Non-stationary Extreme Values Analysis of waves: a simplified approach

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Non-stationary EVA of the wave climate

Why?

Estimation of wave climate design parameters in view of climate change,
e.g. design of coastal infrastructures, probabilistic operational forecasts.

When?

- Change of the frequency of extremes with time and/or space (IPCC 2007);
- Presence of trend at the timeseries;
- Strong interannual, seasonal variability
- Influence of ocean-atmosphere climatic patterns (SOI, ENSO, NAO)

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Development of non-stationary probabilistic models

a) Include the **variability in time** at the GEV or GPD models: usually achieved by means of a MLE on a parametricGEV/GPD

$$Z_t \sim GEV[\mu(a_1, a_2, ..., t), \sigma(a_1, a_2, ..., t), \varepsilon(a_1, a_2, ..., t)]$$

$$Y_t \sim GP[u(a_1, a_2, ..., t), \sigma'(a_1, a_2, ..., t), \varepsilon'(a_1, a_2, ..., t)]$$

Where μ, σ, ε are the time-dependent location, scale and shape parameters, a1, a2 …. are tubable parameters characteristic of system.

b) Include the dependence on **covariates** such as climate indices at the location parameter:

$$Z_t \sim GEV(\mu(t), \sigma, \varepsilon)$$

$$\mu(t) = \beta_0 + \beta_1 SOI(t)$$

Sources:
Coles, 2001, Springer London
Mendez, 2006, JGR
…

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An alternative methodology, basic concept

- transform the non stationary signal into a stationary one
- execute a stationary EVA on the transformed signal
- back transform the stationary EVA into a non stationary one

The simplicity of this approach is that it deduces the non-stationary parameters from a stationary MLE which is simpler to implement.

A key aspect is how transformation and back-transformation are carried out.

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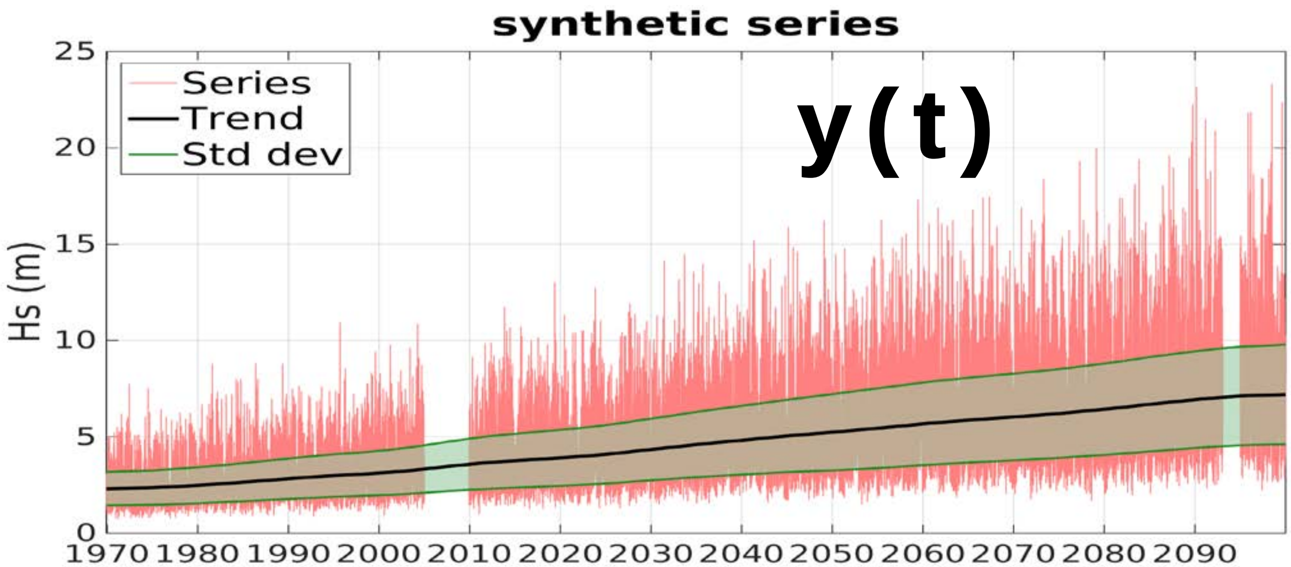
Proposed transformation: a time varying normalization

$$x(t) = \frac{y(t) - tr_y(t)}{std_y(t)}$$

- $y(t)$ is the non stationary series
- $x(t)$ is the transformed series
- $tr_y(t)$ is the slow varying trend of $y(t)$
- $std_y(t)$ is the slow varying standard deviation of $y(t)$



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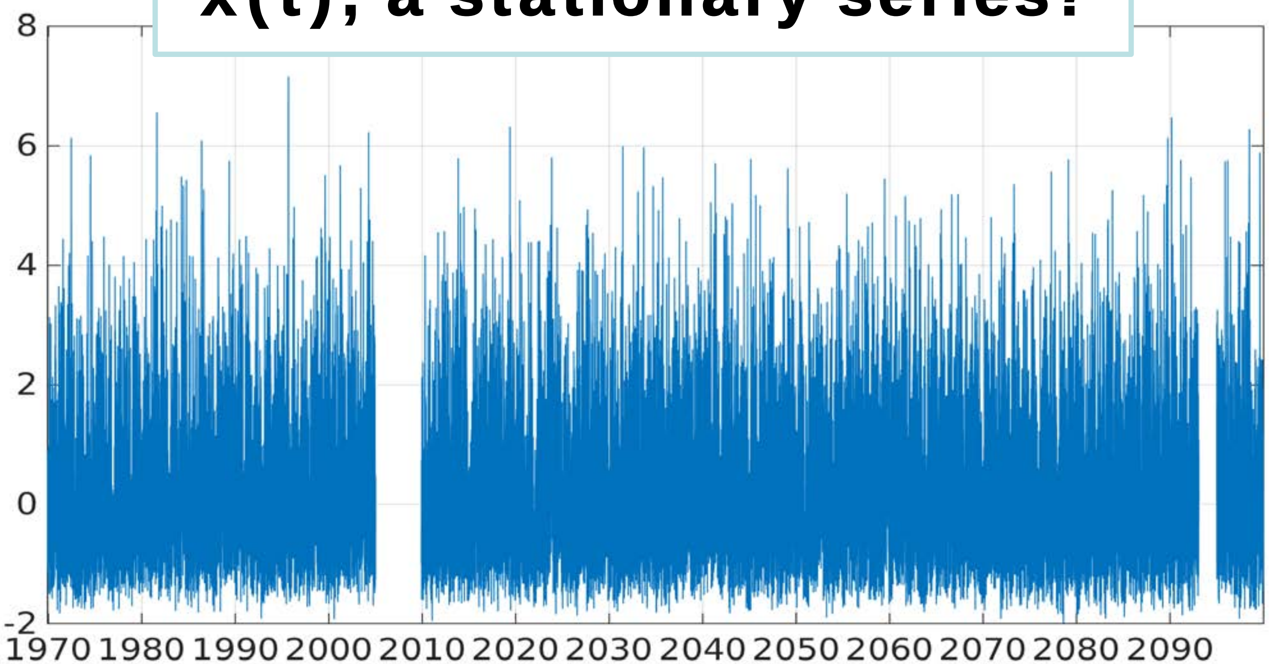
There are a lot of possible ways for estimating $tr_y(t)$ and $std_y(t)$

For example Fourier transform, polynomial regression, low pass filters, employment of climatic indexes …
We used a running mean and a “running standard deviation” on a time window of 20 years.

x(t), a stationary series?

The transformed series x(t) in this case looks stationary, but a stationarity test is needed.
Check the higher order statistics



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We can now find the GEV G_x best fitting the annual maxima of x(t), typically through a Maximum Likelihood Estimator (MLE)

$$G_X(x) = \Pr(X < x) = \exp\left\{ -\left[1 + \varepsilon_x \left(\frac{x - \mu_x}{\sigma_x} \right) \right]^{-1/\varepsilon_x} \right\}$$

How can we backtransform G_x into a non-stationary distribution G_y?
If we call f the transformation x(t)→y(t),

$$f(x,t) = y(t) = std_y(t) \cdot x + tr_y(t)\,,$$

$$f^{-1}(y,t) = x(t) = \frac{y(t) - tr_y(t)}{std_y(t)}\,,$$

$$G_Y(y,t) = \Pr[Y(t) < y] = \Pr[f(X,t) < y] = \Pr[X < f^{-1}(y,t)] = G_X[f^{-1}(y,t)]$$

We can always compute G_y this because g(x,t) is a monotonically increasing function of *x* for every time t, so we can always invert it.

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We find that G_y is a time varying GEV with

- Shape parameter $\varepsilon_y = \varepsilon_x$,
- Scale parameter $\sigma_y(t) = std_y(t) \cdot \sigma_x$,
- Location parameter $\mu_y(t) = std_y(t) \cdot \mu_x + tr_y(t)$

Relationship with the usual approach

It is like best fitting through MLE a non stationary GEV G_{ns} with parametric ε, σ, μ given by

$$\varepsilon_{ns} = const.,$$
$$\sigma_{ns} = std_y(t) \cdot a ,$$
$$\mu_{ns} = std_y(t) \cdot b + tr_y(t)$$

for varying parameters a and b.

The MLE in facts returns: $a = \sigma_{x,} \quad b = \mu_x$

Extension to GPD distribution

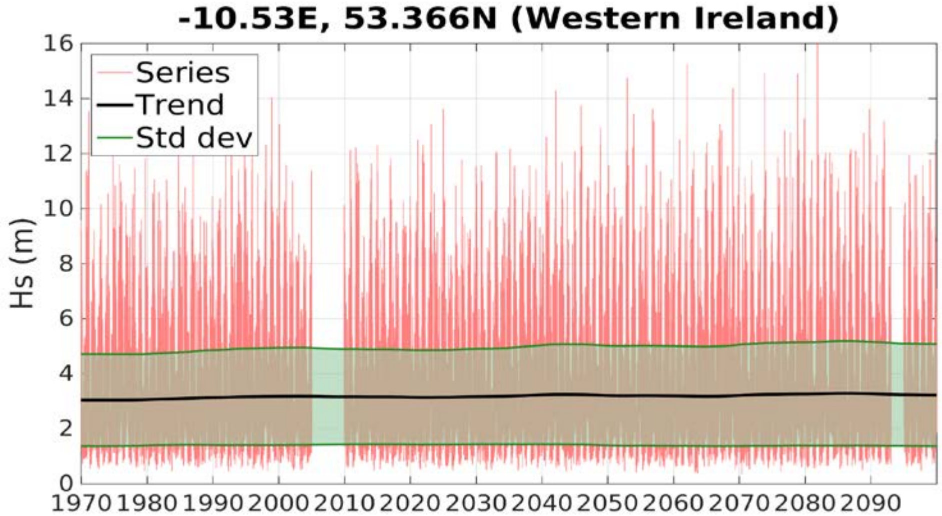
Generalized Pareto Distribution (GPD) is derived from the GEV as the conditional probability that an observation beyond a given threshold *u* is greater than x. **Therefore the arguments valid for GEV are valid also for GPD.**

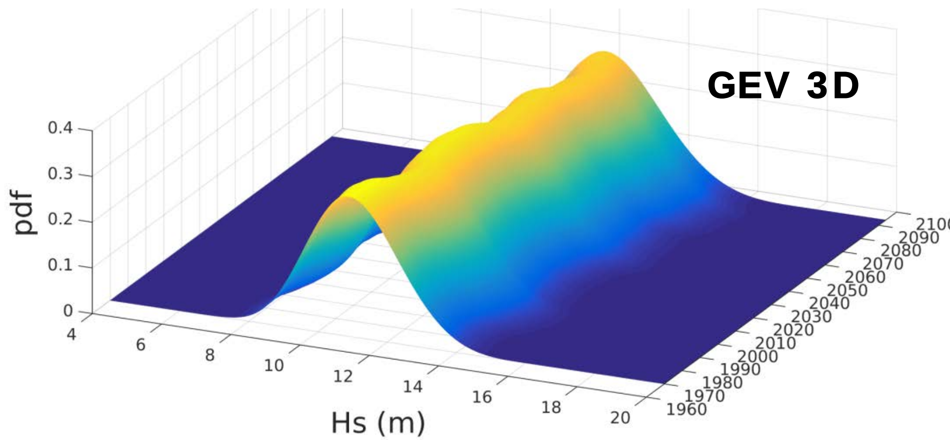
The parameters of the non stationary GPD are:

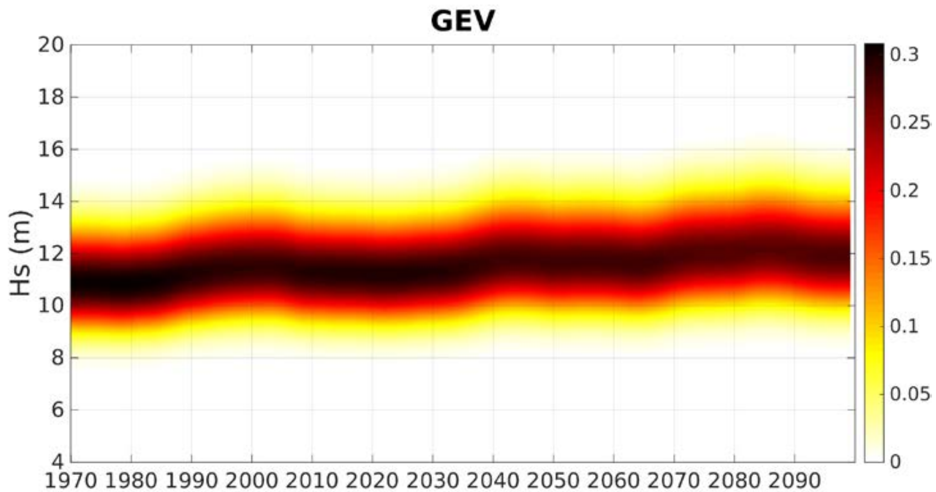
- threshold: $u_y(t) = std_y(t) \cdot u_x + tr_y(t)$,
- shape parameter: $\varepsilon_y = \varepsilon_x = const.$,
- scale parameter: $\sigma_{GPD\,y}(t) = \sigma_y(t) + \varepsilon_y[u_y(t) - \mu_y(t)] = std_y(t) \cdot \sigma_{GPD\,x}$

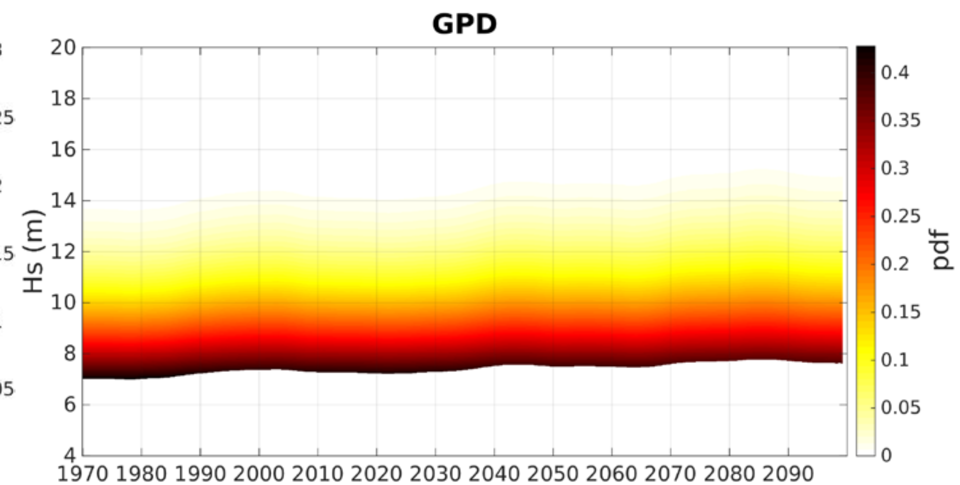
Case study 1

wind data: GFDL-ESM2M, scenario: RCP85, wave model: WWIII



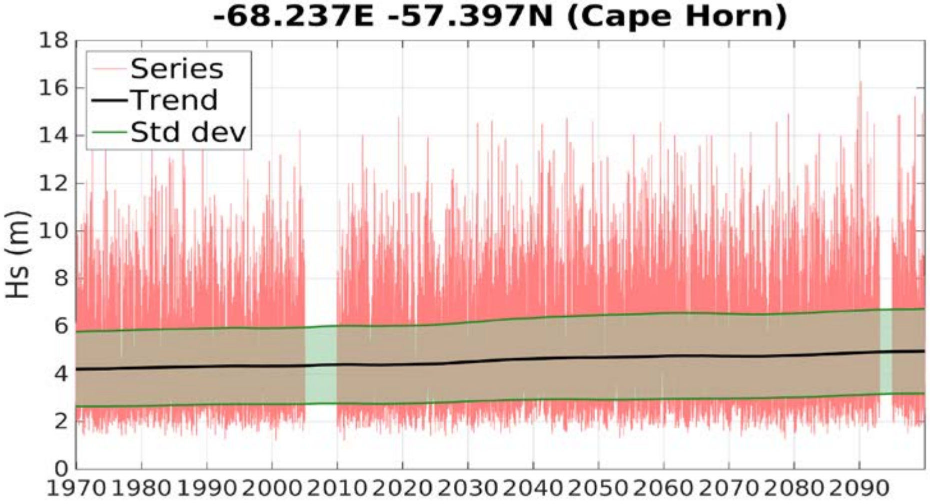


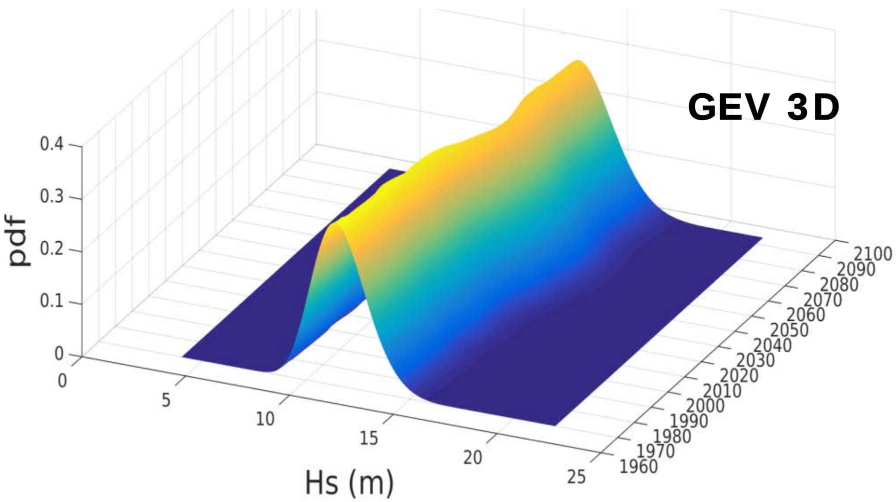


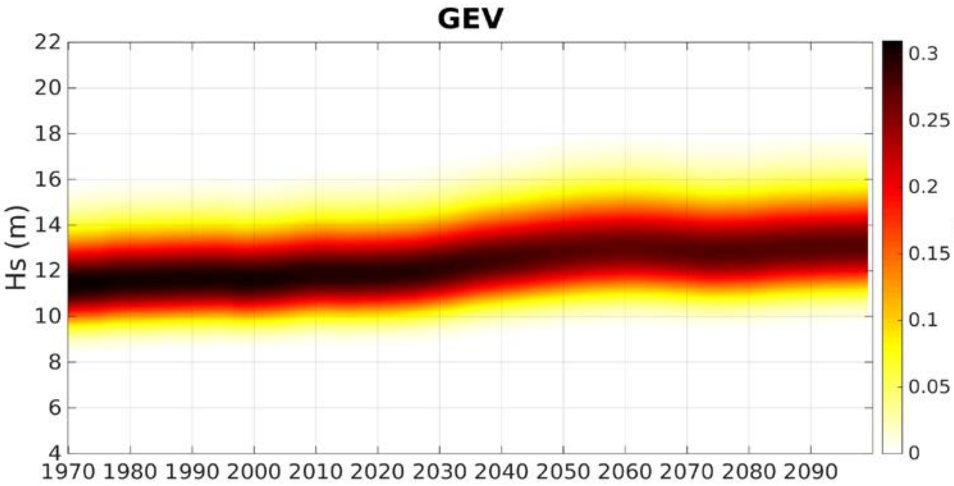


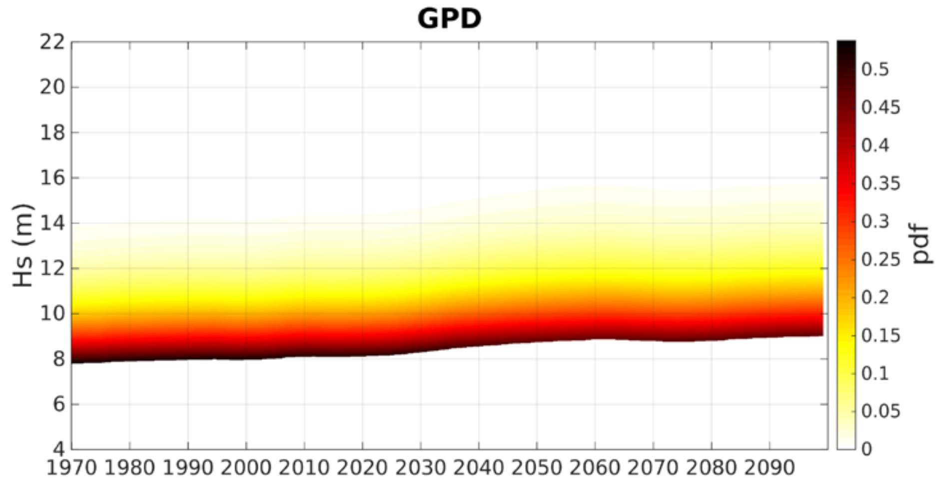
Case study 2

wind data: GFDL-ESM2M, scenario: RCP85, wave model: WWIII









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Adding the seasonality

The transformation y(t)-->x(t) is modified to:

$$x(t)=\frac{y(t)-tr_y(t)-sn_{tr}(t)}{std_y(t)\cdot sn_{std}(t)}$$

where $sn_{tr}(t)$ and $sn_{std}(t)$ are the seasonality of the trend and of the standard deviation respectively. The parameters of the GEV get:

- Shape parameter $\varepsilon_y=\varepsilon_x$,
- Scale parameter $\sigma_y(t)=std_y(t)\cdot sn_{std}(t)\cdot\sigma_x$,
- Location parameter $\mu_y(t)=std_y(t)\cdot sn_{std}(t)\cdot\mu_x+tr_y(t)+sn_{tr}(t)$

The seasonality coefficiets $sn_{tr}(t)$ and $sn_{std}(t)$ can be estimated from the monthly means respectively of the detrended series and of the ratio between a monthly-varying standard deviation and the slow-varying standard deviation.

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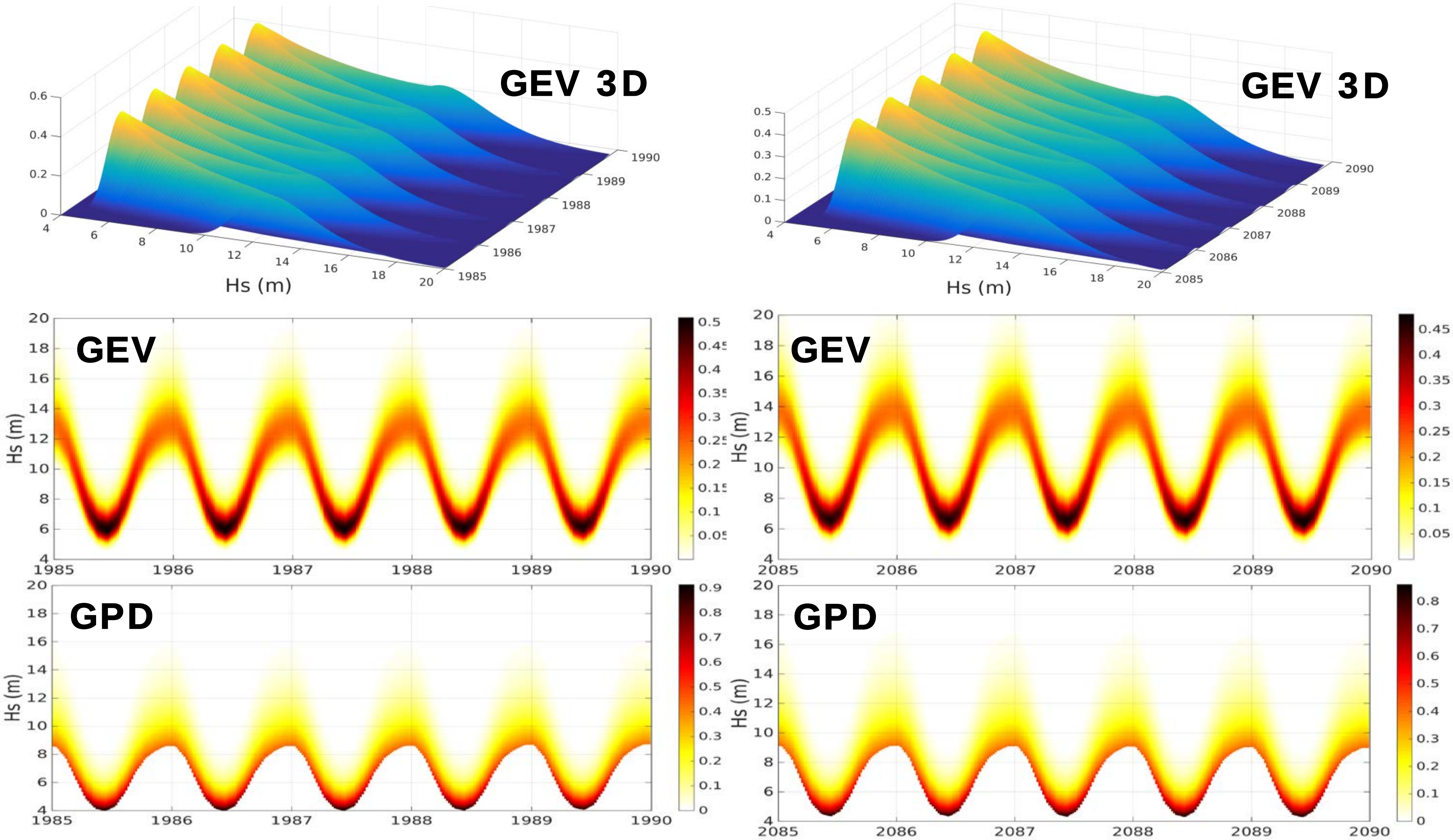
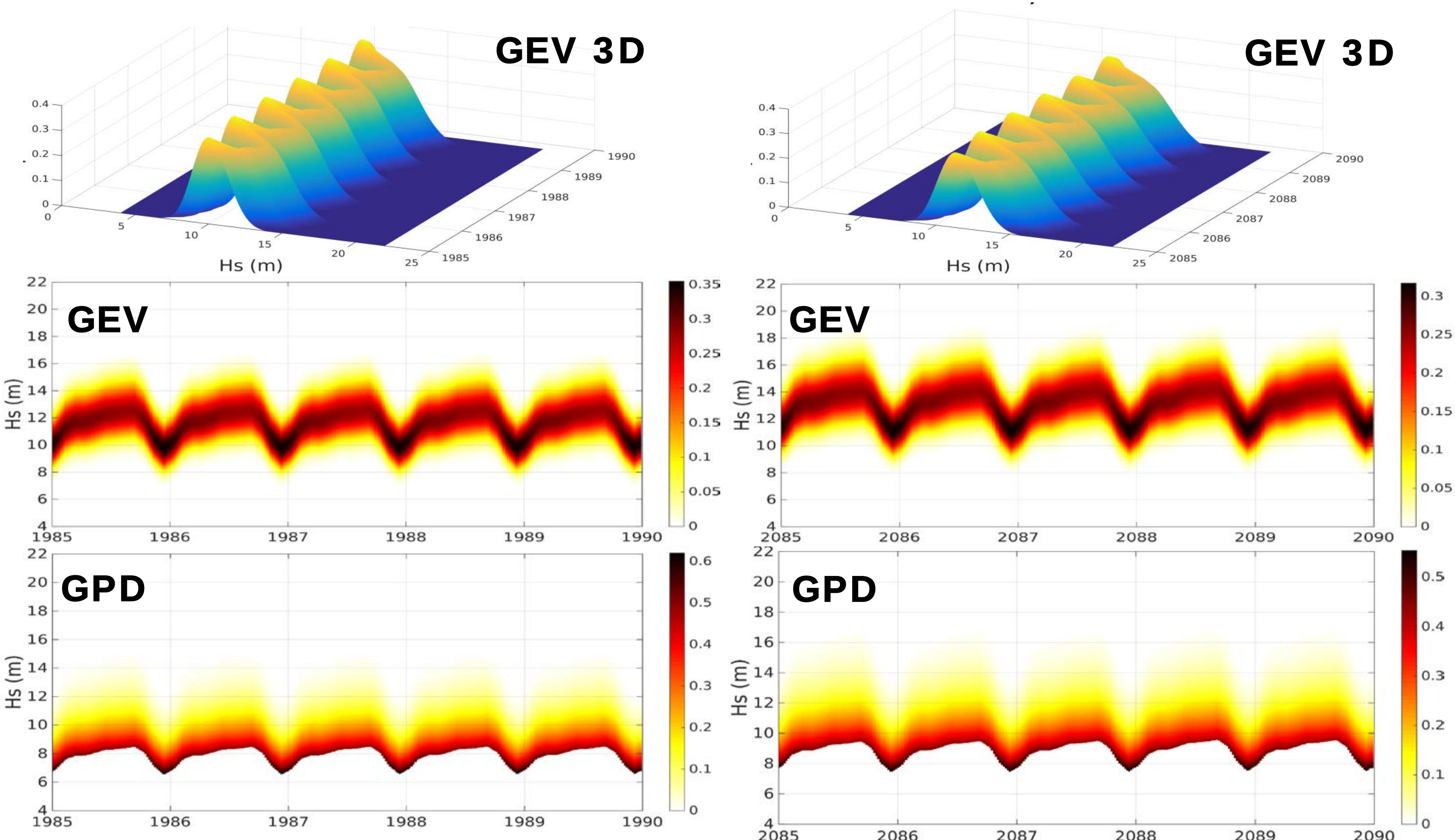
Case study 1: Western Ireland



Case study 2: Cape Horn



Advantages of this approach

• simple to implement and fast to run (with the formulations of tr(t) and std(t) illustrated in these slides)

• all you need is the series itself

• the transformation of the series to stationary makes it possible to verify the appliability of EVA and MLE

Disavantages

• the current implementation is not so general as the usual approach, you limit your analysis to 2 parameters. However some resources (e.g. Coles 2001) suggest that simple models should be preferred to complex ones

Possible generalization?

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Thank you!

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