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Measured crest height distribution compared to second order distribution

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Introduction

- Fixed drag dominated structure is commonly designed using the so called “design wave approach”. Normally specified by a long crested Stokes 5th order wave.
- The dynamic of the structure is normally considered for extreme wave loading when the natural period of the structure is above 3 second.
- In addition the airgap of installation has to be checked. This loading has an on-off nature related to the air gap.
- A central question is then , What is the maximum q-annual crest height ?

Crest probability distribution

- Short term ?
 - Based on numerical simulation of surface elevation ?

- Long Term ?
 - All seas approach ?
 - POT ?

SECOND ORDER WAVE CREST DISTRIBUTION

- The sea surface elevation is now commonly described by a second order process.
- Based on second order time domain simulations, Forristall established a short term distribution of the crest heights.

$$F_C(x) = 1 - \exp \left[- \left(\frac{x}{\alpha_C H_S} \right)^{\beta_C} \right]$$

- The Weibull location and scale parameter is a function of the water depth(Ursell number) and the wave steepness.

$$U_{rs} = \frac{H_S}{k_1^2 d^3}$$

$$S_1 = \frac{2\pi}{g} \frac{H_S}{T_1^2}$$

SECOND ORDER WAVE CREST DISTRIBUTION, CONT.

- For short crested seas the Forrsitall estimated the Weibull parameters:

$$\alpha_{C.3D} = 0.3536 + 0.2568S_1 + 0.0800U_{rs}$$

$$\beta_{C.3D} = 2 - 1.7912S_1 - 0.5302U_{rs} + 0.2824U_{rs}^2$$

- A stationary sea state of 3-hour duration is often considered. The 3-hour extreme crest distribution can then be estimated by

$$F_{C3h}(x) = \left(1 - \exp \left[- \left(\frac{x}{\alpha_C H_S} \right)^{\beta_C} \right] \right)^{n_{3h}}$$

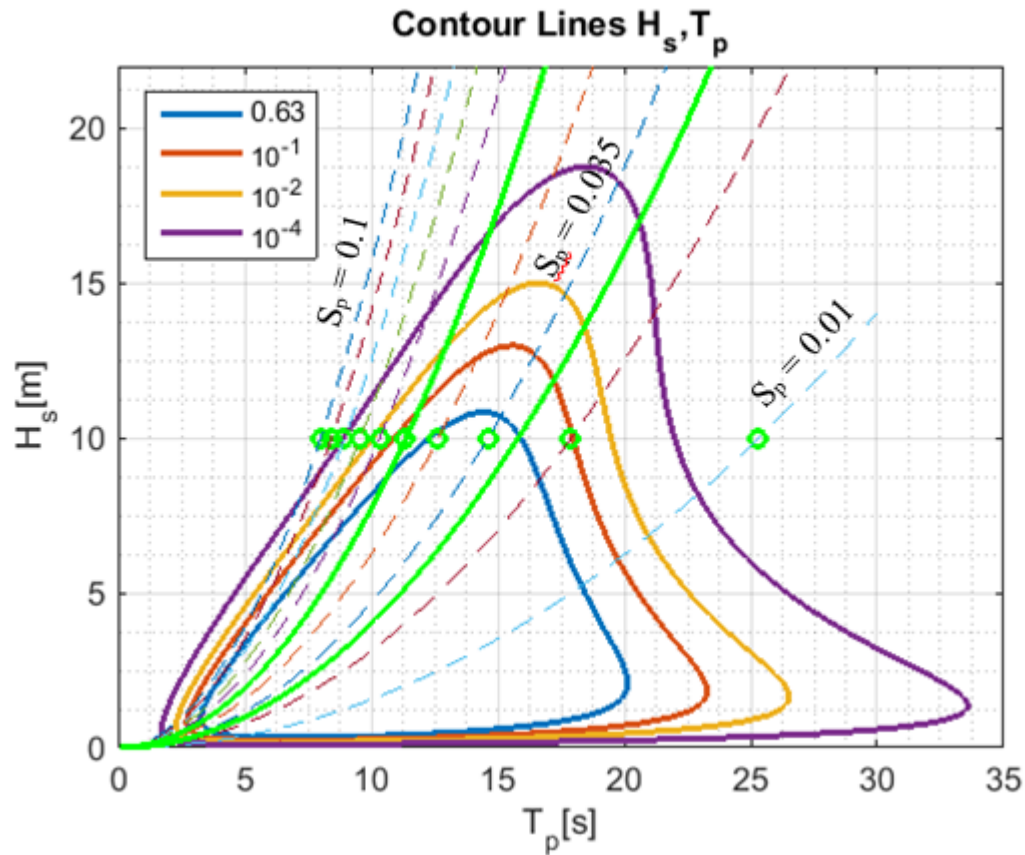
Where n_{3h} is the expected number of global crests in 3-hour

q-annual extreme crest height

- The q-annual extreme crest can be estimated using a long term analysis. Alternatively, it can be estimated using the contour line method.
- The sea state at the peak of the contour and a fractile of 85%-90% for ULS and 90%-95% for ALS should then be used.

$$x_p = \alpha_C H_S \left[-\ln \left(1 - F_{C3h}(x_p)^{\frac{1}{n_{3h}}} \right) \right]^{\frac{1}{\beta_C}}$$

Contour lines



The JONSWAP spectrum is a reasonable model for

$$3.6 < T_p / \sqrt{H_s} < 5$$

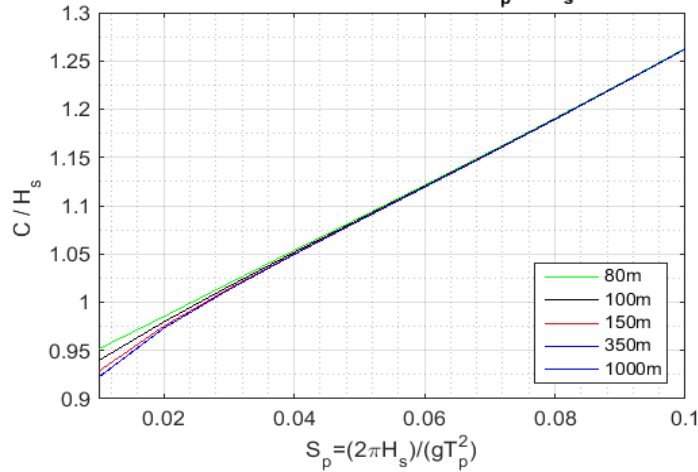
Steepness S_p is defined by

$$S_p = \frac{2\pi}{g} \frac{H_s}{T_p^2}$$

Sensitivity to steepness, water depth and 2D&3D

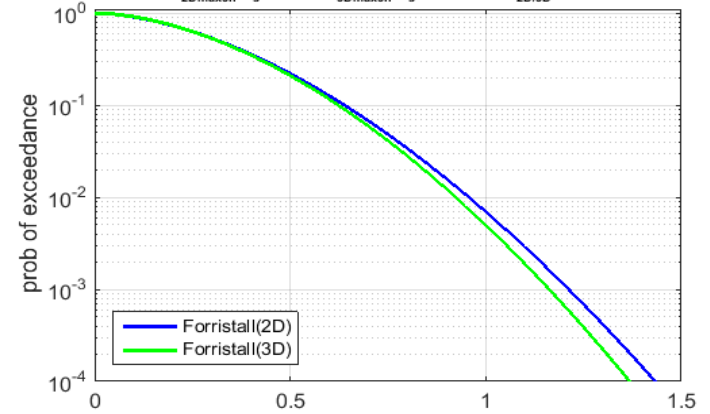
Sensitivity of Forristall distribution to water depth and steepness

Long-crested, $\gamma = \max(1, 42.2 S_p^{6/7})$, $H_s = 10\text{m}$



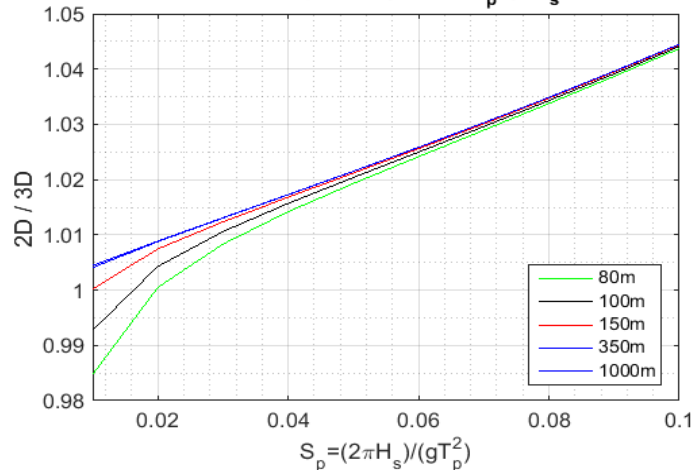
$H_s = 10\text{m}$, $T_p = 8\text{s}$, $S_p = 0.1$, Depth=100m

Ursell=0.0014342, $S_1 = 0.1329$, $n_{3h} = 1653.5$, $\lambda_1 = 75.2\text{m}$, $\lambda_p = 100.1\text{m}$
 $C_{2Dmax3h}/H_s = 1.263$, $C_{3Dmax3h}/H_s = 1.209$, Ratio_{2D/3D} = 1.044



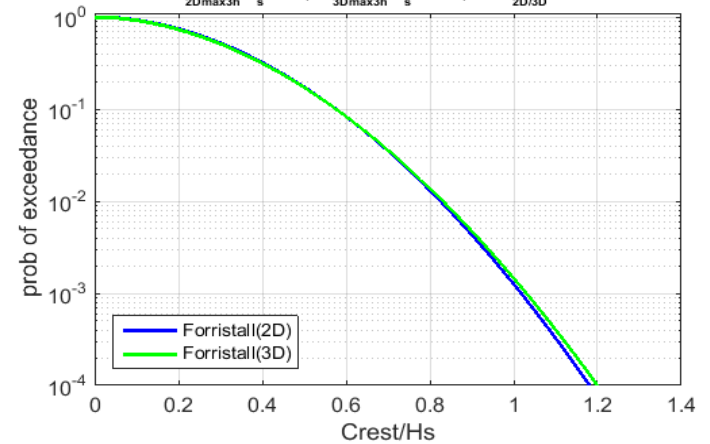
Sensitivity of Forristall distribution to water depth and steepness

Ratio 2D/3D, $\gamma = \max(1, 42.2 S_p^{6/7})$, $H_s = 10\text{m}$



$H_s = 18\text{m}$, $T_p = 24\text{s}$, $S_p = 0.02$, Depth=100m

Ursell=0.10698, $S_1 = 0.032$, $n_{3h} = 616.7$, $\lambda_1 = 484.4\text{m}$, $\lambda_p = 664.3\text{m}$
 $C_{2Dmax3h}/H_s = 0.98$, $C_{3Dmax3h}/H_s = 0.991$, Ratio_{2D/3D} = 0.989



FULL SCALE MEASUREMENTS

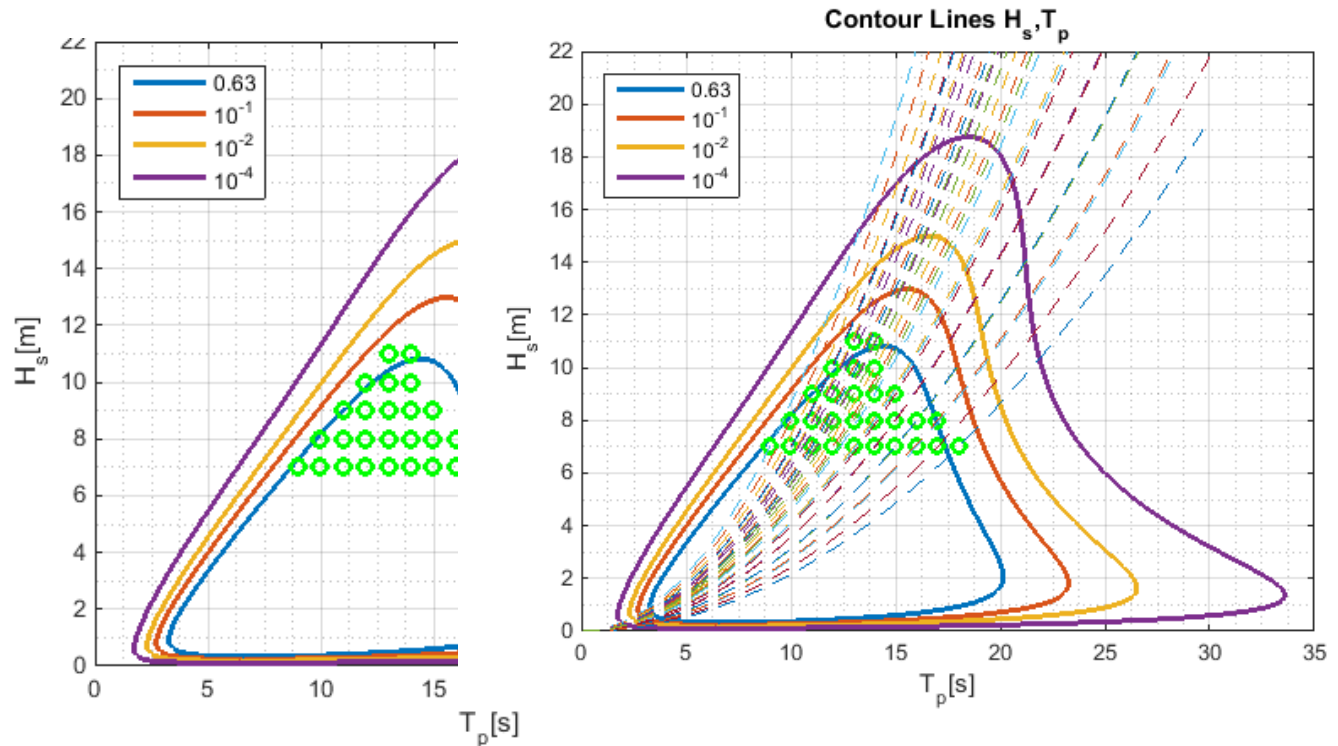
- The measurement is performed by a wave radar type WaveRadar REX manufactured by SABB/Rosemount.
- The sampling frequency is 7.68Hz.
- Measured since 2004
- Total available time series are 132752 x 20 min = 44250 hours
- 2116 hours are in sea states above H_s 6.5m
- 61 hours above H_s 10.5m
- Highest measured 20 min. sea state is $H_s=14.1$ m
- Highest measured crest was 15.1m. (In sea state of $H_s=12.1$ m)

FULL SCALE MEASUREMENTS

- 2028 hours of measured surface elevation has been compared to the Forristall second order crest distribution
- The comparison has in general been limited to sea state class with minimum 10 hours of observations
- Steepness from 0.018 to 0.055 are covered, as indicated by the shaded area in Table:

		Tp[s]									
		9	10	11	12	13	14	15	16	17	18
Hs [m]	7	0.055	0.045	0.037	0.031	0.027	0.023	0.020	0.018	0.016	0.014
	8	0.063	0.051	0.042	0.036	0.030	0.026	0.023	0.020	0.018	0.016
	9	0.071	0.058	0.048	0.040	0.034	0.029	0.026	0.023	0.020	0.018
	10	0.079	0.064	0.053	0.045	0.038	0.033	0.028	0.025	0.022	0.020
	11	0.087	0.071	0.058	0.049	0.042	0.036	0.031	0.028	0.024	0.022

Total hours of measurement of each sea state class.



		Tp[s]									
		9	10	11	12	13	14	15	16	17	18
Hs [m]	7	41.7	187.0	374.7	281.7	130.7	83.7	53.3	32.0	29.0	18.0
	8		32.3	149.7	175.3	57.0	33.0	17	13.3	16.7	
	9			27.7	61.7	58.3	25.3	13.3			
	10				23.7	37.0	27.3				
	11					13.7	14.0				

Distribution of the largest crests

- The Gumbel extreme distribution is used to estimate the distribution of the largest crests

$$F_{X|HpTp}(x) = \exp \left\{ -\exp \left[-\left(\frac{x - \alpha}{\beta} \right) \right] \right\}$$

β is the scale parameter and α the location parameter

mean and variance is given by

$$E(X) = \mu_X = \alpha + 0.57722\beta$$

$$\sigma_X^2 = \frac{\pi^2}{6} \beta^2 = 1.64493\beta^2$$

The location and scale parameters is estimated by the methods of moment

Comparison measurements and 2nd-order

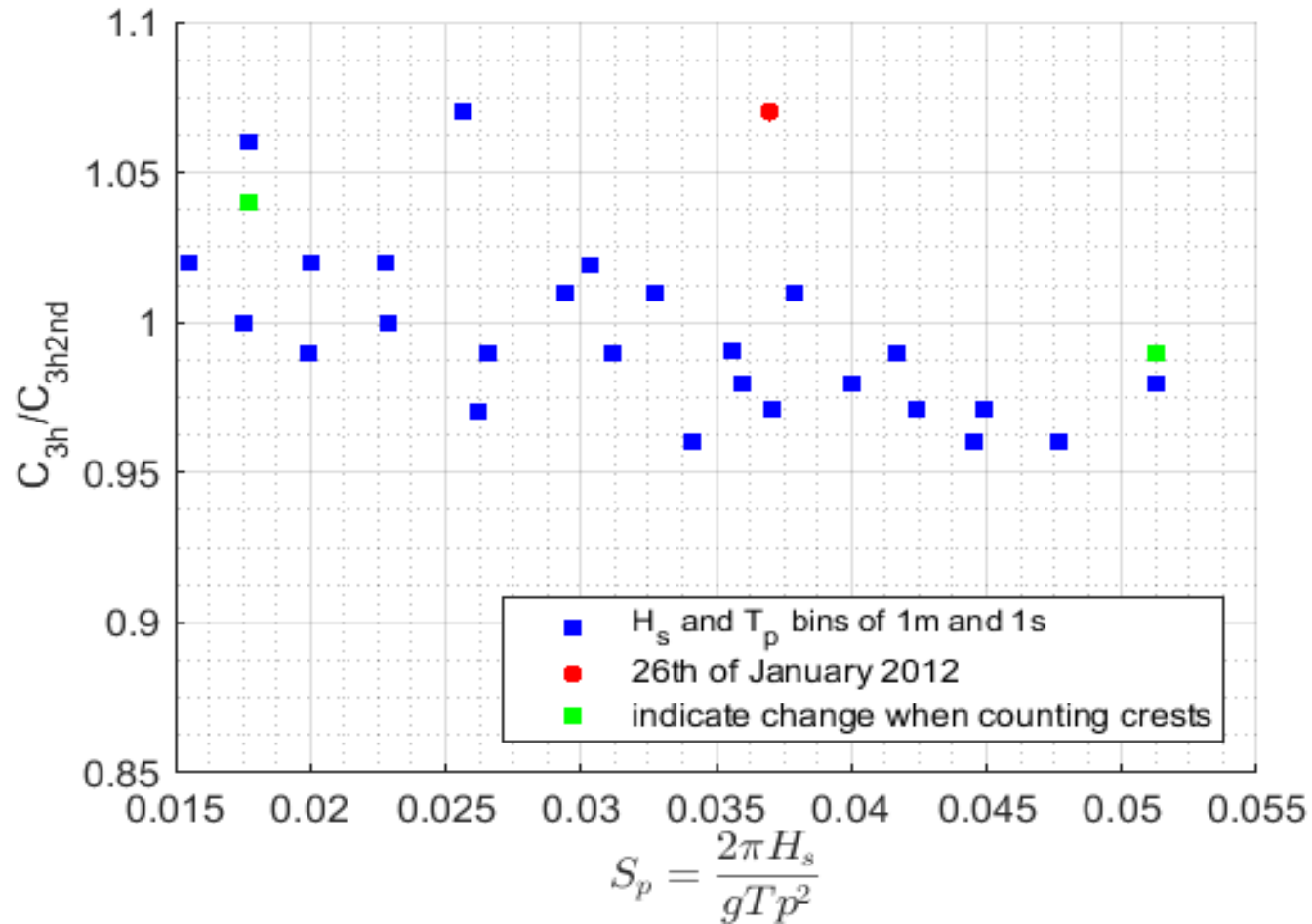
- Comparison is made between the characteristic largest in 3-hour from measurements and second order.
- The characteristic largest from measurements is estimated by the most probably maximum (mpm) i.e the 37% fractile from the 3-hour Gumbel extreme distribution

$$F_{X3h|HpTp}(x) = \{F_{X20min|HpTp}(x)\}^9$$

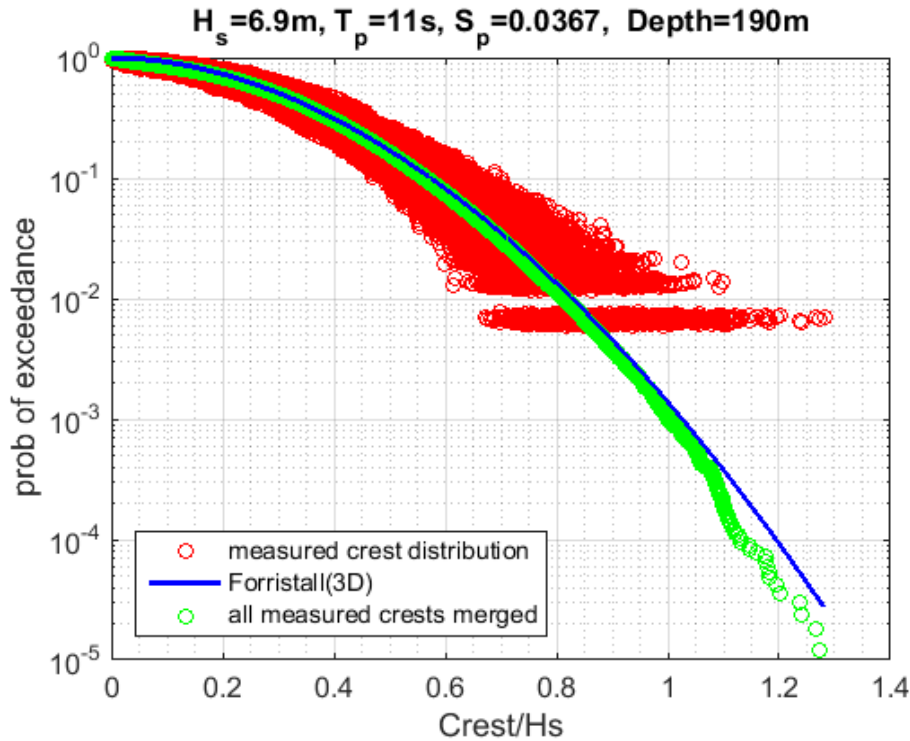
$$c_{n3hG} = \alpha_{20min} + \ln(9)\beta_{20min}$$

- This is compared to the characteristic largest estimated from the Forristall 3D distribution.

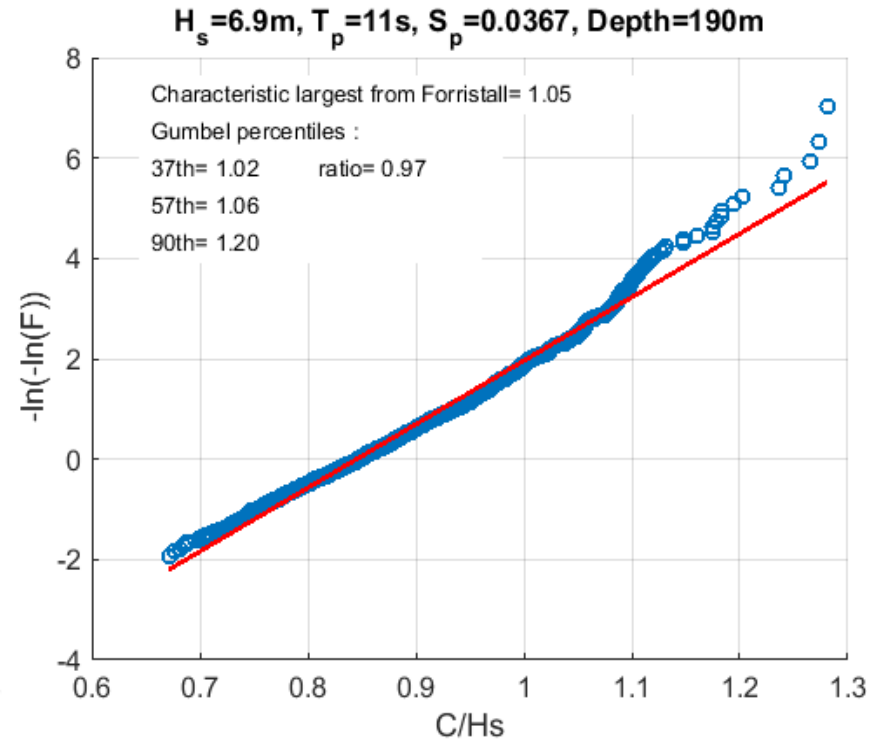
Comparison measurements and 2nd-order



Comparison at Steepness $S_p=0.037$

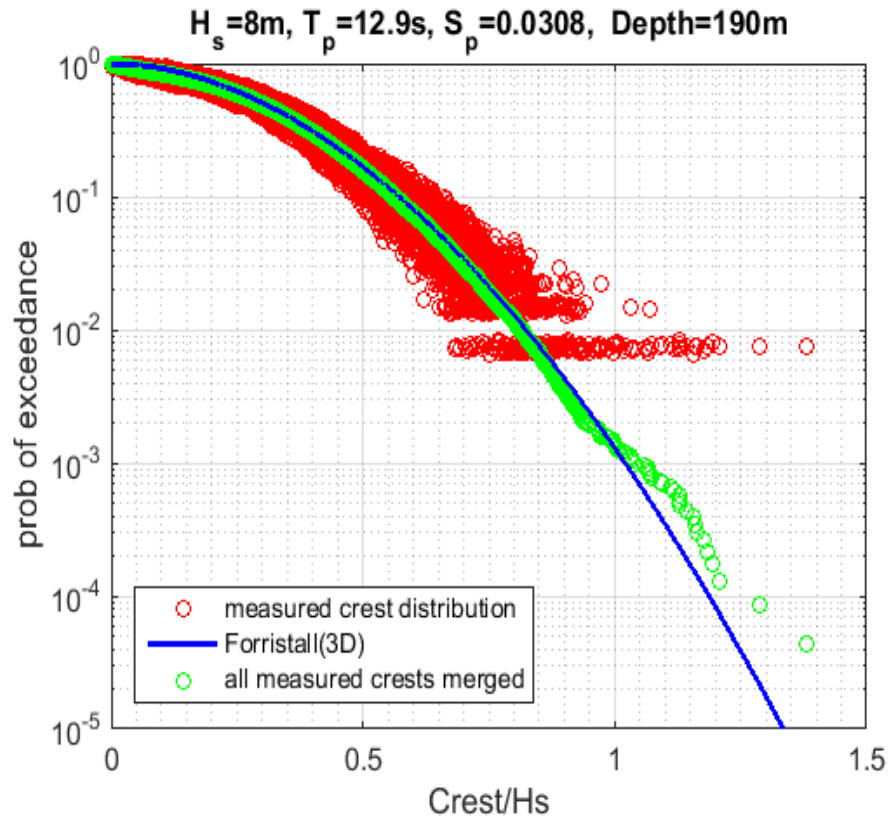


375 hours of measurements.

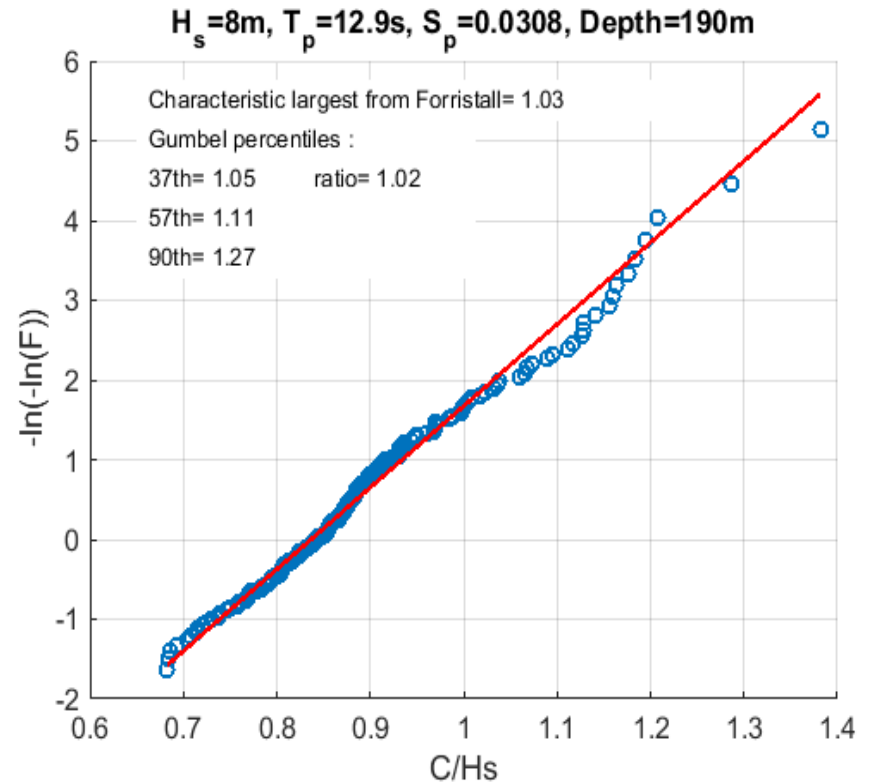


Gumbel distribution of 1124
20 minutes extremes.

Comparison at Steepness $S_p=0.031$

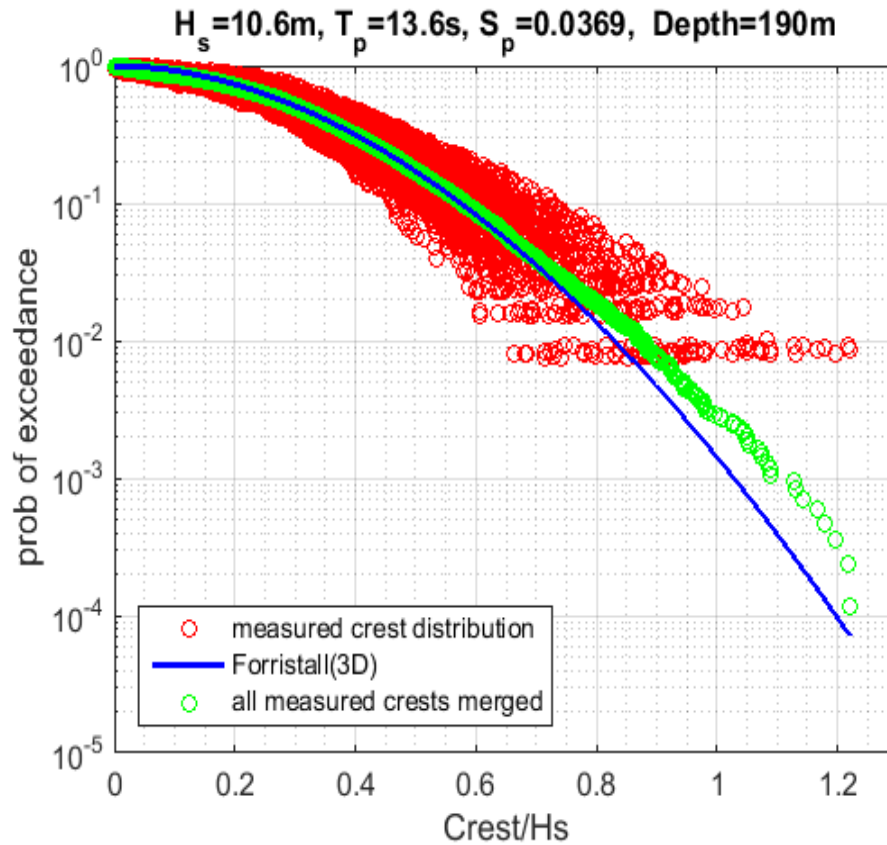


57 hours of measurements

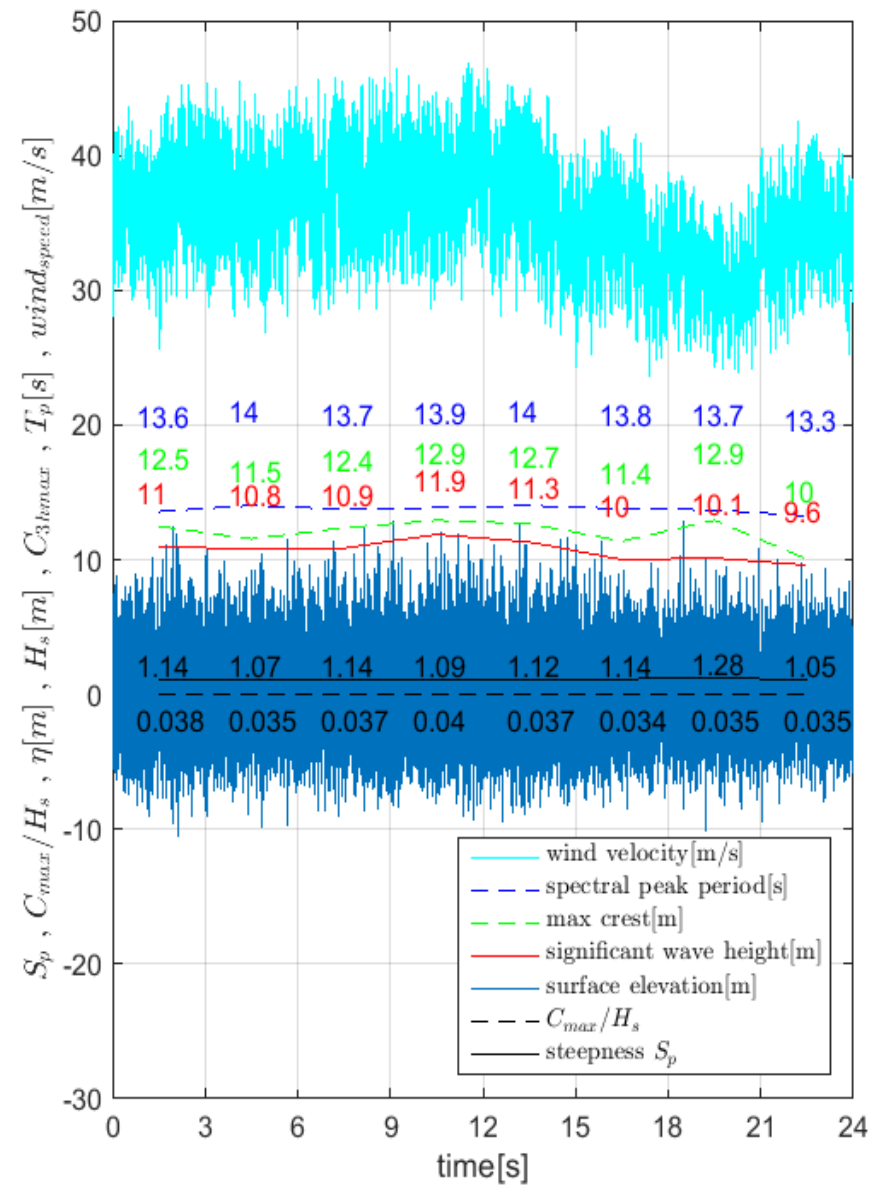


Gumbel distribution of 171
20 minutes extremes

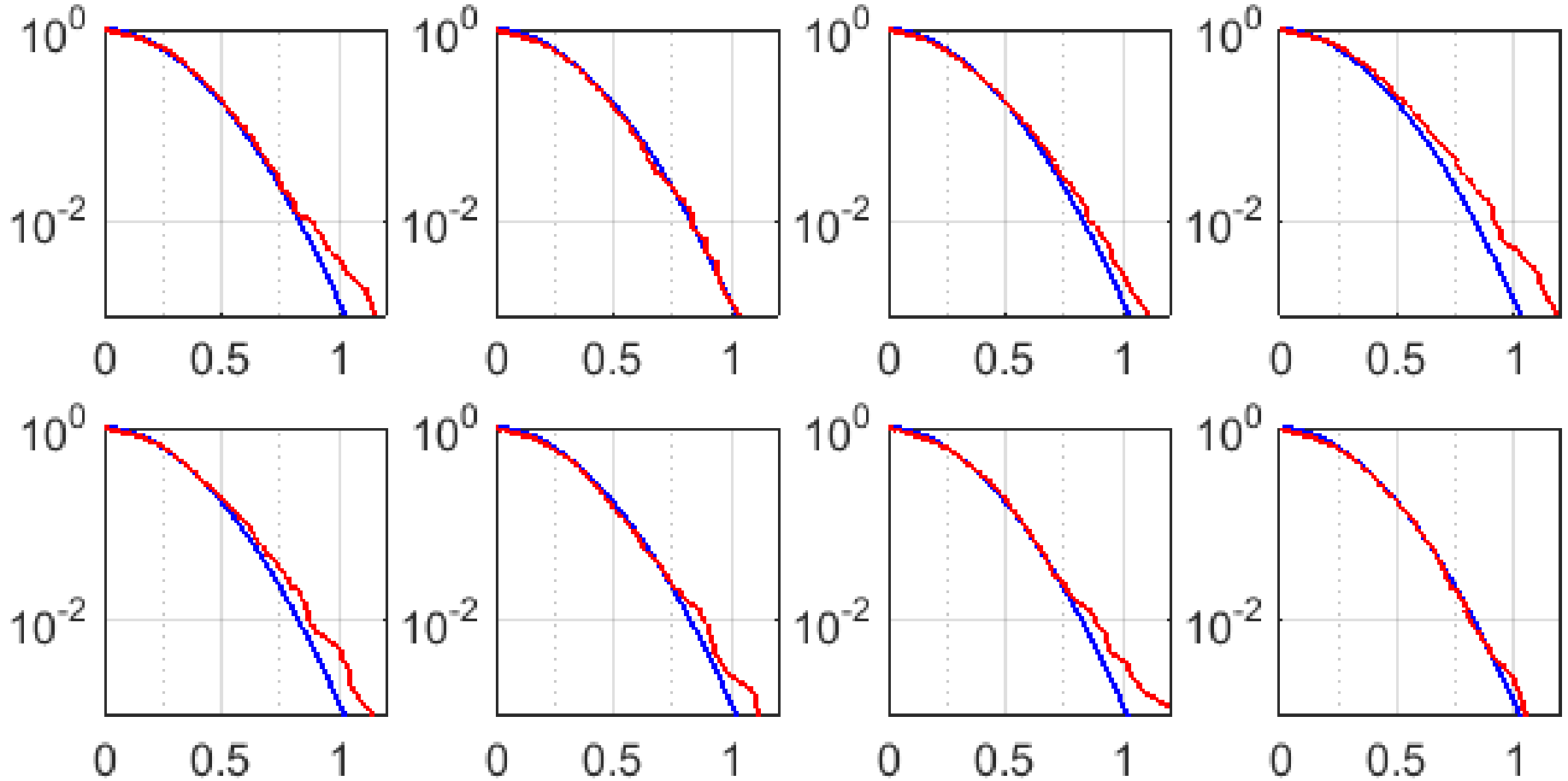
26th of January 2012



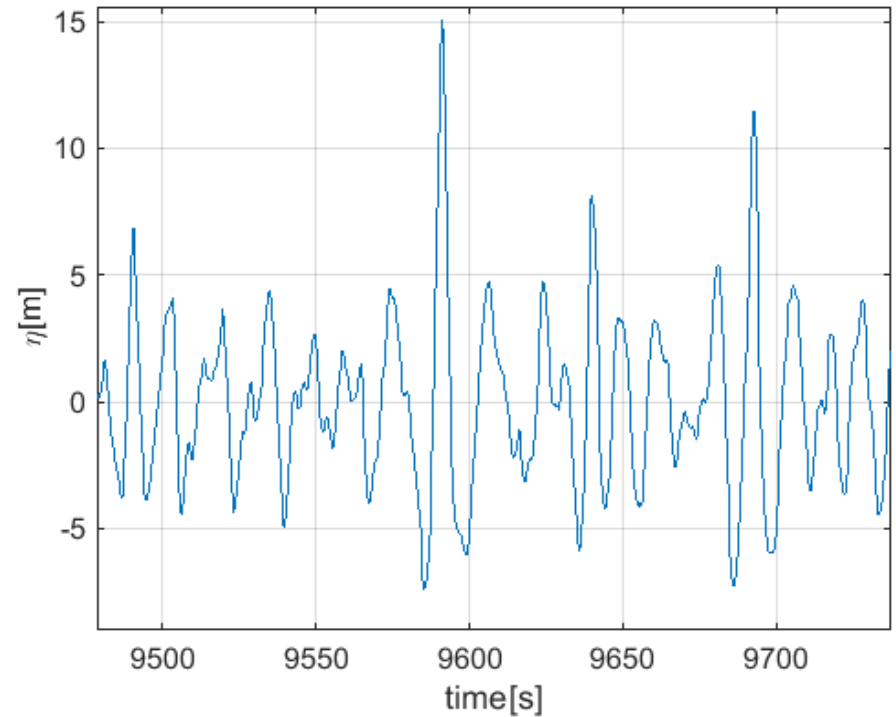
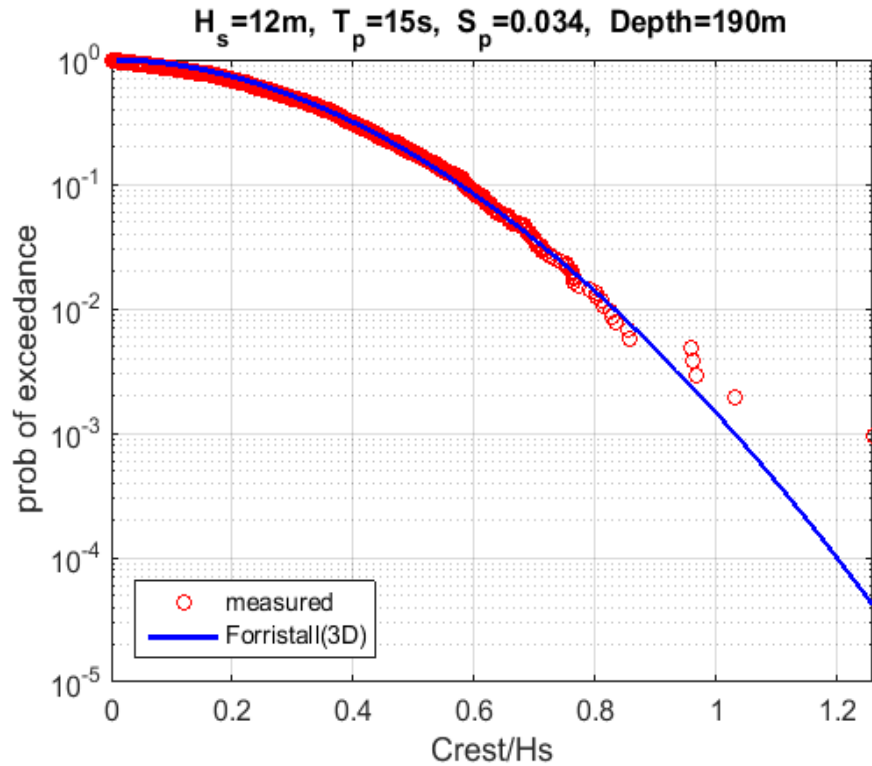
The characteristic largest is 7% higher than estimated by second order.



3-hour crest distributions the 26th of January 2012



Highest crest in the measuring period 15m. 12 January 2015 (typo error in paper)



Conclusion

- In general the wave crest distribution is found to be very well described by the second order crest distribution by Forristall.
- However, during a storm that lasted for more than 24 hours, significantly higher crests than by second order were measured.
- Although there are events exceeding second order distribution, it seems reasonable to assume second order wave distributions when performing a long term analysis.
- An additional requirement could be to check that the probability of exceedance of the q-annual crest height should be less than a given percentage during a 3-hour storm.