

Improving wave model validation based on RMSE

L. Mentaschi¹, G. Besio¹, F. Cassola², A. Mazzino¹

¹ Dipartimento di Ingegneria Civile, Chimica ed Ambientale (DICCA),
Università degli studi di Genova

² Dipartimento di Fisica (DIFI), Università degli studi di Genova

Motivation

- This work has been developed during a tuning of WWIII in the Mediterranean sea.
- RMSE, NRMSE and SI provided unsatisfactory indication of wave model performances during a validation of WWIII in the Mediterranean sea in storm conditions.
- **A best fit of the model based on RMSE led to parametrizations affected by strong negative bias.**

Conclusions

- RMSE, NRMSE and SI tend to be **systematically better for simulations affected by negative bias**. This is mostly evident when:

- We are tuning parameters involving an amplification of the results.
- Correlation coefficient appreciably smaller than 1 , $\rho < 0.9$.
- Standard deviation of the observations of the same order of the average,
 $\sigma_o / \bar{O} \sim 1$.

- The indicator HH introduced by Hanna and Heinold (1985), defined by:

$$HH = \sqrt{\frac{\sum (S_i - O_i)^2}{\sum S_i O_i}} = \sqrt{\frac{(S - O)^2}{SO}}$$

provides more reliable information, being minimum for simulations unaffected by bias.

Validation of wave model in the Mediterranean sea

Model Wavewatch III

- Ardhuin et al. (2010) source terms (41 parameterizations).
- Tolman and Chalikov (1996) source terms, not tuned to the Mediterranean sea conditions.

Statistical indicators used for validation

Normalized Bias (NBI)

$$NBI = \frac{(\bar{S} - \bar{O})}{\bar{O}}$$

Root Mean Square Error

$$RMSE = \sqrt{\overline{(S - O)^2}}$$

Normalized Root Mean Square Error (NRMSE)

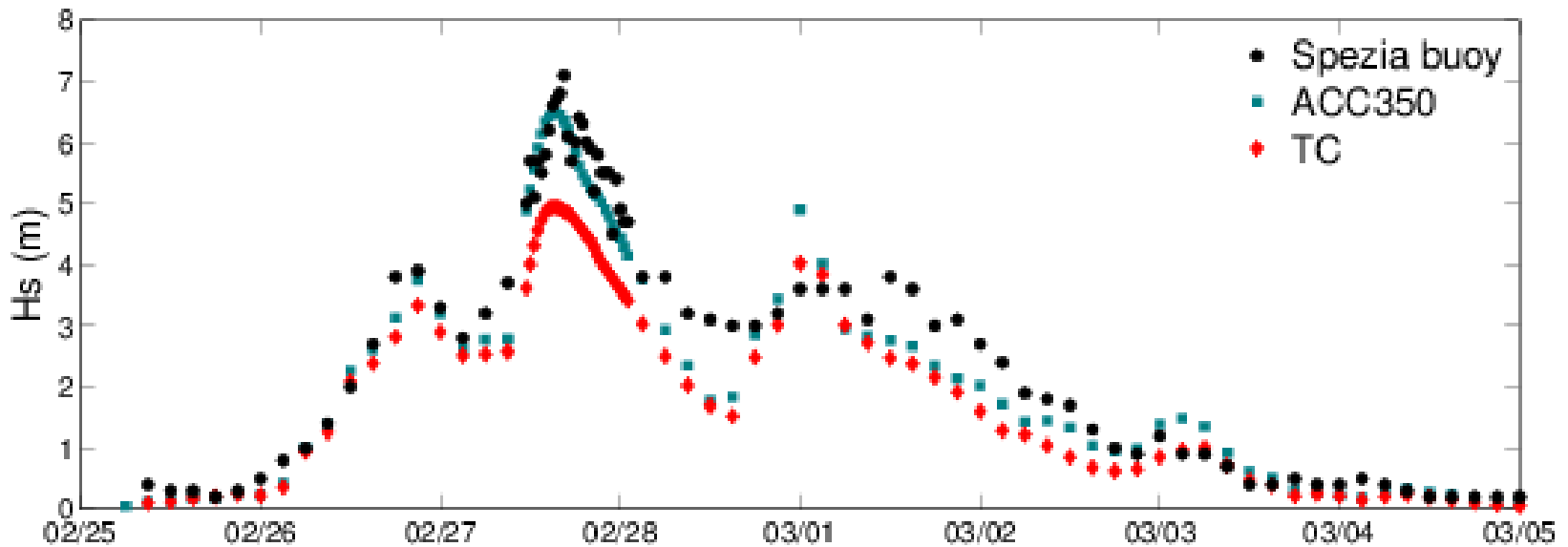
$$NRMSE = \sqrt{\frac{\overline{(S - O)^2}}{\bar{O}^2}}$$

Correlation coeff. (ρ)

$$\rho = \frac{\sum (S_i - \bar{S})(O_i - \bar{O})}{N\sigma_s\sigma_o}$$

Statistics on 17 storms and 23 buoys

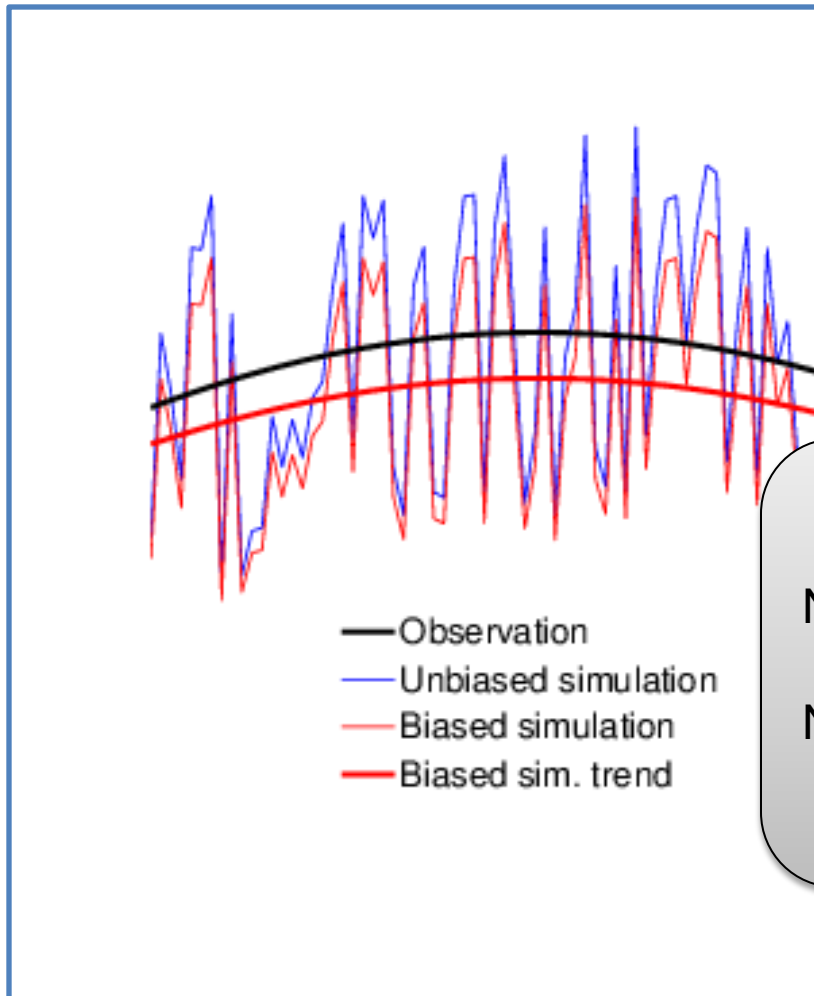
	ACC350	BJA	T & C
NBI	2.1%	-4.6%	-11.2%
ρ	0.889	0.885	0.883
NRMSE	0.2864	0.2800	0.2798 (-2.3%)



February 1990 storm

Drawback of using RMSE

A numerical example



$\rho=0.614$ for both of the simulations

NBI = -12% for the red simulation

One would say the best simulation is the blue one.

NRMSE(blue) = 0.384

NRMSE(red) = 0.356 (~ -7.2%)

Geometrical decomposition of RMSE

Scatter component

$$SC = \sqrt{\sum [(S_i - \bar{S}) - (O_i - \bar{O})]^2}$$

Bias component $BI = \bar{S} - \bar{O}$

$$RMSE^2 = SC^2 + BI^2$$

Geometrical decomposition of NRMSE

Scatter component (also called Scatter Index)

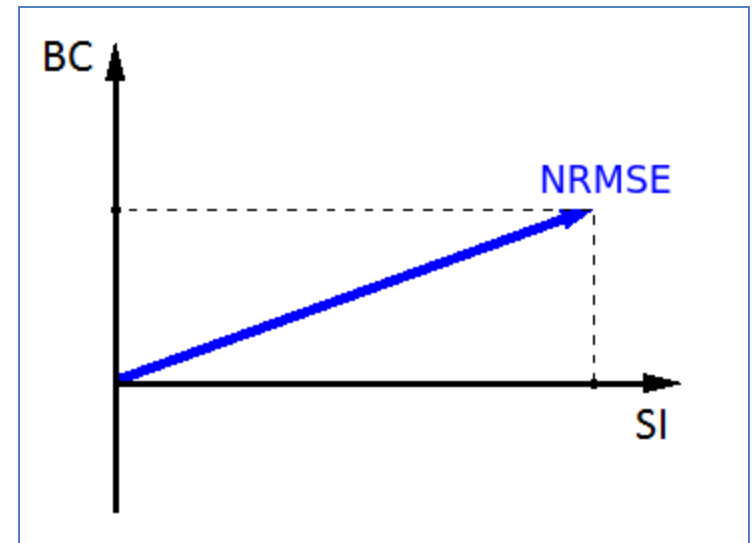
$$SI = \sqrt{\frac{\sum [(S_i - \bar{S}) - (O_i - \bar{O})]^2}{\sum O_i^2}}$$

Bias component

$$BC_{NRMSE} = \sqrt{\frac{\bar{O}^2}{\bar{O}^2 + \sigma_o^2}} NBI$$

$$NRMSE^2 = SI^2 + BC_{NRMSE}^2$$

Are SI and BC independent?



In general σ_s and \bar{S} are not independent.

Let's assume this relation:

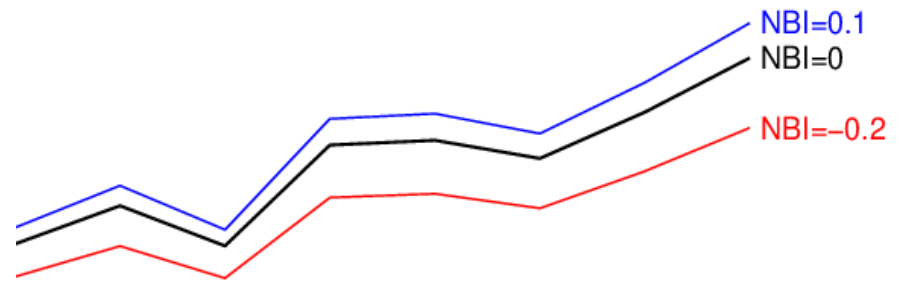
$$\frac{\sigma_s}{\bar{S}} \sim \text{const.}$$

Holds for amplifications

Example of set of simulations with constant ratio σ_s/\bar{S} :

$$S_{NBI\ i} = (1 + NBI)S_{0\ i}$$

where $S_{0\ i}$ is the unbiased simulation.



Amplification factor: $\alpha = 1 + NBI$

$$\frac{\sigma_s}{\bar{S}} \sim \text{const.}$$



$$\rho \sim \text{const.}$$

Relationship SI – NBI

We can express both \bar{S} and σ_s as functions of NBI

- $\bar{S} = \bar{O}(1 + NBI)$
- $\sigma_s / \bar{S} = \text{const.} \Rightarrow \sigma_s = \sigma_{s0}(1 + NBI)$

Also SI can be expressed as a function of NBI

$$\sigma_{s0} \sim \sigma_0$$



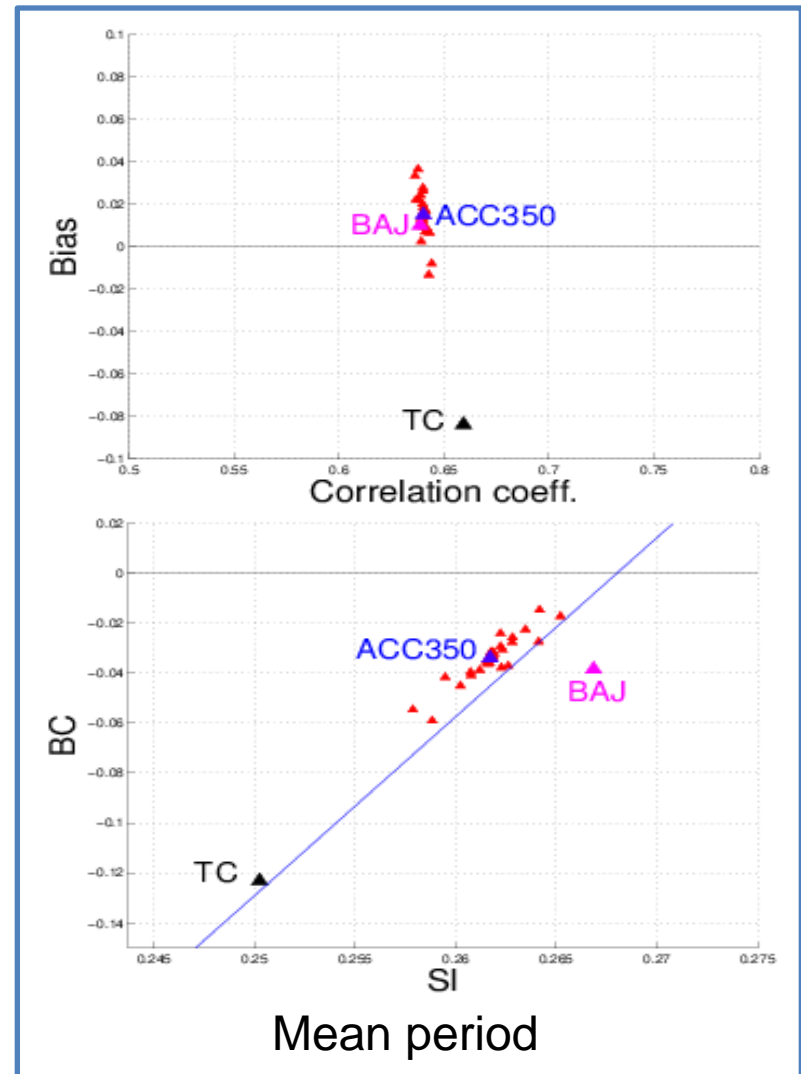
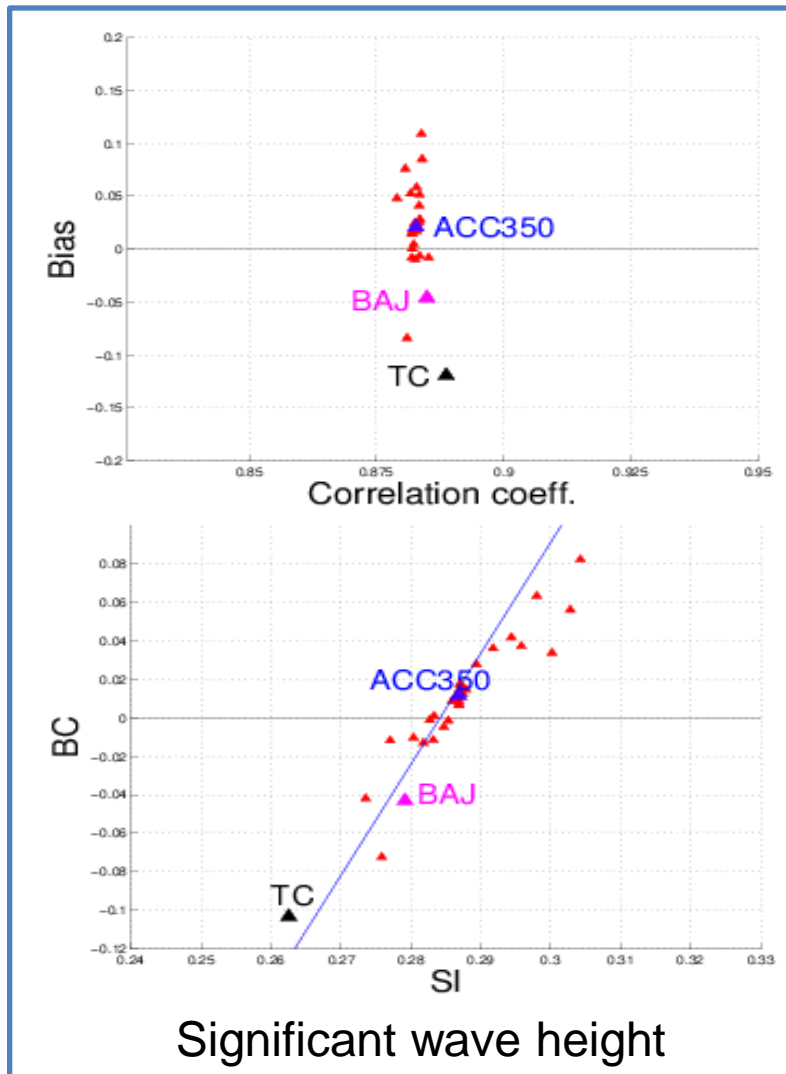
$$SI \sim SI_0 \left(1 + \frac{1}{2} NBI \right)$$

$$\left. \frac{\partial SI}{\partial NBI} \right|_{NBI=0} \sim \frac{1}{2} SI_0 > 0$$

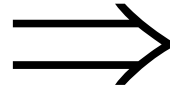
SI grows linearly in NBI around NBI=0

$$SI \sim SI_0 \left(1 + \frac{1}{2} NBI \right)$$

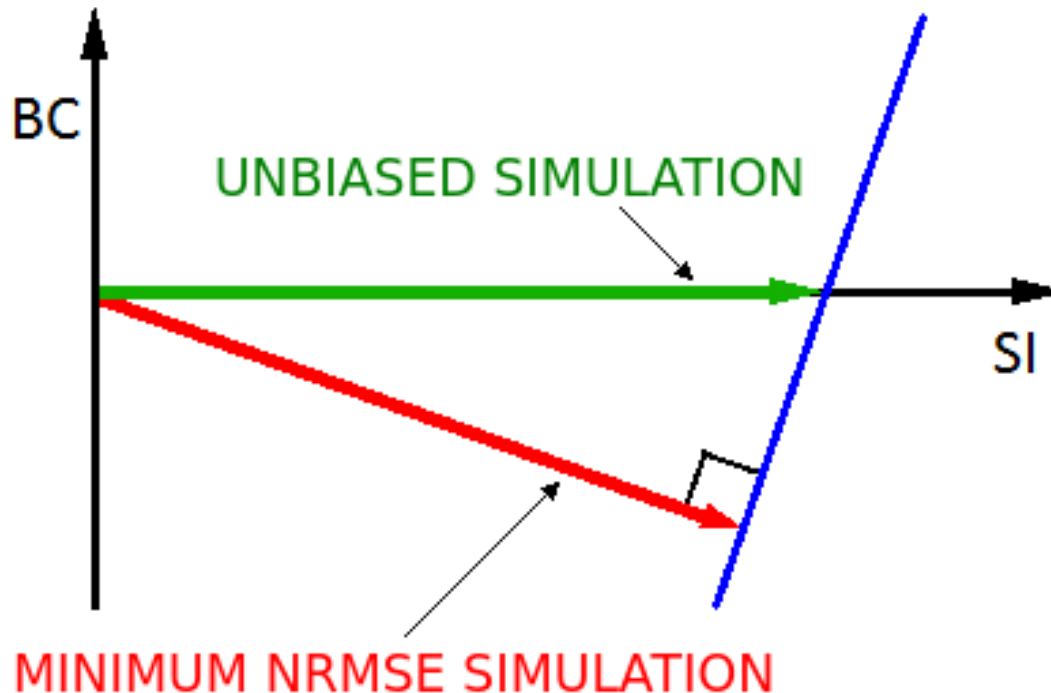
Fits well real world data



$$SI \sim SI_0 \left(1 + \frac{1}{2} NBI \right)$$



The simulation with the minimum value of RMSE/NRMSE underestimates the average value



Conditions when this effect is most evident

- Standard deviation of the same order of the mean

$$\frac{\sigma_o}{O} \approx 1$$

- Correlation appreciably smaller than 1 (0.7 - 0.9), since SI is minimum for

$$NBI = 1 - \rho$$

How to overcome this problem?

Hanna and Heinold (1985) indicator:

$$HH = \sqrt{\frac{\sum (S_i - O_i)^2}{\sum S_i O_i}} = \sqrt{\frac{(S - O)^2}{SO}}$$

Property of HH:

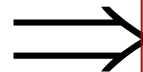
ρ constant  HH minimum for null bias

$$HH^2 = \frac{\sum (S_i - O_i)^2}{\sum S_i O_i} = \frac{\bar{S}^2 + \sigma_S^2 + \bar{O}^2 + \sigma_O^2}{\bar{S}\bar{O} + \rho\sigma_S\sigma_O} - 2$$

- $\bar{S} = \bar{O}(1 + NBI)$
- $\sigma_S / \bar{S} = \text{const.} \Rightarrow \sigma_S = \sigma_{S0}(1 + NBI)$
- $\sigma_{S0} \sim \sigma_O$

$$\left. \frac{\partial HH^2}{\partial NBI} \right|_{NBI=0} \sim 0$$

$$\left. \frac{\partial^2 HH^2}{\partial NBI^2} \right|_{NBI=0} \sim 2 \frac{\bar{O}^2 + \sigma_O^2}{\bar{O}^2 + \rho\sigma_O^2} > 0$$



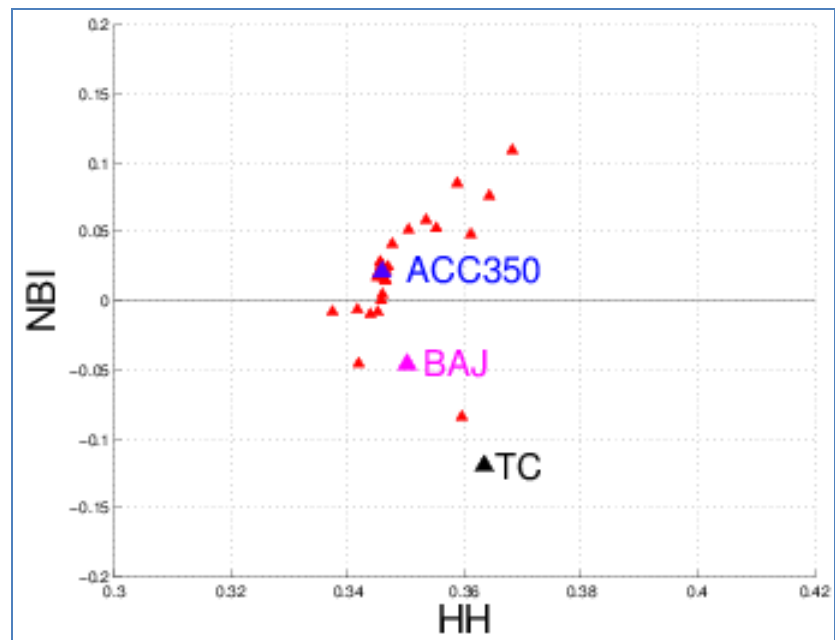
HH has a minimum
for null bias

Wavewatch III validation on the Mediterranean sea.

	ACC350	BJA	T & C
NBI	2.1%	-4.6%	-11.2%
ρ	0.889	0.885	0.883
NRMSE	0.2864	0.2800	0.2798 (-2.3%)
HH	0.3459	0.3502	0.3634 (+4.8%)

HH has a minimum
for null bias

Mentaschi et al. 2013



Conclusions

- RMSE, NRMSE and SI tend to be **systematically better for simulations affected by negative bias**. This is mostly evident when:
 - We are tuning parameters involving an amplification of the results.
 - Correlation coefficient appreciably smaller than 1, $\rho < 0.9$.
 - Standard deviation of the observations of the same order of the average,
$$\sigma_o / \bar{O} \sim 1$$
- HH indicator overcomes this problem introducing a different normalization of the root mean square error.

Thank you!