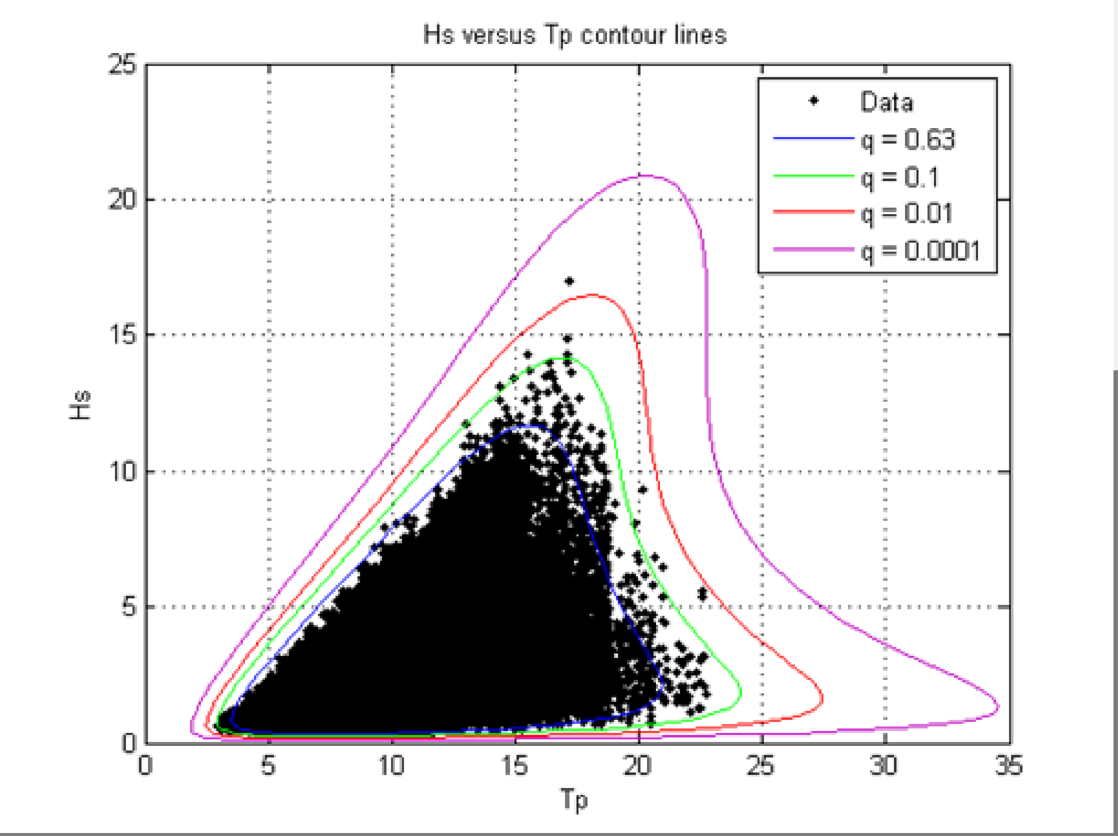
Environmental contour method: An approximate method for obtaining characteristic response extremes for design purposes

Sverre Haver & Kjersti Bruserud, Statoil ASA
Gro Sagli Baarholm, Det Norske Veritas



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Rule requirements for characteristic design response

- **Characteristisk response, x_c, are specified by requirements regarding the annual probability, q, of exceeding the characteristic value.**

- **Ultimate limit state (ULS):** $q \leq 10^{-2}$ **(per year)**

- **Accidental limit state (ALS):** $q \leq 10^{-4}$ **(per year)**

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Sources of inherent randomness

- **Long term variability of slowly varying weather characteristics, e.g. significant wave height, H_s, and spectral peak period, T_p.**

Possible description: $$f_{H_s T_p}(h,t) = f_{H_s}(h)\, f_{T_p|H_s}(t|h)$$

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$$\boldsymbol{F_{X_{3h}|H_s T_p}(x|h,t)}$$

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- Short term variability of 3-hour (or 30 minute) maximum given the weather condition, i.e.:

  $$F_{X_{3h}|H_s T_p}(x|h,t)$$

- **Long term distribution of X_{3h}:**

$$F_{X_{3h}}(x) = \int_h \int_t F_{X_{3h}|H_s T_p}(x|h,t)\, f_{H_s T_p}(h,t)\, dt\, dh$$

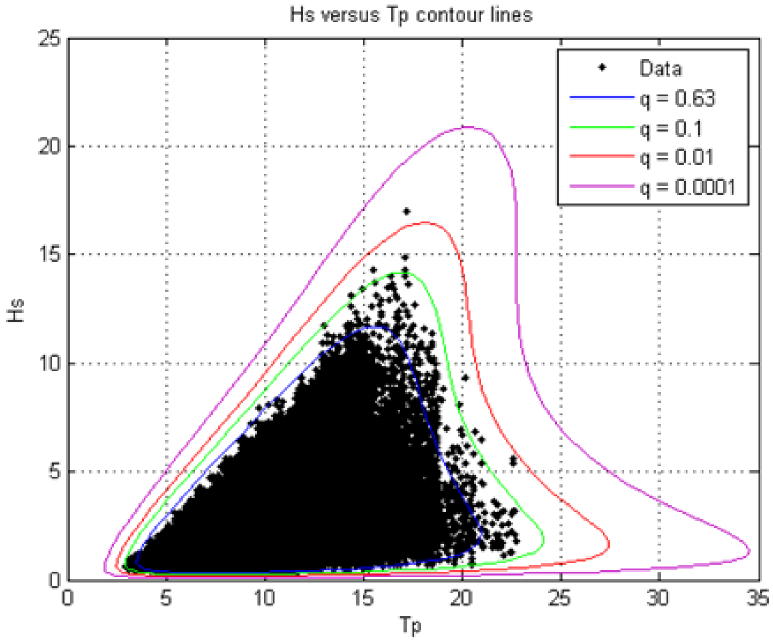
Target response:

$$x_q = F_{X_{3h}}{}^{-1}\left(1 - \frac{q}{2920}\right)$$

Environmental contour method

The challenge!

1. **Determine contours from** $f_{H_s T_p}(h,t)$ **.**



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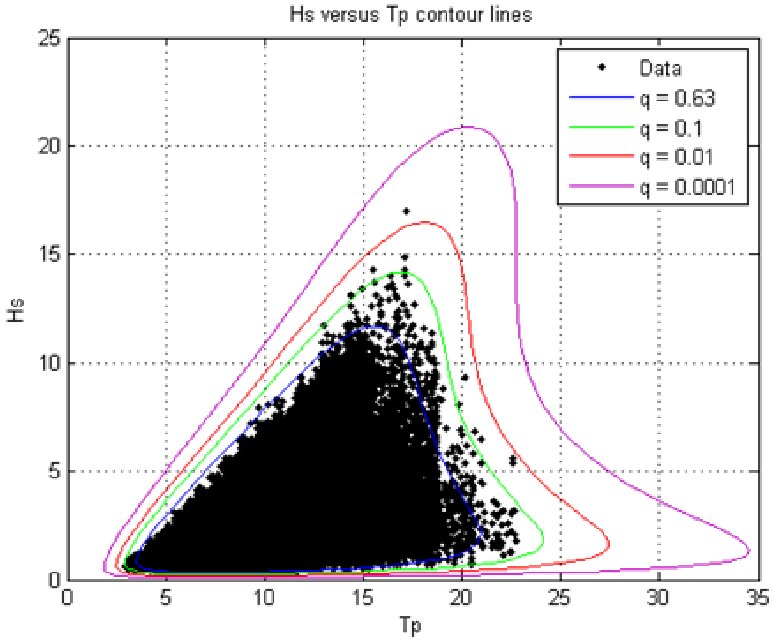
2. Find worst sea state along e.g. 10^{-2} - annual probability contour for response under consideration.



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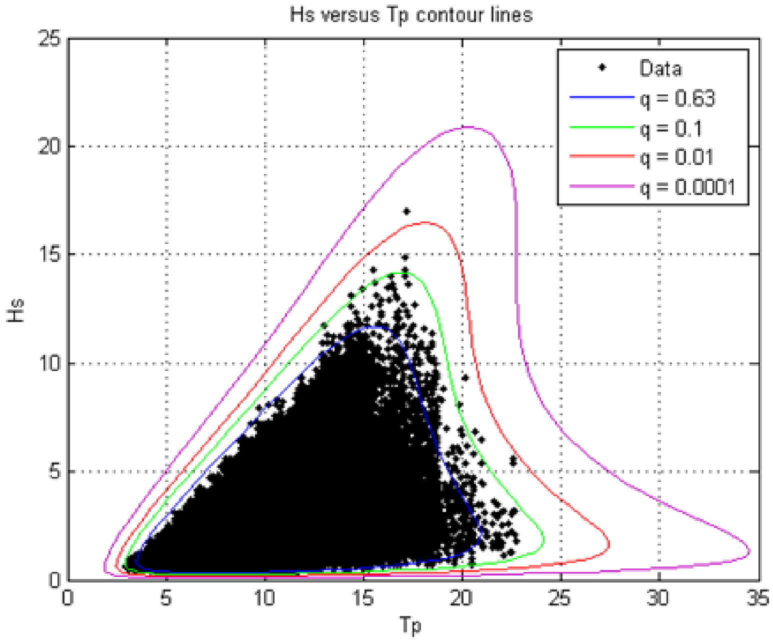
$$F_{X_{3h}|DSS}(x|DSS).$$



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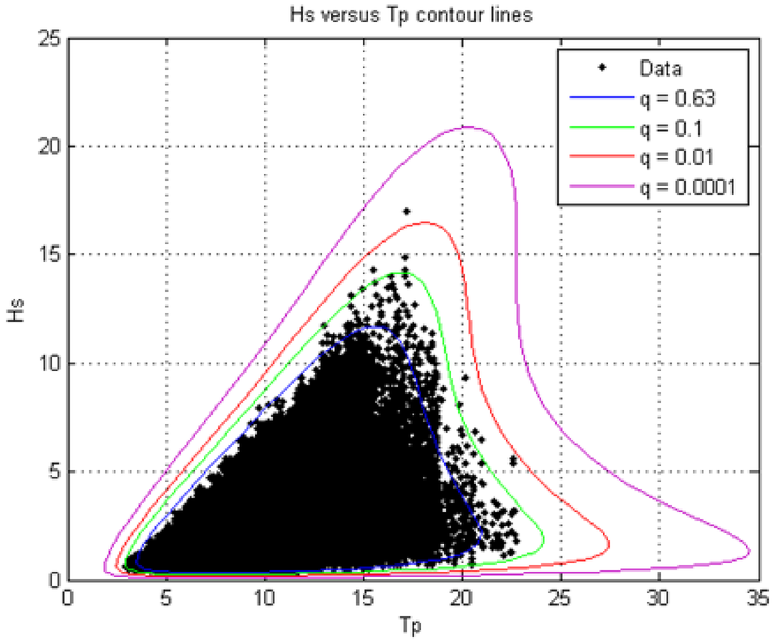
4. **Estimate $x_{0.01}$ by:**

$$x_{0.01} = F_{X_{3h}|\,DSS}{}^{-1}(\alpha)$$

where typically $\alpha = 0.85 - 0.90$



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Why should it work?

- **Let us assume that the 3-hour maximum response is a deterministic function of significant wave height and spectral peak period:**

$$x_{3h} = g(h, t)$$

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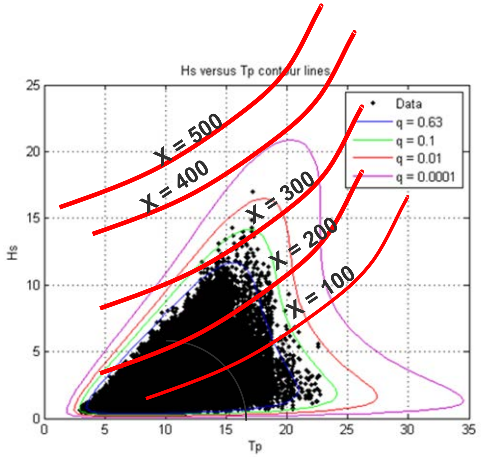

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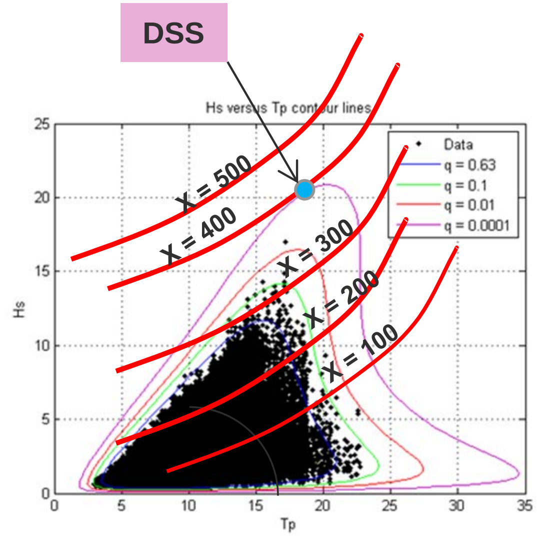
- ➔ $x_{0.0001}$ = 400 (we can think of this as the median response in a vary narrow extreme value distribution). Design sea state (DSS) is shown on contour.



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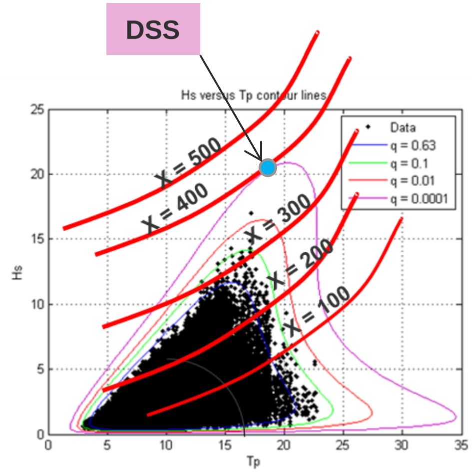

- **In reality, the 3-hour extreme will be of an inherent random nature. The median will be too small. We have to go to a higher percentile. How high depends on the relative importance of the short term variablity. Experiences indicate that this is rather similar for a broad range of problems. Good estimates are often obtained selecting the 0.90-0.95 fractile (for $q = 10^{-4}$).**

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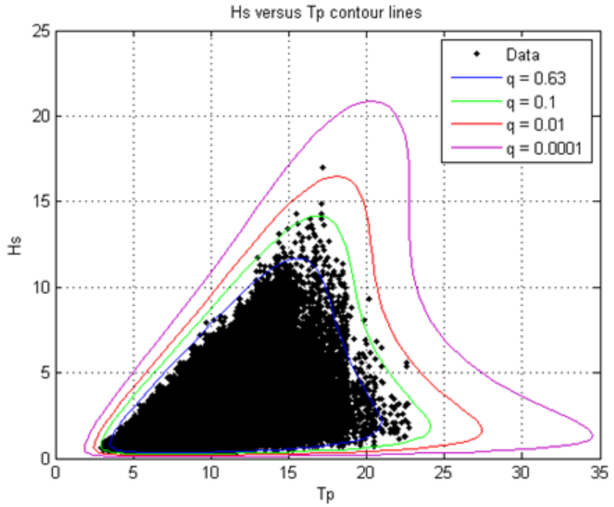
What must be fulfilled for the method to work?

- **The dominant part of long term variability must be carried by the selected weather characteristics.**



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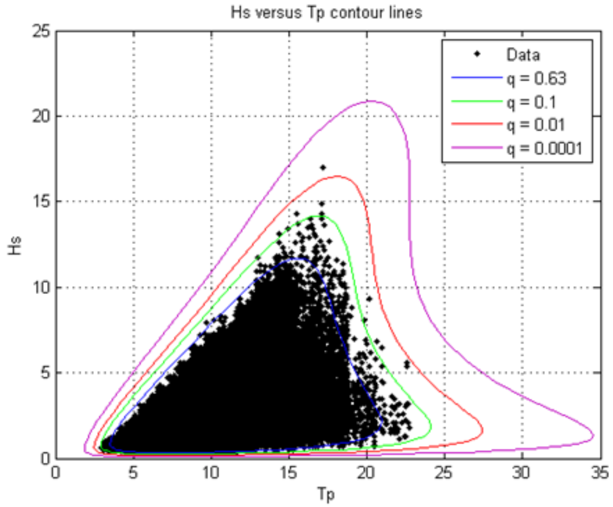
- The extreme response along the q_1 - contour must be larger than the extreme response along the q_2 - contour if $q_1 < q_2$.

  If this is not fulfilled, some sort of a full long term analysis should be preferred.



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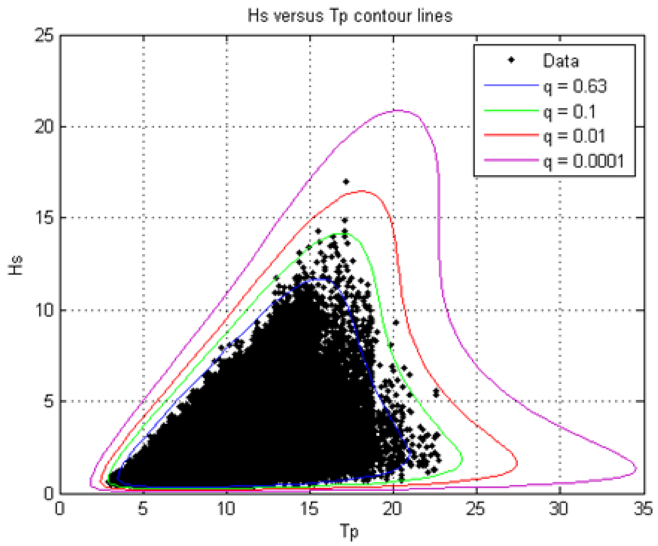
- **For a typical response problem cov for X_{3h} is in the 0.1 – 0.25. $\alpha = 0.85 - 0.9$ often ok when q = 10^{-2}.**

  For loads from breaking wave impacts, the cov of X_{3h} is 0.5 – 1.0 !!! Method may work – but one will most proably have to adopt high fractiles.



➔ A long term analysis is possibly to be preferred?

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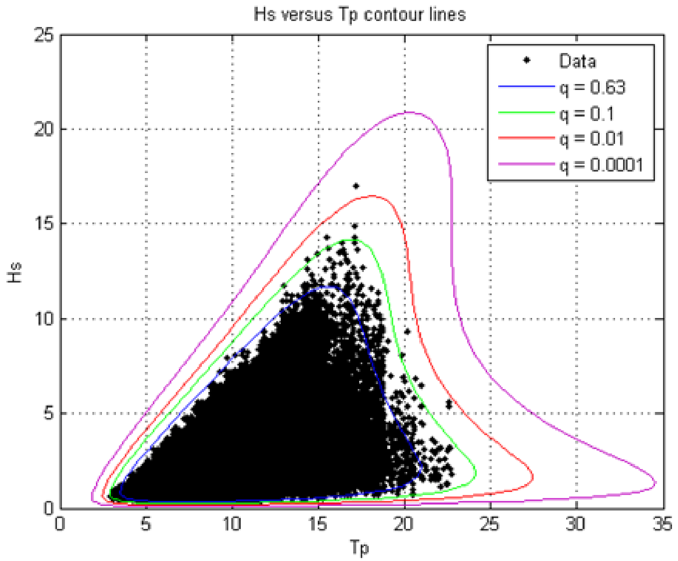
Challenge: Modelling T_p conditionally on H_s

- 

- **T_p given H_s is assumed to follow a log-normal model, parameters are $\mu = E(\ln T_p|H_s)$ and $\sigma^2 = Var(\ln T_p|H_s)$.**

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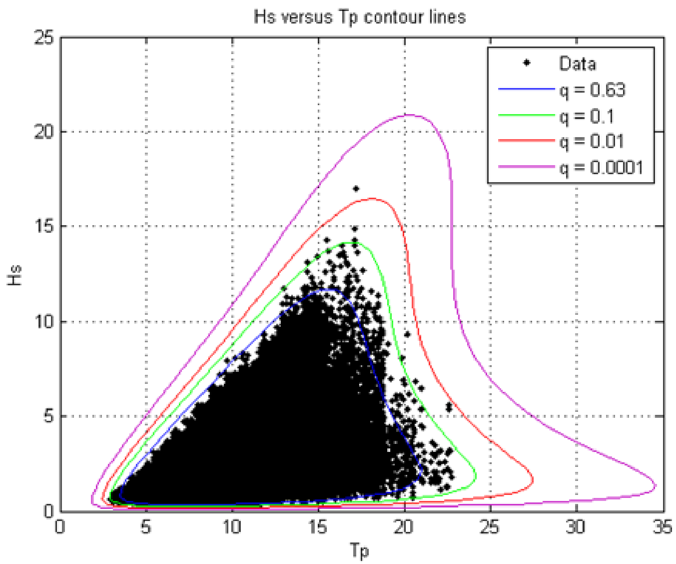
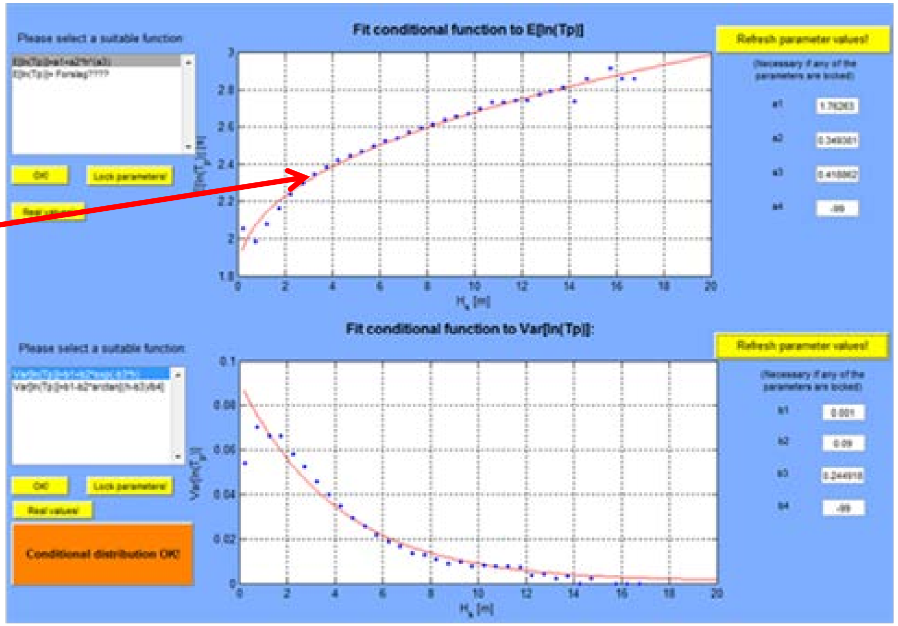


- T_p given H_s is assumed to follow a log-normal model, parameters are μ and σ^2.
- **Estimating μ is not to critical, but uncertainties are introduced.**



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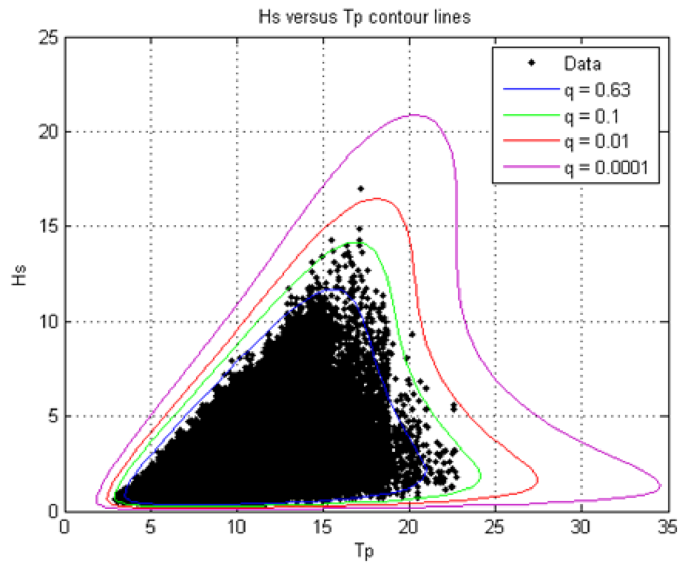
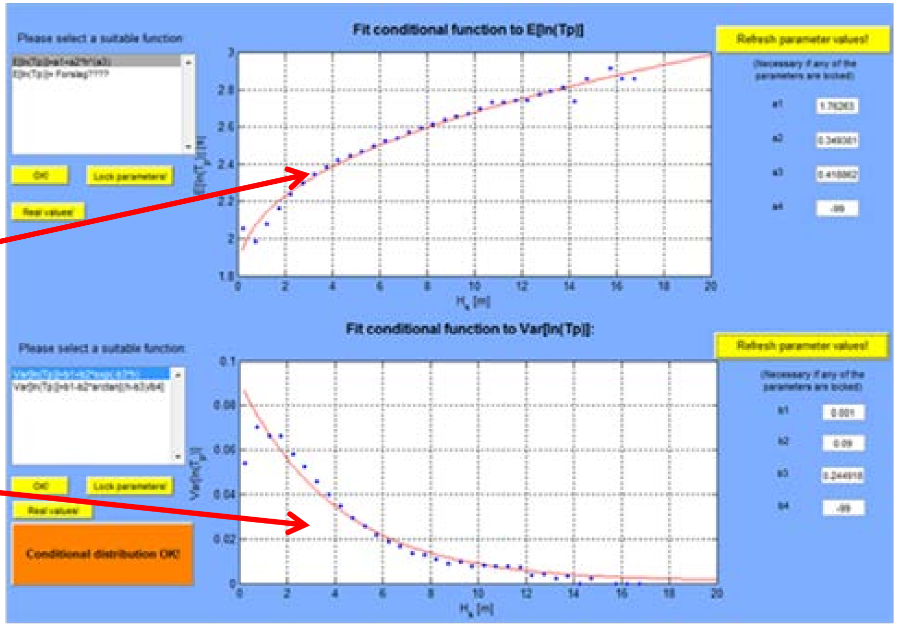
- 

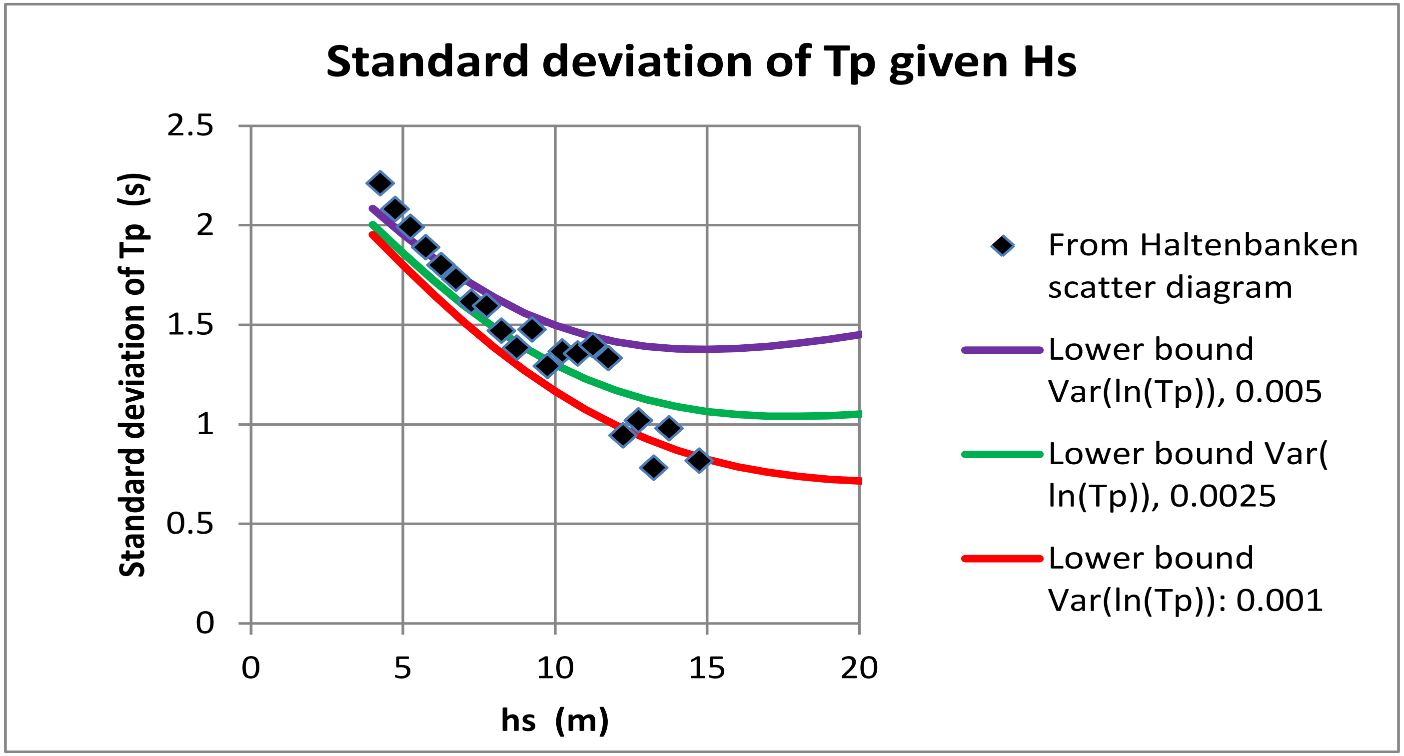
$$\bar{t_p} = exp\{\mu + 0.5\sigma^2\}$$

$$\sigma_{T_p} = \bar{t_p}\sqrt{exp\{\sigma^2\} - 1} \approx \bar{t_p}\,\sigma$$

- T_p given H_s is assumed to follow a log-normal model, parameters are μ and σ^2.
- Estimating μ is not to critical, but uncertainties are introduced.
- **Estimating σ^2outside range of data is a challenge!**



Uncertainties in standard deviation of T_p given H_s



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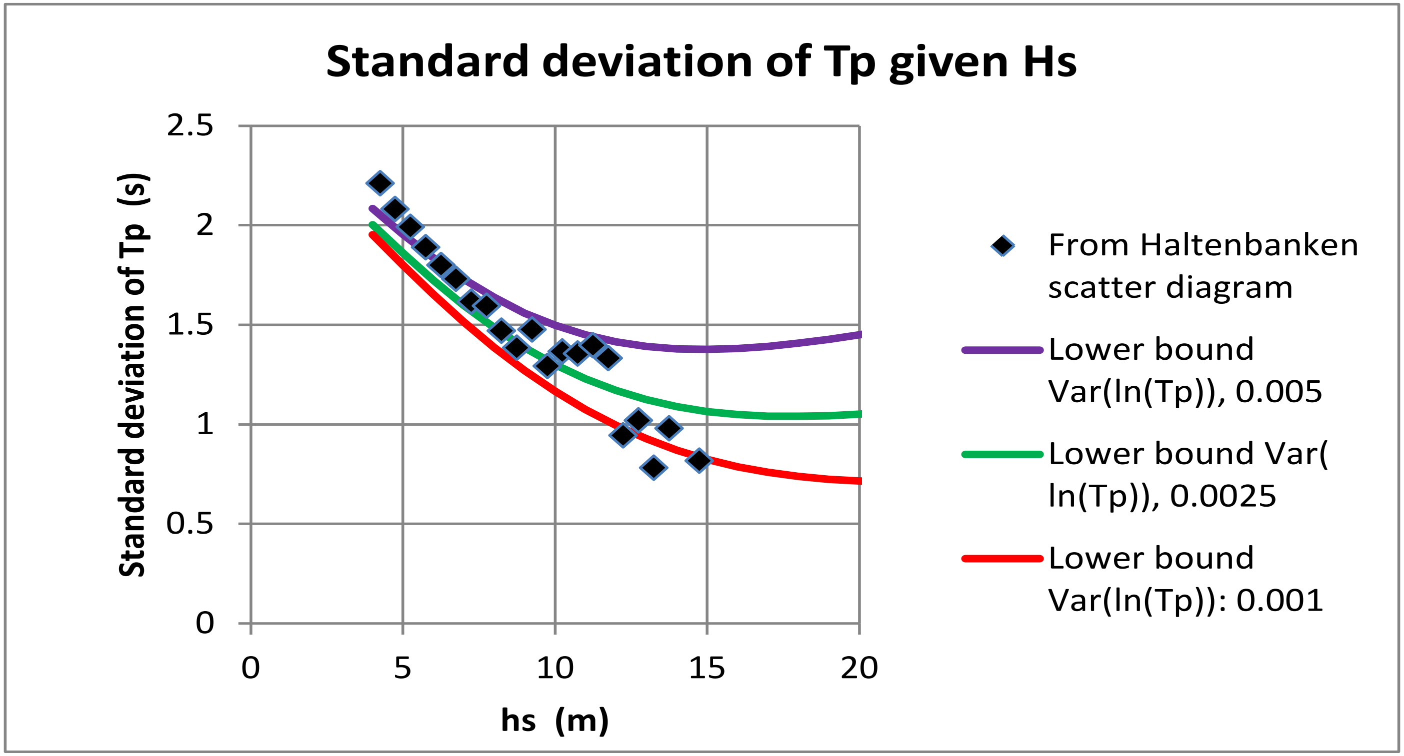

➔ **We need:**

*** More data of extreme sea states (not so easy).**

*** Better understanding of accuracy of hindcast T_p.**

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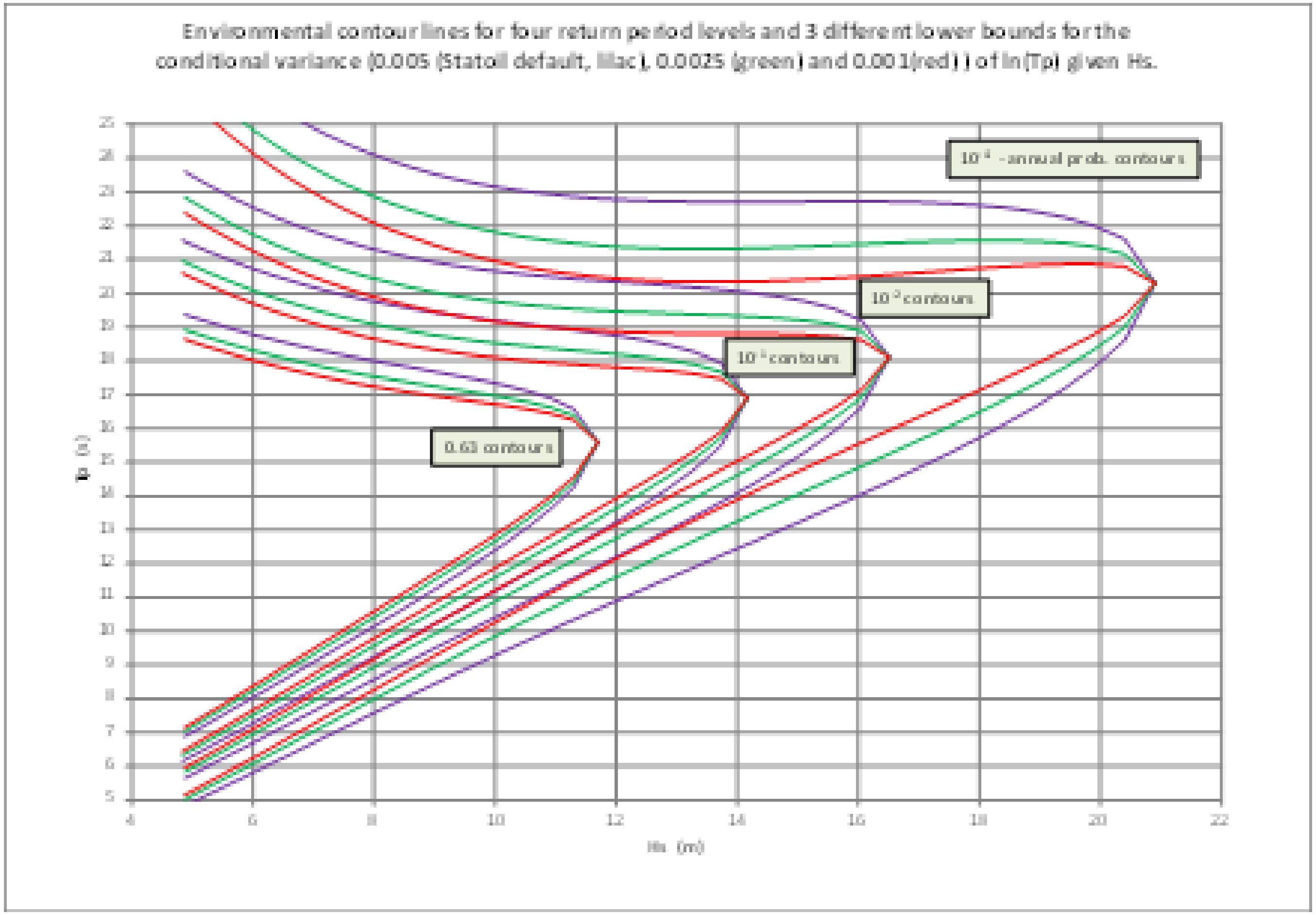
Consequence of spreading uncertainty

Spectral peak period (s)



Significant wave height (m)

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Part II: Can contour method be used in a GoM hurricane climate ?

- **We consider all hurricanes exceeding some threshold.**

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$$F_{Y \mid hurricane\ characteristics}(y \mid hurricane\ characteristics)$$

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- **When applying the environmental contour method we would characterize a hurricane (for the purpose of a analysis of wave induced response) by three parameters:**
  H_{sp} = maximum significant wave height of the storm, T_{pp} = spectral peak period associated with H_{sp} (and D_p = duration of the most severe part of hurricane). These carry the long term variability:

$f_{H_{sp}T_{pp}D_p}(h,t,d)$ **(In long term analysis these are replaced by $\tilde{Y}$ [mpm of Y].)**

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$f_{H_{sp}T_{pp}D_p}(h, t, d)$ (In long term analysis these are replacer $\tilde{Y}$ [mpm of Y].)

- **If the two sources of inherent randomness have the same relative contribution to total variability for a broad range of response cases, the contour method may well be a useful approximate method for hurricane governed areas also.**

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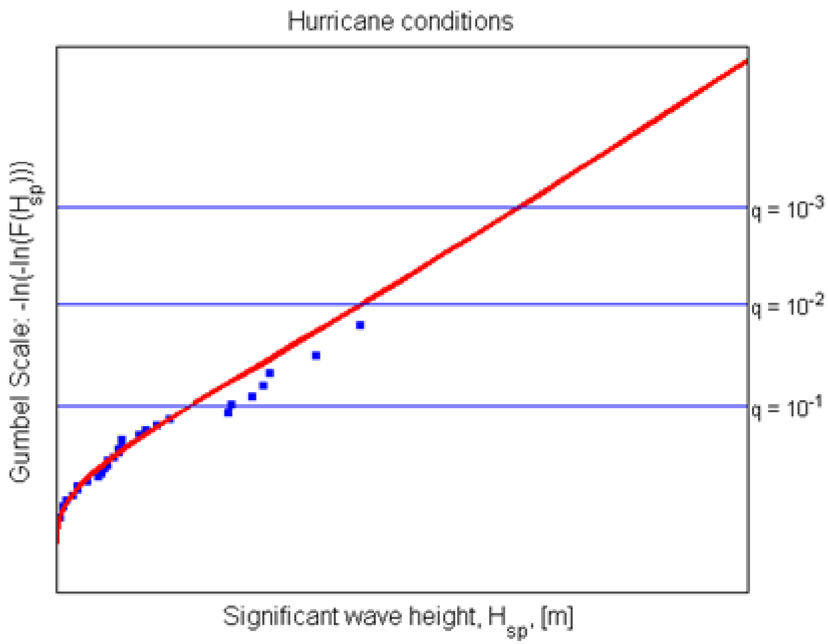
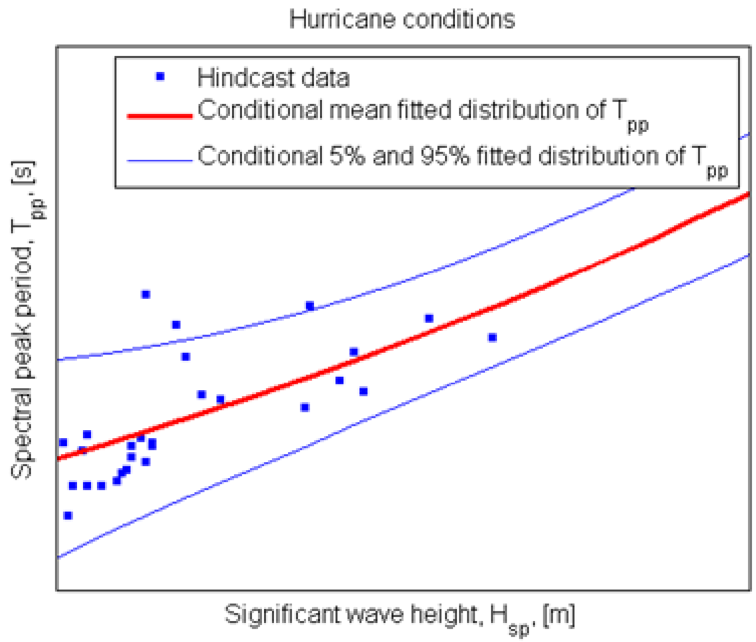
Modelling of joint distribution of H_{sp} and T_{pp}





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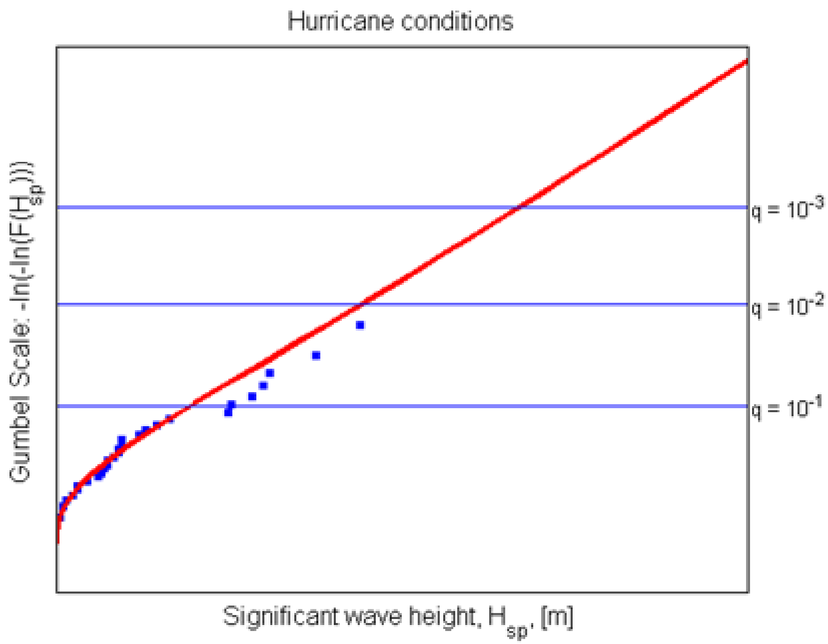

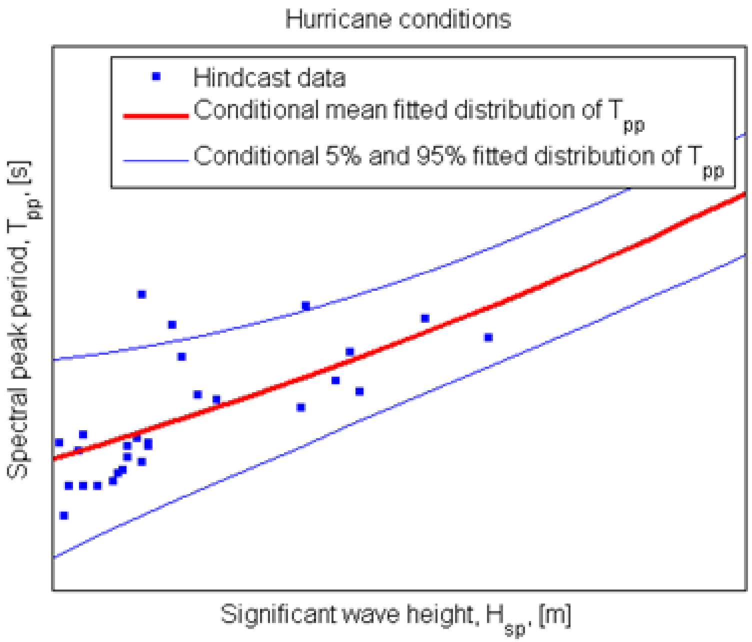

Challenge:

Limited amount of independent hurricane data within an area of say 1º x 1º

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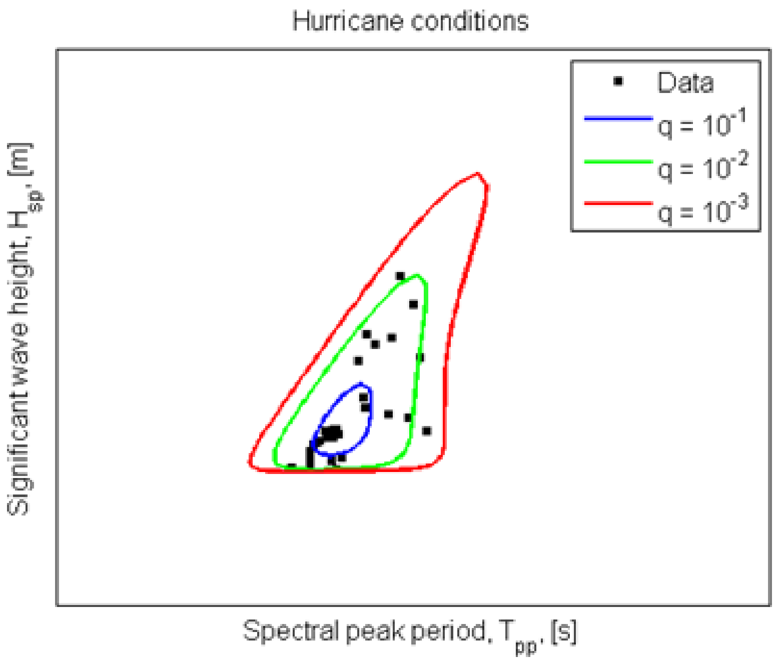
Contour & example results



- **Duration of hurricane maximum is taken to be 30 minutes.**

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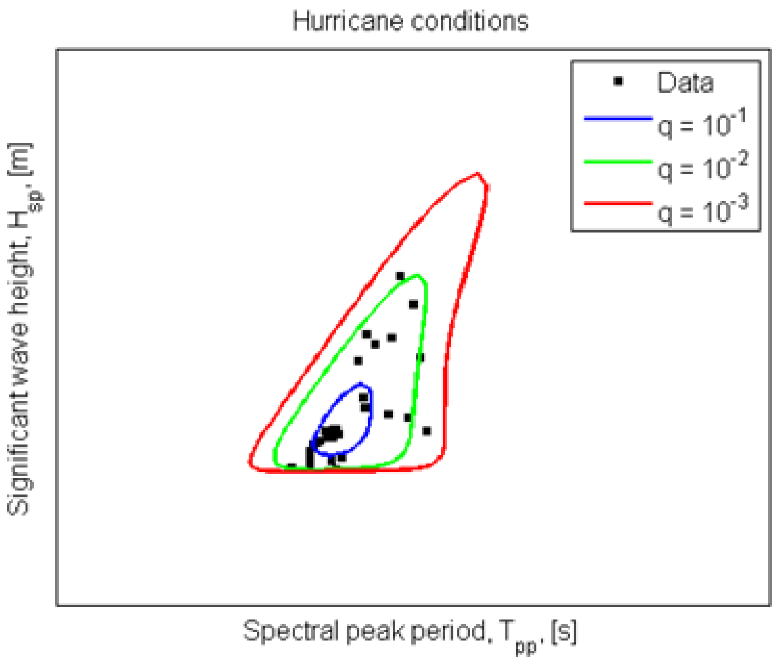

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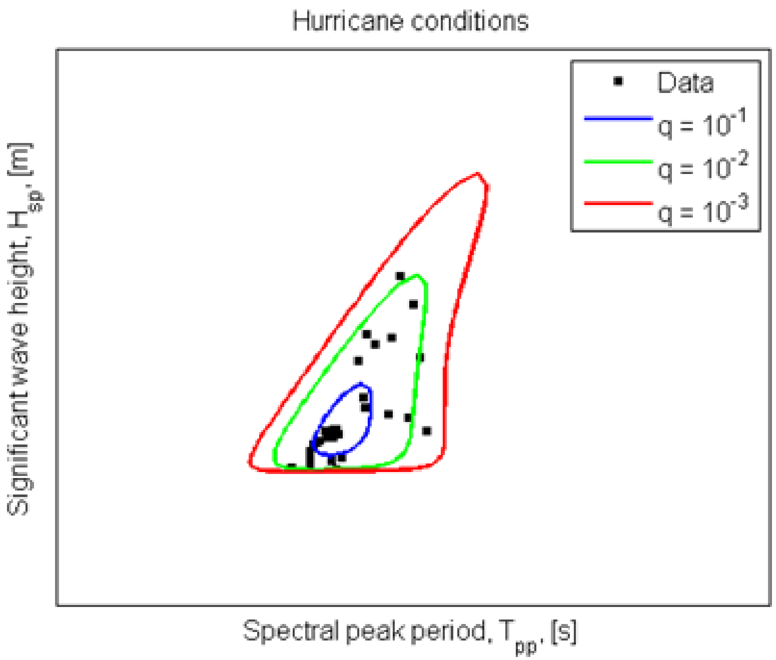

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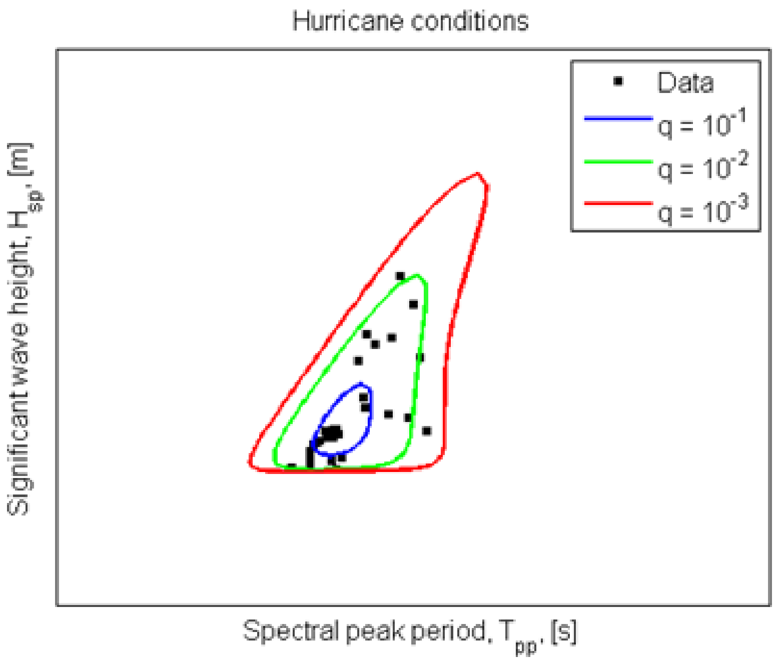

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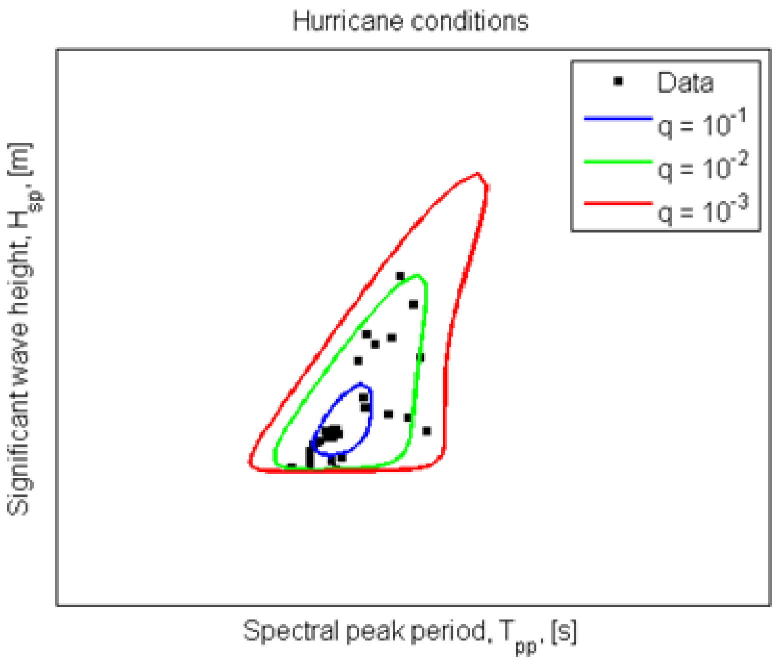

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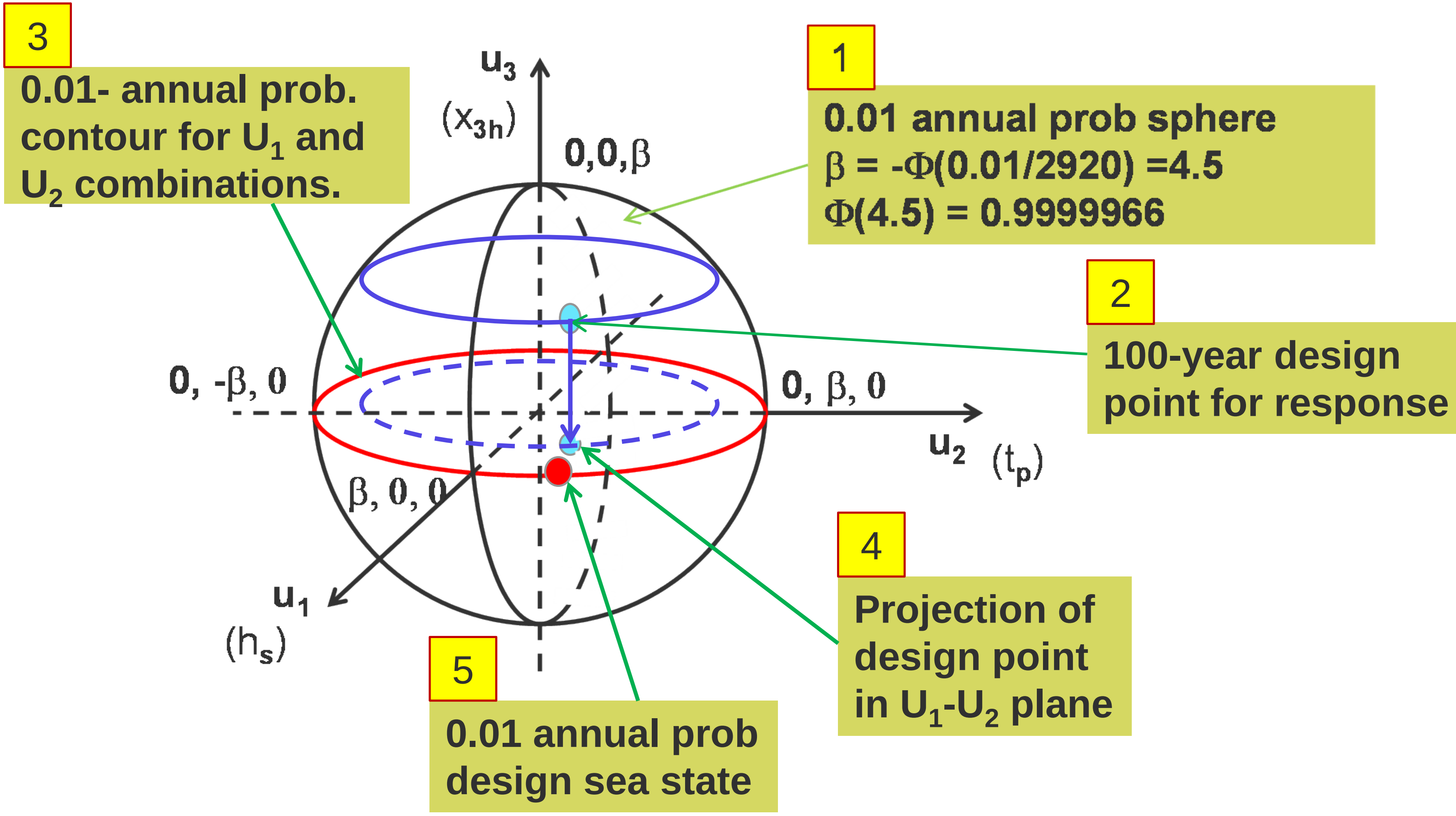
- For the cases we considered, target percentile for obaining 10^{-2} – response was varying from 0.88 to 0.97 with an average value of 0.94.
- If we artificially increase duration of peak event to 3 hours, target percentiles reduces to 0.75 – 0.80 about.

➔ Short term variability is of somewhat less importance in GoM than in North Sea (as expected).

Background IV

(And it is clear why we need a percentile of X_{3h} > 0.5 ??)



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Conclusions

Most important:

A too small amount of data of extreme weather conditions is the largest challenge!

There's never been a better time for good ideas

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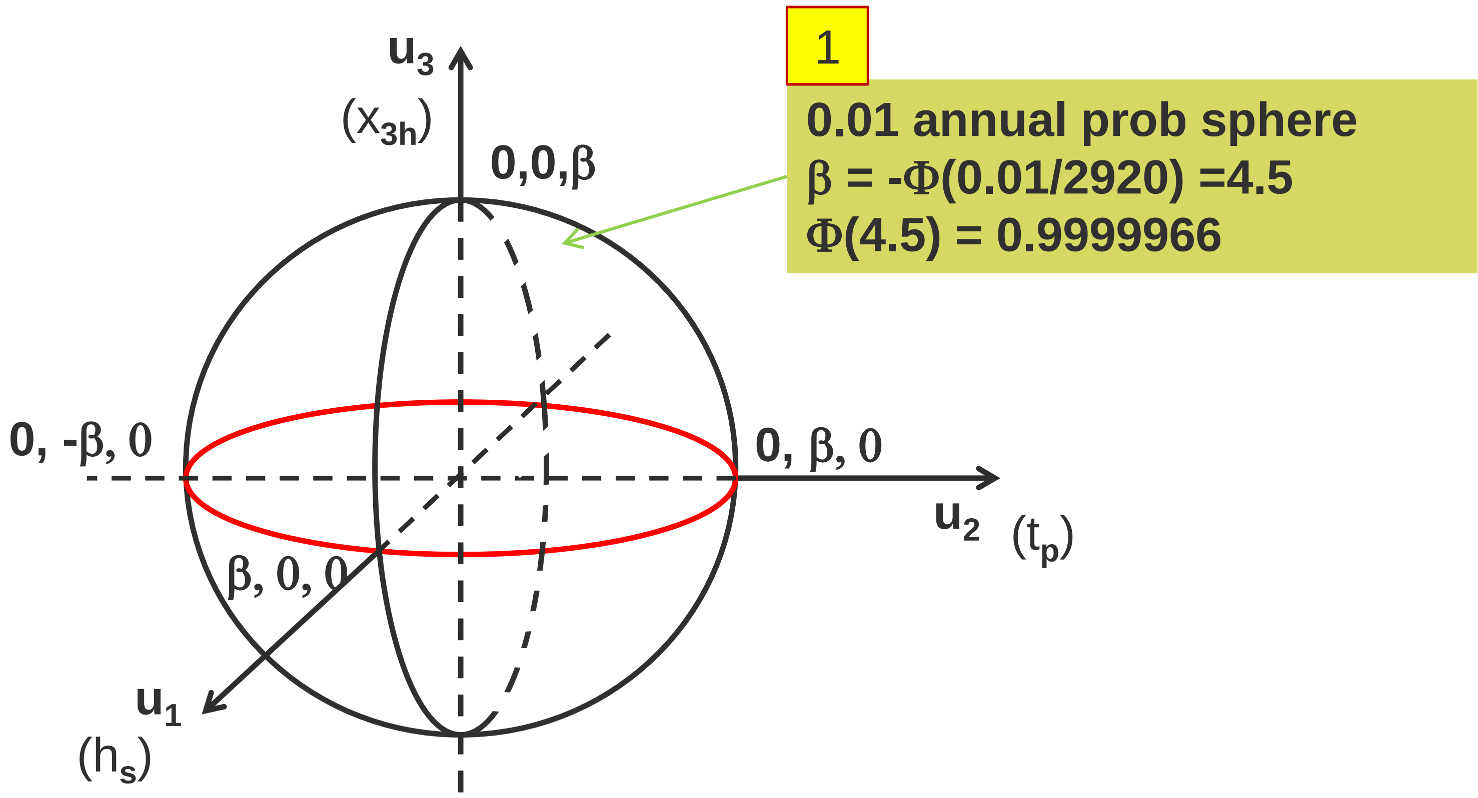
Introduction to background got the environmental contour method I
Problem is transformed to u-space

(u-space consists of independent, standard Gaussian variables)



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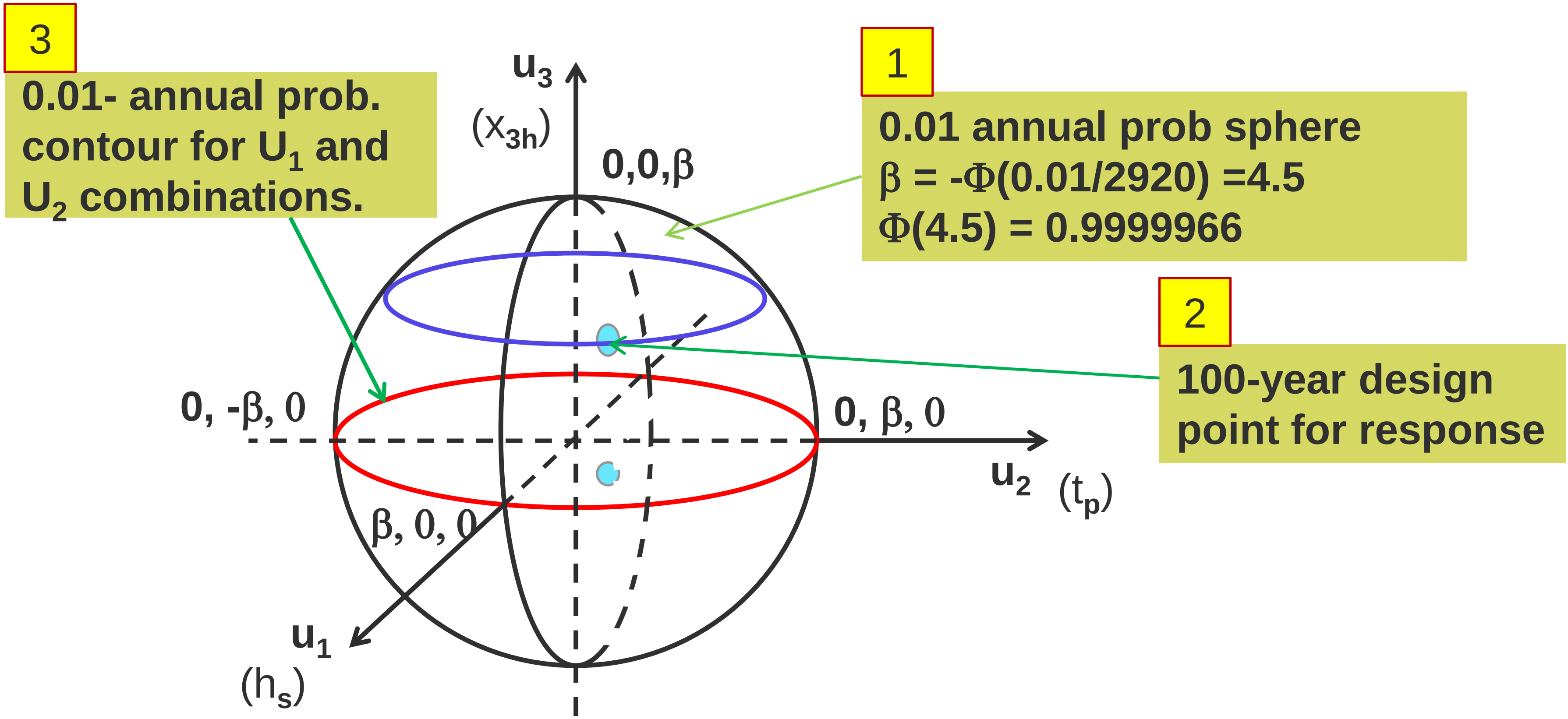
Background II



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Background III