THE SOURCE TERM BALANCE AND CONSISTENT SPECTRAL SHAPES IN 3G MODELS

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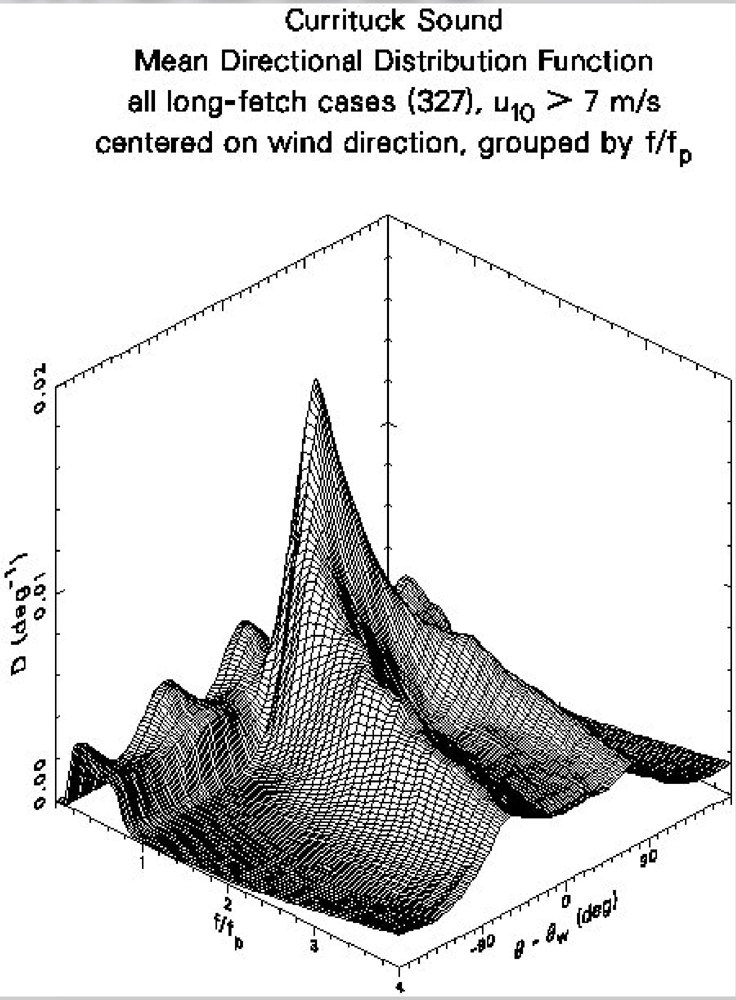
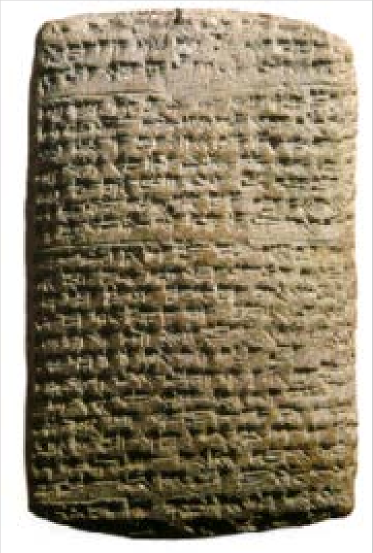


Introduction



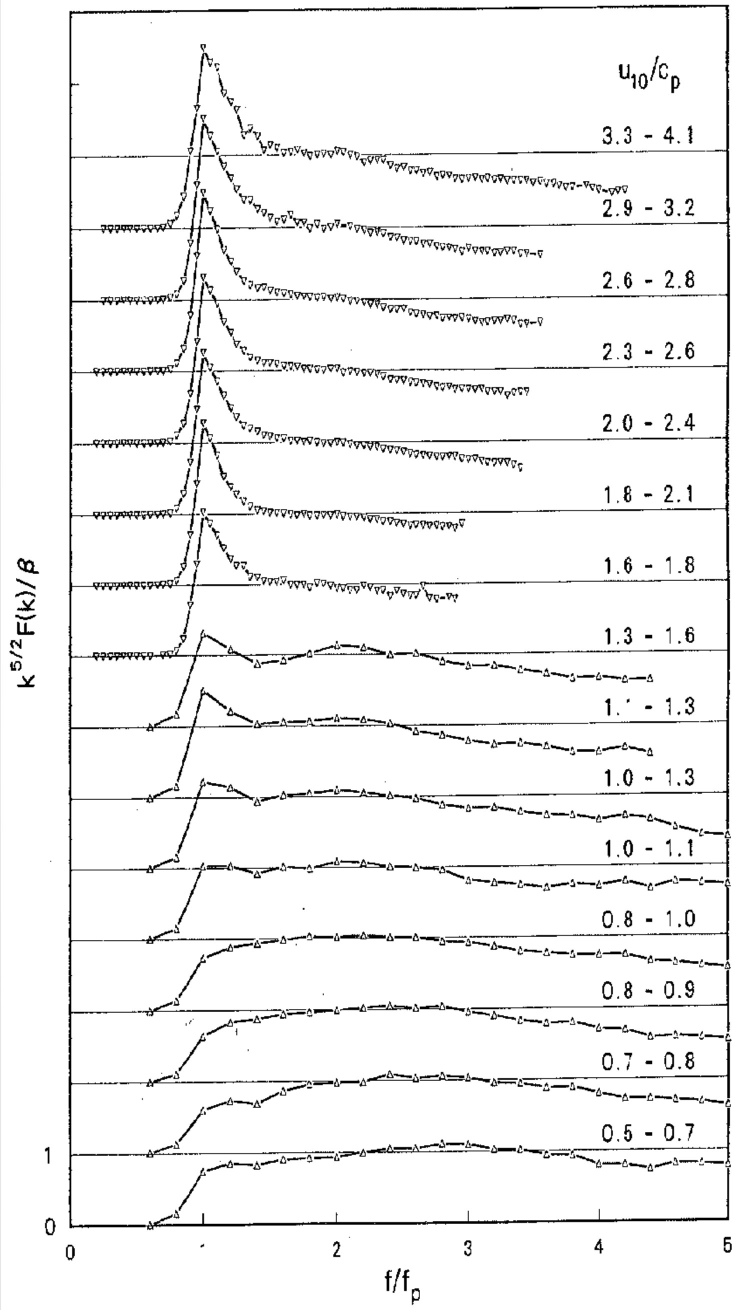
- Today, Spectral shape is becoming increasingly important
  - Affects higher order phenomena in waves
  - Remote sensing applications: often rely on knowledge of a portion of the spectrum to infer desired information
  - Coupled models rely on the detailed-balance shape and source characteristics
    - Surge-wave models
    - Wind-wave coupling
    - Wave transformations near a coast
- Is it time to move past broad spectral shapes with the endpoint of a stationary, fully-developed Pierson-Moskowitz spectrum?
- This presentation will investigate a different approach to this issue

Motivation

- Operational wave modeling has made great strides over that last decade or so
- Skill scores continue to improve in terms of their ability to predict integrated wave parameters such as wave height and mean period

However:

- There are many terms in the operational models which are optimized empirically to match the integrated parameters but not for spectral shape
- Spectral shape continues to elude the model in terms of 1) spectral peakedness, 2) angular distributions, and 3) energy levels and shape of equilibrium range



MOTIVATION

- The initiation of 3G wave modeling was predicated on the need for an improved "detailed-balance" form for source terms
- WAMDIG (1988): "in order to treat all of the complexity of the wave-generation process in critical applications, it is important to examine the detailed balance of energy within each frequency-direction component of the spectrum individually."
- Spectra should evolve into correct shape since there would be no parametric constraints on shape
- Thus, spectral shape provides a critical basis for the examining the correctness of the detailed-balance in model source terms in a 3G context

Conclusions:

- To obtained more accurate spectral shapes in models, they must be built on an accurate detailed-balance Snl form
- Momentum, energy and action fluxes should be constant for modeled spectral shape in the equilibrium range
- Existing source terms do not accomplish this
- New wind input & dissipation terms postulated here appear to provide reasonable agreement with observations
- The stationary, fully-developed sea appear may be a bad paradigm for source term balance: particularly for low-frequency wave energy in the ocean

Methodology

1. Quick review of spectral shapes
2. Problems with existing source terms?
3. Potential new source terms
   - Snl
   - Wind input
   - Dissipation
4. Some test results
5. Conclusions

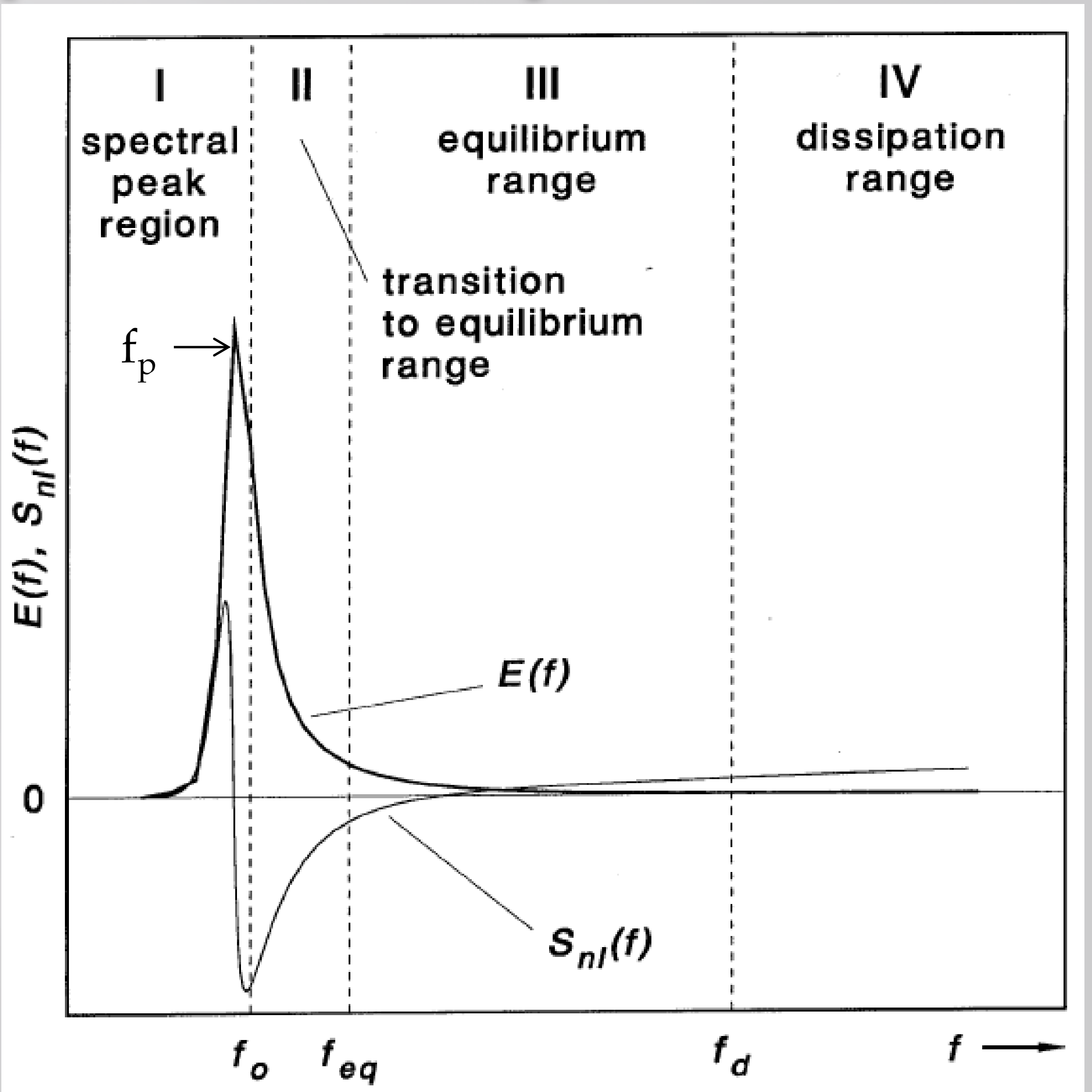
Spectral Shape

Four frequencies:

f_p peak frequency

f_0 "0-flux" frequency

f_d high-frequency region dominated by dissipation



Wave breaking with α = universal constant, JONSWAP: $\alpha=\alpha(gx/u^2)$

Phillips, 1958 $E(f) \sim \alpha_5 g^2 f^{-5}$

Wind input with α_4 = universal constant x energy flux from atmosphere

Toba, 1974 $E(f) \sim \alpha_4 u g f^{-4}$

where

α_4 is the equilibrium range coefficient and

u is term with units of velocity.

Wind input with α_4 = universal constant x momentum flux from atmosphere

Resio, Long, & Vincent 2004 $E(f) \square \alpha_4 (u^2 c_p)^{1/3} f^{-4}$

Where
C_p = phase velocity of spectral peak

Wave breaking (or something) changes the source balance at some transition frequency (f_t) above the spectral peak

Forristall, 1981 $E(f) \sim \alpha_4 u g f^{-4} \quad \rightarrow \quad \sim \alpha_5 g^2 f^{-5}$

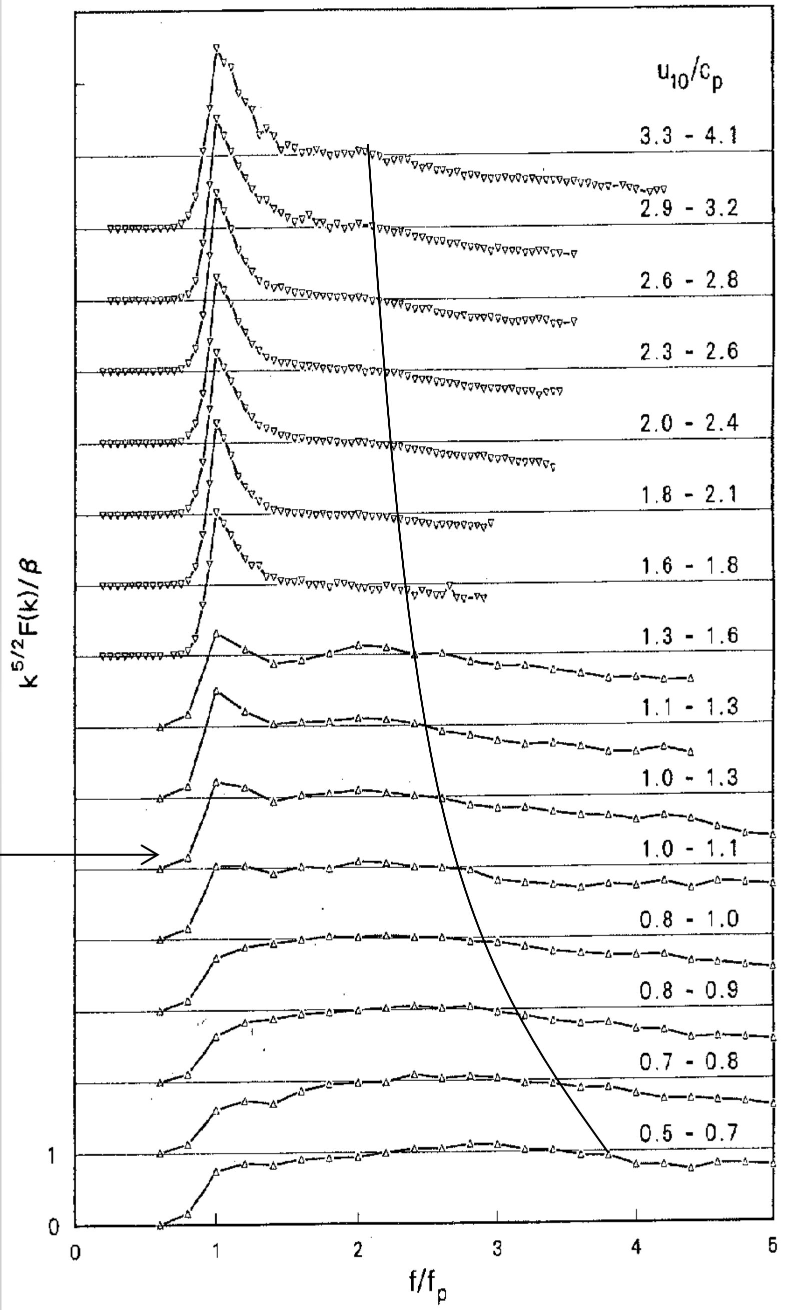
for $\hat{f}\ (= ufg^{-1}) > const.$

Many spectra from around the world are shown here, is what is termed a compensated spectral form.

In deep water this is an f^{-4} form with its energy level scaled by momentum flux

FULLY-DEVELOPED FORM ?

Note that the "fully developed" form fits nicely into this pattern but is not an "end-point" to it.



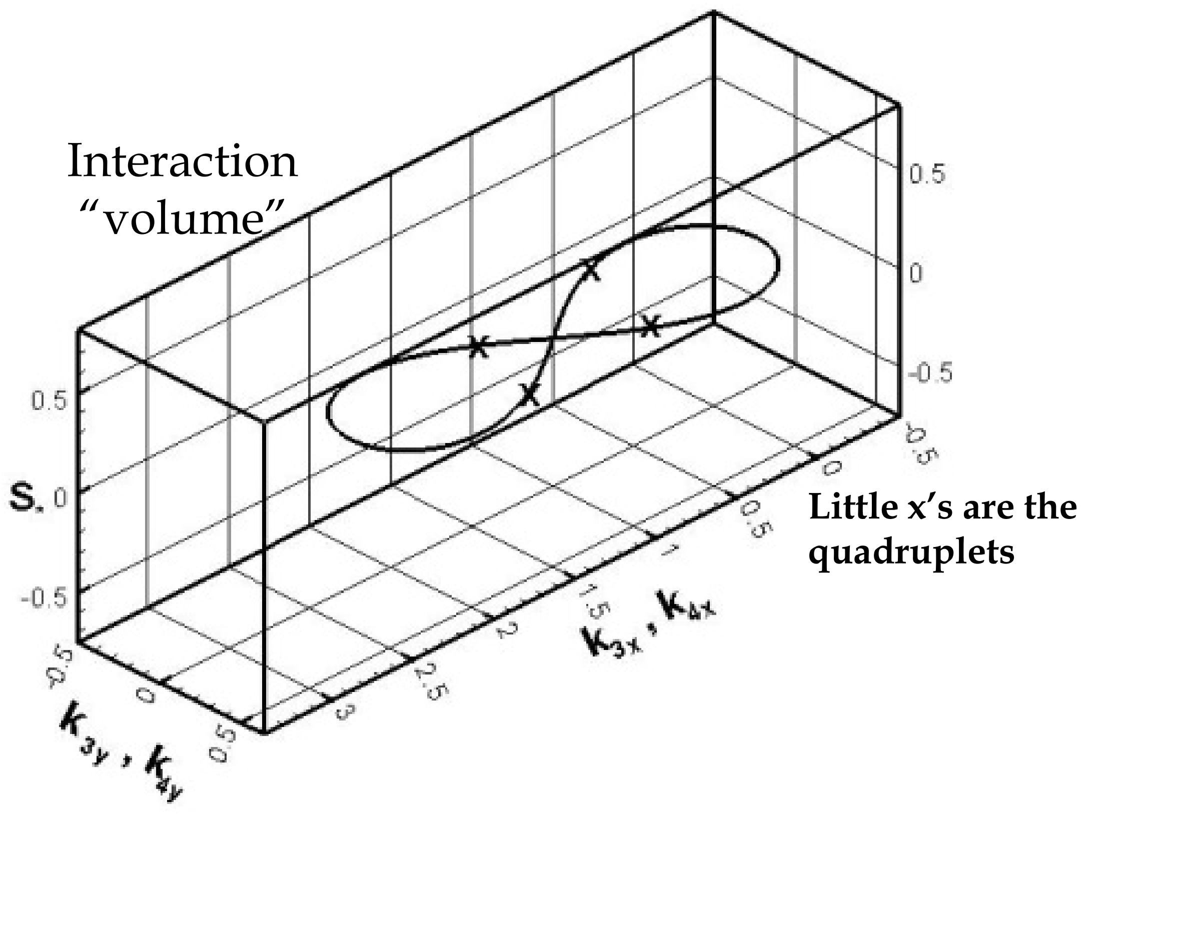
Source Terms

- **Nonlinear wave-wave interactions**
  (4-wave, resonant)
- **Wind input**
- **Wave dissipation (breaking)**

Snl The “figure-8” line used in the DIA, as derived earlier by Phillips (1962), effectively reduces the integral for 3-wave interactions. These do not contribute to resonant interactions - only to quasi-resonant interactions

Phase volume of a line = 0 → means that SNL =0

This wave further reduced by picking only 4 points on the “figure 8” (DIA) or multiple points (MDIA)



So how does it become finite in models?

Discretization loosens the constraint of k1=k2 to |k1-k2|<ε

This produces a <u>volume</u> dependent on the choice of ε typically taken to be the size of the f-theta resolution and an additional wavenumber factor in the scaling coefficient

This yields a finite result but 1) only represents a small sample of the total interactions and 2) contains a fraction of total interactions which depend on spectral shape

DIA and MDIA use empirical coefficient to get a rough agreement with the full integral <u>*but cannot replicate fluxes of energy and momentum through the spectrum, flux divergence will not return spectrum to proper shape, and systematic biases with peakedness are inevitable (since this integral is "weighted" toward local interaction.*</u>

So how do models with the DIA produce reasonable wave height?

The net energy gain contribution due to the total energy can be viewed in terms of the net energy transfer from frequencies higher than a particular frequency, f_0, to frequencies lower than f_0, (i.e. point for which no net flux flows from one side to the other)

As long as the two other source terms are adjusted systematically to compensate for systemic errors in the integrated value of Snl on the low-frequency side of f_0, reasonable behavior for the integrated wave parameters (specifically wave growth) can be optimized parametrically.

But this optimization does not ensure a good relationship for the detailed balance which controls the shape of the directional spectrum

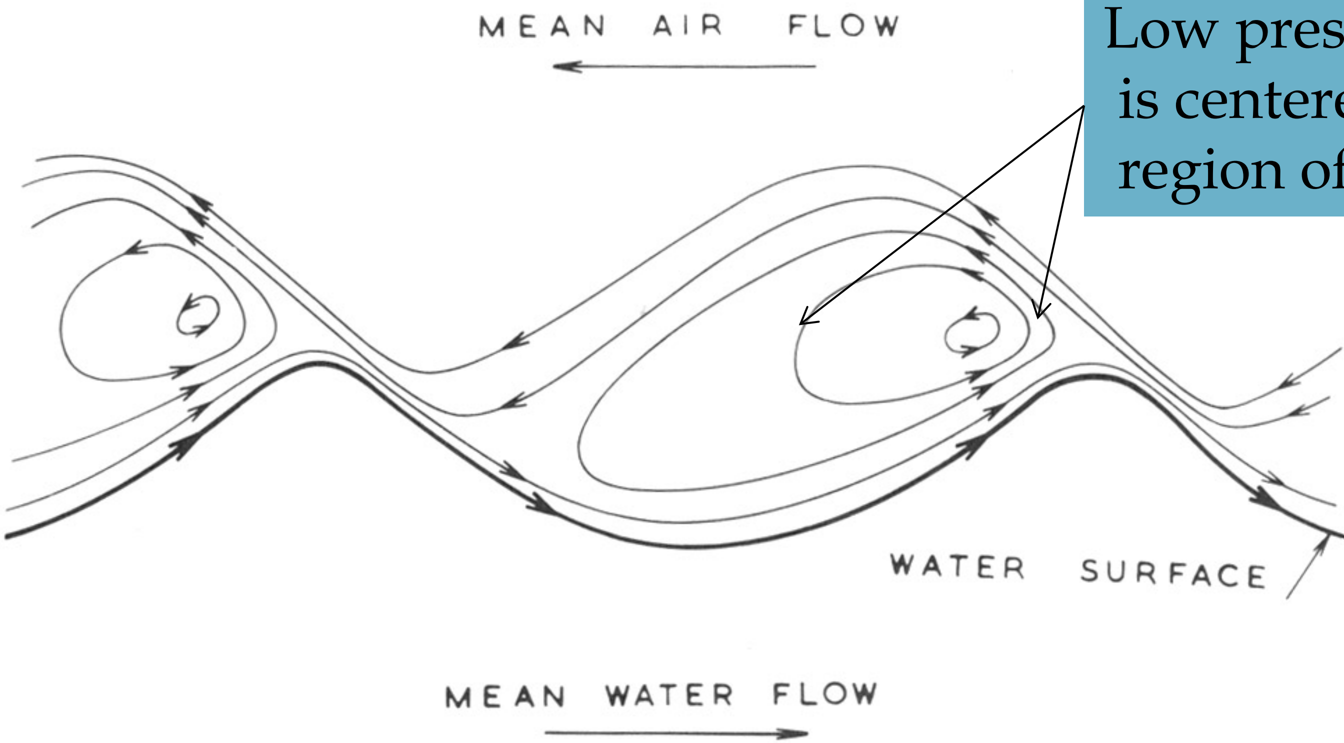
Wind Input

Miles postulated a mechanism by which momentum from a shear flow can transfer momentum from the atmosphere into the irrotational flow in the wave field for a *monochromatic unidirectional* wave

Basic concept is that δp and w must be correlated (where p is pressure and w is the vertical water motion)

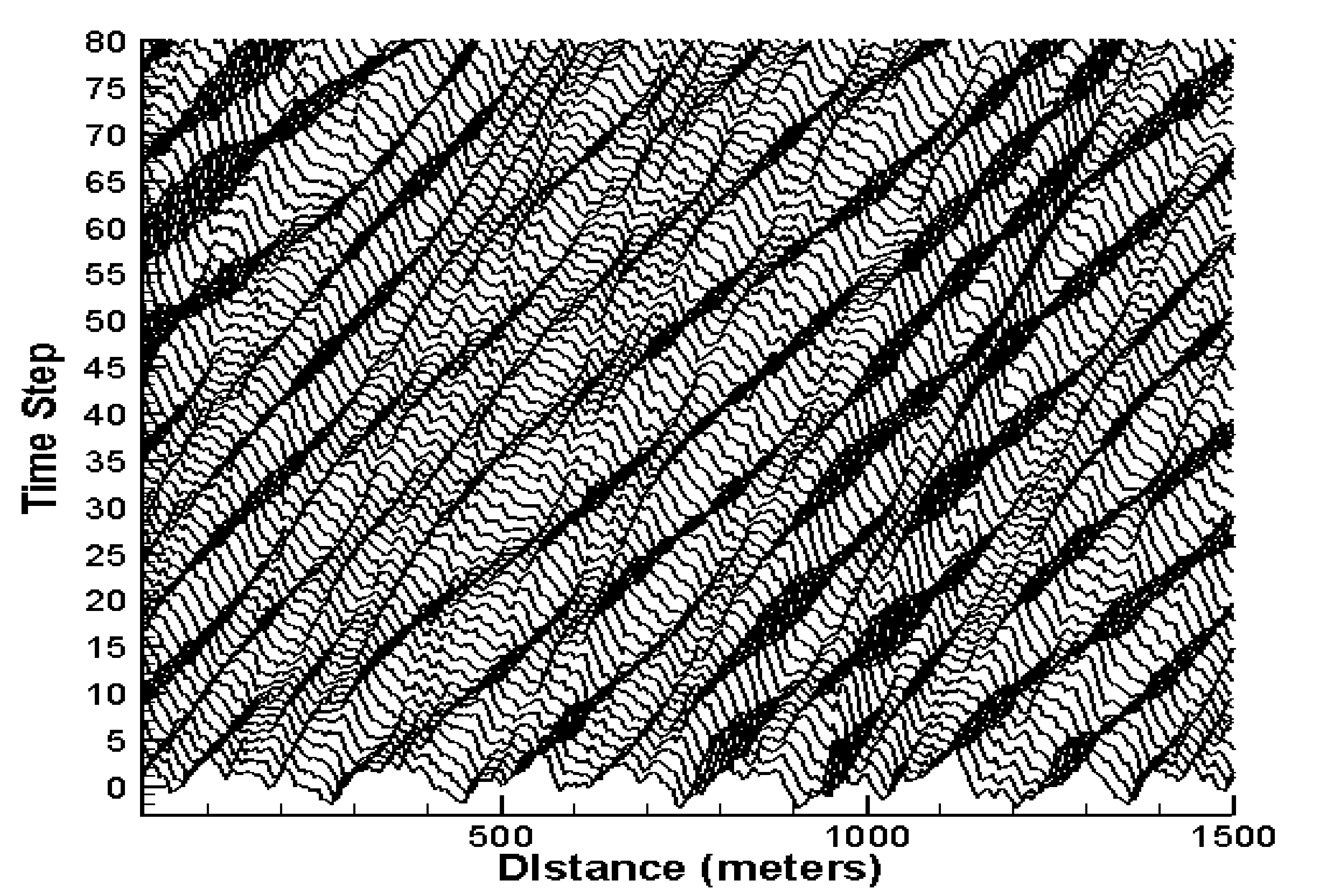
Extension to spectrum was linear superposition – but this assumes that pressure perturbations don't see the real water surface

Low pressure is
is centered on the
region of +w



Flow field in air passing over waves "visualized" from smoke injected into a laboratory flume. Frame of reference is moving with the phase speed of the spectral peak. Note that the "cats eyes" are shifted with respect to the wave crests.

We have formulated a new wind input term which operates on the water surface not individual spectral components



Our new source term estimates the pressure perturbations over moving water surfaces, which varies in time and space. Pressure perturbations are created by the superposition of spectral components not individual components

Wind Input

Monte Carlo simulations of water surface create moving pressure perturbations which are linked primarily to the large waves and travel with these waves for some number of wave periods

Using the moving pressure patterns created by the surface, the covariance of δp and w is calculated for the random phase spectra which create the water surface.

The resulting covariance structure is strongly positive in the vicinity of the spectral peak and concentrated near the direction of the wind

To convert to a wind input we normalize on expected momentum flux (only 1 free parameter – the percentage of total momentum entering the wave field which can be deduced from fetch growth measurements)

Wave Breaking: A new/old approach

- Irisov and others:
- Monte-Carlo simulations of dynamics of 2D, potential, and random surface gravity waves indicate that the dominant physical mechanism causing wave breaking appears to be the "concertina" effect (using the terminology introduced by Longuet-Higgins)

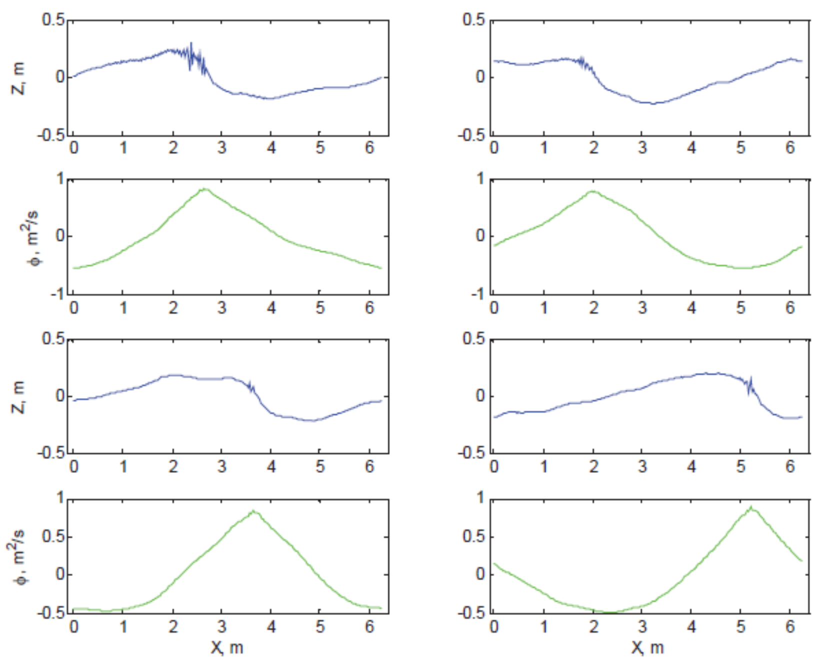


Figure 1. A few examples of the surface profile (blue) and velocity potential (green) at the of instability development.

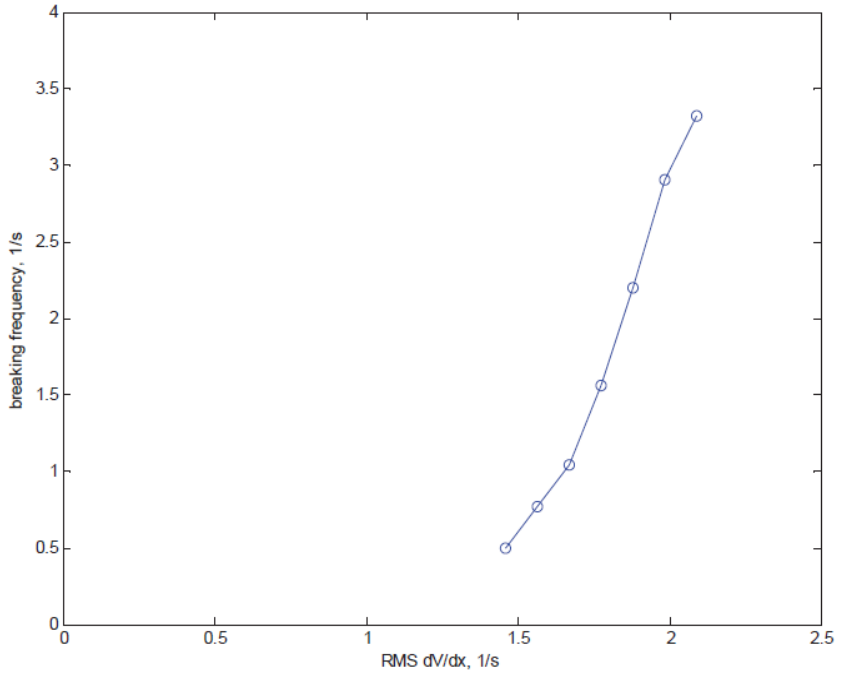


Figure 3. The dependence of the averaged breaking frequency on mean surfac current (negative) gradient.

Needs to be parameterized based on rms current scale

$$\left\langle {c_{x_1}}^2 \right\rangle = \int_0^{f_t} \int_{-\pi/2}^{+\pi/2} E(f,\theta)\cos(\theta)\omega^2 d\theta df \;\square\; \Phi_1(\sigma_\theta^2)\beta F(\gamma)c_p \int_0^{f_t} f^{-4} df$$

$$\left\langle {c_{x_2}}^2 \right\rangle = \int_t^{\infty} \int_{-\pi/2}^{+\pi/2} E(f,\theta)\cos(\theta)\omega^2 d\theta df \;\square\; \Phi_2(\sigma_\theta^2)\alpha \int_{f_t}^{\infty} \omega^2 f^{-5}$$

From this we can show that

$$\frac{f_t}{f_p} = z\hat{f}_p^{-1/3}\left[\frac{\frac{1}{2\pi} - \alpha^{1/2}}{\gamma^{1/2}}\right]$$

This produces the same general pattern of increasing transition frequency with increasing wave age

The only free parameters for this is γ, since α is determined by the constaint that $E_1(f) = E_2(f)$ at f_t

New Source Terms

No tunable coefficients for different basin sizes:

1 free parameter in wind input

1 free parameter in wave breaking

0 free parameters in Snl (Using FBI for these tests)

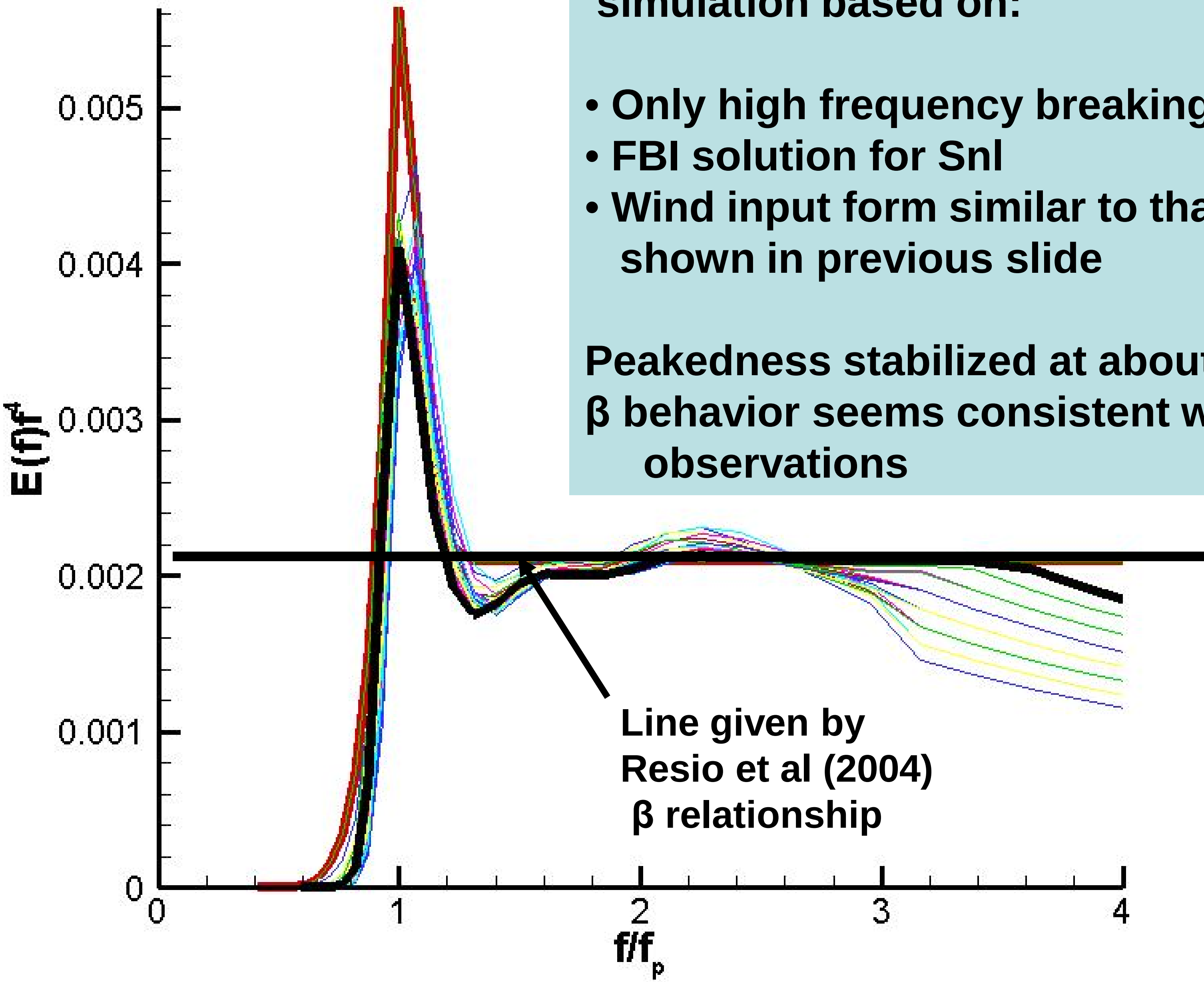
Snl plays the primary role in spectral shape once the somewhat narrower wind input is used and Sds is moved to higher frequencies

Compensated spectra from simulation based on:

- **Only high frequency breaking**
- **FBI solution for Snl**
- **Wind input form similar to that shown in previous slide**

Peakedness stabilized at about 2

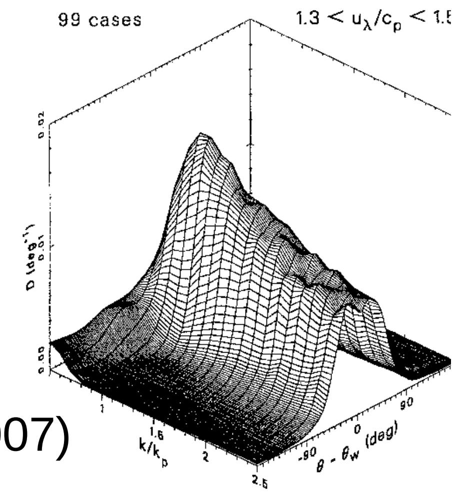
β behavior seems consistent with observations



Characteristics of directional distributions of energy:

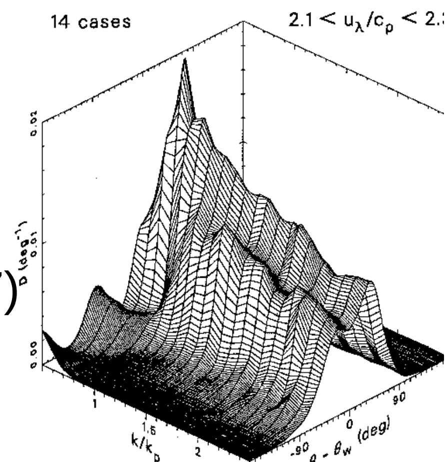
1. **“Young” waves are very bimodal**
2. **“Old” waves approach unimodal**
3. **Both distributions are similar to Hasselmann et al. (1980)**

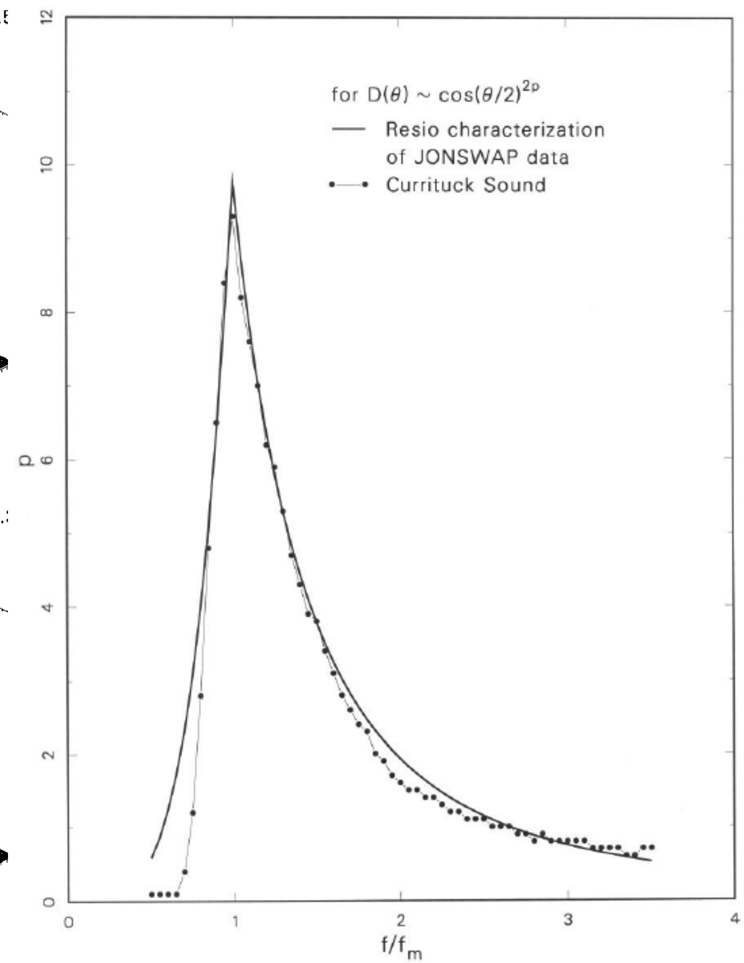
Low inverse wave age (old waves) (Long and Resio, 2007)



Directional form:

High inverse wave age (young waves) (Long and Resio, 2007)

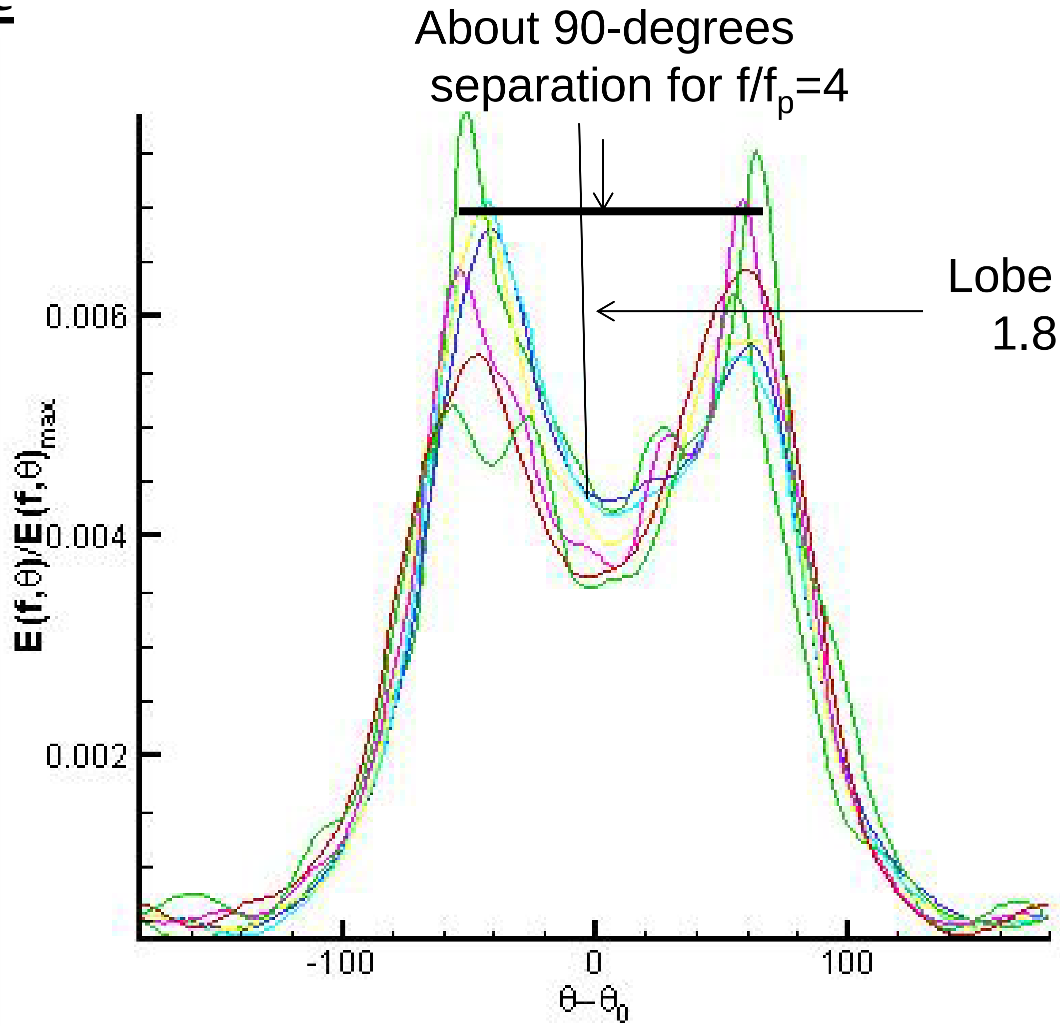




Variation in “n” obtained when fitting a cos^{2n} function compared to the data from Hasselmann *et al* (1980)

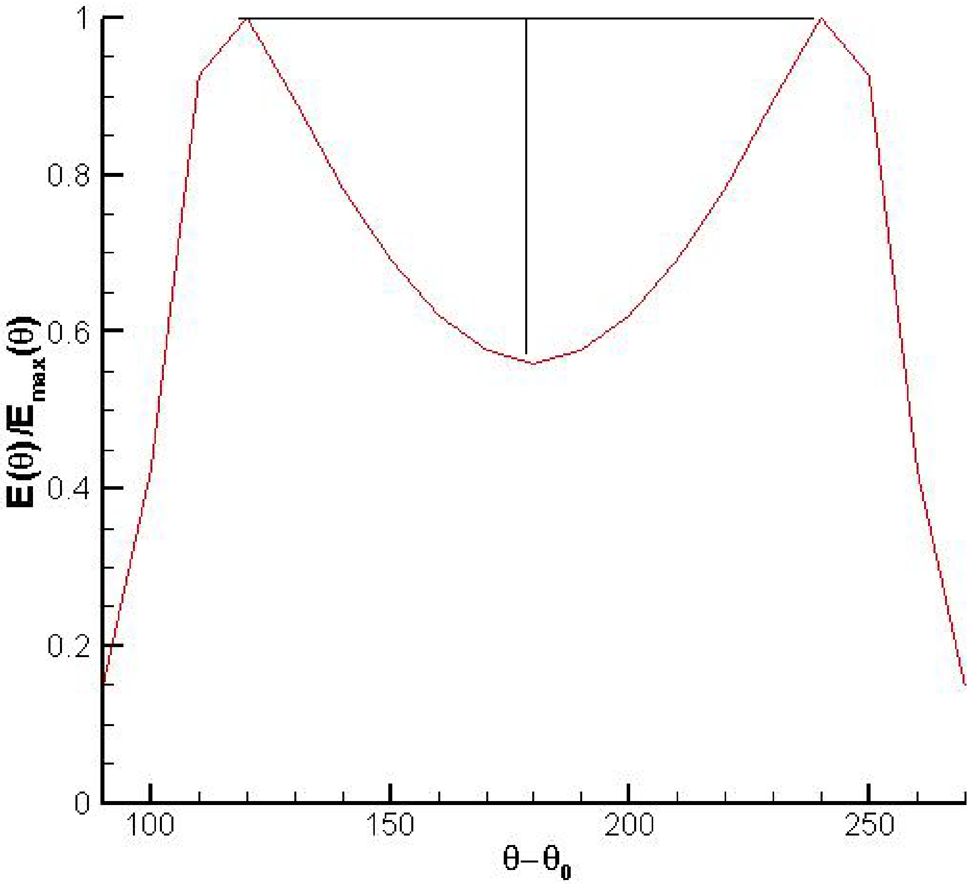
Directional Distribution depends on peakedness….

Essentially all recent studies have shown that the tail of the spectrum bifurcates at about f/f_p=1.4-2.0 Below we see data from Long and Resio (2004) study in Currituck sound for "slices" at $f/f_p>2$. **Not Stratified by wave age**

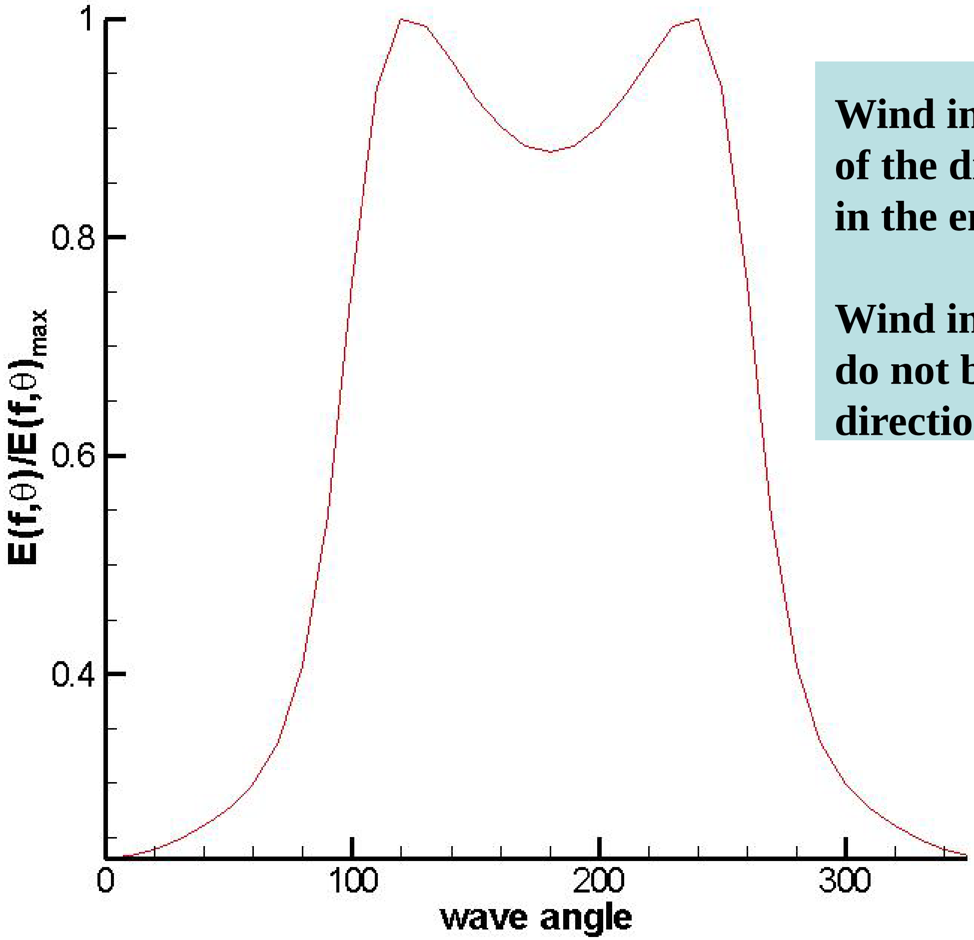


Simulated lobe ratio at $4f_p$ based on new source terms

Lobe ratio = 1.8 and peak separation = 120 degrees



Simulated lobe ratio at $4f_p$ based on WAM4 source terms
Using WRT for Snl

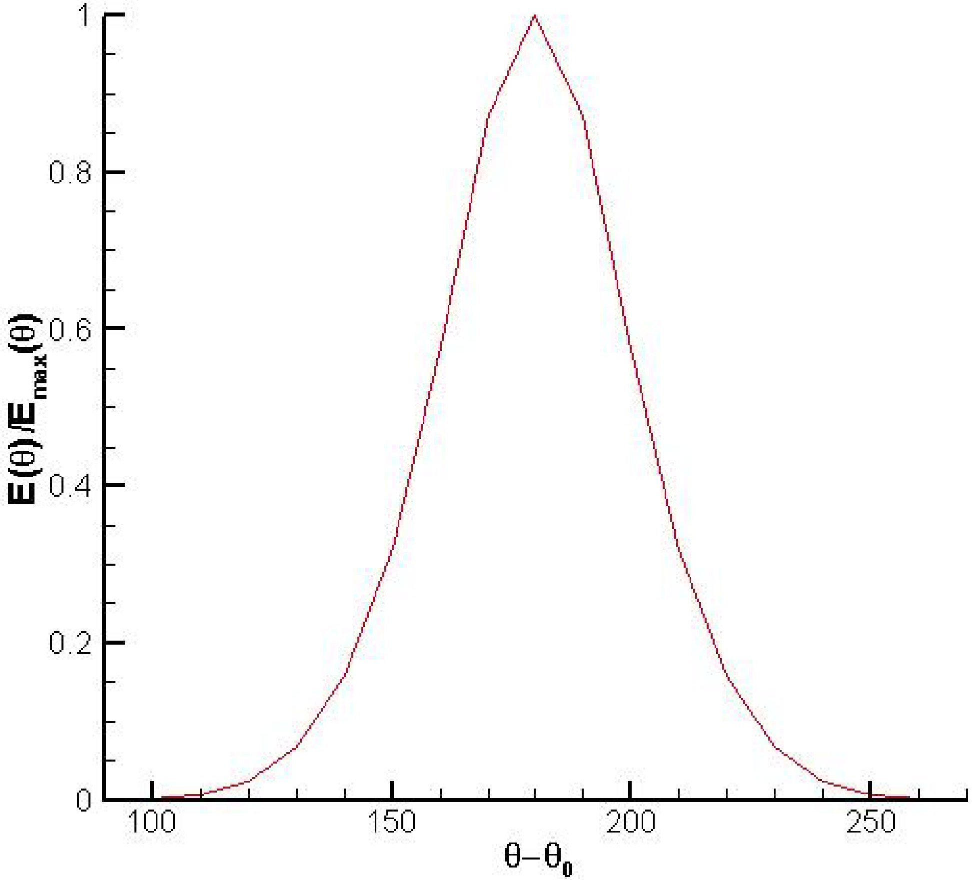


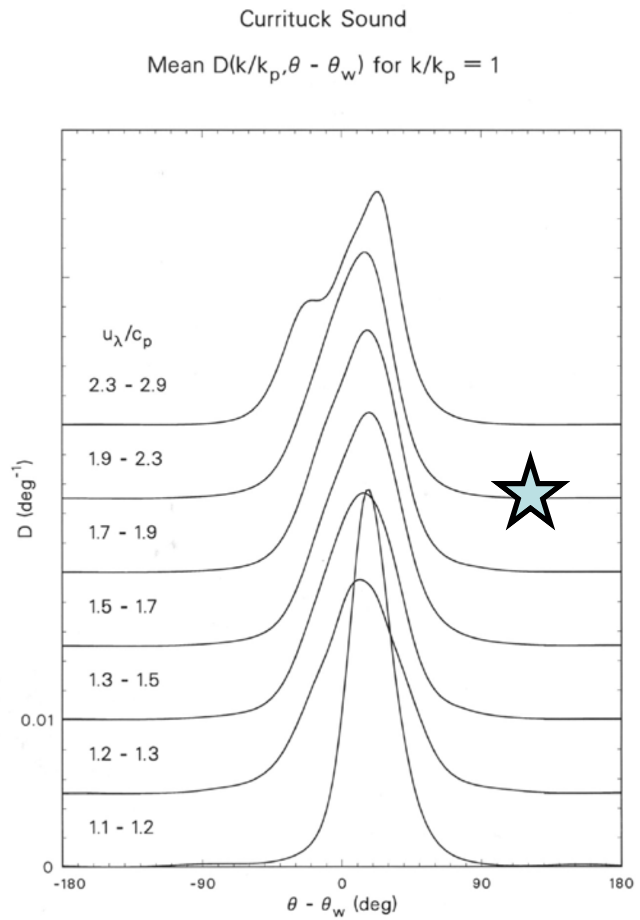
Wind input into the central portion of the directional distribution fills in the energies in this region.

Wind input and wave breaking do not balance with respect to directional characteristics

Lobe ratio is only about 1.12

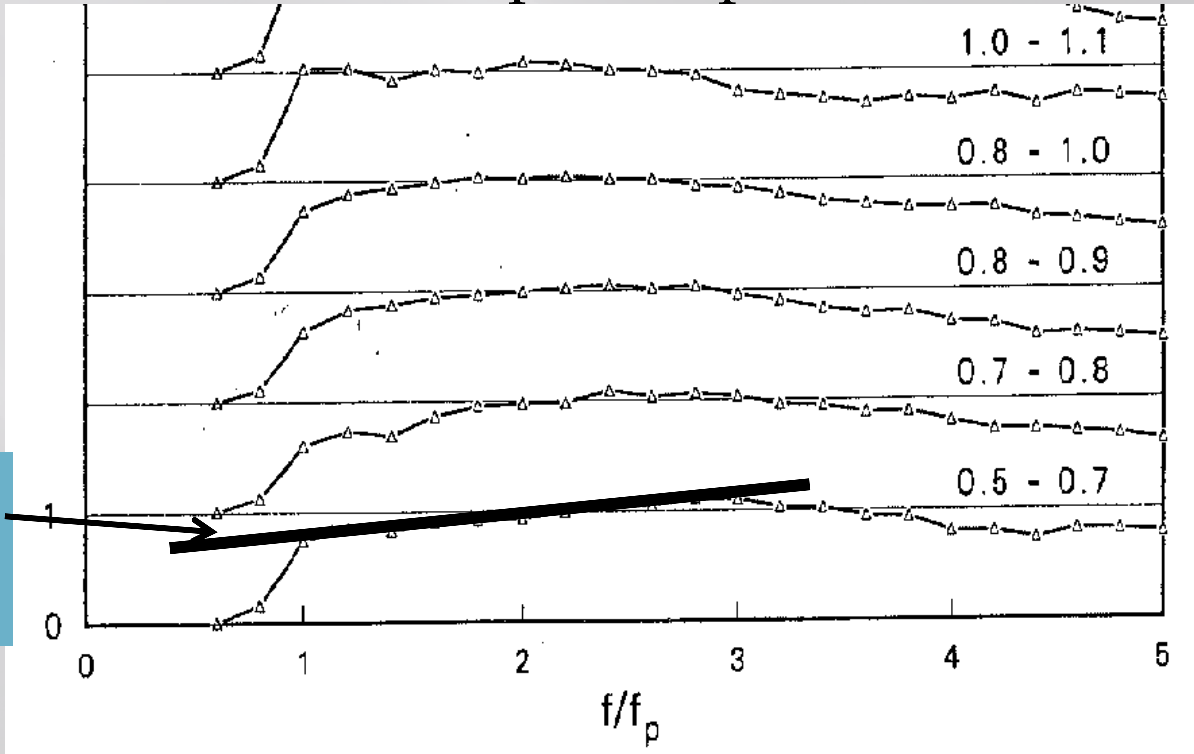
Simulated directional distribution of energy at spectral peak using new source terms





The source term combination of (FBI or TSA)+S_{in}+S_{ds} produce consistent spectral shapes with observations --- Of particular note:

The spectrum continues to evolve when the peak frequency reaches the "Pierson-Moskowitz" limit –more energy in low frequency and a shift in the spectral peak.



Spectral form for constant action flux.

Conclusions:

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QUESTIONS??