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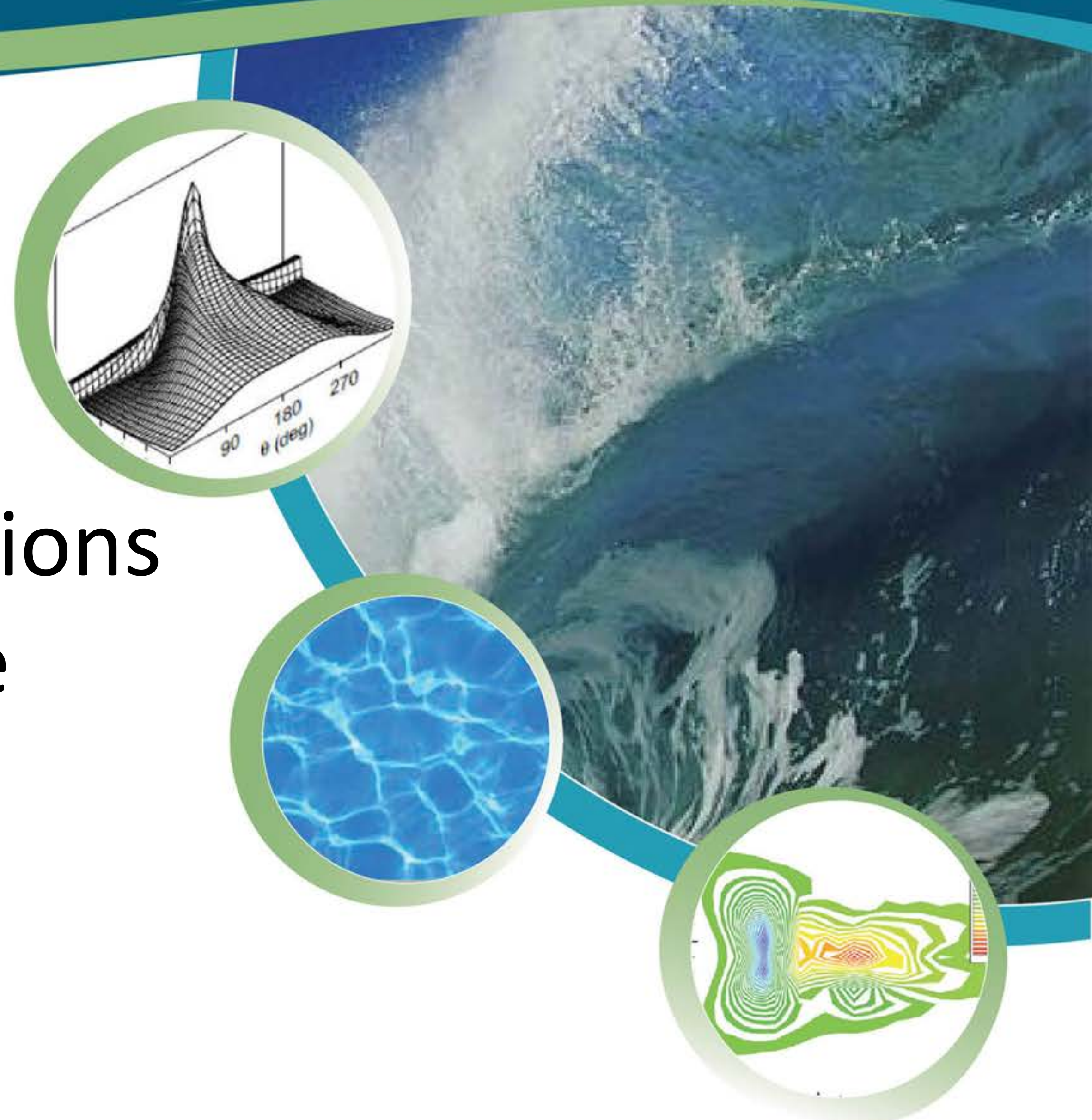
The Two-Scale Approximation: Wave-Wave Interactions For Detailed Balance Models



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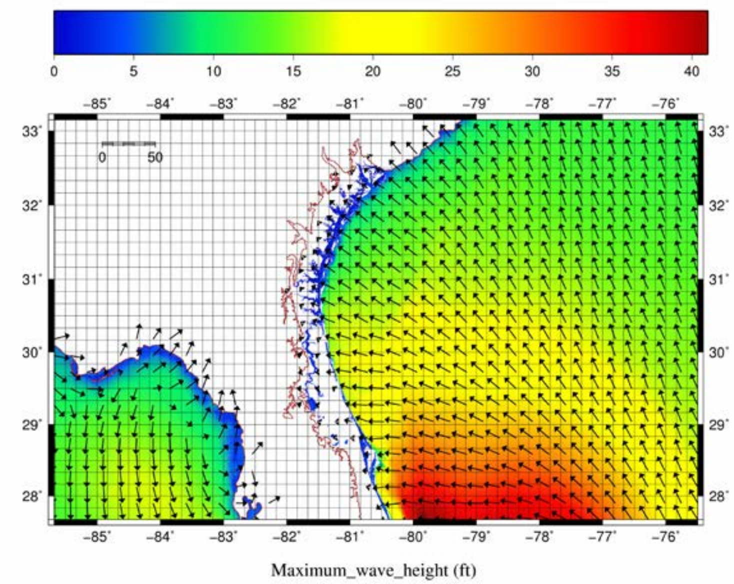
Motivation

- The Two-Scale Approximation (TSA) has been shown to provide robust and efficient estimation of nonlinear, four-wave interactions
- Nonlinear, four-wave interactions are critical to developing realistic 2-D spectra required by operational forecasting models
- Goal: Develop new algorithms and approaches within the TSA to:
  1. increase efficiency
  2. allow application within operational forecasting models



Methodology

- Extend TSA model while maintaining degrees of freedom required of third-generation wave models
- Evaluate TSA results with comparisons to Full Boltzmann Integral (FBI) solution (compare to exact solution)
- Demonstrate TSA model performance for realistic spectra
- Document TSA model execution times in comparison to FBI and Discrete Interaction Approximation (DIA) solutions

Conclusions

- Testing of new TSA approach shows good comparisons to FBI and significant improvement to DIA
- Testing shows reasonable computational speeds and efficiency
- After additional testing, new algorithms should allow implementation in operational spectral wave models

Presentation Overview

- History of TSA development
- Need for improved nonlinear, four-wave interactions within operational models
- Extension of TSA within this effort
- Comparisons to existing methodologies
  - Non-linear source terms
  - Computational speed and cost
- Future efforts

History of TSA Development

- Start with general wind-wave radiative transfer equation:

$$\frac{\partial E(f,\theta)}{\partial t} + \boldsymbol{c}_g \cdot \nabla E(f,\theta) = S_{in}(f,\theta) + S_{nl}(f,\theta) + S_{ds}(f,\theta)$$

- E(f,θ) is wave spectral energy (actually, sea-surface displacement variance) density
- Conventionally, the four-wave interaction is characterized in terms of action density N(k,θ) rather than energy density

$$\frac{c_{g1}}{(2\pi)^2 k_1} S_{nl}(f_1,\theta_1) = \frac{\partial N(k_1,\theta_1)}{\partial t} = \int_0^{2\pi}\int_0^{\infty} T(k_1,\theta_1,k_3,\theta_3) k_3 dk_3 d\theta_3$$

History of TSA Development

- Implement Two-Scale Approximation (TSA) Approach

$$N(\boldsymbol{k}_i) = \widehat{N}(\boldsymbol{k}_i) + N'(\boldsymbol{k}_i)$$

- As shown by Resio and Perrie (2008)
- Original four action density triplets expand to 32 triplets
  - four triplets consisting purely of elements of $\hat{N}(k)$
  - four triplets consisting purely of elements of $N'(k)$
  - 24 triplets involving mixed elements
- The approximation proposed by Resio and Perrie (2008) is based on the argument that triplets containing perturbation elements $N'(k_2)$ and $N'(k_4)$ contribute sufficiently less than the remaining triplets to the locus integral

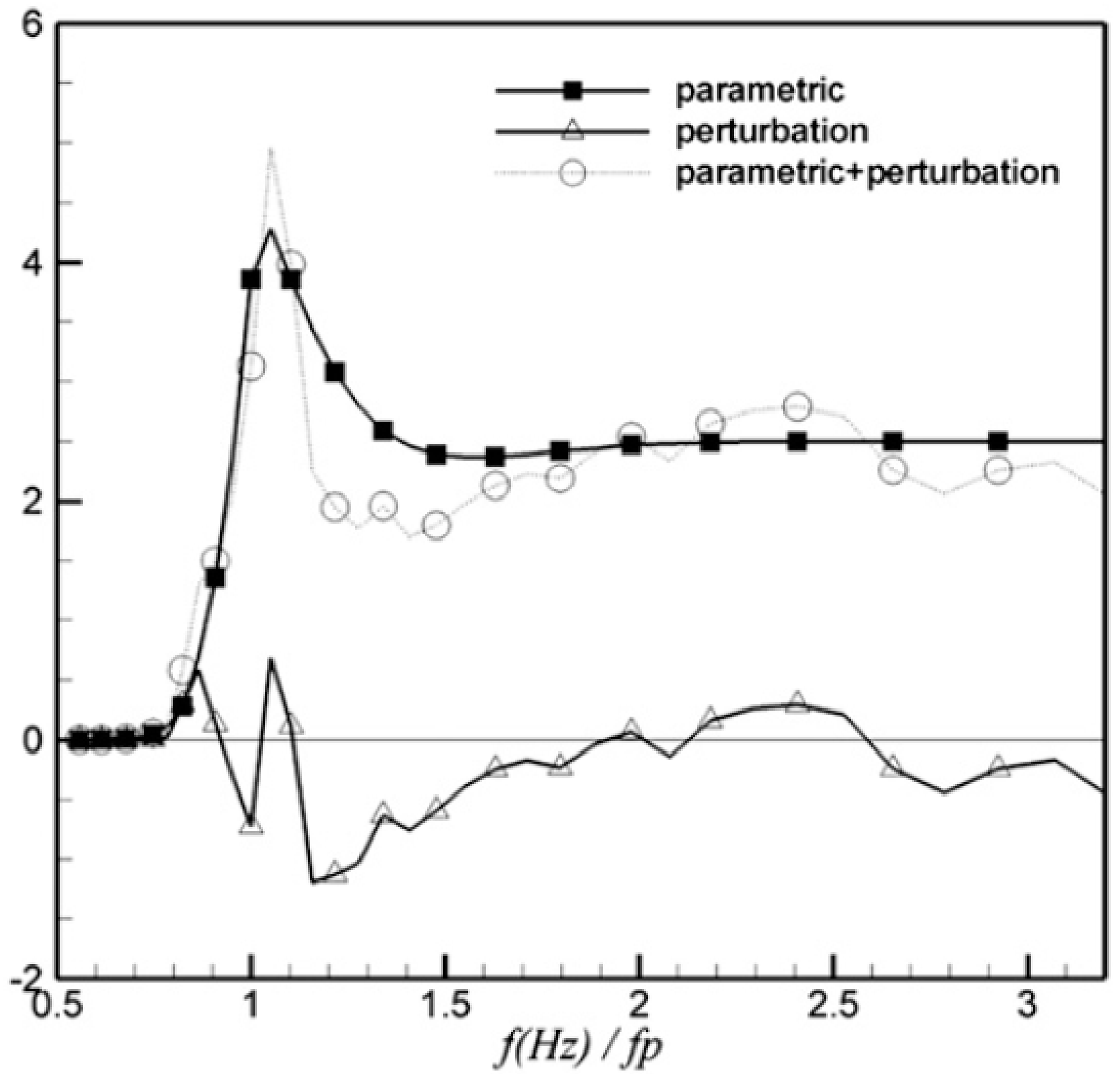
Need for TSA Approach

- Move to 3rd Generation spectral wave models built on the tenet of allowing spectra to freely evolve similar to natural spectra
- This requires the implemented approaches to contain enough degrees of freedom to capture natural spectra evolution
- The TSA approach maintains the required degrees of freedom to allow natural spectral transformation

Need for TSA Approach

- Perrie and Resio (2009)

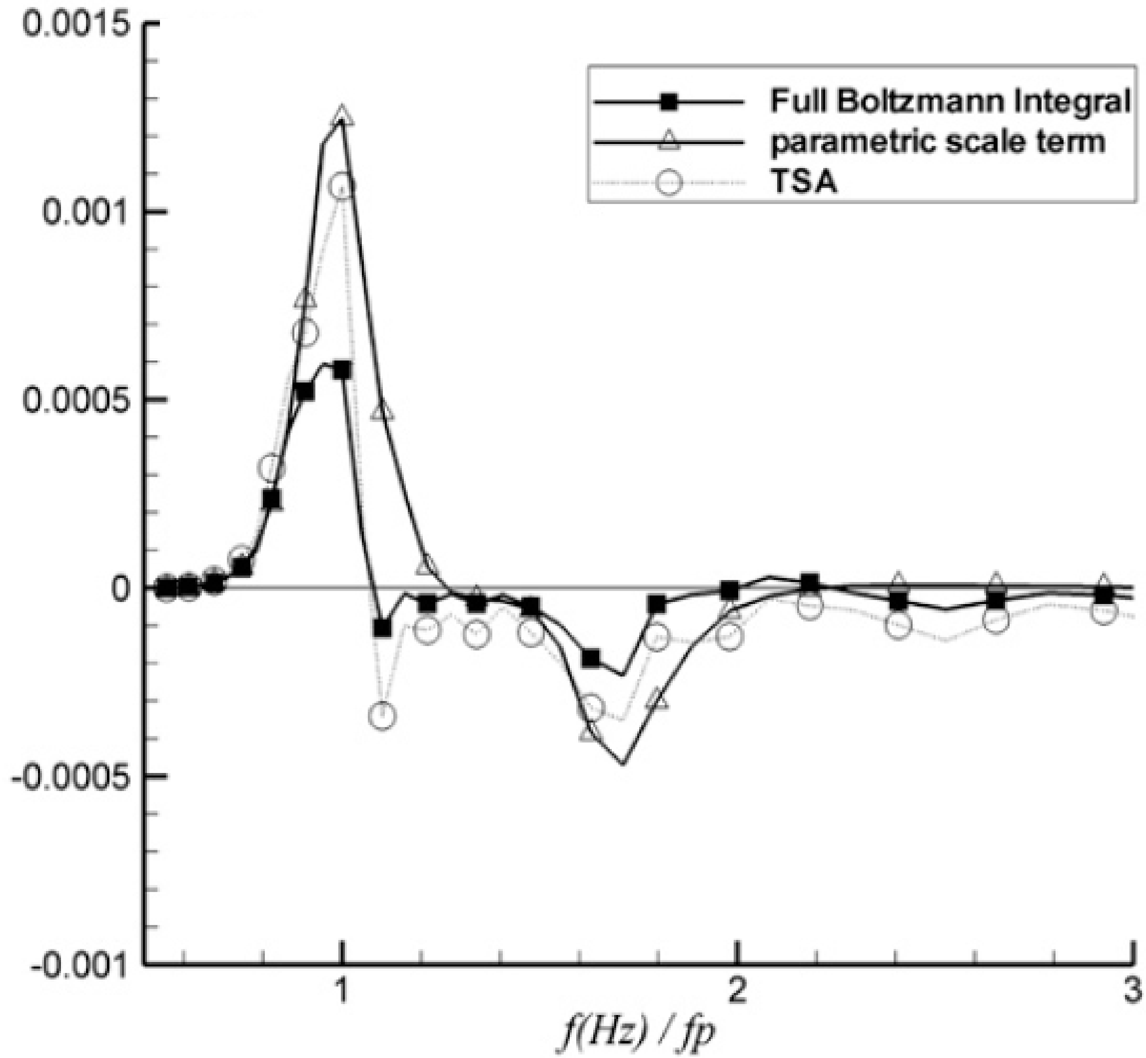
- Hurricane Wilma
- 1D Energy Decomposition
- (normalized by ω^4)



Need for TSA Approach

- Perrie and Resio (2009)

- Hurricane Wilma
- 1D Energy Transfer
- (m^2/Hz/s)



Need for TSA Approach

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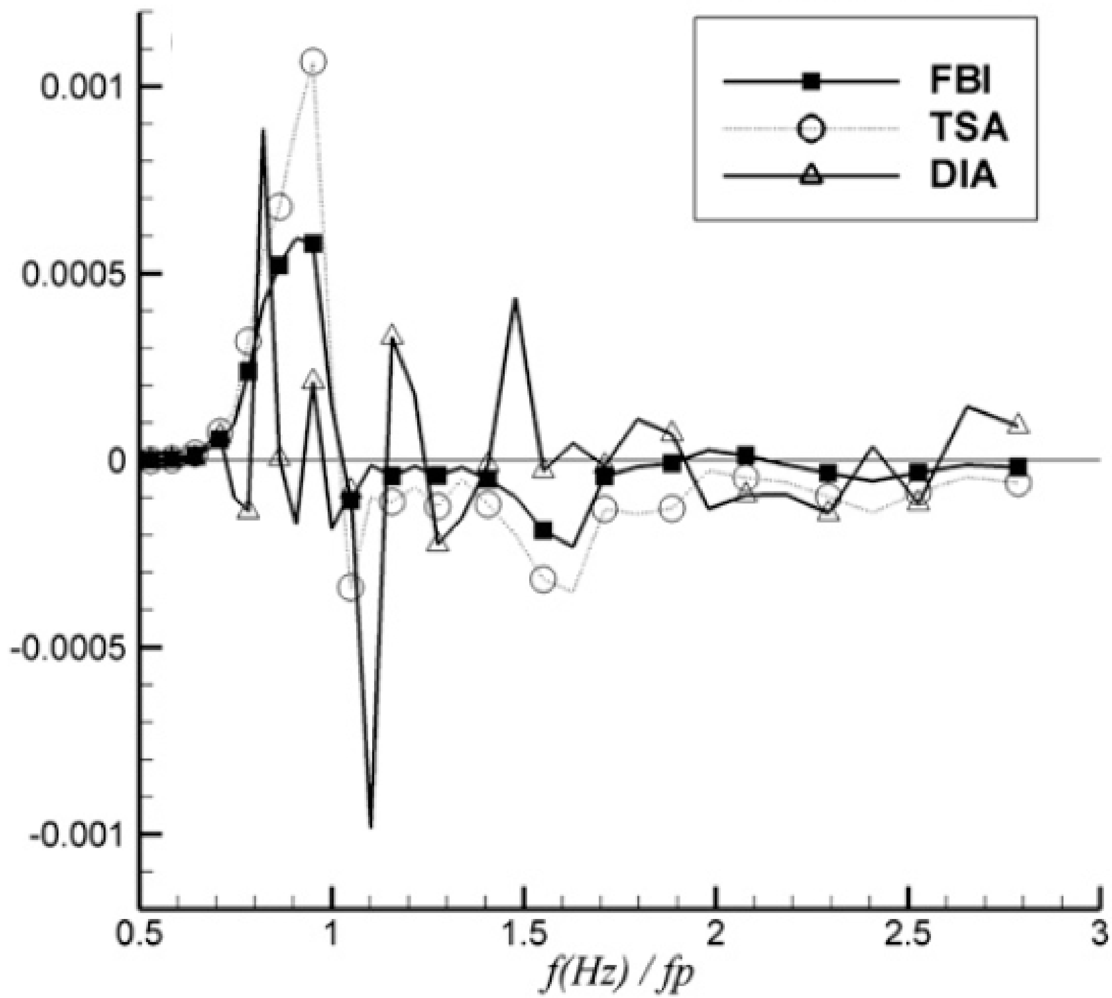

Extension of TSA Model in Current Effort

- TSA applied for more realistic spectra than previous efforts
- Directional distribution of Resio et al. (2011) allows evaluation of model performance for more natural seas
- Natural spectra show azimuthal spreads that are narrowest near spectral peak frequencies and broaden at both lower and higher frequencies
- Broadening found to relate to wave age (or spectral peakedness)

Extension of TSA Model in Current Effort

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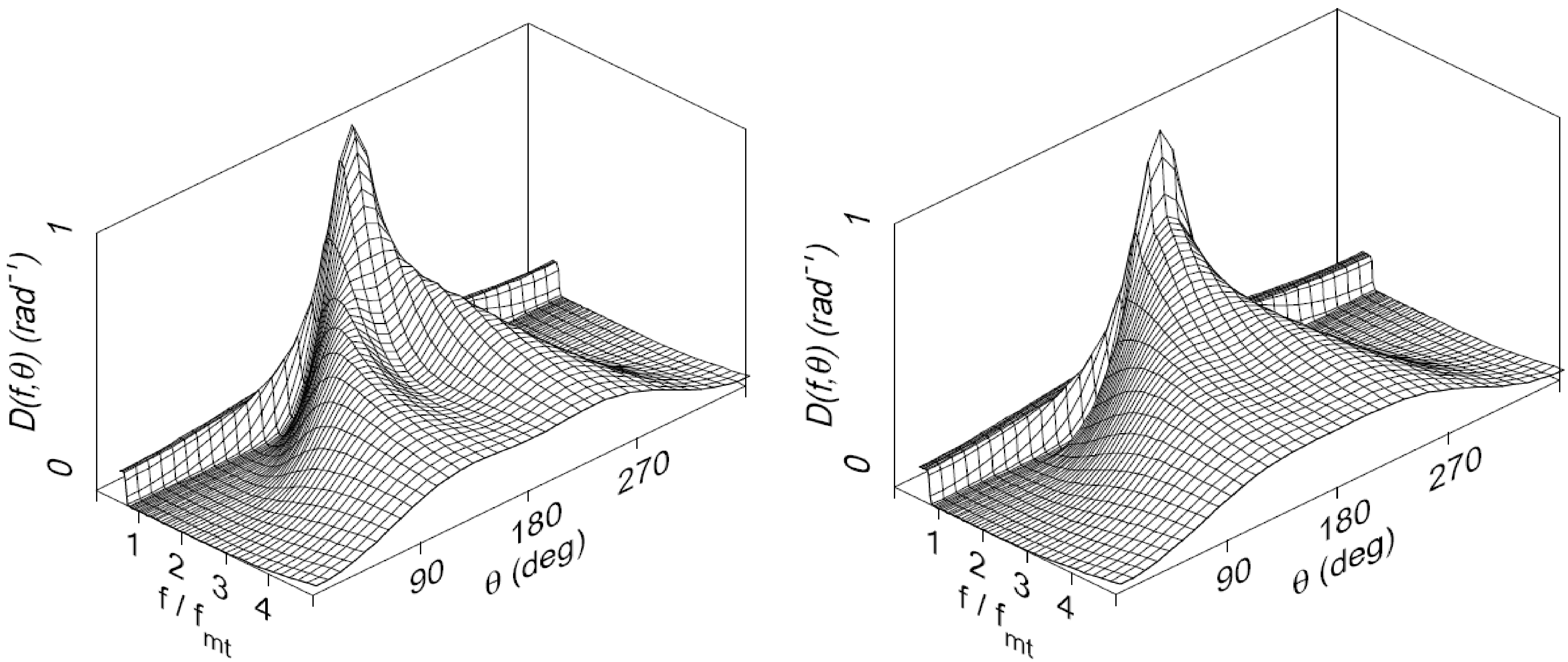
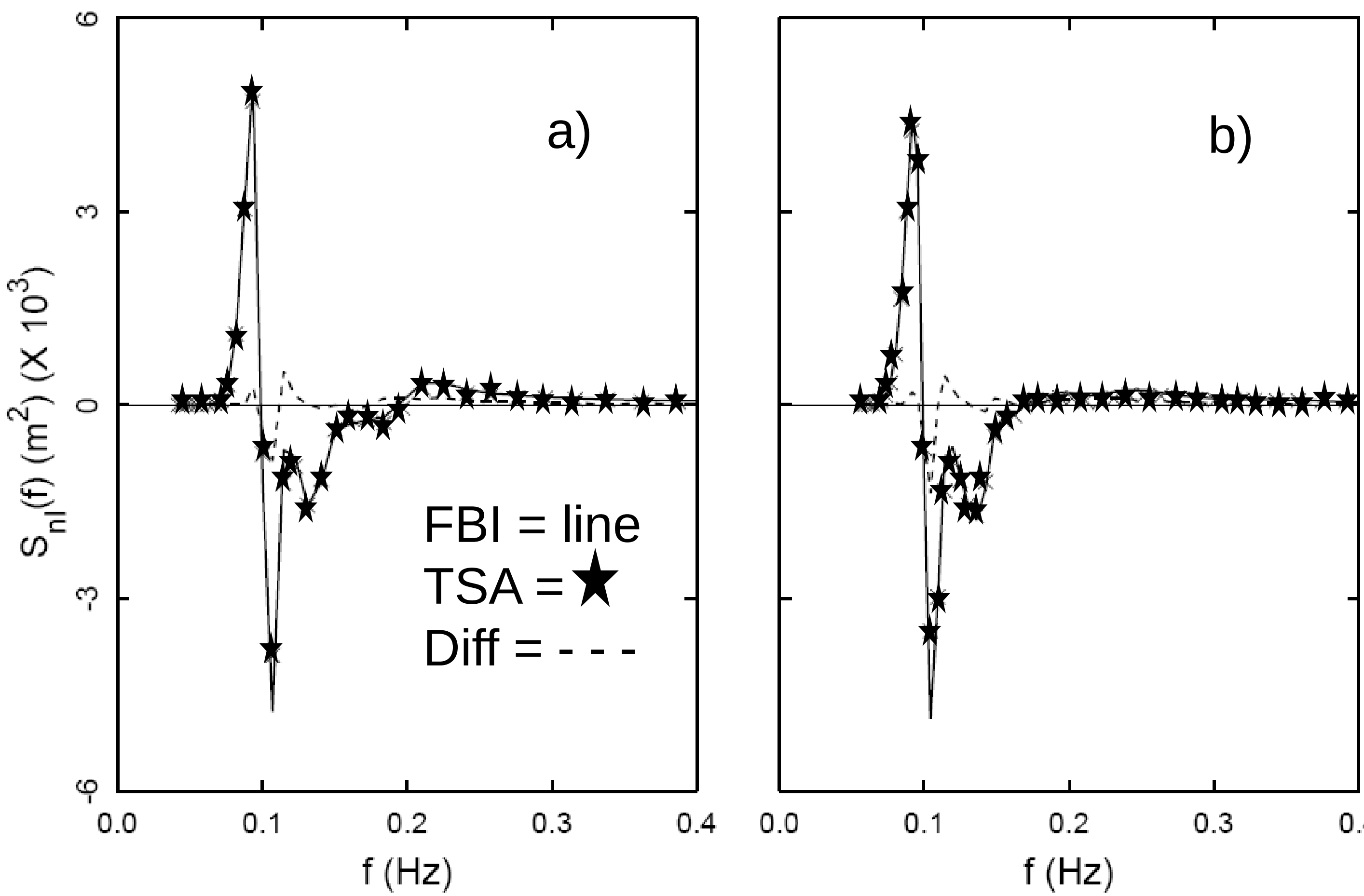


Extension of TSA Model in Current Effort

- Directionally integrated source terms
- E(f) conventional JONSWAP form

$\alpha = 0.012$,
$f_m = 0.1$ Hz,
$\sigma_A = 0.07$,
$\sigma_B = 0.09$,
$\gamma = 3.3$

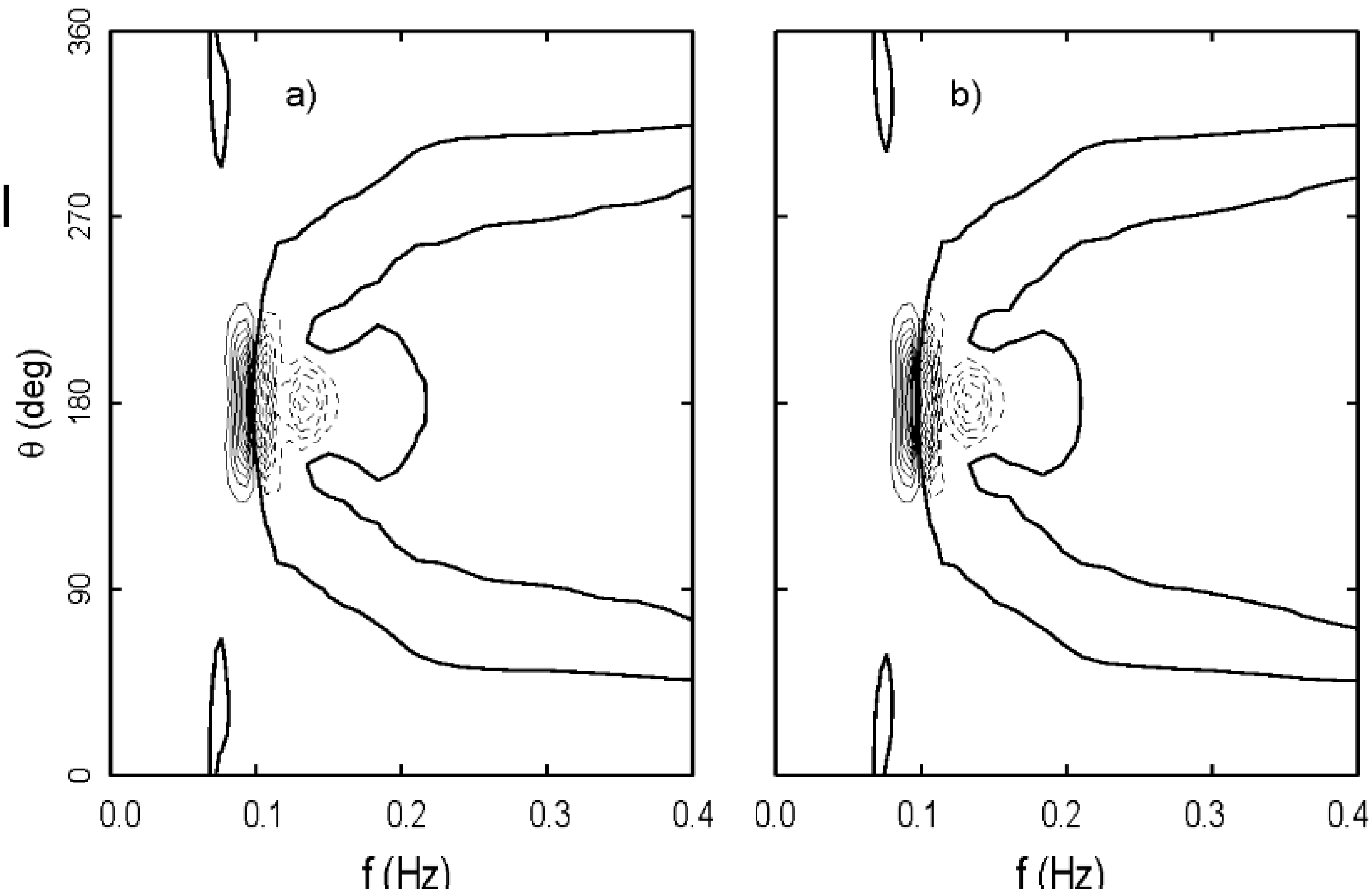


a) Standard template grid and b) Refined Template Grid

Extension of TSA Model in Current Effort

- 2-D nonlinear source terms
- E(f) conventional JONSWAP form

α= 0.012,
f_m= 0.1 Hz,
σ_A= 0.07,
σ_B= 0.09,
γ= 3.3



TSA results in plotted in a) and FBI results plotted in b).
(solid lines = + changes, dashed lines = - changes)

Evolution of Methodology

- Recent efforts have applied a “template” approach to map the spectra
  - Allows application of TSA approach for spectra developed in nature or a numerical models
- Template approach works, but has its limitations in terms of accuracy and efficiency
  - Trouble when spectra peak occurs between two values within template
- Current and future work is taking a new approach that interpolates the spectral shape

Future Efforts

- Recent efforts have shown the TSA to provide accurate and efficient results
- However, accuracy and efficiency improvements can occur moving away from the template-based approach
- In addition, improved efficiency in coding algorithms can provide additional run time improvements
- Dorukhan Ardag will expand on these current and future efforts

Summary

- Testing of new TSA approach shows good comparisons to FBI and significant accuracy improvement compared to DIA
- Testing shows improved computational speeds and efficiency; but one order of magnitude slower than the DIA
- After additional testing, new algorithms should improve efficiency and allow implementation in operational spectral wave models
- More to follow…

THANK YOU