Presented by Dorukhan Ardag
Co-Authors: Donald T. Resio and Christopher Bender
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Problems in detailed-balance calculations of S_{nl} and possible alternative estimation methods.

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College of Computing, Engineering & Construction

Motivation

- The Discrete Interaction Approximation (DIA) is inaccurate for various real-life scenarios.
- The initial Two-Scale Approximation (TSA) has been shown to provide efficient estimation of nonlinear, four-wave interactions however constant $\cos^{2n}(\theta - \theta_0)$ spreading had some serious problems. (Perrie & Resio 2008, Resio et al 2011)
- The initial TSA was about order of a magnitude slower than the DIA in speed. This presentation will show that the TSA can be significantly faster while retaining its accuracy.
- This presentation will address these issues.

Overview

- **Goal:**

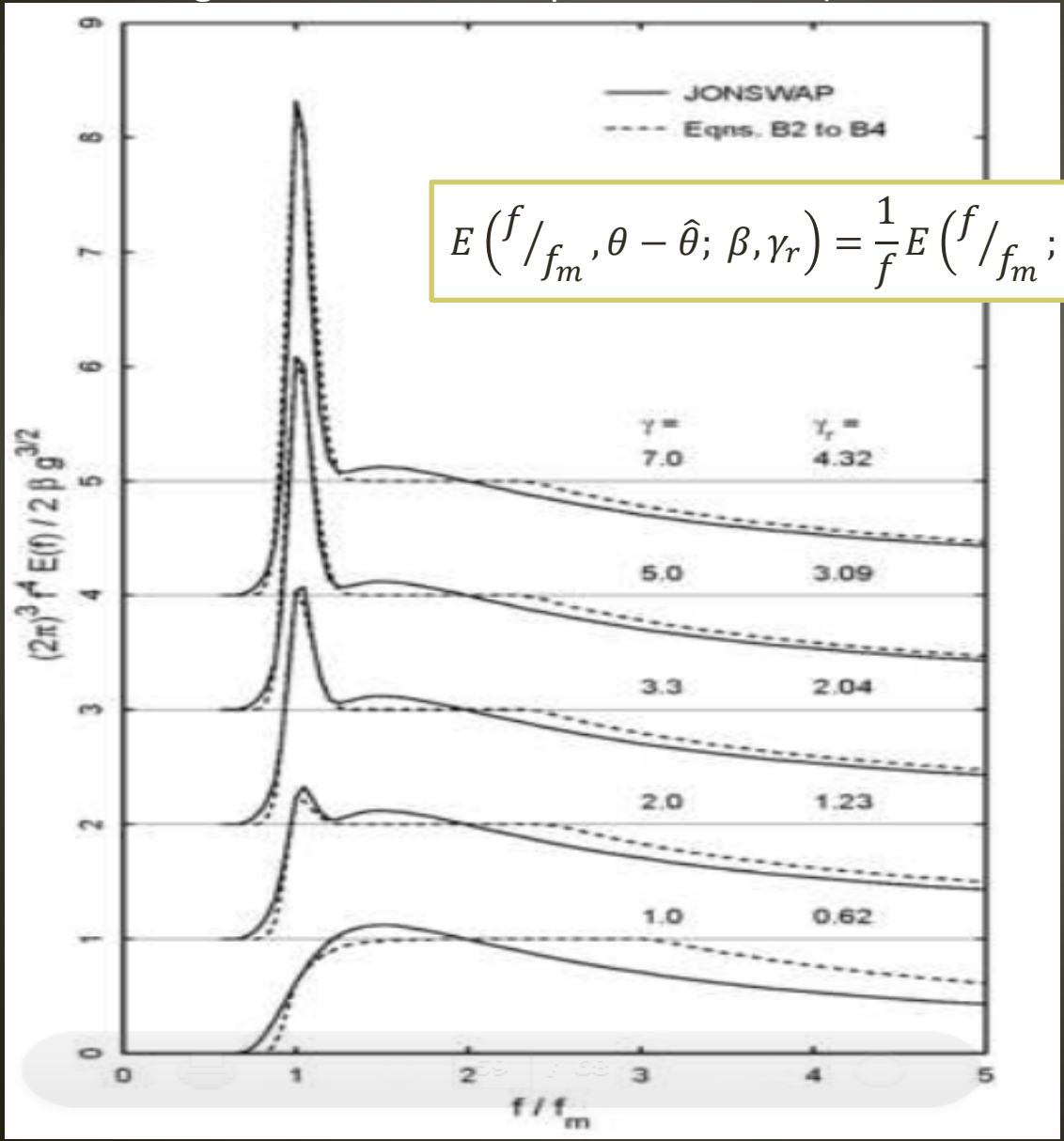
  - Investigate discrepancies between the DIA and the Full Boltzmann Integral(FBI) for the new spectral shape.

  - Compare the speed of the initial TSA to the modified TSA results for the variable angular spreading.

- The DIA – Overview.
- Compare results of the DIA and the FBI in terms of the accuracy.
- Possible reasons for poor performance.
- Show the changes in the modified TSA can improve the speed while maintaining the accuracy.
- Conclusion.

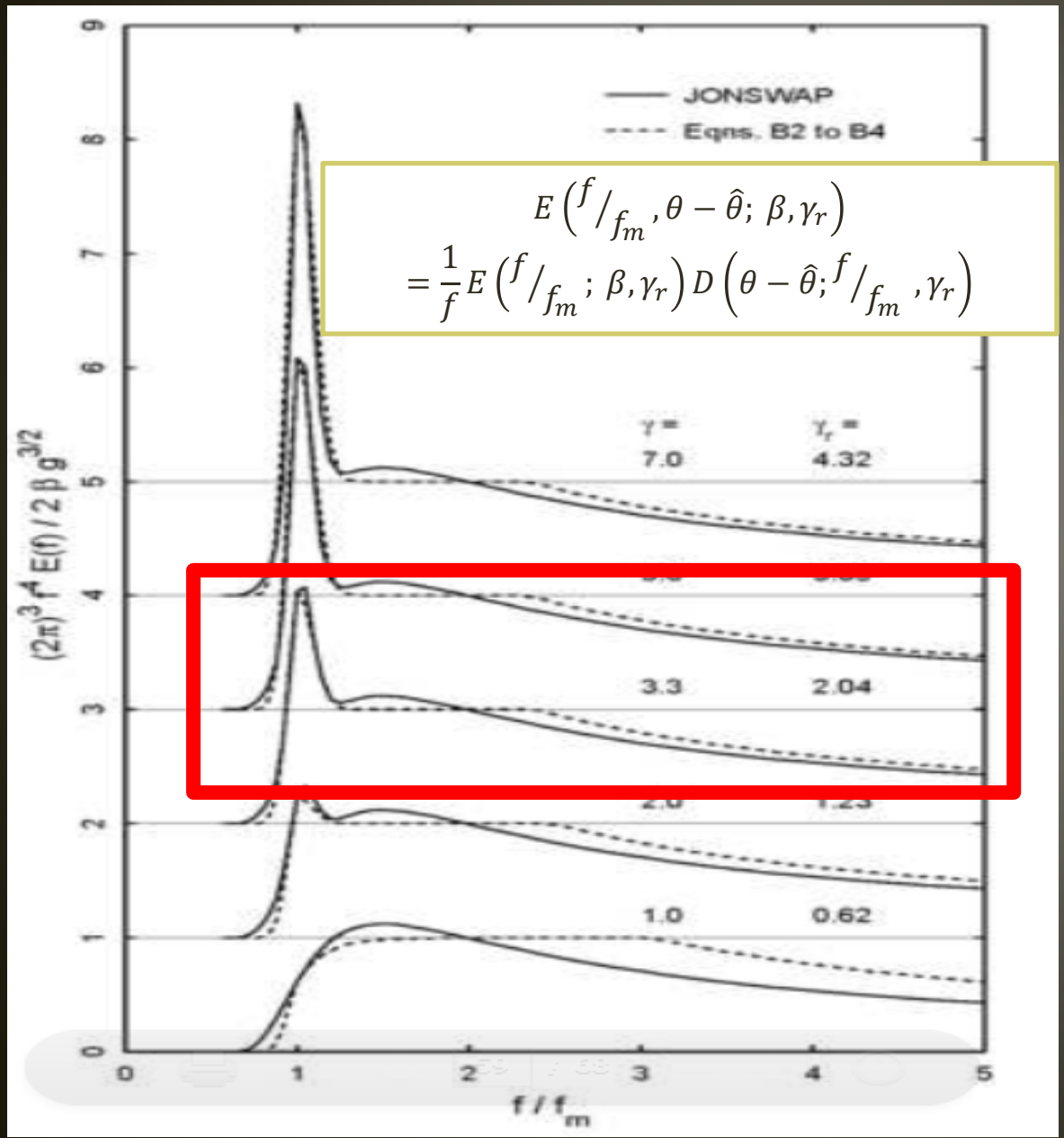
Methodology

- Defining a new reference spectra for TSA (Bender et all, in progress)



$$E\left({}^{f}/_{f_m}, \theta - \hat{\theta};\, \beta, \gamma_r\right) = \frac{1}{f} E\left({}^{f}/_{f_m};\, \beta, \gamma_r\right) D\left(\theta - \hat{\theta}; {}^{f}/_{f_m}, \gamma_r\right)$$

Methodology



New Spectral Form:

-Different basis but similar empirical fit.

-Parameters consistent with dynamics and observations.

-Uses f^{-4} basis that works better representing non-linear fluxes.

Methodology

- Recalibrate the DIA according to the reference spectra for “relative” peakedness (Υ_r)= 2.04 case.

- Compare results of the DIA and the FBI in 1D directionally integrated energy spectra and 2D contour map with 8 different peakedness values.

- Document the modified TSA model execution times in comparison to the initial TSA solutions.

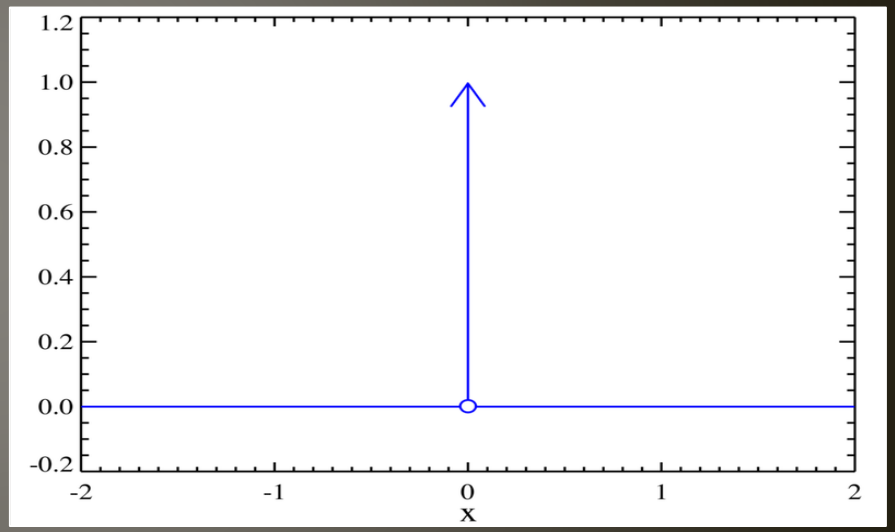
Discrete Interaction Approximation (DIA)

Transfer Integral (Webb,1978):

$$\frac{\partial n(k_1)}{\partial t} = \int_0^{2\pi} \int_0^{\infty} T(k_1, k_3)\, dk_3 k_3 d\theta$$

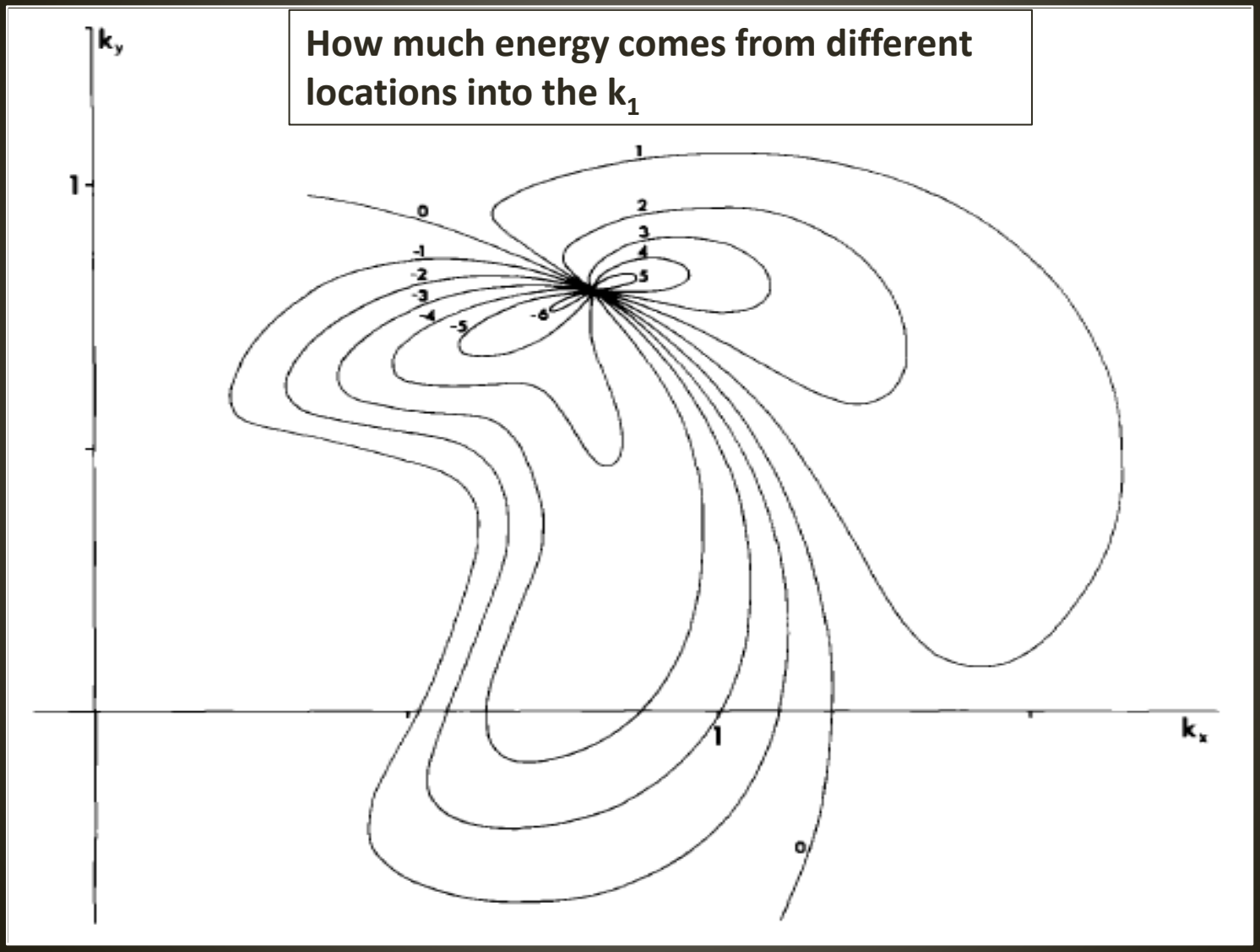
Delta Dirac Function:

$$\delta(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$$



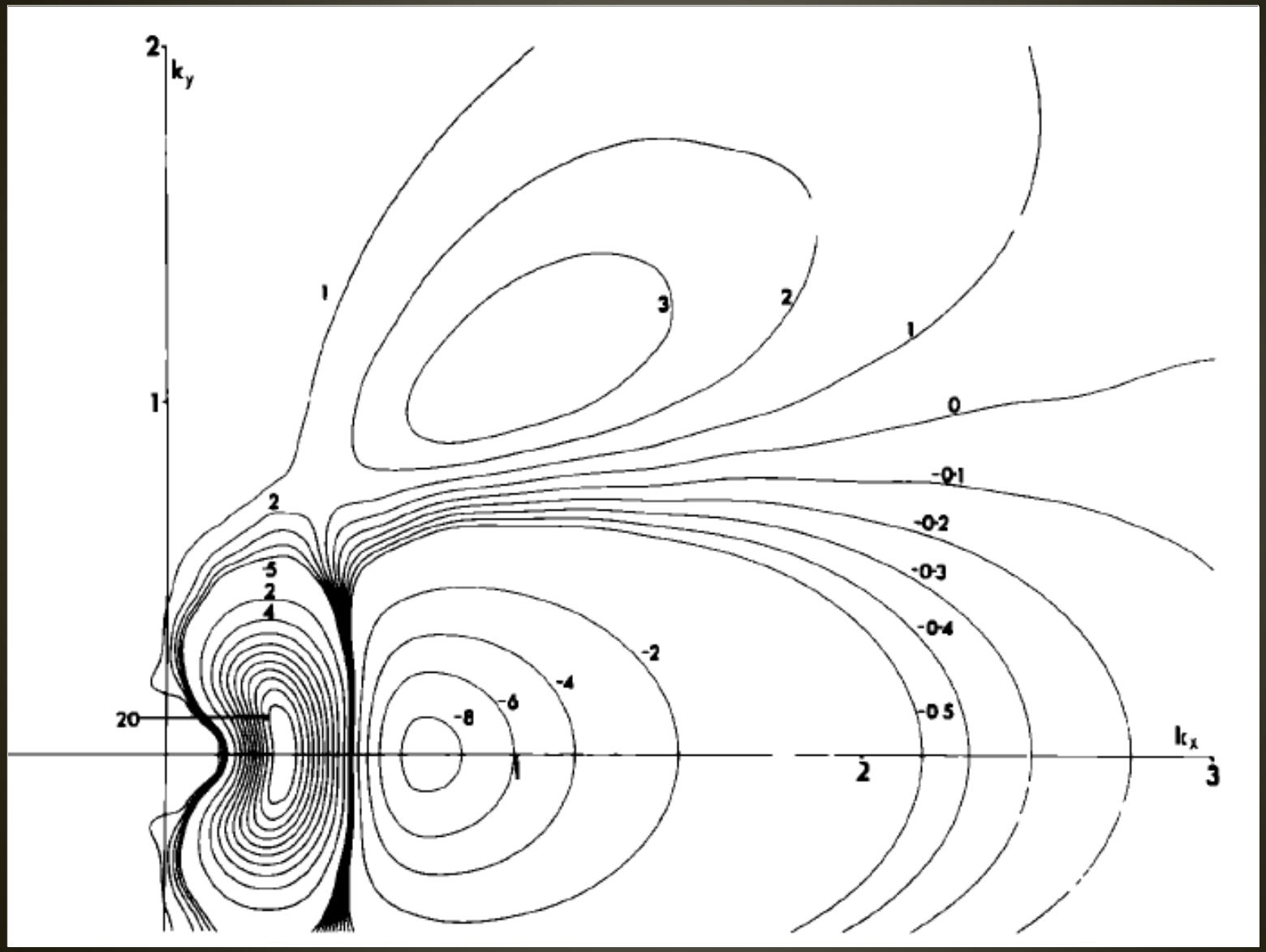
Resonance Condition : $\overrightarrow{k_1} + \overrightarrow{k_2} = \overrightarrow{k_3} + \overrightarrow{k_4}$

Discrete Interaction Approximation (DIA)



The pumped transfer T_p (k_1 ,k_3) for k_1 (0.8, 0.8) (Webb,1978)

Discrete Interaction Approximation (DIA)



The non-linear transfer *dn/dt* as a function of wavenumber (Webb,1978)

Discrete Interaction Approximation (DIA)

The DIA implements an additional Delta function into the transfer Integral ($k_1 = k_2$):

$$\frac{\partial n(k_1)}{\partial t} = \int_0^{2\pi} \int_0^{\infty} T(k_1, k_3)\, \boldsymbol{\delta(k_1 - k_2)} dk_3 k_3 d\theta$$

$\boldsymbol{\delta(k_1 - k_2)}$**=1 when** $|\, \boldsymbol{k_1 - k_2}\, | \leq \in \quad (\in \approx 0)$

Also it was approximated that;

$$\overrightarrow{k_3} = \mathbf{0.75}\,\overrightarrow{k_1}$$

$$\overrightarrow{k_4} = \mathbf{1.25}\,\overrightarrow{k_1}$$

- **Discretized representation.**
- **Conserves action, energy and momentum.**

Discrete Interaction Approximation (DIA)

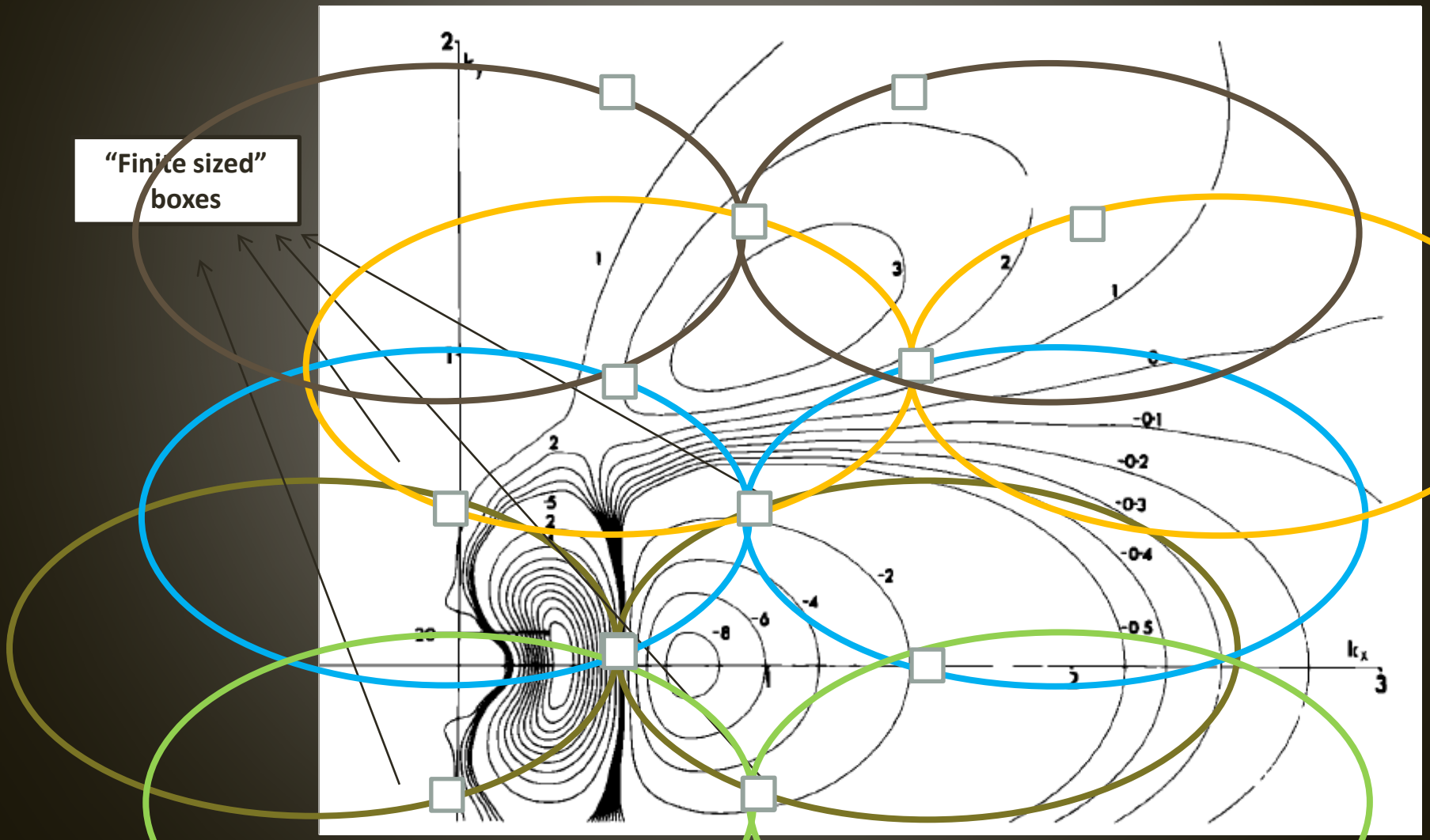
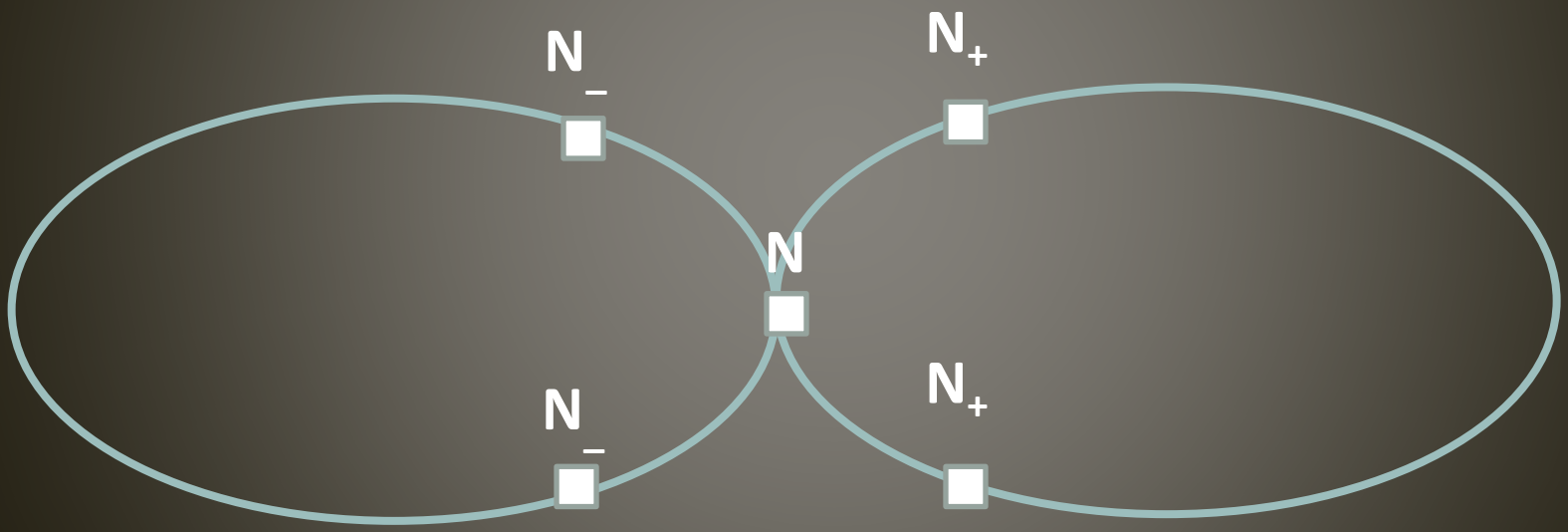


Figure-8 diagram represents the new transfer integral

Discrete Interaction Approximation (DIA)

$$\frac{\partial}{\partial t}\begin{bmatrix} N \\ N_+ \\ N_- \end{bmatrix} = \begin{bmatrix} -2 \\ +1 \\ +1 \end{bmatrix} C g^{-8} f^{19} [N^2 (N_+ + N_-) - 2 N N_+ N_-] \Delta \mathbf{k}$$

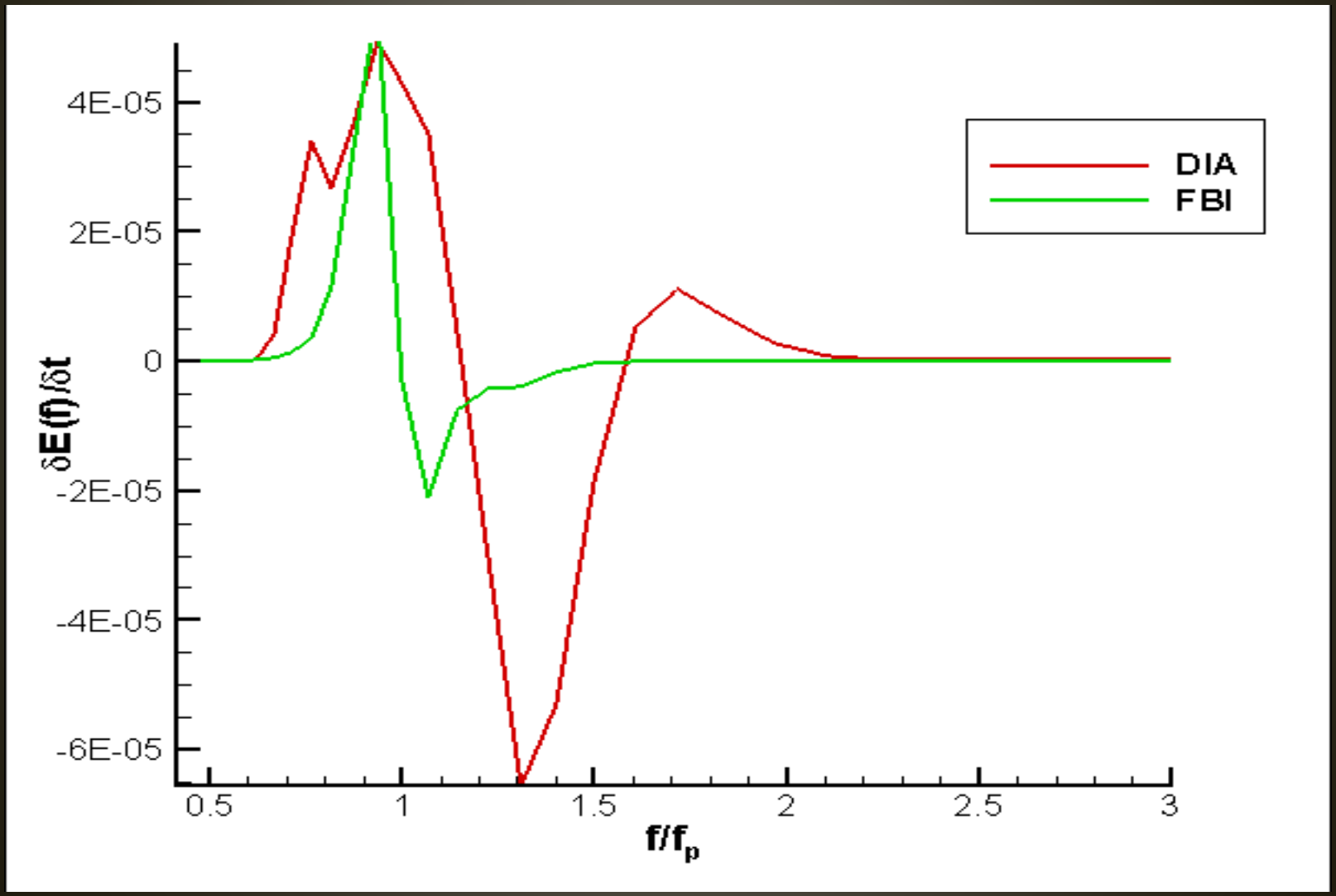


Since we had a different spectral shape we had to recalibrate the DIA and for our calibration case:

$C = 5.5 * 10^{14}$

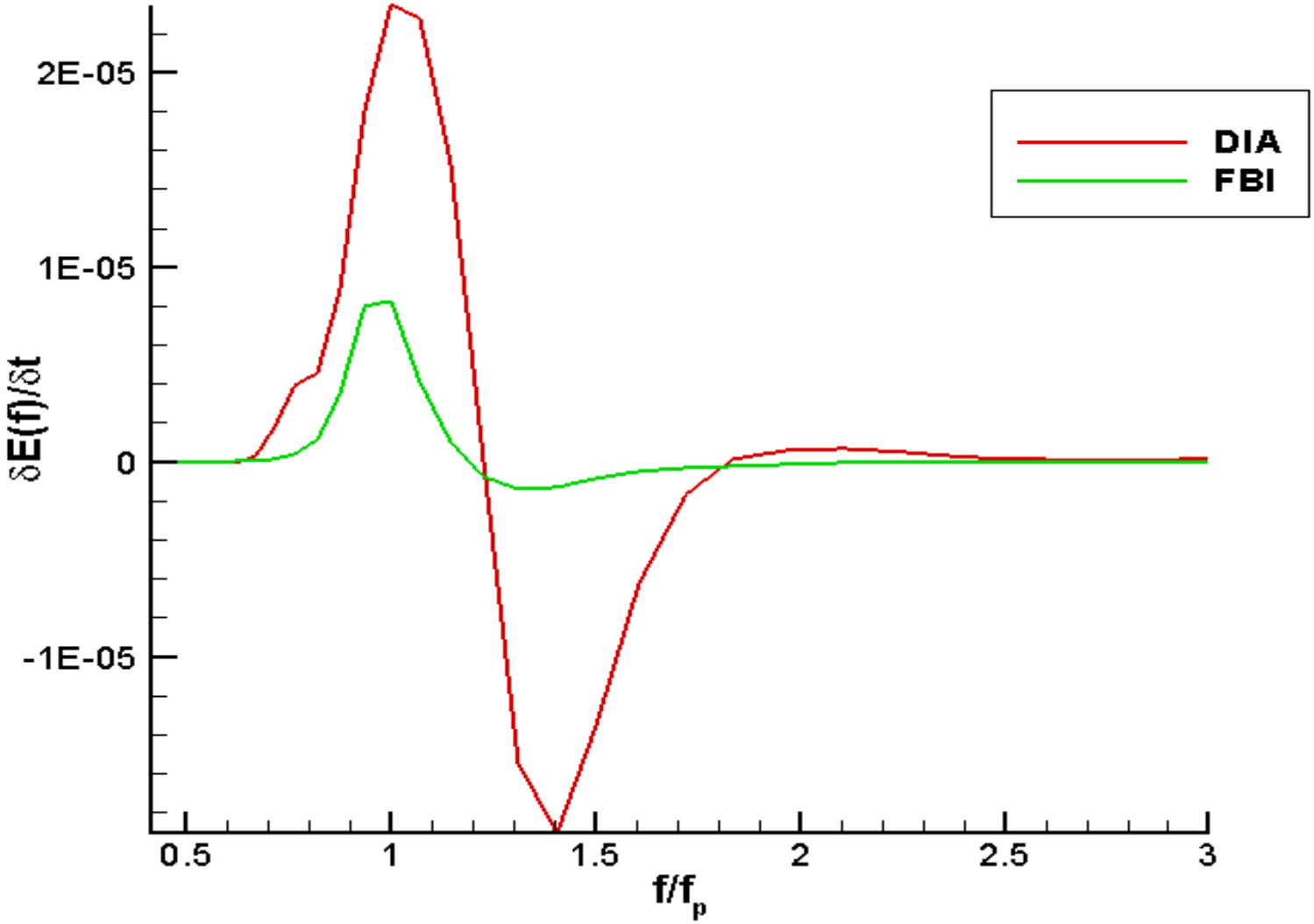
Comparison

- The calibration case ($\Upsilon_r = 2.04$) directionally integrated spectra:



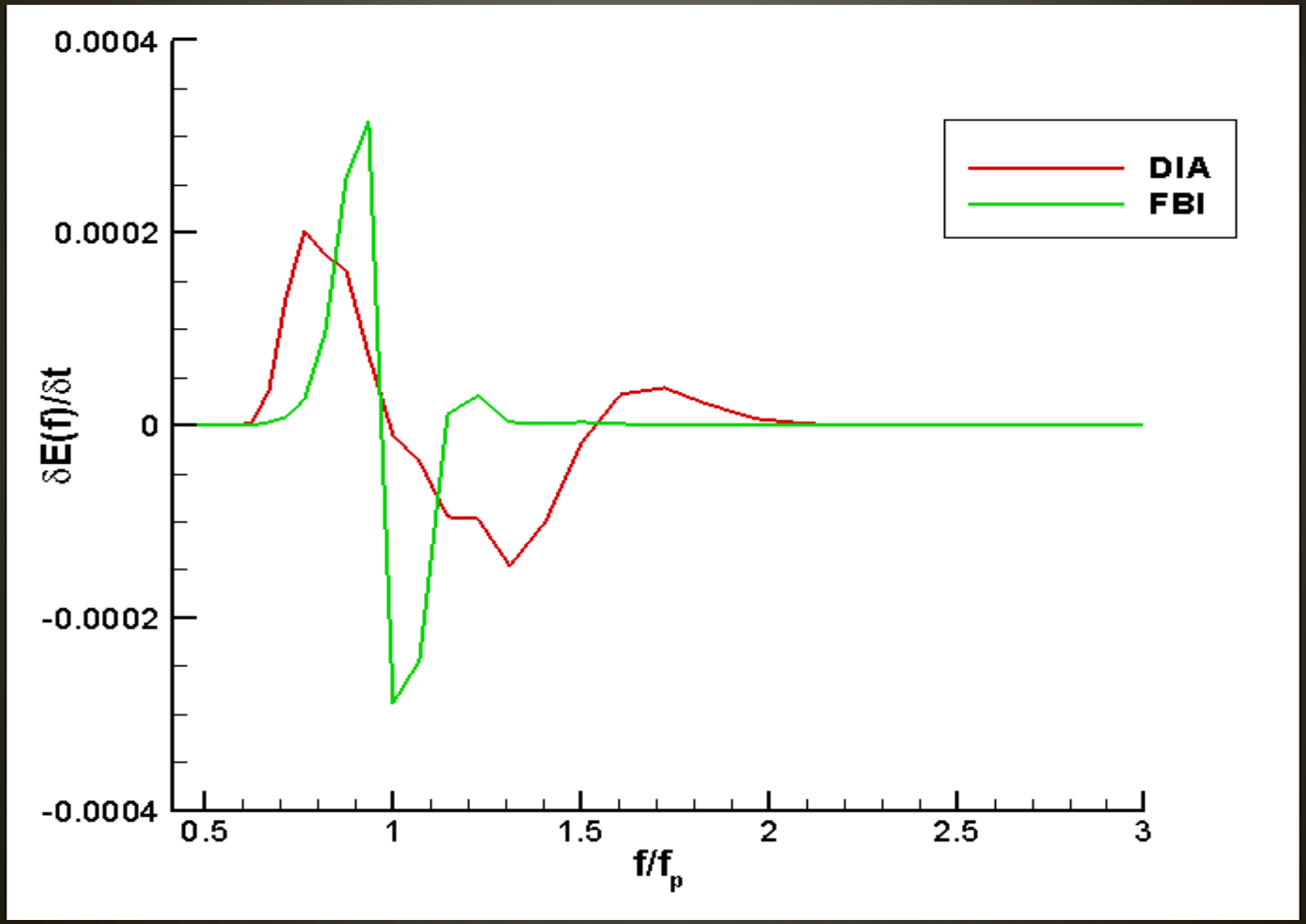
Comparison

- Another case ($\Upsilon_r = 0.7$) directionally integrated spectra:



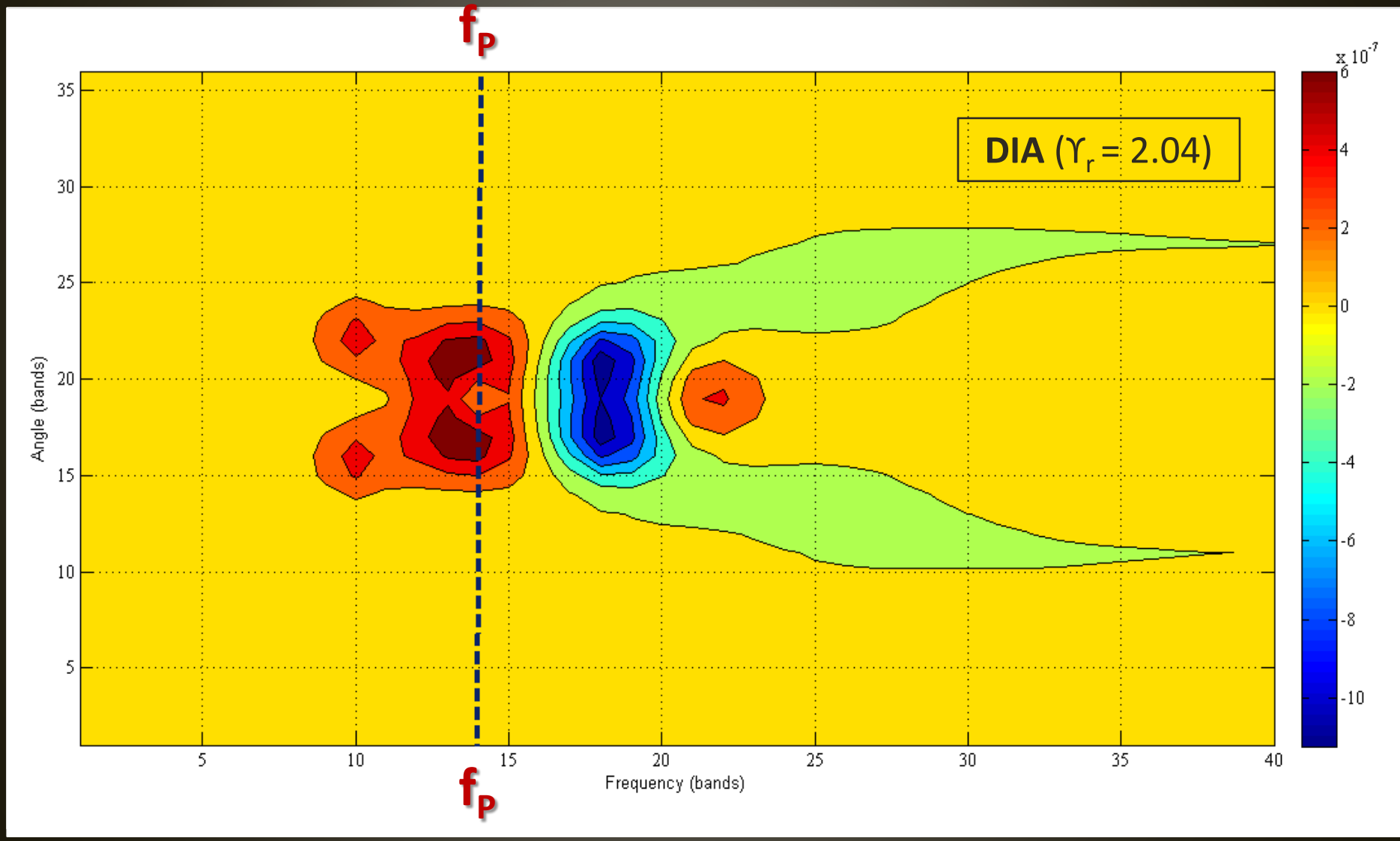
Comparison

- The final case ($\Upsilon_r = 4.32$) directionally integrated spectra:

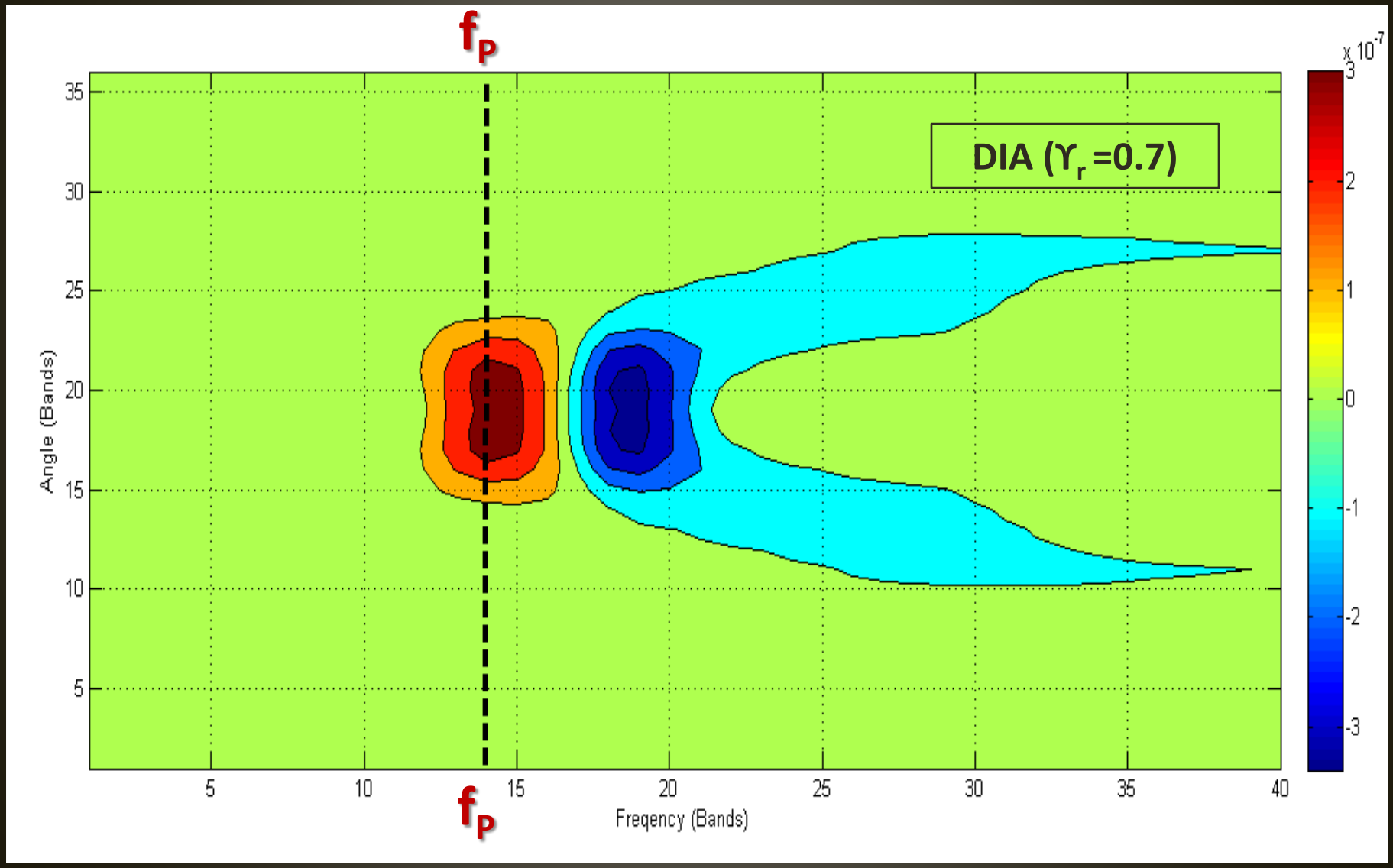


Comparison

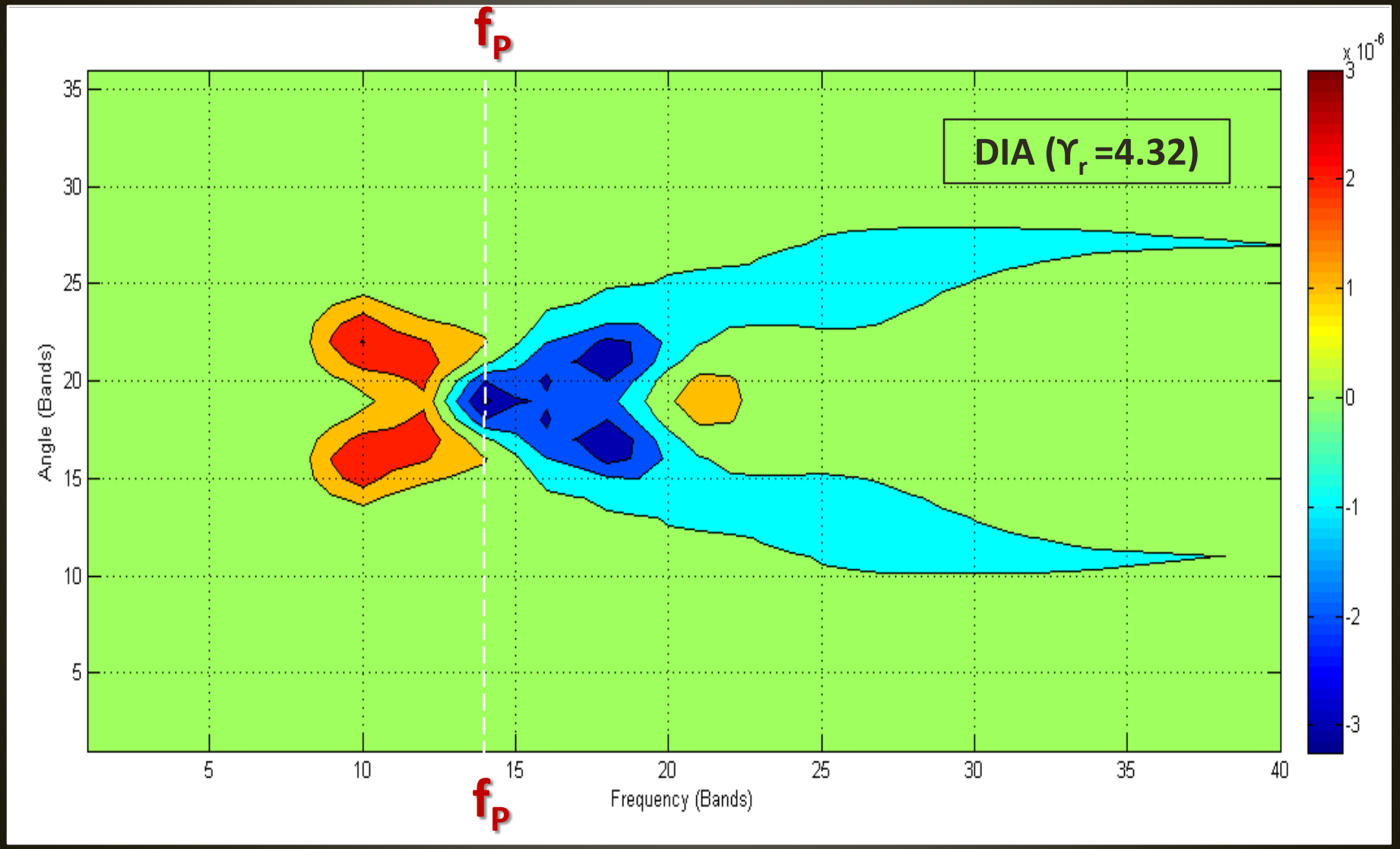
The calibration case with 2D contour maps:



Comparison


f_P
DIA (Υ_r=0.7)
Angle (Bands)
Freqency (Bands)
f_P
x 10^{-7}
3
2
1
0
-1
-2
-3
5
10
15
20
25
30
35
40

Comparison

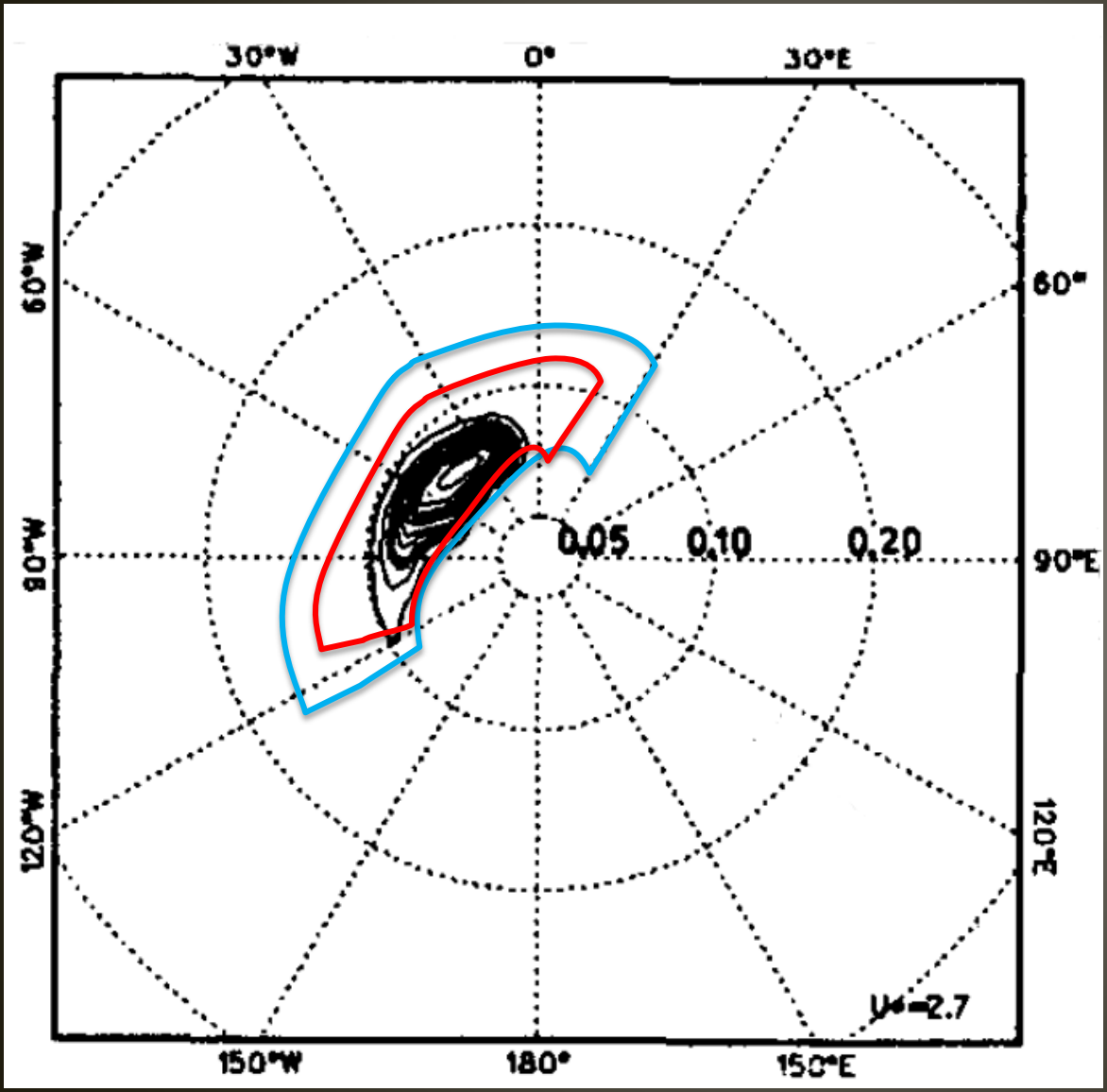

f_P
DIA (Υ_r=4.32)
Angle (Bands)
Frequency (Bands)
f_P
x 10^{-6}
35
30
25
20
15
10
5
5
10
15
20
25
30
35
40
3
2
1
0
-1
-2
-3

Efficiency of the DIA

- Practical consideration within the DIA prevented it from representing the general interactions space, affected the accuracy of the model. Integral in the DIA does not have the same form as the FBI.

- Source functions in the present model will undoubtedly need to be modified and improved as further experience is gained in the operation of the model. (K. Hasselmann,1988).

The Modified TSA



The initial TSA local scale had 11 frequency, 17 angle bands (Blue) = Overkill.

For the initial testing, a crude reduction of this scale to a 9 by 15 grid subset (Red) gives a factor of 1.5 = **([11/9] * [15/17])** increase in comp. speed and did not affect the accuracy significantly.

This graph here is illustrative.

The Modified TSA

- For further testing we reduced angle bands to 5 and frequency bands to 7 also changed resolution of the frequency bands 2 at a time.

  Reduction by limiting the domain:

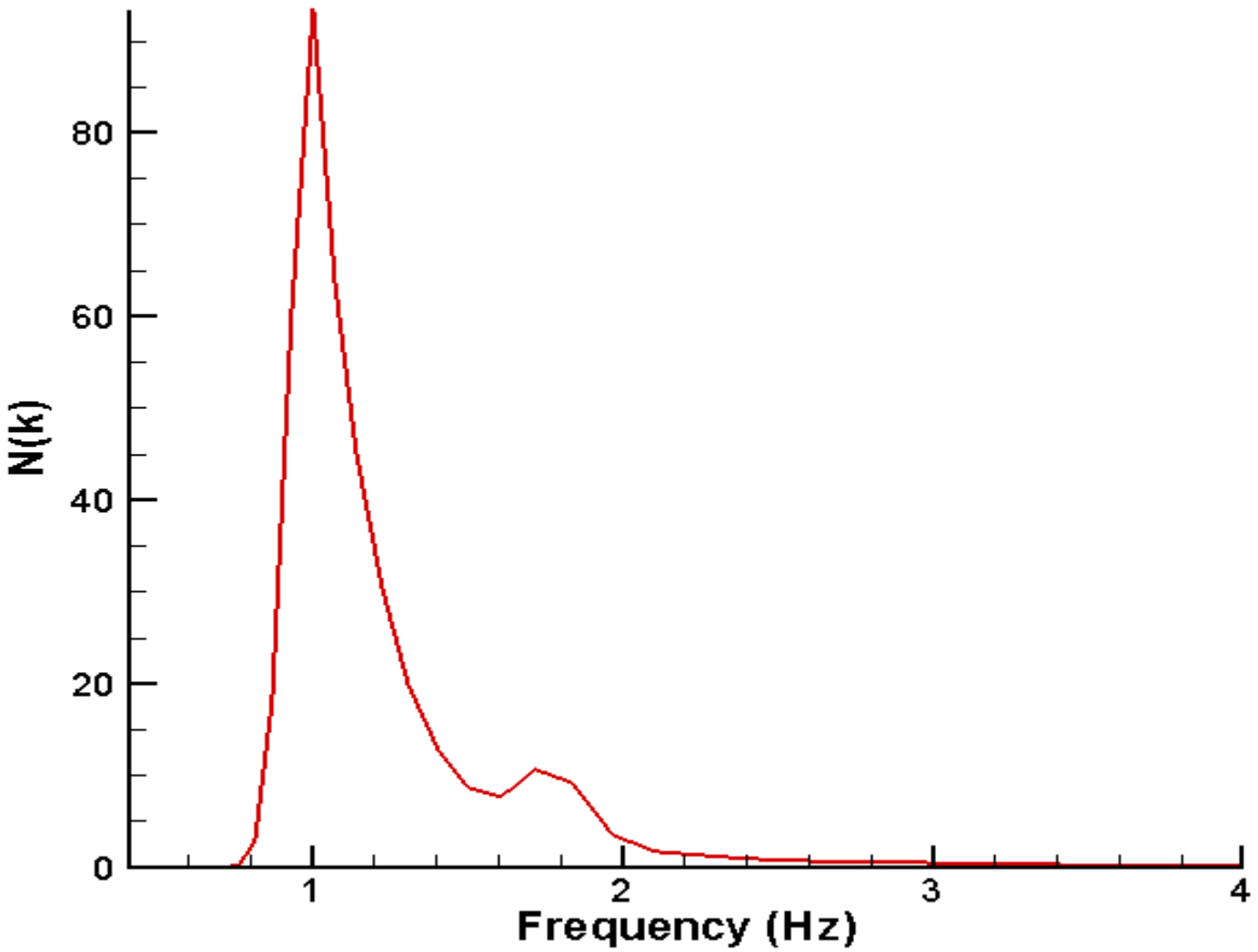
  $$(17/5) * (11/7) = 5.34$$

  Reduction by change in the resolution:

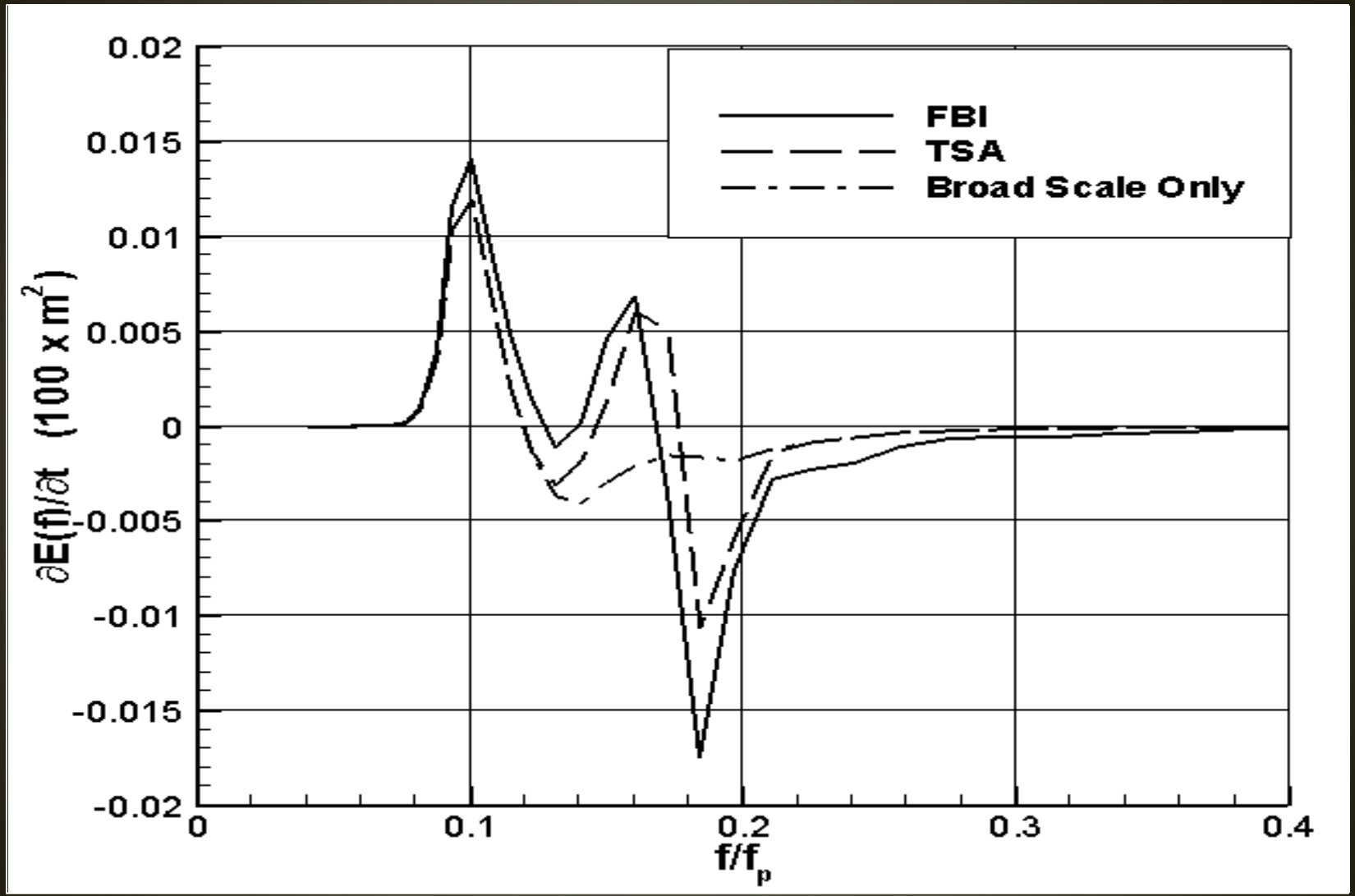
  $$5.34 * 2 = 10.68$$

- So this change gives a dramatic increase in speed by a factor of up to **10.68**.

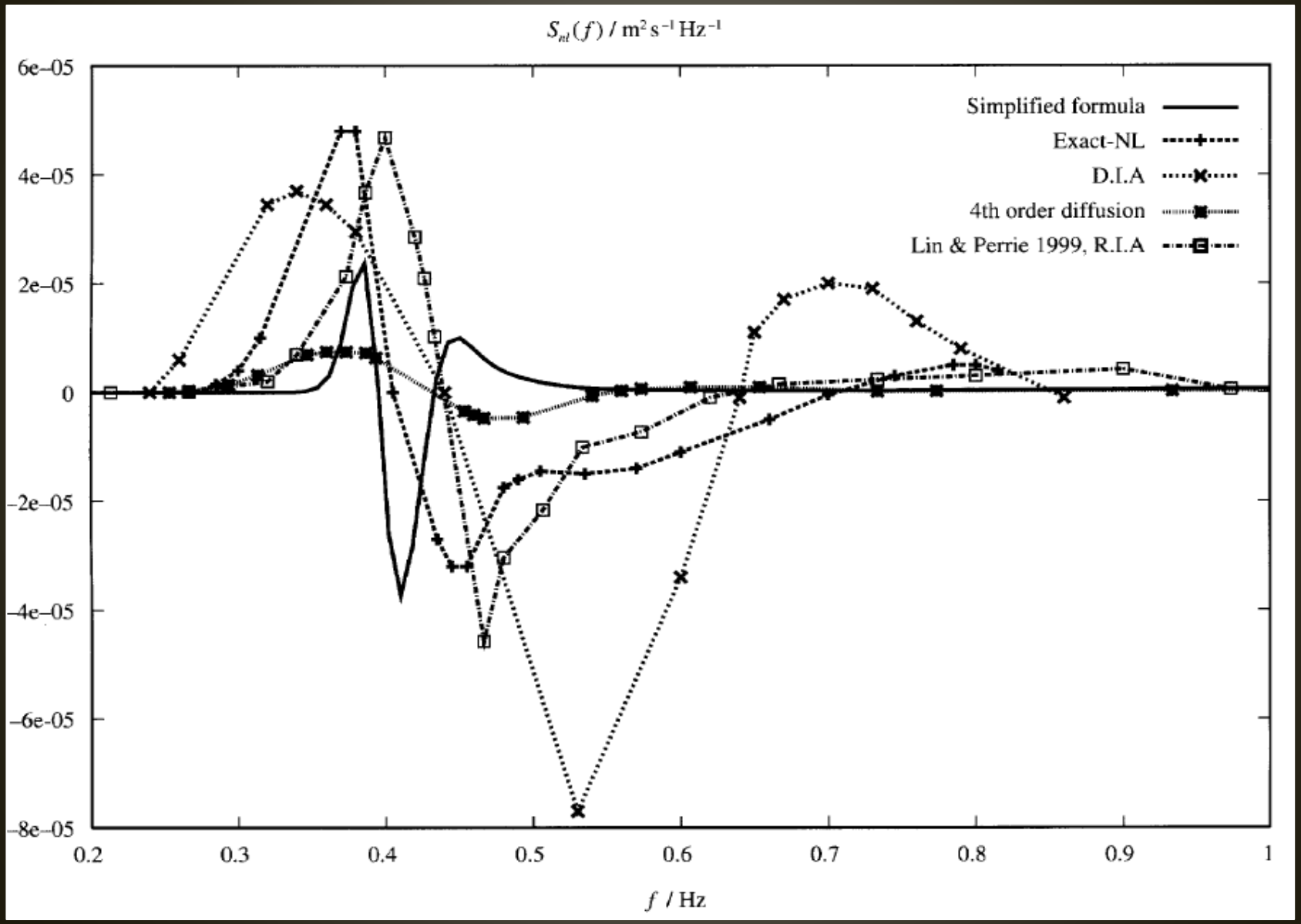
The Modified TSA



The Modified TSA

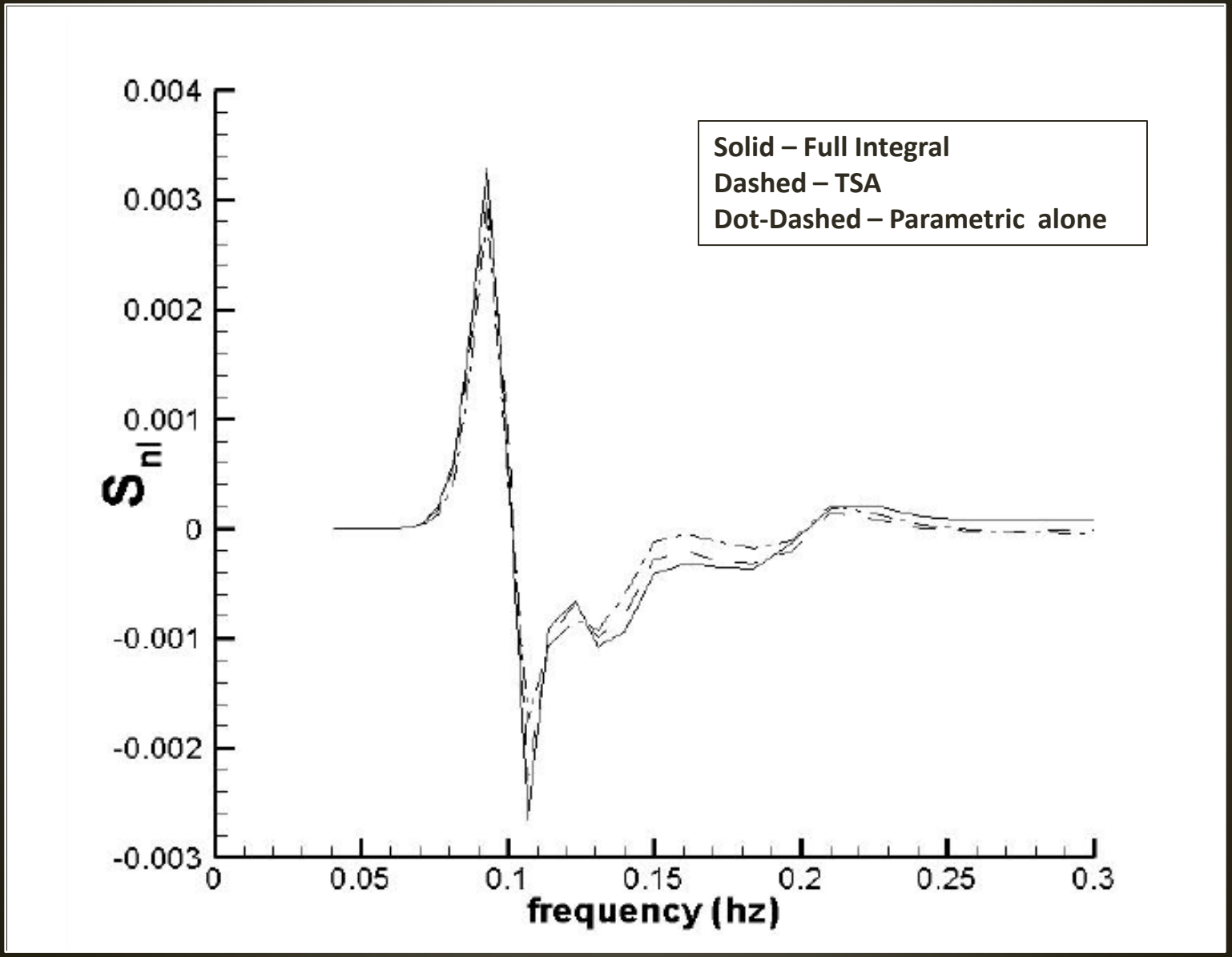


Other Alternatives?



Comparison of 5 Different Methods in Calculating S_{nl} term – *(Jenkins & Phillips, 2001)*

Other Alternatives?



Same case with the TSA method – *(Resio & Perrie, 2009)*

Conclusion

- The DIA has been used for more then 25 years and needs to be replaced with a more accurate method for proper representation of the spectral shape.

- The modified TSA is shown to be more precise than the DIA and practical enough to be used in an operational model.

- Also the TSA is open for further development.

Future Work

-Operating the TSA on a continuous basis rather than discretized. Expecting to see a continuous peak frequency provides a better estimate towards S_{nl} then the previous discretized approach.

-Testing the modified TSA for different scenarios to validate its accuracy.

HARRY BELIEVED IN HAVING THE RIGHT TOOL FOR THE WRONG JOB

Questions?

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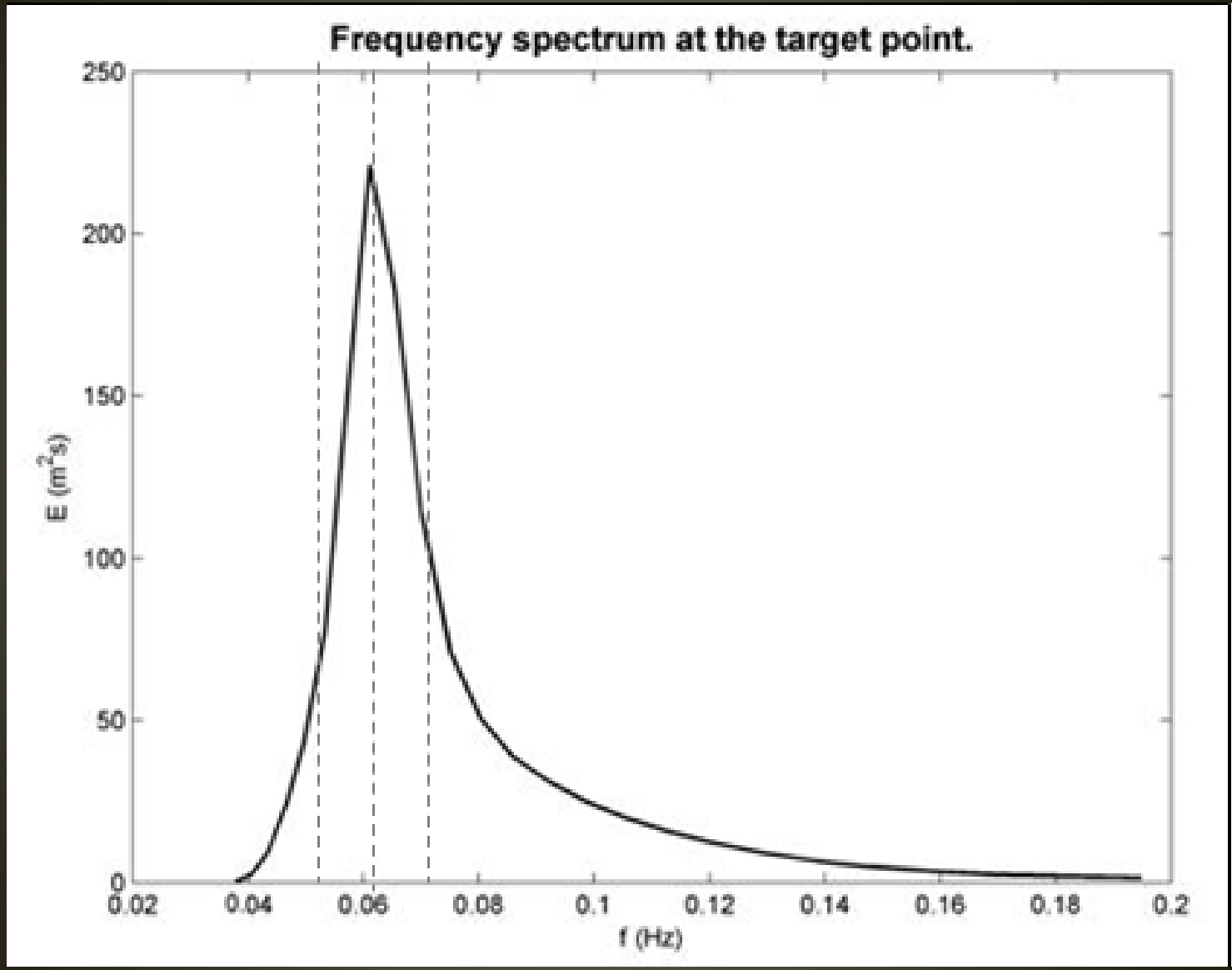
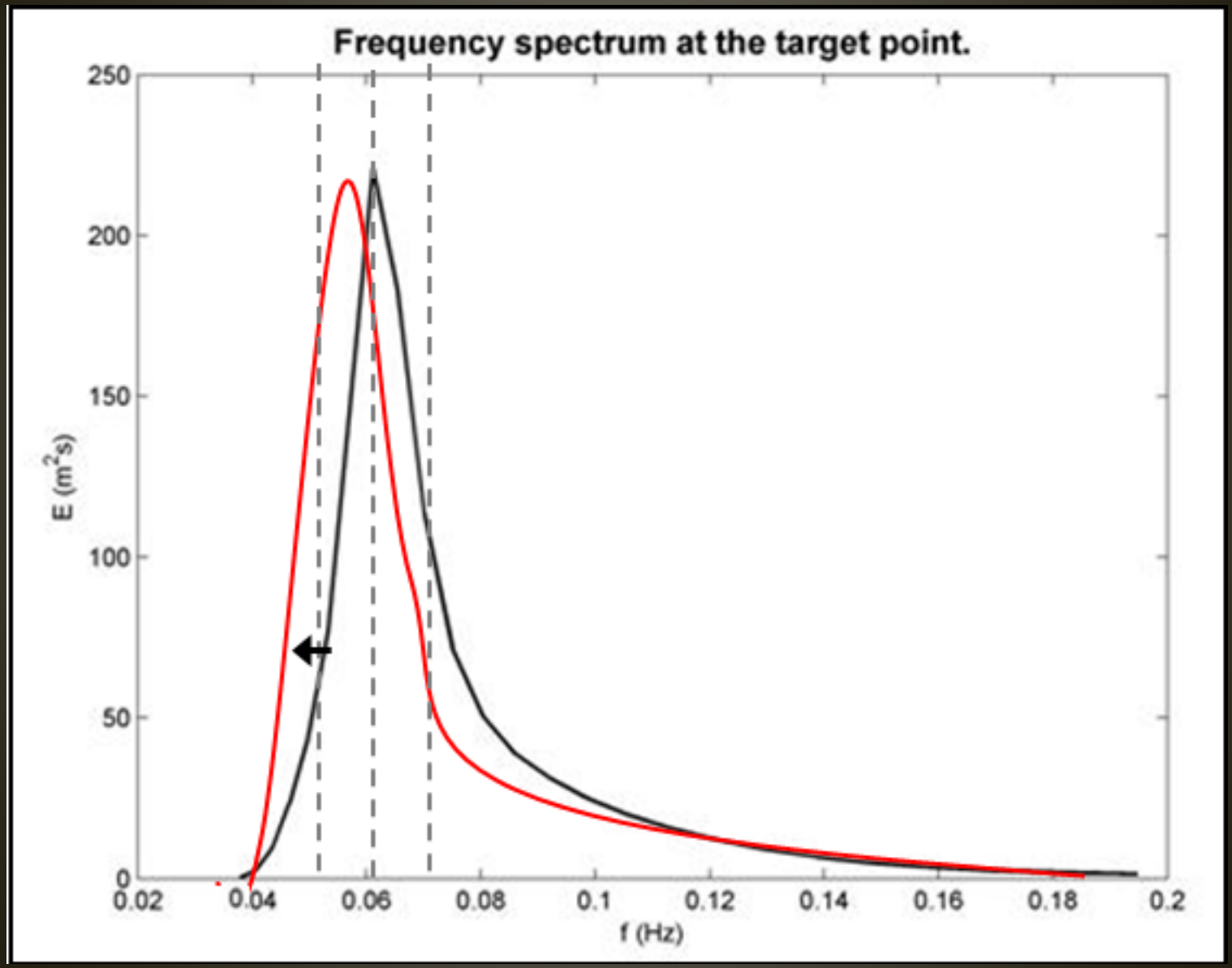

Frequency spectrum at the target point.
250
200
150
100
50
0
E (m²s)
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
f (Hz)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