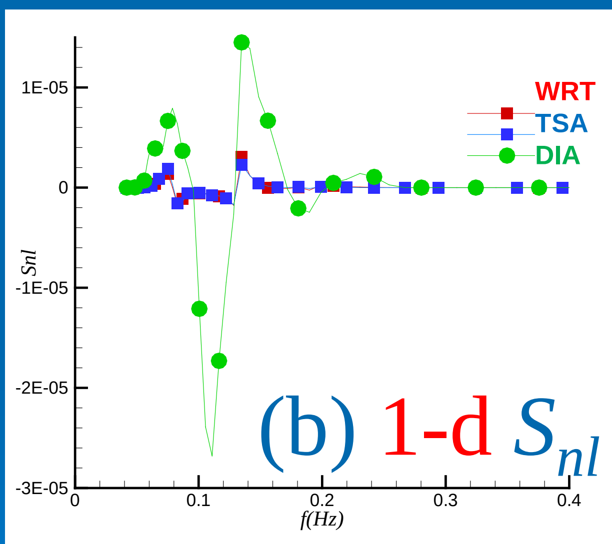
[Resio and Perrie 2008; Perrie & Resio 2009]

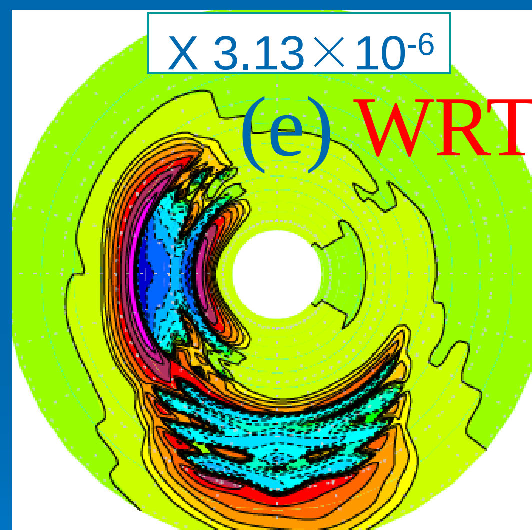
JONSWAP test spectra

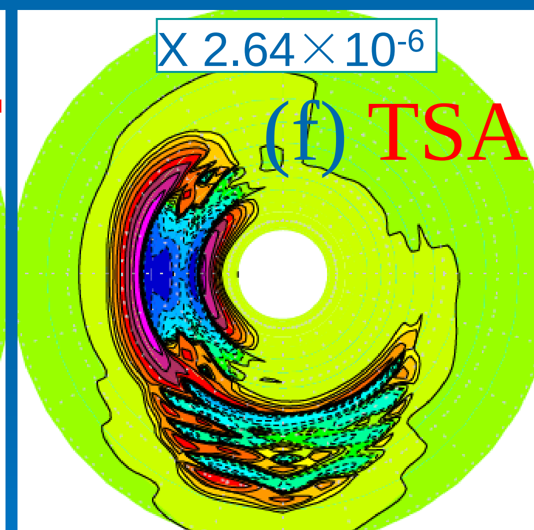


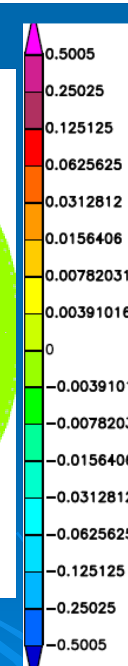












JONSWAP **sheared** spectrum with Hasselmann-Mitsuyasu directional:

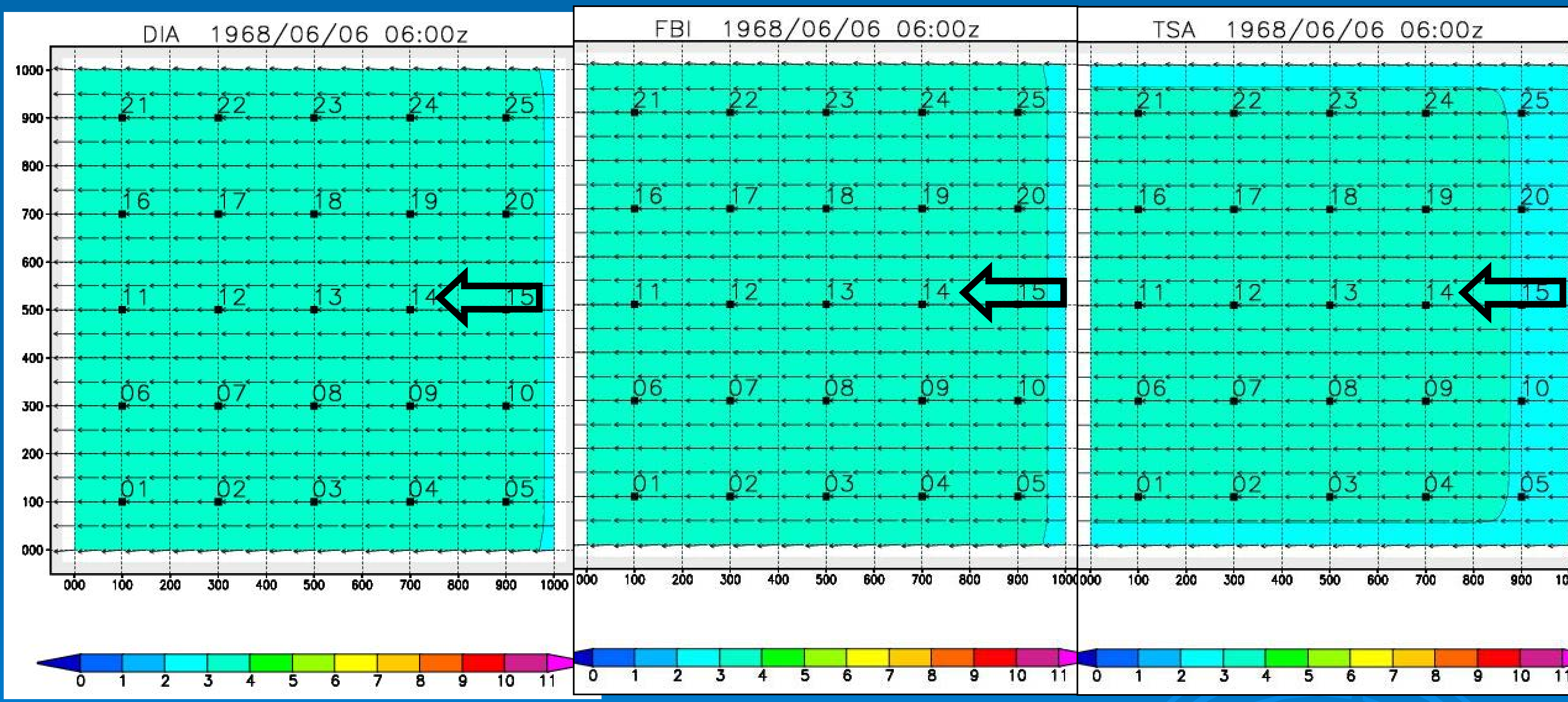
(a) broad- and local-scale terms normalized by f^{-4},

(b) 1-d comparison of DIA, WRT and TSA, (c) 2-d action density n_i,

(d) $S_{nl}(f,\theta)$ results from DIA (e) WRT (f) TSA. f_p=0.1, α=0.0081, σ_A=0.07, σ_B=0.09.

WW3 + ST1 source: fetch-limited growth

6 hr



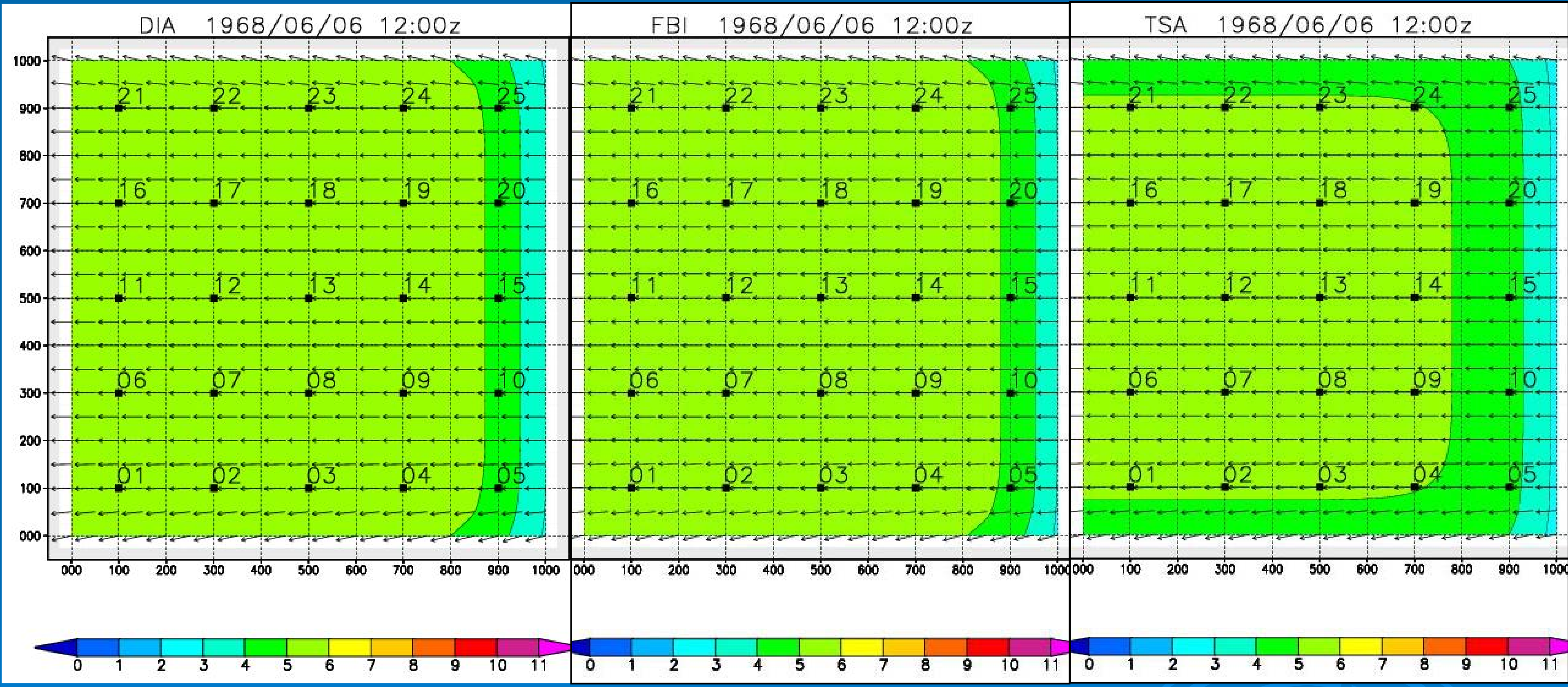
DIA

FBI

TSA

WW3 + ST1 source: fetch-limited growth

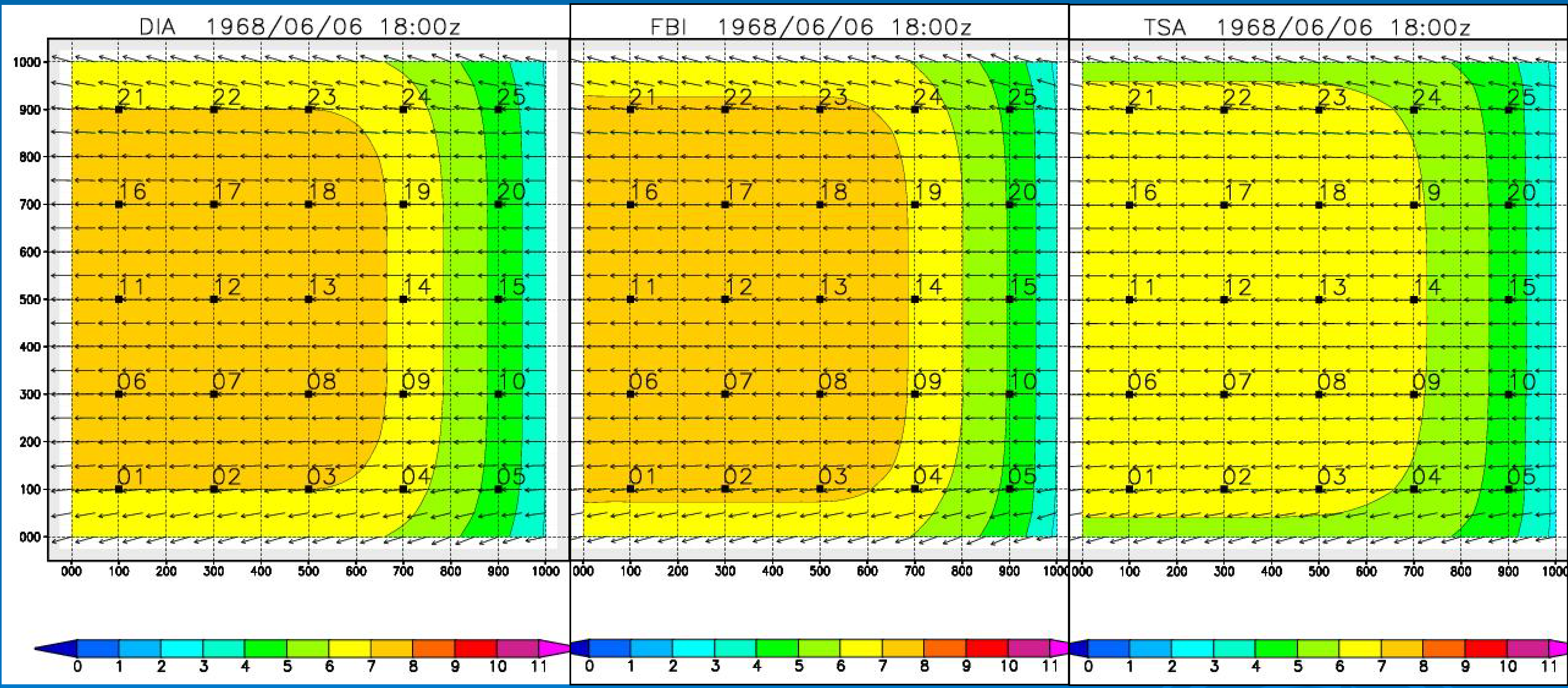
12 hr



DIA FBI TSA

WW3 + ST1 source: fetch-limited growth

18 hr



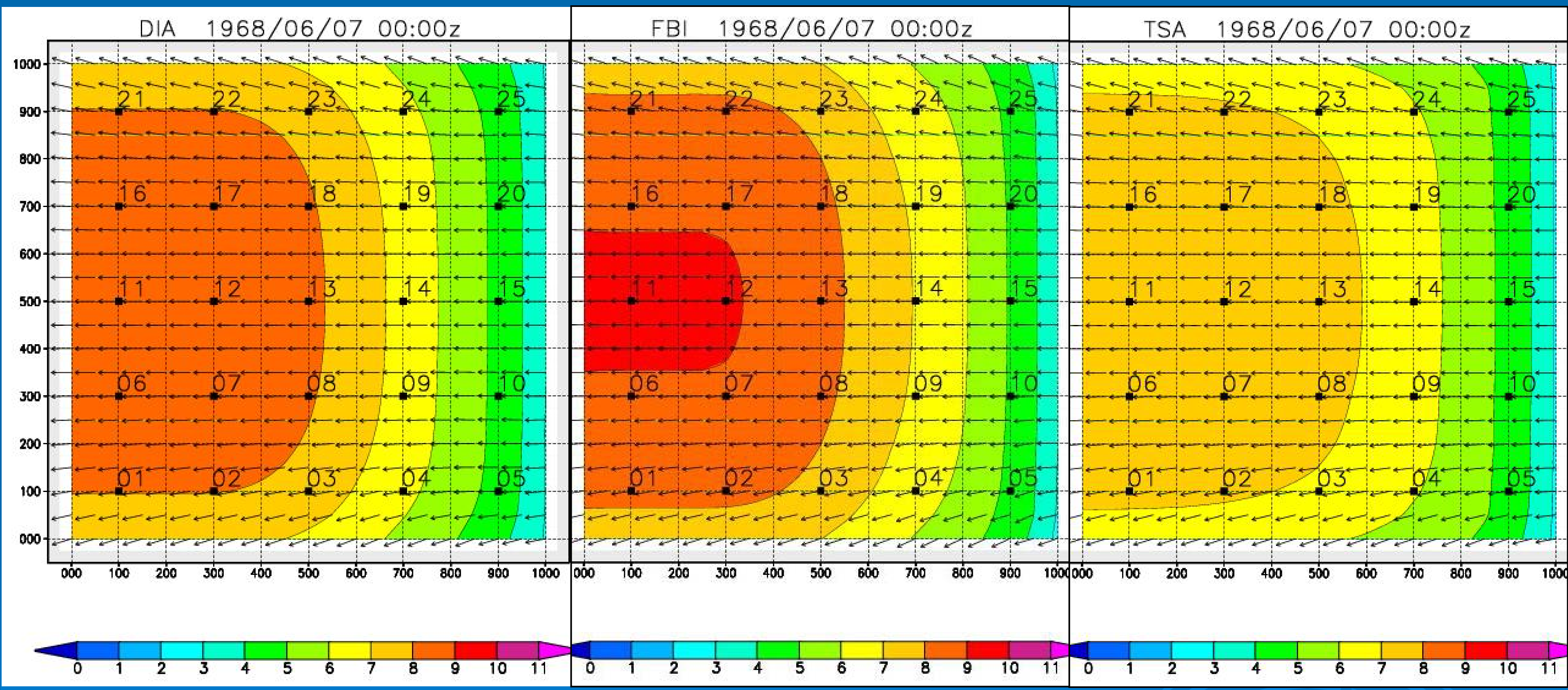
DIA

FBI

TSA

WW3 + ST1 source: fetch-limited growth

24 hr



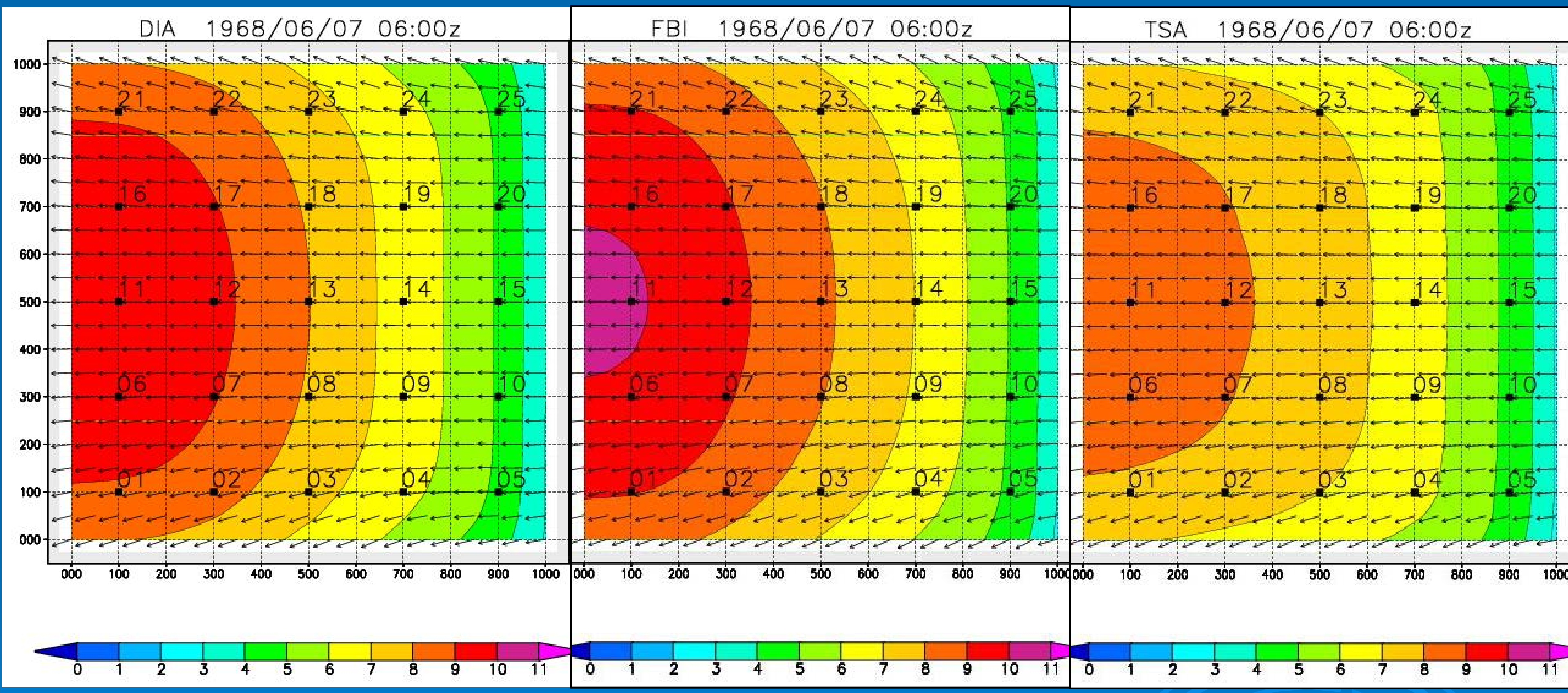
DIA

FBI

TSA

WW3 + ST1 source: fetch-limited growth

30 hr

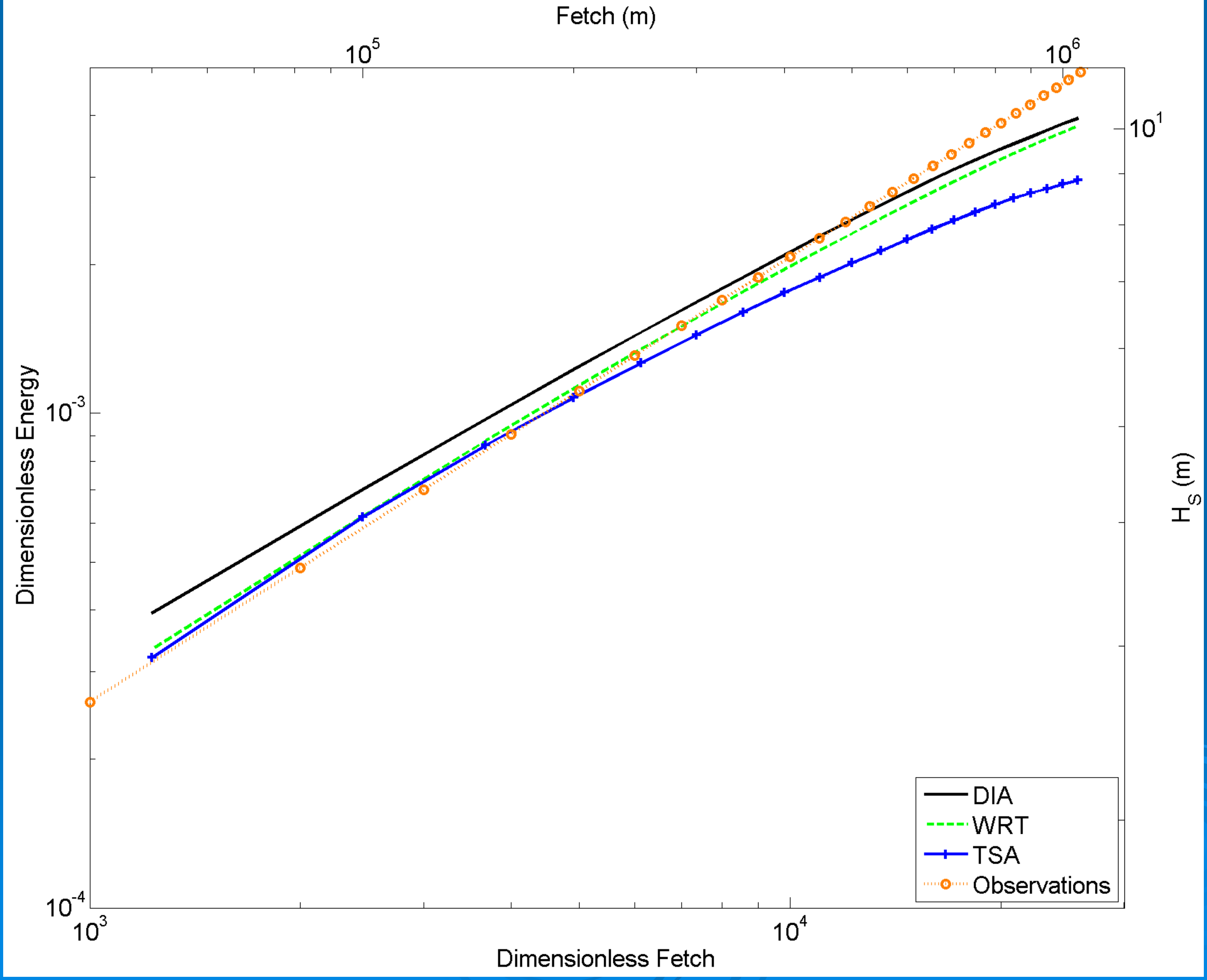


DIA

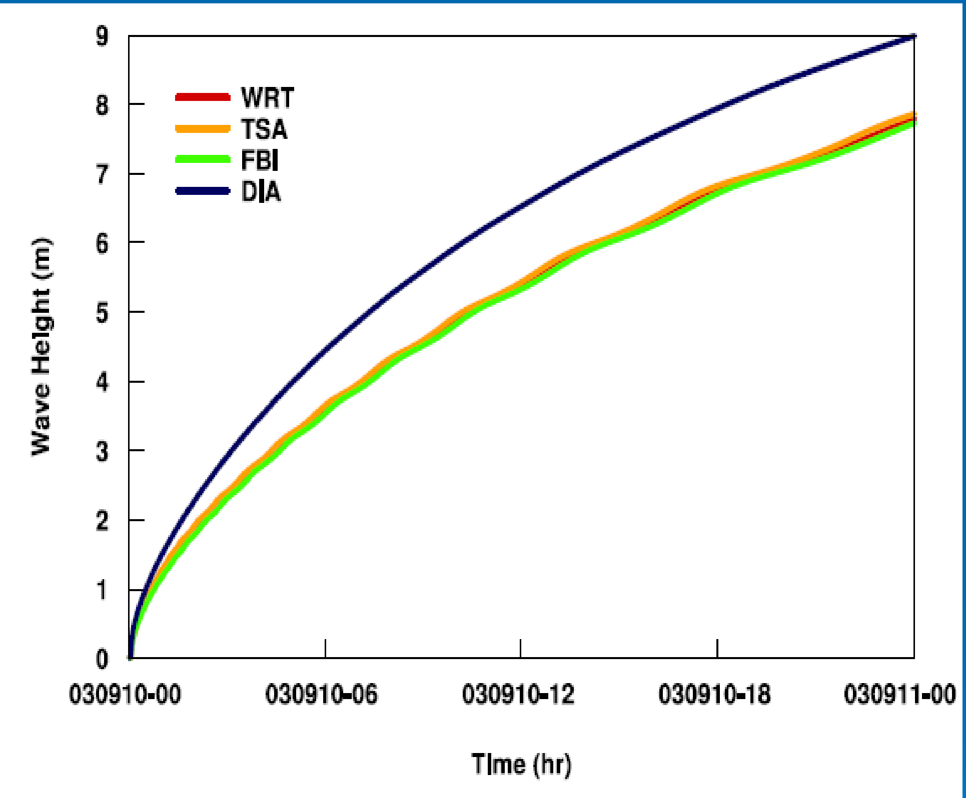
FBI

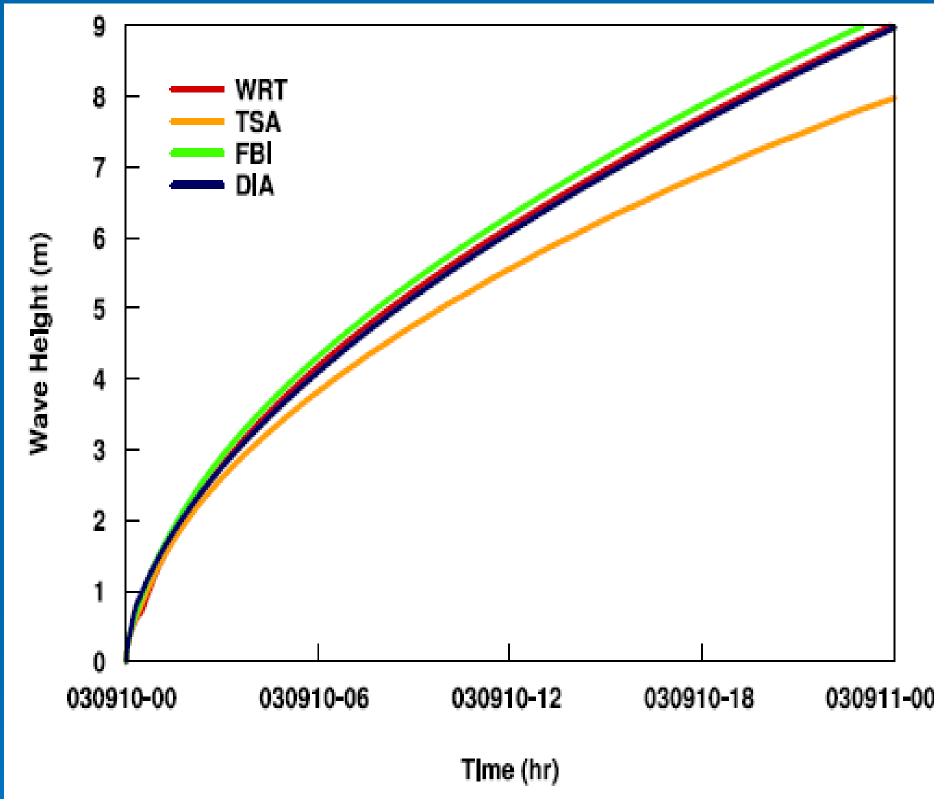
TSA

WW3 – with ‘old’ ST1: fetch-limited growth



1-point time integration

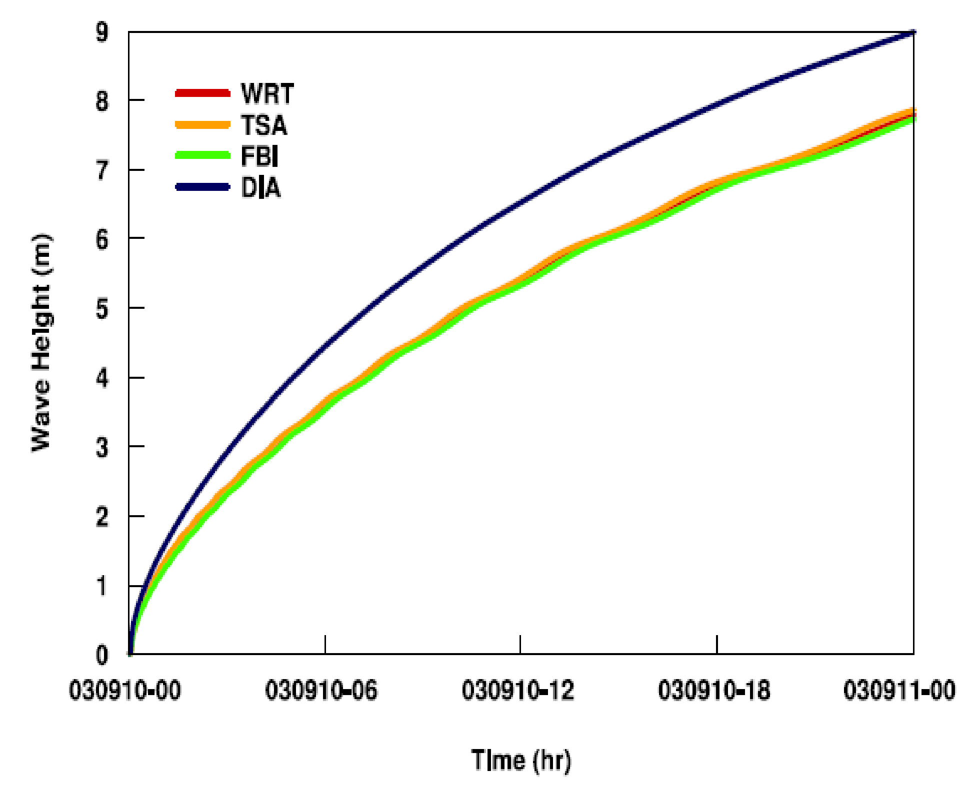


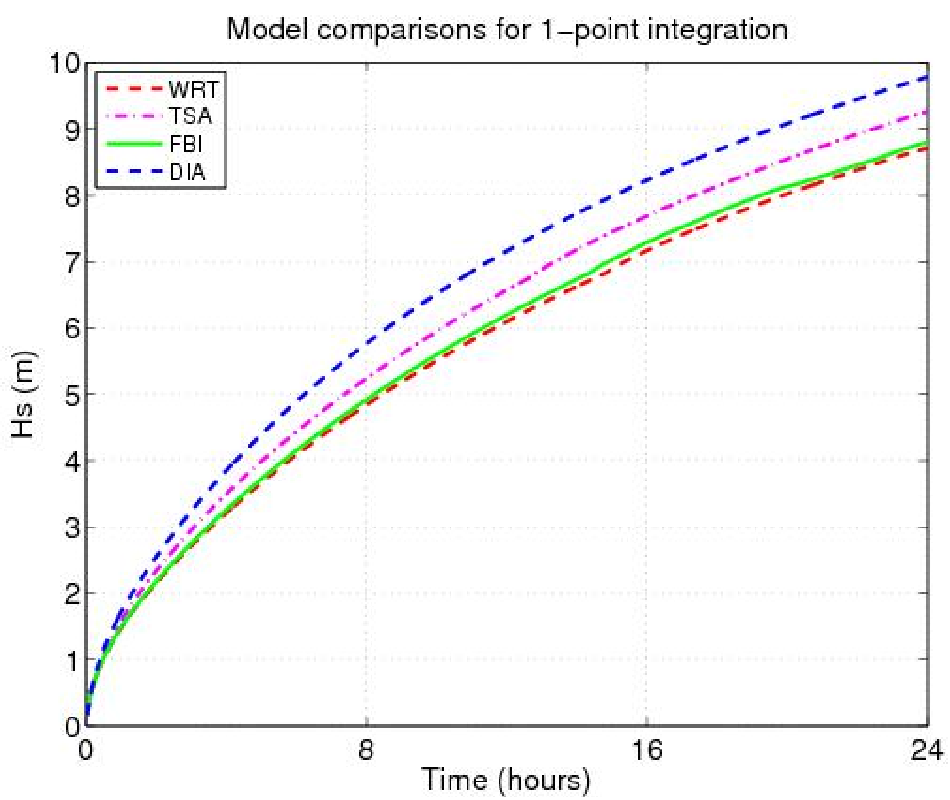


WWM (Roland et al., 2012) w. early TSA version + **ST4** From Ardhuin et al. (2010)

WW3 (Tolman, 2009) with **ST2** From Tolman + Chalikov (1996)

1-point time integration

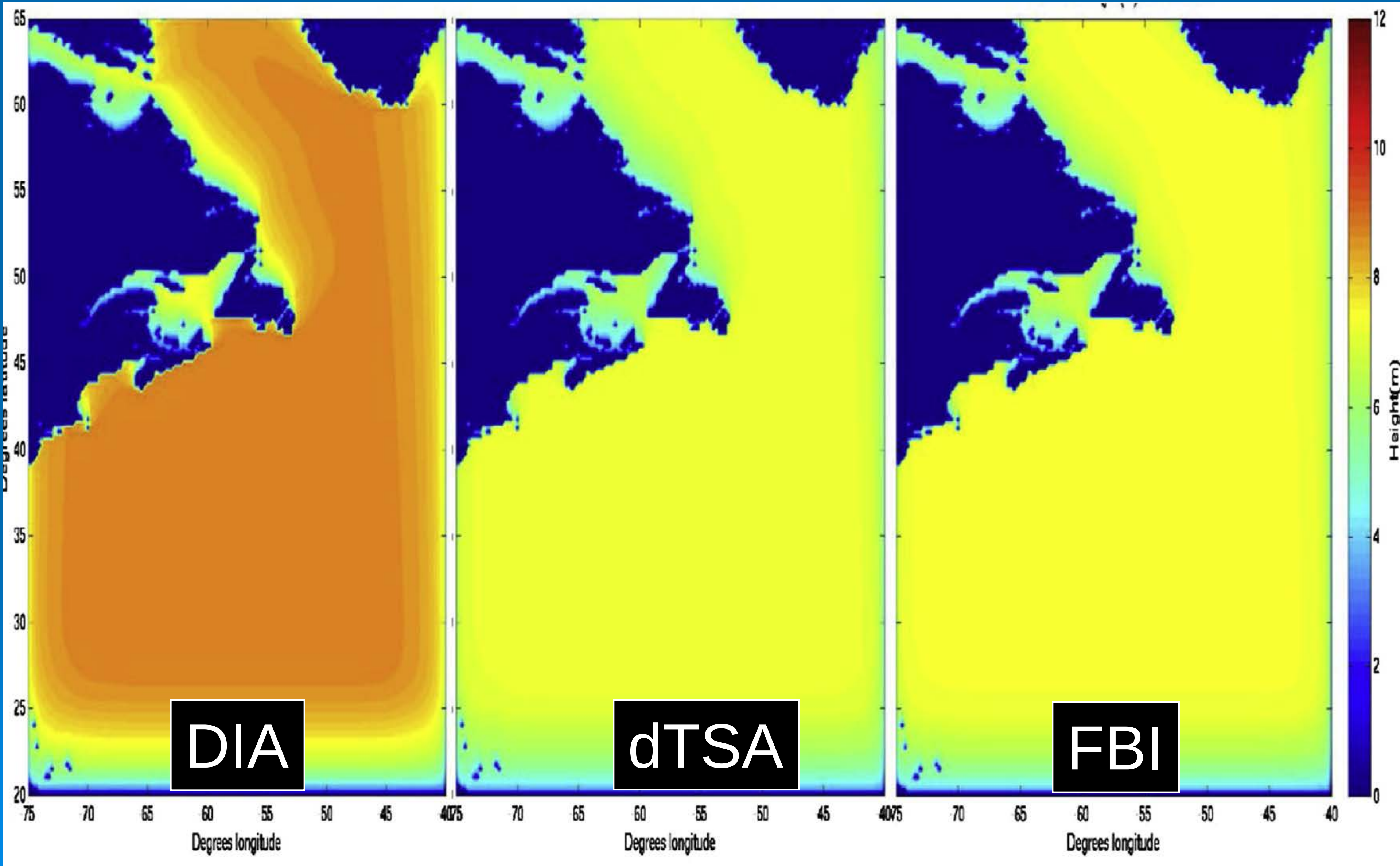




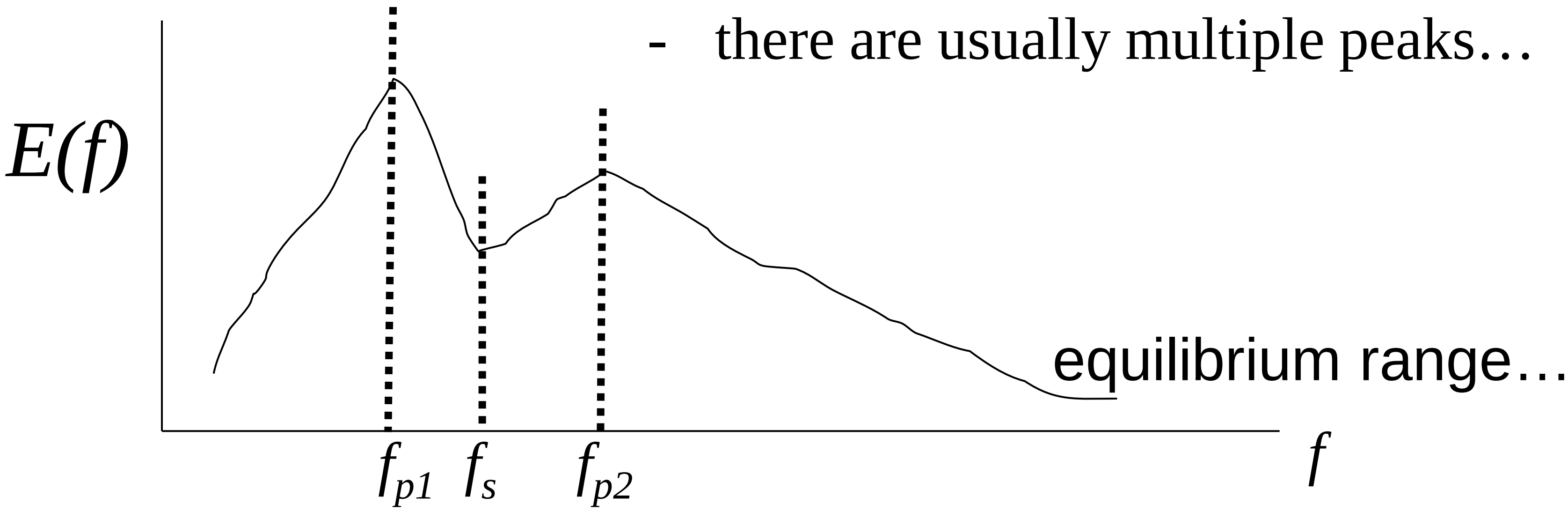
WWM (Roland et al., 2012) with **ST4**

WW3 (v4.18) with **ST4**

WW3 – constant U10 – 48+hr ST4



1. Multiple spectral peaks - mTSA



Broad-scale term parameterization…?

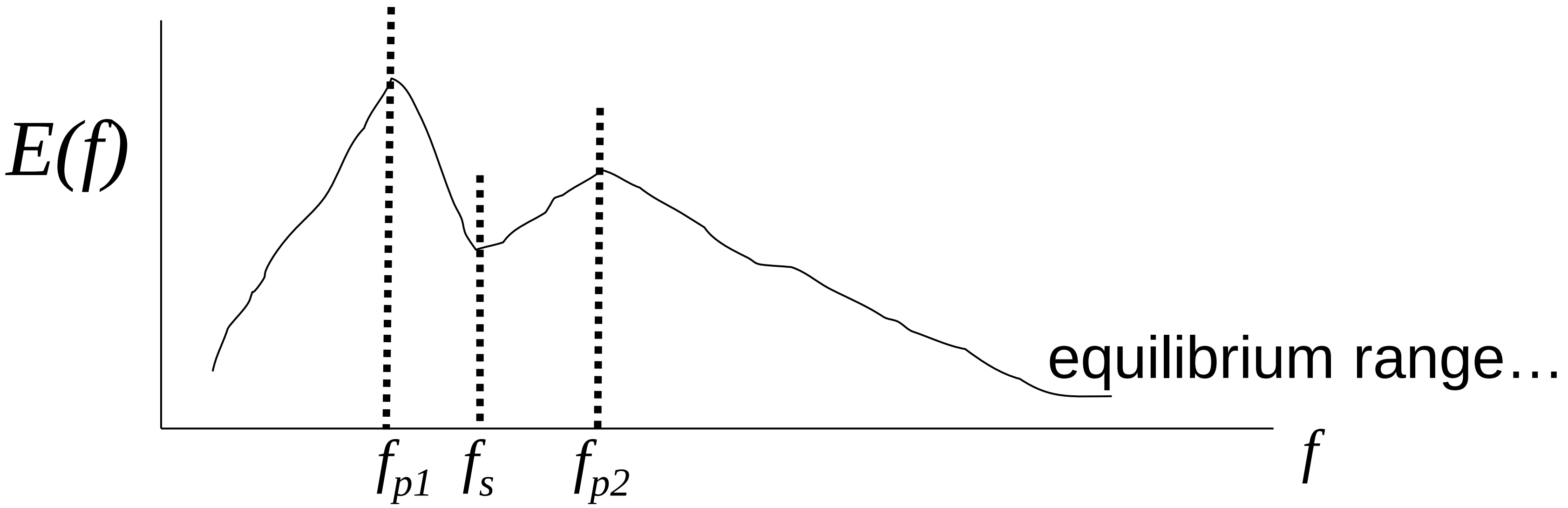
$F(k)_{Norm} = F(k) \times k^{2.5} / \beta$ [Resio&Perrie, 1989; Resio et al. 2004…]

Should be $\beta \sim 1/\Delta f \sum [F(k) \times k^{2.5}]|_{\text{equilibrium range}}$

But equilibrium range is hard to define when f_{p1} and f_{p2} are close…

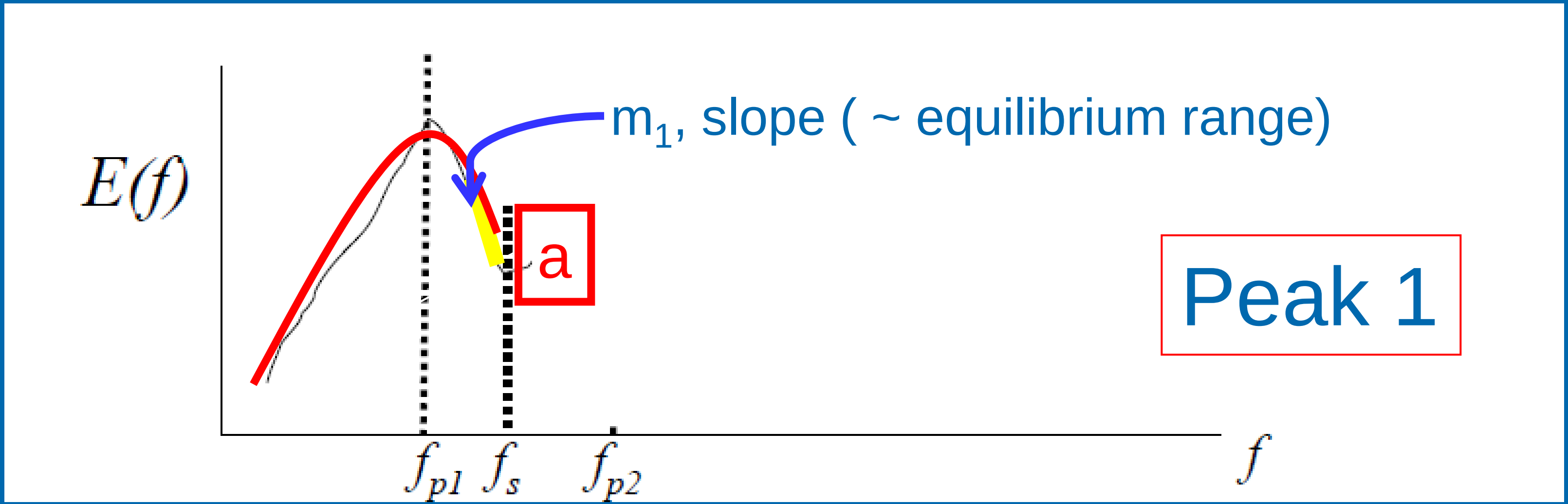
So *let* $\beta = F(k) \times k^{2.5} |f_s$ … *for the first peak … second etc.*

1. Multiple spectral peaks - mTSA

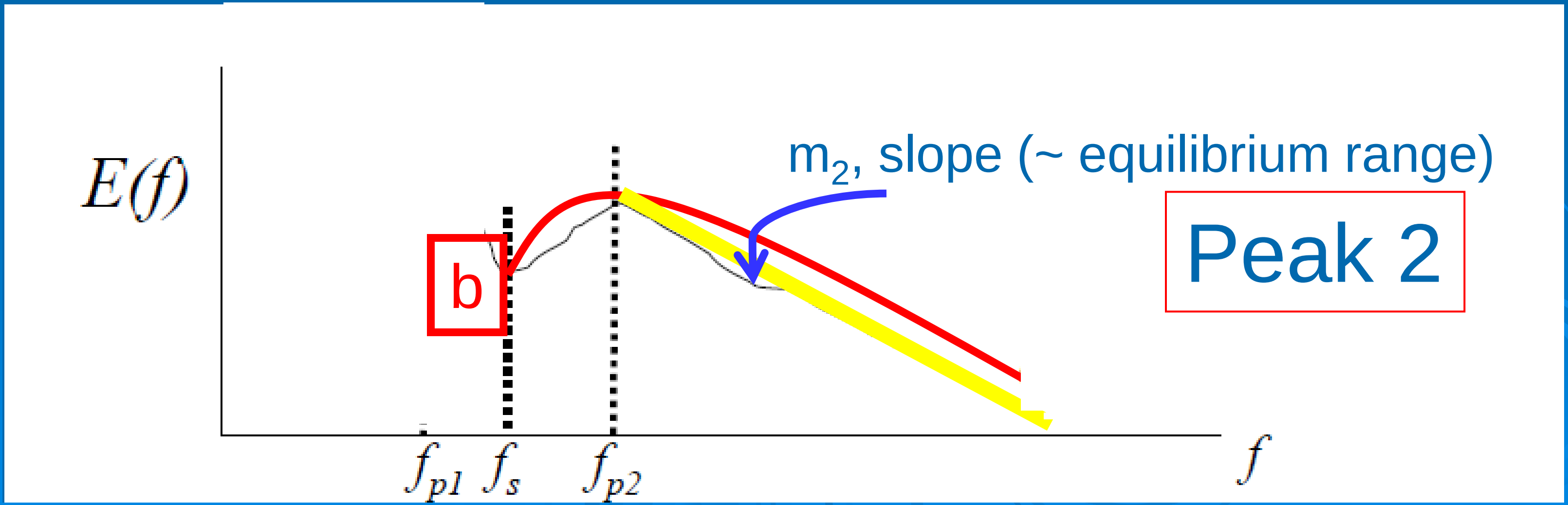


1. Find the peaks; if "many" then choose the largest 2…
2. Fit each peak separately: f_{p1} and f_{p2}
3. Define a separation frequency: e.g. take the ***mean***, or maybe the ***minimum*** in the spectrum, f_s
4. Make the fit for JONSWAP-type parameters for each peak: f_p, α, σ θ γ

Multiple TSA



Have to match or smooth… spectral energy at points a and **b**



mTSA

<u>Equilibrium</u> <u>range</u> problem…

- usually understood as (Resio et al., 2004)

  $1.5\, f_p < f < 2.5\, f_p$

- *in practice* must find at least 1 frequency bin
- this depends on the grid spacing λ where

  $f_{n+1} = \lambda\, f_n$

- <u>require</u> in each peak region to have enough bins so that

  $f_{equilibrium\ range} = \lambda^n\, f_{peak}$ <u>*or*</u> $2 \times f_{peak}$

then: $\lambda^n - 0.025 < f_{equilibrium\ range} / f_{peak} < \lambda^n + 0.025$

- Otherwise TSA is not called for 2 peaks!

2. Alternate the loops

5 loops in frequency, direction and interaction locus…

$$\frac{\partial n_1}{\partial t} \equiv S_{nl} = \int\int T(\mathbf{k}_1, \mathbf{k}_3)\, d\mathbf{k}_3$$

where

$$T(\mathbf{k}_1, \mathbf{k}_3) = 2\oint [n_1 n_3 (n_4 - n_2) + n_2 n_4 (n_3 - n_1)] C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)\, \theta(|\mathbf{k}_1 - \mathbf{k}_4| - |\mathbf{k}_1 - \mathbf{k}_3|)\, |\partial W/\partial n|^{-1}\, ds,$$

Do “loops”, 2

- interpolate to fill frequency and angle bins
- multiple × 2, because still have original “ds” on locus of interactions

$$\overrightarrow{k_1} + \overrightarrow{k_2} = \overrightarrow{k_3} + \overrightarrow{k_4}$$
$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

3. TSA “zone of influence” calculation

1. Given (*f,θ*), the ‘zone of influence’ is about ± 6 frequency or angle bins, for λ in this grid…
2. So the **cross-terms** in the main TSA calculation between broad-scale and local-scale are only allowed in the ‘zone of influence’

k_1 → 1 to $n_{\text{frequency bins}}$
k_3 → starts at bin of k_1,
goes to ‘zone of influence’,
k_{zone}; no need for more…

$$\frac{\partial n_1}{\partial t} = B + \int\int\oint N_*^3 C \left|\frac{\partial W}{\partial n}\right|^{-1} ds\, k_3\, d\theta_3\, dk_3$$

$$\begin{aligned} N^3 = {} & \hat{n}_1\hat{n}_3(\hat{n}_4 - \hat{n}_2) + \hat{n}_2\hat{n}_4(\hat{n}_3 - \hat{n}_1) \\ & + n_1' n_3'(n_4' - n_2') + n_2' n_4'(n_3' - n_1') \\ & + \hat{n}_1\hat{n}_3(n_4' - n_2') + \hat{n}_2\hat{n}_4(n_3' - n_1') \\ & + n_1' n_3'(\hat{n}_4 - \hat{n}_2) + n_2' n_4'(\hat{n}_3 - \hat{n}_1) \\ & + \hat{n}_1 n_3'(\hat{n}_4 - \hat{n}_2) + \hat{n}_2 n_4'(\hat{n}_3 - \hat{n}_1) \\ & + n_1'\hat{n}_3(\hat{n}_4 - \hat{n}_2) + n_2'\hat{n}_4(\hat{n}_3 - \hat{n}_1) \\ & + \hat{n}_1 n_3'(n_4' - n_2') + \hat{n}_2 n_4'(n_3' - n_1') \\ & + n_1'\hat{n}_3(n_4' - n_2') + n_2'\hat{n}_4(n_3' - n_1') \end{aligned}$$

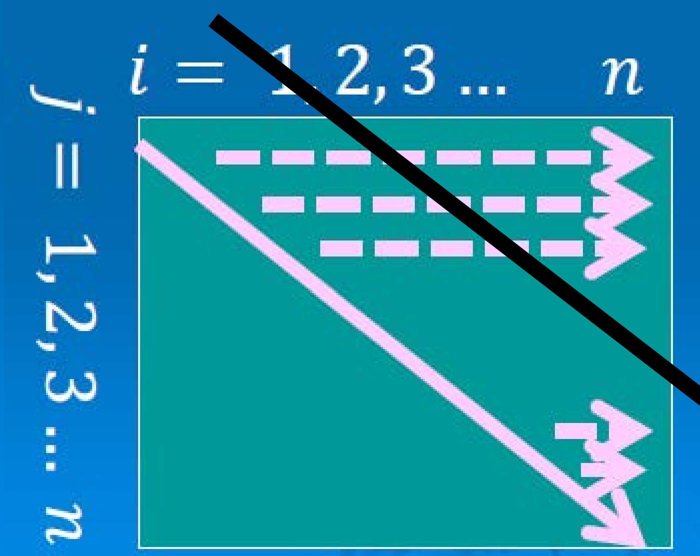
To compute TSA, need grad function $|{}^{\partial W}/_{\partial n}|$, weighting functions, spectra terms …

$$\frac{\partial n_1}{\partial t} \equiv S_{nl} = \int\int T(\mathbf{k}_1, \mathbf{k}_3)\, d\mathbf{k}_3$$

where

$$T(\mathbf{k}_1, \mathbf{k}_3) = 2\oint [n_1 n_3 (n_4 - n_2) + n_2 n_4 (n_3 - n_1)] C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \theta(|\mathbf{k}_1 - \mathbf{k}_4| - |\mathbf{k}_1 - \mathbf{k}_3|) |\partial W/\partial n|^{-1}\, ds,$$

if $f(k_3) > 2 \times f(k_1)$ skip…



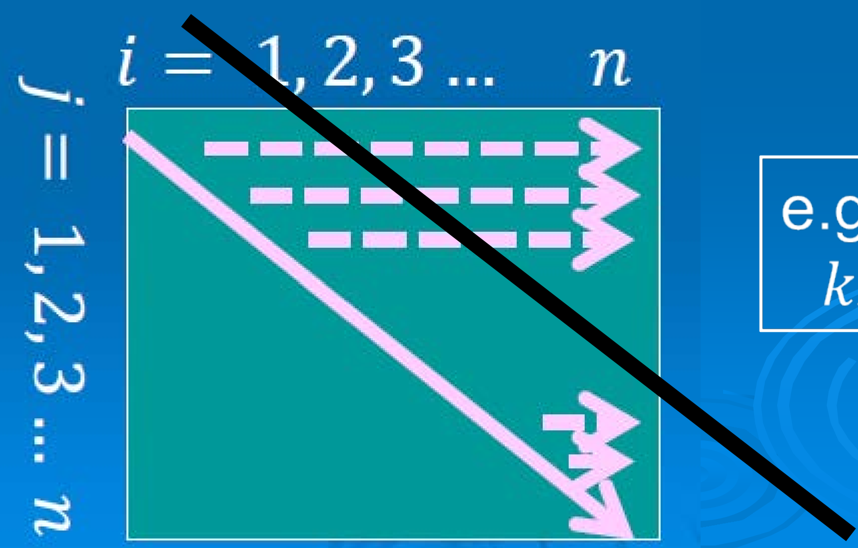
$$nzz = n(n+1)/2$$

Limit the loops – no need to go to *n:*

and because $f_n = f_0 \times \lambda^n$ we can write

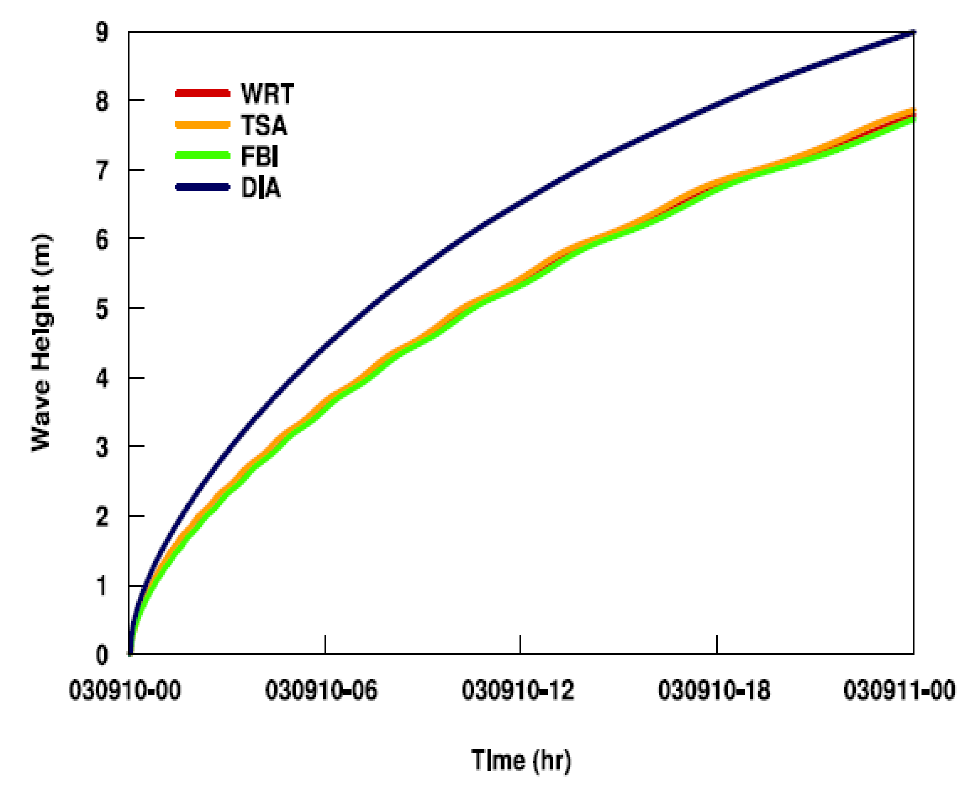
$$kzone = INT\ (log2/\log\lambda)$$

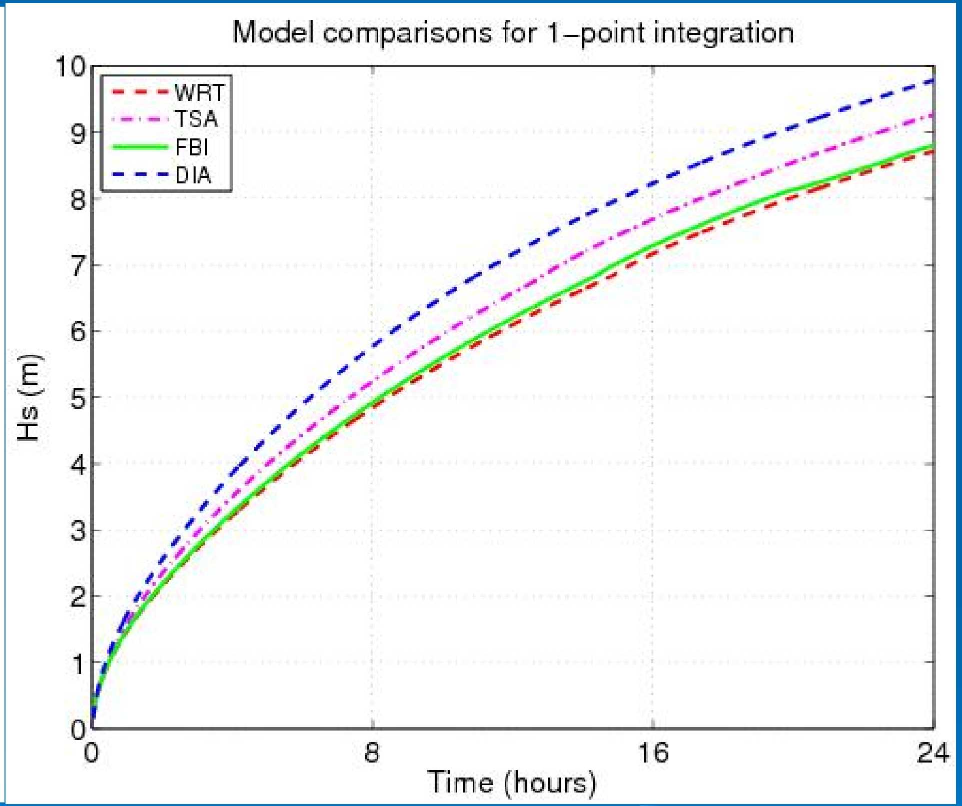
We tried “3, 4, …” and found “2” is enough…



e.g. $\lambda = 1.05$
$kzone = 14$

1-point time integration





WWM (Roland et al., 2012) with **ST4**

WW3 (v4.18) with **ST4**

Depth calculations… and look-up tables: … ongoing work

Terms of the Snl:

$$\frac{\partial n_1}{\partial t} \equiv S_{nl} = \int\int T(\mathbf{k}_1, \mathbf{k}_3)\, d\mathbf{k}_3$$

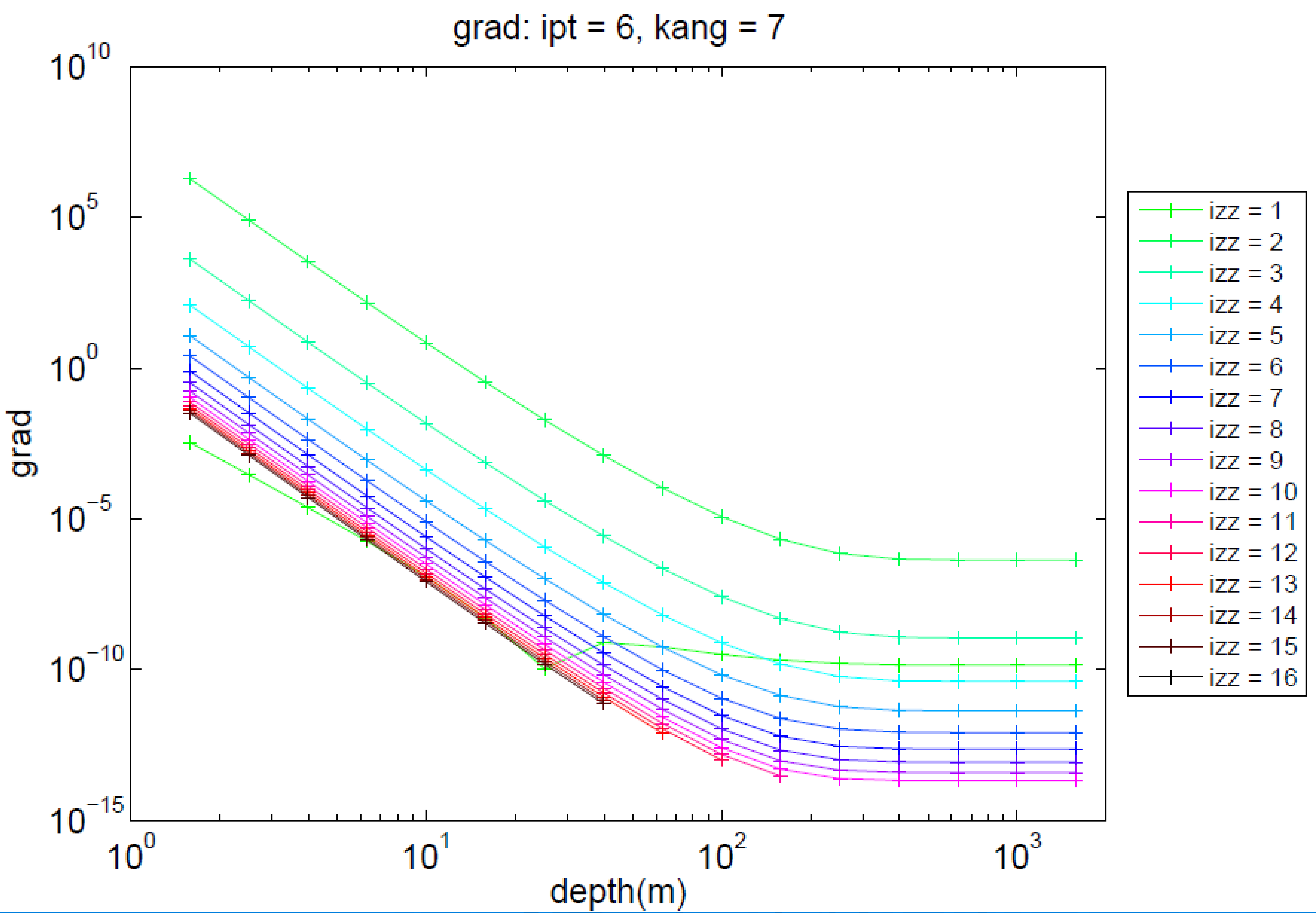
where

$$T(\mathbf{k}_1, \mathbf{k}_3) = 2\oint [n_1 n_3 (n_4 - n_2) + n_2 n_4 (n_3 - n_1)] C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \theta(|\mathbf{k}_1 - \mathbf{k}_4| - |\mathbf{k}_1 - \mathbf{k}_3|) |\partial W / \partial n|^{-1}\, ds,$$

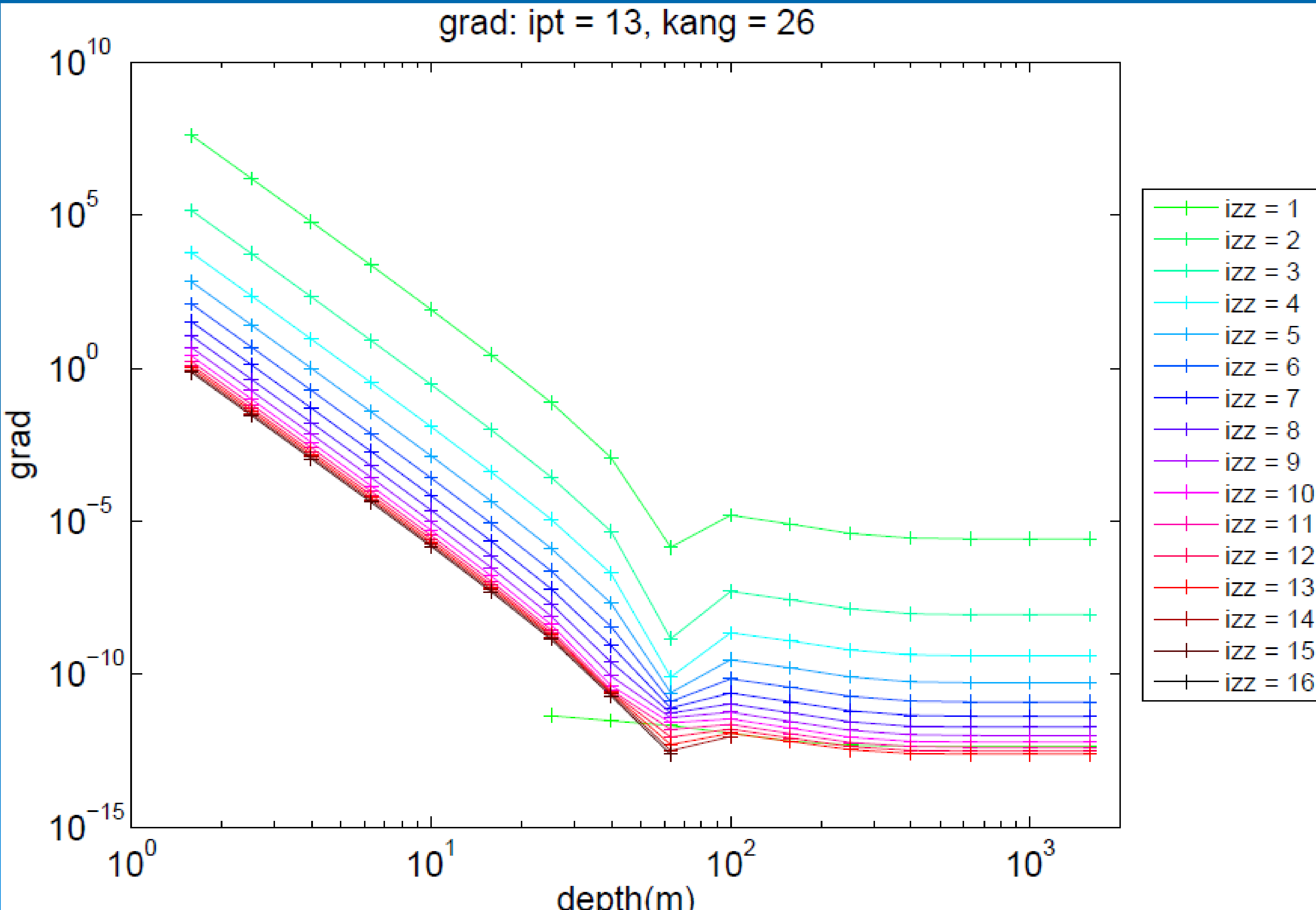
are functions of depth…..

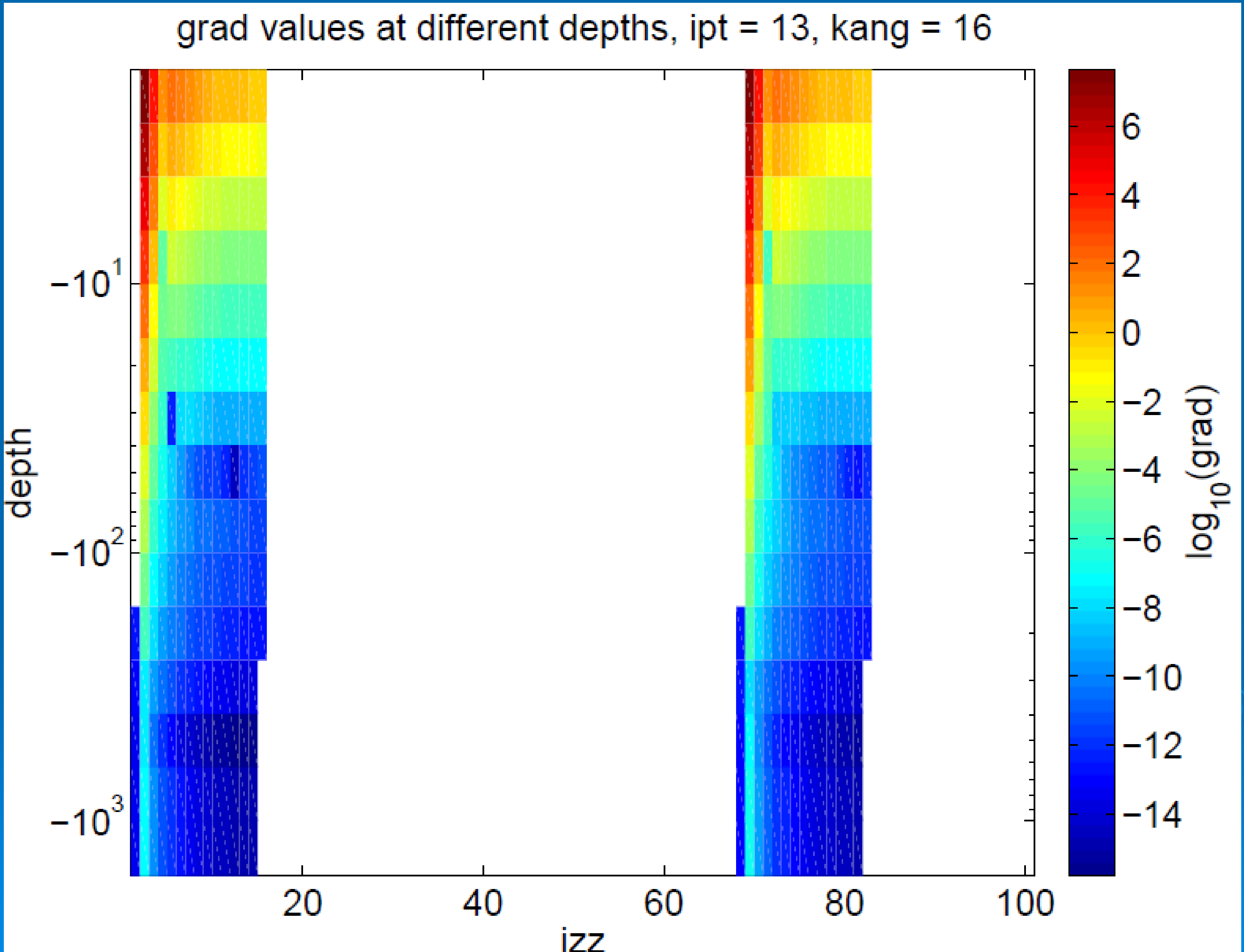
→ pre-compute these for each depth…

For example the "$grad$" $=$ $|\partial W / \partial n|$ term…

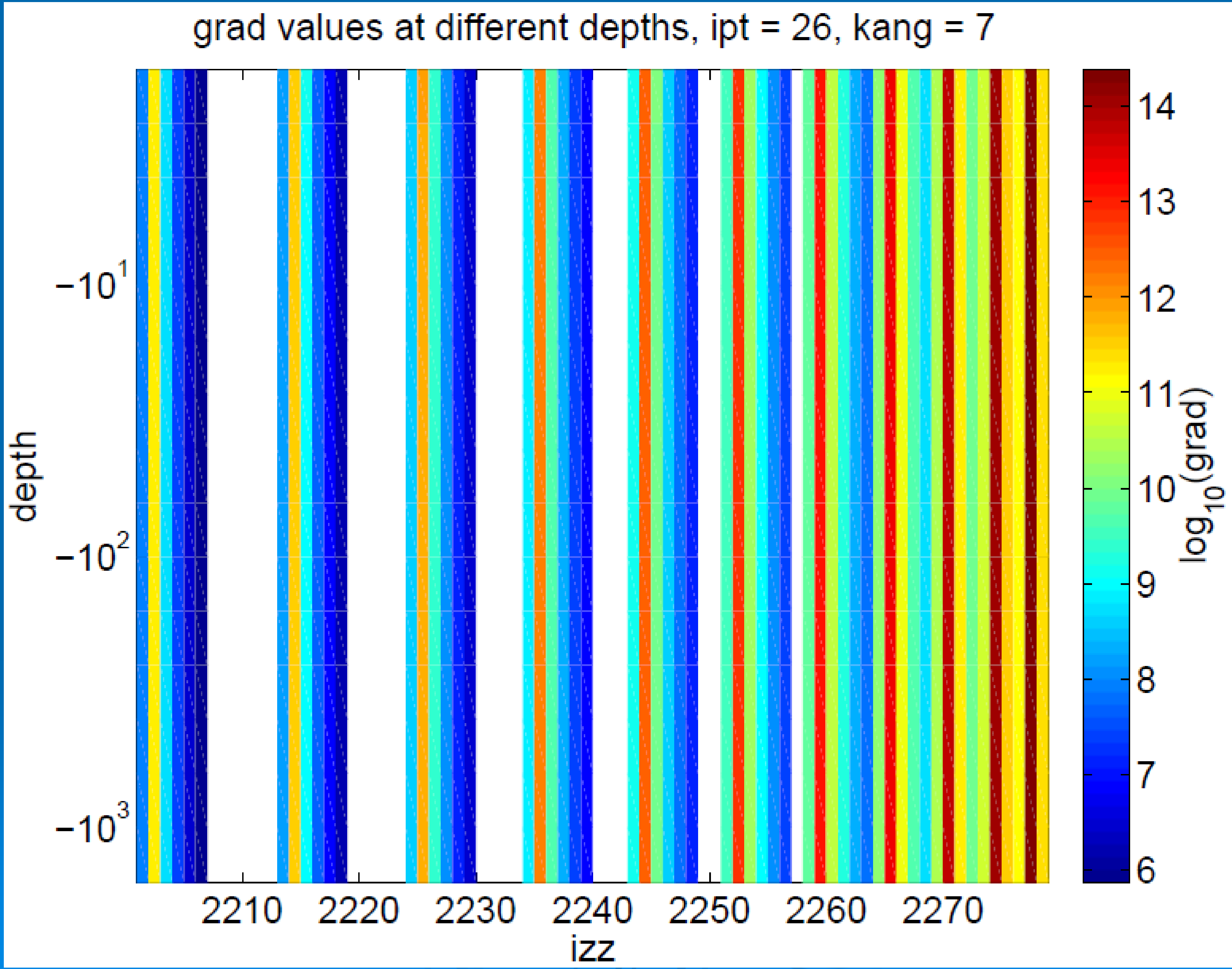


grad: ipt = 6, kang = 7
grad
depth(m)
10^{10}
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{0}
10^{1}
10^{2}
10^{3}
izz = 1
izz = 2
izz = 3
izz = 4
izz = 5
izz = 6
izz = 7
izz = 8
izz = 9
izz = 10
izz = 11
izz = 12
izz = 13
izz = 14
izz = 15
izz = 16


grad: ipt = 13, kang = 26
grad
depth(m)
10^{10}
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{0}
10^{1}
10^{2}
10^{3}
izz = 1
izz = 2
izz = 3
izz = 4
izz = 5
izz = 6
izz = 7
izz = 8
izz = 9
izz = 10
izz = 11
izz = 12
izz = 13
izz = 14
izz = 15
izz = 16

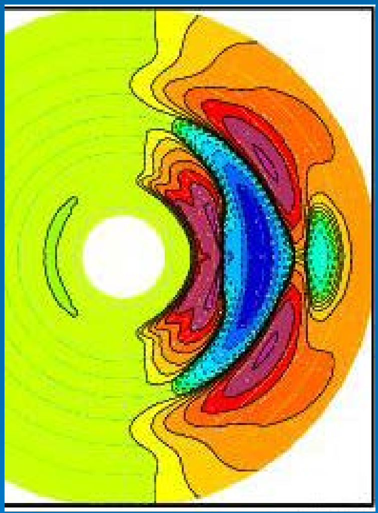


grad values at different depths, ipt = 13, kang = 16
depth
−10^1
−10^2
−10^3
izz
20
40
60
80
100
log10(grad)
6
4
2
0
−2
−4
−6
−8
−10
−12
−14

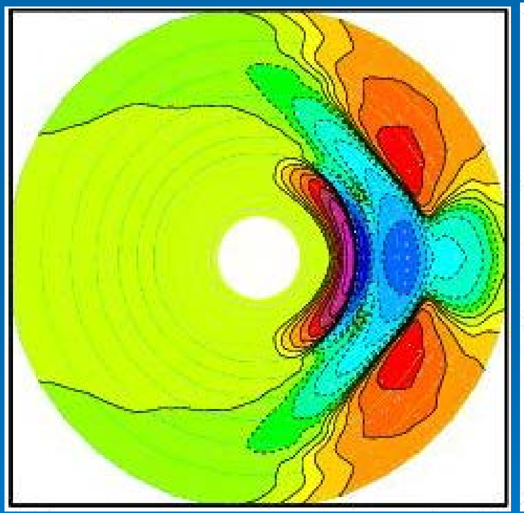

grad values at different depths, ipt = 26, kang = 7
depth
-10^1
-10^2
-10^3
2210
2220
2230
2240
2250
2260
2270
izz
14
13
12
11
10
9
8
7
6
$log_{10}(grad)$

Computational time

- Alternate frequency, angles & interaction points can speed up dTSA about 30-40 × or more;
- Including ‘zone of influence’ computation;
- Best accurate results are 20 × slower than DIA

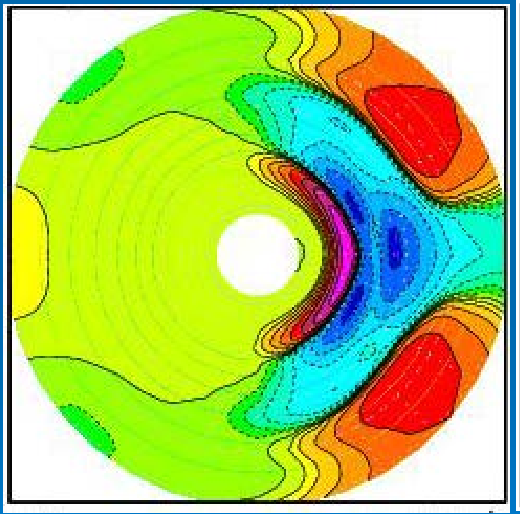


DIA



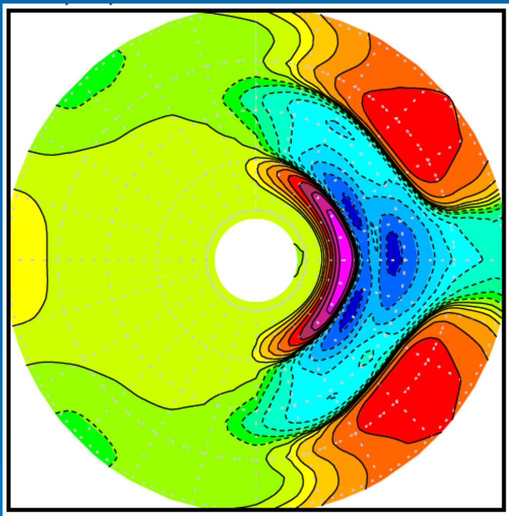
WRT

×1000 DIA in speed



dTSA

×40DIA; ~ 2013



mTSA

20×DIA; 2015

Summary

1. Implemented TSA in WWM and WW3
2. Reliable results for ‘academic’ JONSWAP tests
3. “ " fetch- and duration-limited growth
4. Optimization of TSA code is ongoing.
5. MPI + OpenMP may make faster TSA runs
6. Finite water depth methodology … ongoing…

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