Environmental contour method: An approximate method for obtaining characteristic response extremes for design purposes

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1. INTRODUCTION

The design environmental load is given as the product of a characteristic load, $x_{e,q}$ and a partial safety factor γ_f. Governing rules and regulations give values to be used for γ_f and they also define characteristic environmental loads (and load effects) by specifying the maximum permissible annual exceedance probabilities, q. For ultimate limit state, ULS: $q = 10^{-2}$. In addition, Norwegian regulations also require the accidental limit state, ALS, to be applied for environmental loads. For this limit state, γ_f is in most cases set equal to 1.0, while $q = 10^{-4}$ per year when applied to an intact platform.

If the load pattern is not dramatically altered by approaching more severe environmental conditions, ULS will typically govern the design. An exception to this could be the case where the area exposed to environmental loads is dramatically increased as more rare conditions are faced. An example of this is wave-in-deck impacts. The ULS wave crest may be below deck level. For a higher wave crest, but less than the ALS crest height, the deck structure may be exposed to wave loads. This will result in enormous increase of the load level and ALS may be governing. This is illustrated in Fig. 1. This sort of scenario should be avoided at original design, but may possibly be faced when reassessing older structures.

The purpose of this paper is to review methods being available for a consistent estimation of the target loads or load effects.

2. TARGET VARIABLES AND SOURCES OF VARIABILITY

Target quantities are defined in terms of their annual exceedance probabilities. In order to estimate such quantities consistently, some sort of long term analysis should be carried out, *i.e.* we should add up annual exceedance probabilities for all possible combinations of important weather characteristics and ensure that this sum is lower than or equal to the target annual exceedance probability. We will review methods for long term analyses below.

When estimating characteristic values, all important inherent (aleatory) variability should be accounted for in the long term analysis. Epistemic (lack of knowledge) type of uncertainties will in most cases be assumed covered by the partial safety factor. Whether or not this is a good idea can be debated, because a considerable part of the partial safety factor, γ_f, is utilized to account for the inherent variability of the largest weather induced load/response experienced during structural life time.

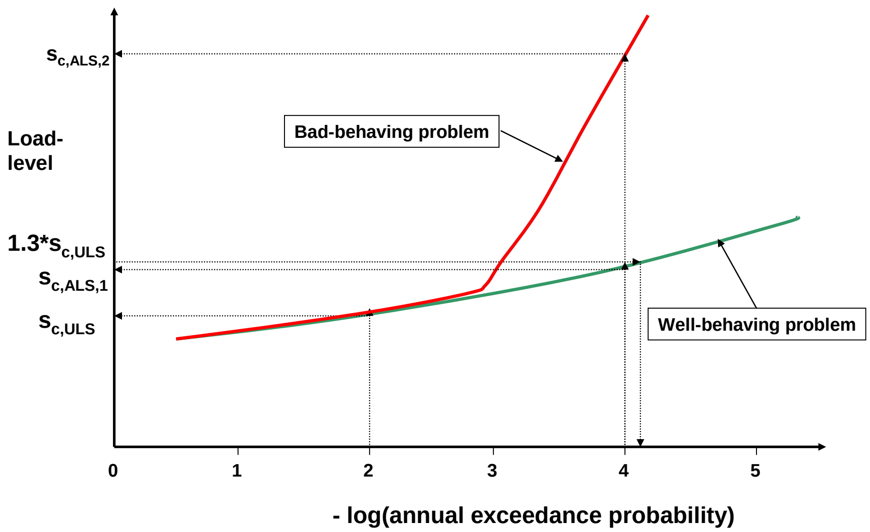

Fig. 1 Illustration of a load problem with an abrupt change of pattern for an annual exceedance probability larger 10^{-4}, but lower than 10^{-2} per year ($s_{C,ALS,1}$ is characteristic ALS response for a well behaving system, while $s_{C,ALS,2}$ is denoting the ALS response for a bad behaving system).

Gross errors or human errors are not covered by the safety factors and need to be eliminated by adequate quality assurance procedures during the design and operational phase.

For very complex structural problems it may be time consuming and costly to perform a full long term response analysis – if at all possible. An example where this could be the case is a structural response problem being of such a complexity that extensive model testing is required to short probabilistic behavior of the response quantity under consideration. For such cases a simplified approach may be required. The environmental contour method may represent one possibility and will be discussed later in this paper.

3. LONG TERM RESPONSE ANALYSES

3.1 NORTH SEA TYPE OF WAVE CLIMATE

In the following we will consider wave induced load effects. We will for illustrative purposes assume that a short term sea state is defined in terms of significant wave height, H_s, and spectral peak period, T_p. More parameters could in principle be included in the analyses, say wave direction, wind speed and wind direction, but establishing an accurate joint probabilistic model of the variables will represent a challenge. If many parameters are expected to be important other formulations of the long term analysis should be preferred.

The long term analysis could either be done by selecting the individual global response maxima (largest maximum between zero-down-crossings), X, as target variable. Alternatively, we can use the largest response maximum in a short term sea state of 3-hour duration, X_{3h}, as our target variable. The modeling of the long term variation of sea states as a piecewise stationary step function with step length of 3 hours is of course an abstraction, but seems to work rather well for typical North Sea/North Atlantic type of wave climate.

Adopting X_{3h} as target variable, the long term distribution of X_{3h}, $F_{X_{3h}}(x)$ is given by:

$$F_{X_{3h}}(x) = \int_h \int_t F_{X_{3h}|H_s,T_p}(x|h,t)\, f_{H_sT_p}(h,t)\, dt dh \,, \tag{1}$$

where $F_{X_{3h}|H_sT_p}(x|h,t)$ is the short term distribution of 3-hour maximum response given sea state characteristics and $f_{H_sT_p}(h,t)$ is the long term distributions of sea state characteristics.

It should be noted that Eq. (1) gives the probability of not exceeding the response level x in an arbitrary 3-hour sea state. Rules define characteristics in terms of an annual exceedance probability, q. Using the long term distribution given by Eq. (1) to estimate the q-probability response (*i.e.* the response corresponding to an annual exceedance probability of q), the target response, x_q, is found by solving the following equation:

$$1 - F_{X_{3h}}(x_q) = \frac{q}{m_{3h}} \,, \tag{2}$$

where m_{3h} is the annual number of 3-hour events.

With the availability of good quality hindcast data for about 50 years, a reasonable good joint probabilistic model for H_s and T_p can be obtained. For complex response systems, a larger challenge is to establish an accurate estimate for the short term distribution of the 3-hour maximum for all possible (or rather all important) combinations of H_s and T_p. If the underlying problem can be solved by numerical time domain simulations, a possible approach could be to assume that the 3-hour extreme value, for a sea state defined by $H_s = h$ and $T_p = t$, follow a Gumbel distribution with location parameter α and scale parameter β:

$$F_{X_{3h}|H_sT_p}(x|h,t) = exp\left\{- exp\left\{- \frac{x - \alpha(h,t)}{\beta(h,t)}\right\}\right\} \tag{3}$$

Doing a rather large number of 3-hour simulations – say 30 – for a large number of sea states, point estimates for α and β covering the sample space of h and t can be found by fitting the Gumbel model to the samples of X_{3h} for the various sea states. By fitting response surfaces, $\alpha(h,t)$ and β(h,t), to the point estimates, Eq. (1) can be solved. An example of such an approach is found in Baarholm et al.(2010).

If one needs to include more than – say 3 – environmental characteristics, the approach above will not be attractive. If this is the case, one should rather perform a long term analysis based on a peak-over-threshold (POT) formulation of the problem. Such an approach will of course be an adequate alternative for North Sea/North Atlantic environmental climate, but the approach will be briefly presented in connection with describing long term response analysis for hurricane governed offshore areas.

3.2 GULF OF MEXICO TYPE OF CLIMATE

For a hurricane governed area, the long term weather consists of two populations:

a) A population describing the hurricane condition

b) A population describing the non-hurricane conditions (winter storms)

The severity of long return period hurricanes are so much worse than the worst non-hurricane conditions that regarding extreme value predictions we can limit the consideration to the hurricane population.

Target characteristic response quantities are defined in terms of their annual exceedance probabilities suggesting that our preferred approach should be some sort of long term response analysis. For this purpose, the hurricane maximum response will be selected as the target response quantity. We will denote this quantity by Y. Furthermore it is assumed that the conditional distribution of Y given the most probable largest response of the storm $\tilde{Y} = \tilde{y}$ is reasonably well approximated by a Gumbel distribution function:

$$F_{Y|\tilde{Y}}(y\,|\tilde{y}\,) = exp\left\{-\,exp\,\left\{-\,\frac{y-\alpha_V\tilde{y}}{\beta_V\tilde{y}}\right\}\right\} \quad (4)$$

α_V and β_V are found by fitting a Gumbel model to a sample of normalized storm maximum response, $v_i = y_i/\tilde{y}_i,\ i\ =\ 1,2,\ldots.,N$, where N is the number of storms.

If we know the long term distribution of most probable largest hurricane response, $f_{\tilde{Y}}(\tilde{y})$, the long term distribution of *Y* is given by, Tromans et al. (1995):

$$F_Y(y) = \ \int_{\tilde{y}} F_{Y\,|\,\tilde{Y}}(y\,|\tilde{y})\, f_{\tilde{Y}}(\tilde{y})\, d\tilde{y} \quad (5)$$

$\tilde{Y}$ is a measure of the severity of a hurricane with respect to the target response quantity. Thus $f_{\tilde{y}}(\tilde{y})$ therefore represent the long term distribution of hurricane severity for this particular response quantity. It is analogue to the long term joint distribution of H_s and T_p in the North Sea section. The disadvantage of the approach the long term modeling of storm severity becomes response specific.

The q-probability response, y_q, is given by:

$$1 - F_Y\left(y_q\right) = \ \frac{q}{m_q}, \quad (6)$$

where m_q is the expected number of hurricanes above selected threshold at the target site during $1/q$ years. Similar approaches are also discussed by Haring and Heideman (1978).

Let us assume that we have measured m hurricanes during T years at an offshore site. We approximate the hurricane history as a step function. Step duration is constant and equal to τ. Within each step we assume all slowly varying hurricane characteristics to be constant, *i.e.* the response process is assumed to be stationary within each step. We will assume that problem can be solved by time domain analyses, *i.e.* for each step we carry out a reasonably large number, say k, of time domain simulations of length equal to τ. From the k observed τ-hour largest values we can fit a proper extreme value distribution. For illustrative purposes, we will here assume the Gumbel distribution. Denoting the response extreme value distribution for step no. s with $F_{X_s}(x)$, $s\ =\ 1,2,\ldots,smax$, and assuming step extremes to be statistically independent, the distribution function for storm maximum response Y reads:

$$F_Y(y) = \ \prod_{s=1}^{smax} F_{X_s}(y) = exp\left\{-\ \sum_{s=1}^{smax} exp\left\{-\,\frac{y-\delta_s}{\varepsilon_s}\right\}\right\} \quad (7)$$

Eq. (7) gives numerical numbers for the hurricane maximum distribution. By determining the double derivative of this and estimating the value of y making this function equal to zero, the most probable largest hurricane response is determined. We will denote this value for a particular storm by $\tilde{y}$. By doing this procedure for all available storms, we will obtain a sample of most probable largest storm maximum representing the severity of the observed hurricanes. By fitting an adequate distribution function to this sample of most probable largest hurricane maxima, long term distribution of severity of hurricanes in view of the response under consideration (*i.e.* long term distribution of $\tilde{Y}$ is obtained.

It is seen from Eq. (4) that we need to estimate the parameters α_V and β_V before we can estimate the long term distribution of hurricane maximum response. We will here utilize a recommendation from Tromans et al. (1995) that the ratio variable $Y/\tilde{y}$ follows a Gumbel distribution independent of storm severity.

From the selected hurricane that corresponded to a particular realization of $\tilde{y}$ we simulate one possible realization for each (important) step of the storm using Monte Carlo and the distribution function for the step extreme value. When this is done for all steps we have one observation of the hurricane maximum response. After repeating this process of many times, we have a reasonably large sample for the ratio variable V, $v_i = \frac{y_i}{\tilde{y}}, i = 1,2, \dots, I$. The parameters α_V and β_V are estimated by fitting *e.g.* a Gumbel model to this sample. This model is assumed to be independent of storm severity as measured by $\tilde{y}$, i.e. we know the conditional distribution of Y given $\tilde{Y}$, Eq. (4).

The approach is illustrated for one set of step simulations for one particular storm and a particular response quantity in Fig. 2. The most probable largest response shown in Fig. 2 is a simplified approach as compared to text above, but the figure is primarily meant to indicate the variability of the step peak response. For another set of simulations, the "observed" step extremes would of course be different, *i.e.* the storm maximum response will be different from previous simulation and it is likely to appear for another step of the storm.

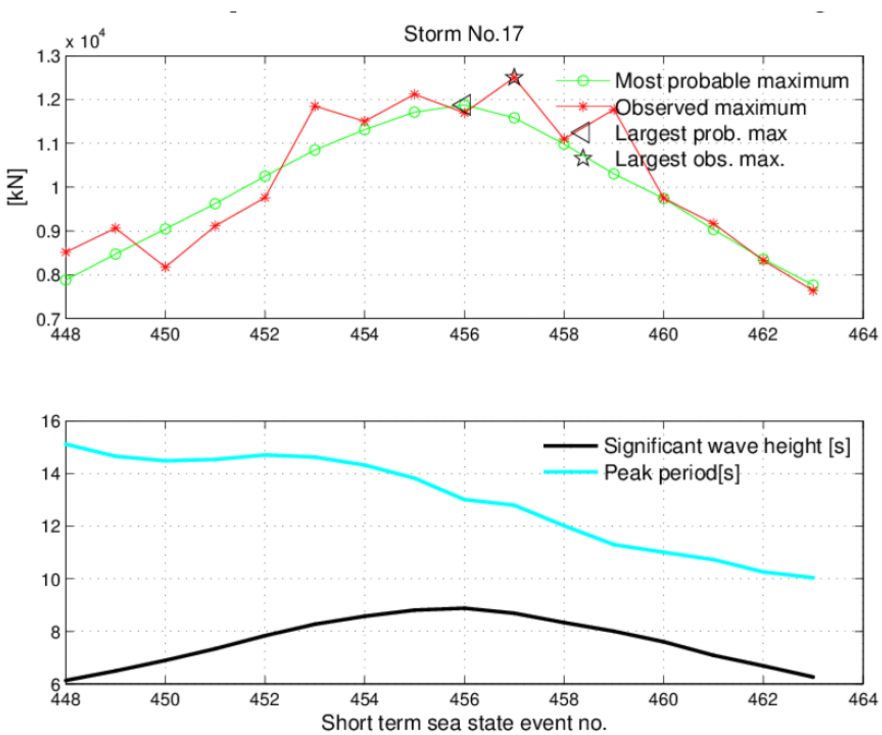

Fig. 2 Most probable hurricane maximum and observed hurricane maximum for an example case, Baarholm (2012).

4. ENVIRONMENTAL CONTOUR METHOD FOR WAVE DOMINATED PROBLEMS

4.1 NORTH SEA TYPE OF WAVE CLIMATE

For very complex response problems it is rather time consuming to establish the short term distribution of the 3-hour maximum response given the sea state characteristics. For such cases it would be useful to have a method where we could estimate long term extremes from a carefully selected short term sea state. The environmental contour method may represent a possible approach. In particular, this can be useful for cases where we have to estimate q-probability response extremes directly form model tests.

4.1.1 FORM AND IFORM

As an introduction to the environmental contour method, we will briefly illustrate how to utilize methods from structural reliability field for long term response analyses. We consider the 3-hour maximum response, X_{3h}, for a response problem where weather is characterized in terms of H_s and T_p. The boundary between the safe and unsafe domain of the data space is given by a limit state function, *i.e.*:

$$g(X_{3h}, H_s, T_p\,; x_{crit}) = x_{crit} - X_{3h}(H_s, T_p) \tag{8}$$

x_{crit} is the response level causing failure and thus $g(\ ) < 0$ defines failure. The failure probability can be found by integrating the joint probability density function of the 3 variables over the failure domain, i.e. all combinations of the variables making $g() < 0$. An alternative approach is to transform the problem to a standard Gaussian variable space, u-space, defined by three independent standard Gaussian variables U_1, U_2 and U. A transformation from physical space, x-space, to the u-space ensuring that the transformation is of a one-to-one nature is the Rosenblatt transformation, Madsen et al. (1986). The transformations ensure that percentiles of the one-dimensional distribution functions are transformed in a unique way from x-space (the physical parameter space) to the u-space:

$$\begin{aligned} &F_{H_s}(h) = \Phi(u_1) \\ &F_{T_p|H_s}(t|h) = \Phi(u_2) \\ &F_{X_d|H_sT_p}(x|h,t) = \Phi(u_3) \end{aligned} \tag{9}$$

$\Phi(\ )$ denotes the distribution function for a standard Gaussian variable. Solving Eq. (9) with respect to u_1, u_2 and u_3 yields the transformations required to go from x-space to u-space. Inverting these equations with respect to h, t and x yield the transformations for going back to the physical parameter space.

It is seen that the transformations conserve probabilities. In u-space the points of constant exceedance probability are located on sphere. The larger the radius, the lower is the exceedance probability. In the physical parameter space, the boundary between safe domain and unsafe domain is a plane normal to the X_{3h}-axis for $x = x_{crit}$. Transforming the limit state function to the u-space using the transformations given by Eq. (9), the limit state surface in the u-space will be of a more complicated shape. The point on the failure surface in u-space being closest to the origin is referred to as the design point. It is the most likely combination of variables as failure occurs, *i.e.* as $g(\ )$ becomes negative or, in other words, X_{3h} exceeds x_{crit}.

Denoting the design point by $(\hat{u}_1, \hat{u}_2, \hat{u}_3)$, the distance to the origin is given by:

$$\beta = \sqrt{\sum_i \hat{u}_i^2} \tag{10}$$

Since we are interesting in the most extreme response levels, the major contribution to the failure probability is typically within a domain rather close to the design point. Thus we can approximate the true failure boundary with a tangent plane in this point. This linearization is referred to as first order reliability method (FORM). Using this approach the failure probability is estimated by:

$$\hat{p}_f(x_{crit}) = \Phi(-\beta) \tag{11}$$

This probability represents an approximation of the long term probability obtained from Eq. (1) as $1 - F_{X_{3h}}(x_{crit})$. The approximation lies in the utilization of FORM, which is typically a very good approximation for low exceedance probabilities. An illustration of FORM is shown in Fig. 3, where – for illustrative purposes - the importance of the second variable is assumed to be negligible. It must be noted that Eq. (11) gives exceedance probability per 3-hour.

In Eq. (11) the failure probability per 3-hour corresponding to a given capacity is estimated. This is typically the target quantity for a reliability assessment. The time consuming part of this analysis is to identify the location of the design point.

For the design process we are aiming towards a response level corresponding to a given annual exceedance probability (failure probability), q. Thus we do know the target probability per 3-hour, $q/2920$ (2920 is the number of 3-hour events in one year), but we do not know the corresponding response level. However, since we know the right hand side of Eq. (11), we can estimate the sphere on which the design point is located by inverting the standard Gaussian distribution function. Introducing $q = 10^{-2}$, the distance to the design point, $\beta_{0.01}$, is found to be 4.5.

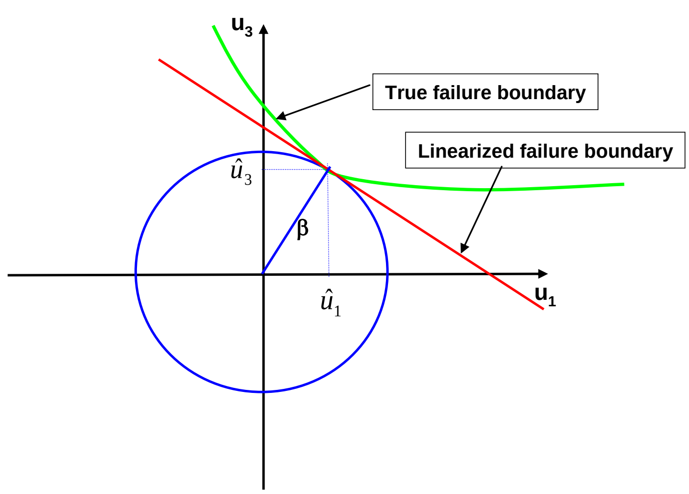

Fig. 3 Illustration of failure surface and linearized failure surface in u-space for a case where the design point of U_2, $\hat{u}_2$, is equal to 0.

This means that we know the sphere on which the design point corresponding to an annual exceedance probability of 10^{-2} must be located – no matter which response problem is considered. This approach is often referred to as Inverse FORM (IFORM), for more details on FORM & IFORM reference is made to e.g. Madsen et al. (1986) and Winterstein et al. (1993). The target response quantity, x_q, can be found by searching the sphere for the point (the design point) maximizing x_{3h} and this maximum value will be an estimate for the 10^{-2}– annual probability response. Alternatively, one can transform the sphere to the physical parameter space, x-space, using Eq. (9). The maximum value of x_{3h} of the surface in x-space will then represent the target response quantity, $x_{0.01}$.

4.1.2 FROM IFORM TO ENVIRONMENTAL CONTOURS

For very complex structural problems, the design point would be a possible design sea state. But this requires that we know the q-probability response and if we do that there is no reason for any further analysis. One could argue that based on previous experience one knows roughly which percentile to expect for $\hat{u}_3$, i.e. we know that the target response value could be found as the maximum x_{3h} along the blue circle in Fig. 4. The percentile of $\hat{u}_3$ is related to the relative importance of the inherent variability carried by U_3 versus the inherent variability carried by the transformed sea state variables U_1 and U_2. If we assume or illustrative purposes that the 3-hour maximum response was almost a deterministic function of significant wave height and spectral peak period, *i.e.* the conditional density function of X_{3h} given H_s and T_p was very narrow, $\hat{u}_3$= 0.0 would be the value of U_3 in the design point, i.e. the target percentile will be the median. As the relative importance of the 3-hour response maximum increases, the design point to higher values of $\hat{u}_3$ and smaller values for $\hat{u}_1$ and $\hat{u}_2$. This means that the design point in terms of $\hat{u}_1$ and $\hat{u}_2$ will be somewhere along the dotted blue circle in Fig. 4. For most practical problems, the percentile to be associated with $\hat{u}_3$ is well above the median say - 0.95 or higher. If we should search for the worst location along the full blue line using model test experiments and with target value corresponding to a percentile well above 0.95 with a reasonable accuracy the amount of testing would have to be extensive. It is from a practical and cost perspective not an attractive approach.

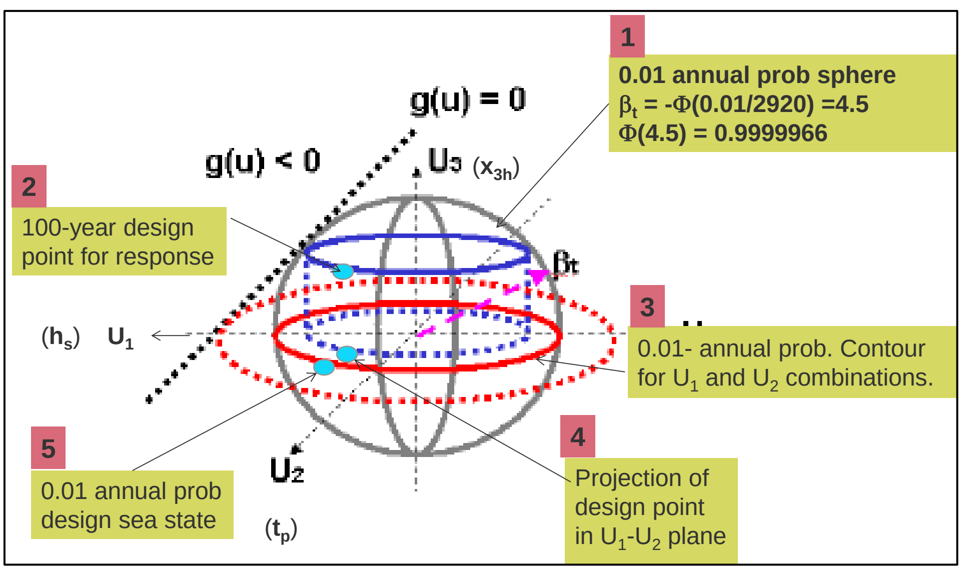

Fig. 4 From IFORM to design sea state.

It is interesting to note that if we project the 10^{-2} – annual probability design point (Step 2 in Fig. 4) down on the sea state plane (Step 4 in Fig. 4), the projection is well inside the 10^{-2}– annual probability contour for H_s and T_p (actually the figure shows the transformed sea state contour), see dotted blue line in Fig. 4. This means that the 10^{-2}– annual probability response is most likely to be realized in a sea state of much less severity than the sea states along the 10^{-2}– annual probability contour. This should be kept in mind if evacuation procedures are implemented.

Using the full blue line as a contour to be searched for the target response value, the procedure couples weather severity and response under consideration. It is much more attractive if we can establish an environmental contour line being decoupled from the response problem. This we will achieve if we rather use the full red line (10^{-2}– annual probability contour of H_s and T_p) as the contour to be searched. Let us therefore assume that we as a short term design sea state for complex problems select the worst sea state along the 10^{-2}- annual probability contour for H_s and T_p (Step 5 in Fig. 4).

If this sea state should represent the underlying design point, the short term variability must be negligible, *i.e.* $u_3 = 0$. For such a case, the target percentile would be the median since $\tilde{u}_3 = 0$ for the design point. The Rosenblatt transformation conserves probability and thus the associated value for X_{3h} will also be the median. For realistic cases, the underlying design point would be found somewhat higher up on the sphere. In order to obtain a reasonable estimate for the underlying long term 10^{-2} annual probability response using a design sea state along the $u_1 - u_2$ contour line, we must find an equivalent percentile of the 3-hour maximum response distribution. All we can say without further verification is that the target percentile must be larger than the median.

Above discussion has been referring to contour in u-space. Physical contours can be obtained by transforming the circles in u-space to the physical parameter space (x-space) using the Rosenblatt transformations, Eq. (9). Examples of contour for a site on the Norwegian Continental shelf are shown in Fig. 5.

4.1.3 UNCERTAINTIES IN JOINT DISTRIBUTION FOR $\boldsymbol{H_s}$ AND $\boldsymbol{T_p}$ FOR EXTREME SEA STATES

The joint density function for H_s and T_p can be written on the following form:

$$f_{H_sT_p}(h,t) = f_{H_s}(h)\, f_{H_s|T_p}(h|t) \tag{12}$$

The marginal distribution for H_s is typically modeled by a 3-parameter Weibull distribution or a hybrid model based on a log-normal distribution for small significant wave heights and a 2-parameter Weibull for larger significant wave heights. Uncertainties will be associated with the upper tail of this model, but these uncertainties we can to a large extent quantify. A less important – but more challenging regarding quantification – source of uncertainty is the uncertainties of the conditional distribution for T_p given H_s.

The conditional density function for T_p given H_s reads:

$$f_{T_p|H_s}(t|h) = \frac{1}{\sqrt{2\pi}\, t\sigma(h)} exp\left\{-\frac{1}{2}\left[\frac{\ln t - \mu(h)}{\sigma(h)}\right]^2\right\} \tag{13}$$

Where the distribution parameters are modeled by the following functions:

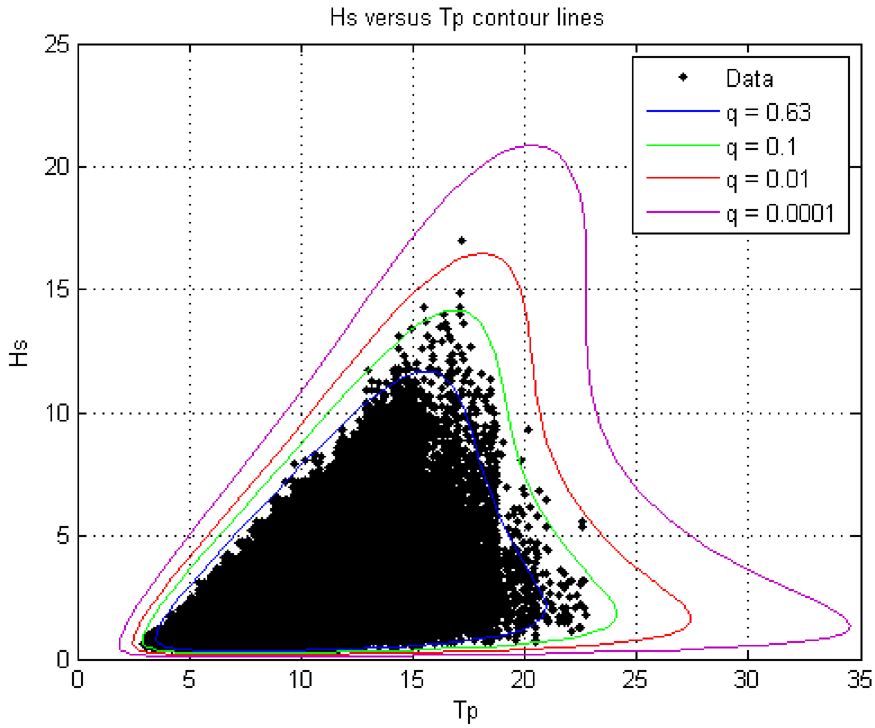

Fig. 5 Environmental contour lines for Haltenbanken, Statoil(2012)

$$\mu(h) = E\left[\ln\left(T_p\right) | H_s\right] = a_1 + a_2 h^{a_3}$$

(14)

$$\sigma^2(h) = Var\left[\ln\left(T_p\right) | H_s\right] = b_1 + b_2 exp\{-b_3 h\}$$

Since the eighties, Statoil have used $b_1 \equiv 0.005$, while the remaining parameters are determined by a least square fit to available data. It is seen that the value for b_1 represents a lower bound for the conditional variance of T_p given H_s for very large significant wave heights. Examples of fitted curves are shown in Fig. 6

The value of b_1 was originally based on fitting the joint model to available wave measurements mid-eighties. Now we typically use hindcast data as our major source for wave data. Fitting the joint model to data from the Norwegian hindcast data base, NORA10, suggests that $b_1 = 0.001$may be a better choice. This is indicated in Fig. 7. What we see from Fig. 7 is that the value we have used for three decades seems to be somewhat on the high side. $b_1 = 0.001$ seems more adequate regarding for large significant wave heights. However, but the data for the largest significant wave heights are possibly corresponding to merely one or two storm events. Therefore we do hesitate to give full weight to this result. A value in between, say $b_1 = 0.0025$, may possibly be a more “correct” value. But until we have considered this for several locations and investigate the accuracy of the hindcast spectral peak period for individual storms, we have decided to maintain $b_1 = 0.005$.

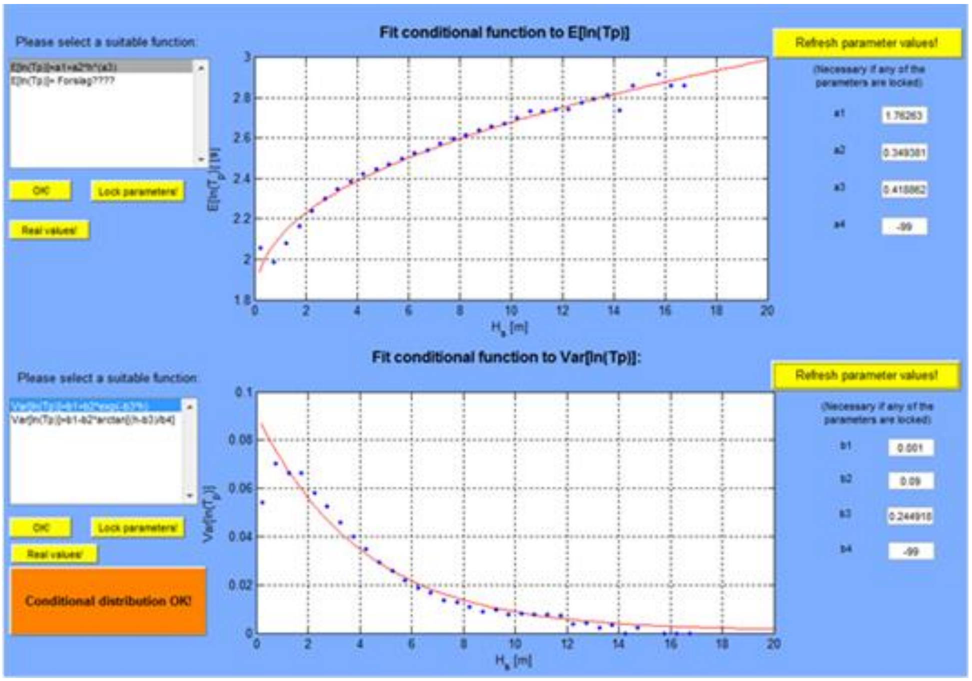

Fig. 6 Examples of fitted functions for μ and σ^2 (Here b_1 = 0.001).

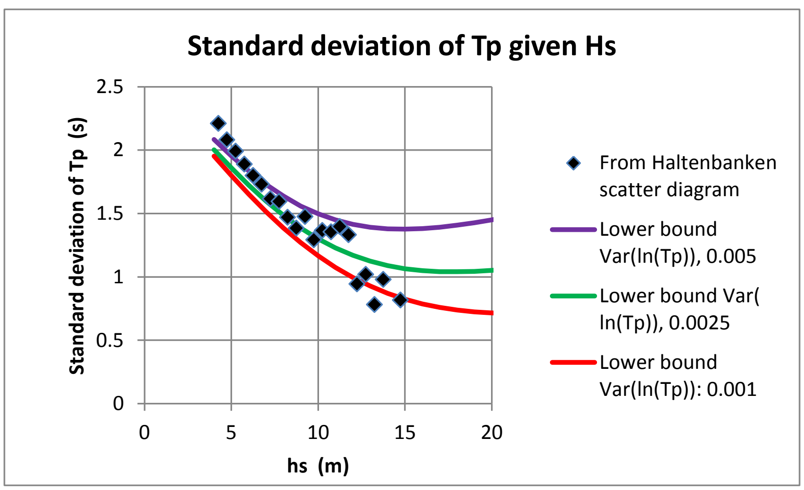

Fig. 7 Standard deviation of T_p given H_s for various models of $\sigma^2(h)$

Regarding a full long term analysis we are not very concerned about the uncertainty in b_1. But it is an important parameter regarding the established contours. So in that sense further work on the “true” value of b_1 should be carried out. In Fig. 8 contours corresponding to the various models for $\sigma^2(h)$ are shown. If the most important sea states along the contours for a given response quantity are close to the modes of the contour (peak of the contour), which for most practical problems will be the case, the b_1 uncertainty is not very important. However, for response problems being governed by either very steep sea states (or very non-steep sea states) this can be a crucial parameter regarding the adequacy of the contour method. It is seen from Fig. 8 that if $b_1 = 0.001$ is the true value, the 10^{-2} – annual probability steep part of contour obtained using $b_1 = 0.005$ is actually corresponding to a 10^{-4}– annual probability contour.

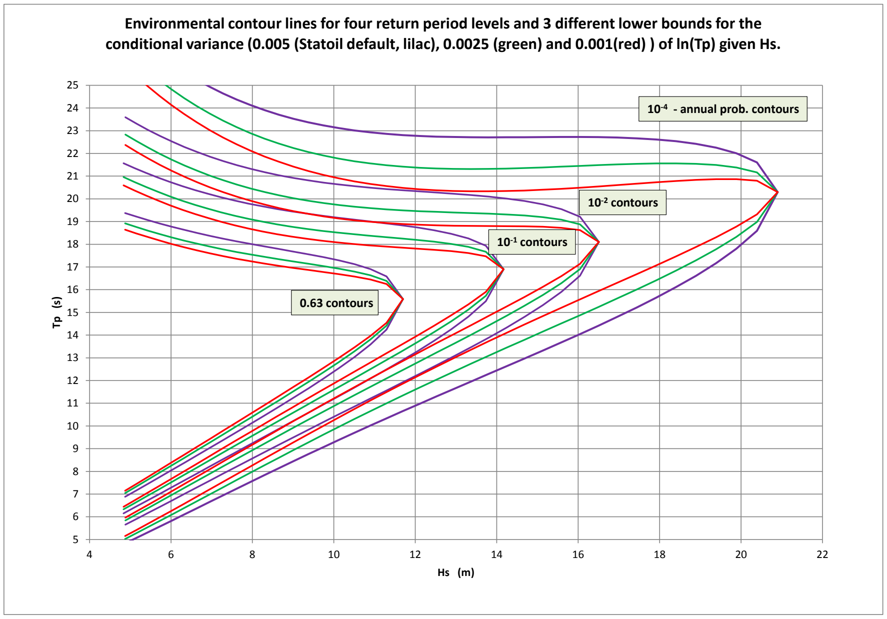

Fig. 8 Environmental contours for various lower bounds for the conditional variance of $\ln(T_p)$.

4.1.4 TARGET PERCENTILES FOR EXAMPLE CASES FROM NORTH SEA/NORWEGIAN SEA

It is not easy to give a theoretical argument for what percentile should be selected. Based on experience with problems where the contour method results are compared to results from full long term analyses, a reasonable percentile for $q = 10^{-2}$ is 90%. For $q = 10^{-4}$, there seem to be a tendency of going to a slightly higher percentile and a possibility could be 95%.

The reason for why we can recommend a percentile that can be expected to result in a reasonable estimate for the target extreme response value for a rather broad class of practical response problems is an apparent similarity among response cases when it comes to the relative importance of the variability of the extreme response versus the variability carried by the weather characteristics. The coefficient of variation (CoV) of X_{3h} is typically from slightly less than 0.1 to around 0.2. As a consequence we do not expect a large variation in the target percentile. But this is an approximation and exceptions can be found, i.e. it must be remembered that the contour method is an approximate method.

In Table 1, percentiles obtained for some structural problems are shown. These examples are obtained by doing a full long term analyses. The target percentiles are then found by identifying which percentile of the 3-hour extreme value for the worst sea state along the contour yields a response equal to the long term response. The numbers in Table 1 represent rounded off percentiles. Other examples are found in Baarholm and Haver (2009).

The environmental contour method has been useful for many applications in the North Sea. In particular in connection with planning and execution of model test experiments the technique has proven useful. Following this approach we can ensure that sea states are selected such that we can expect to see response realizations corresponding to the target return period values during the test program. However, there are certain assumptions that need to be fulfilled in order for the method to behave as expected. These assumptions will be discussed in the next chapter.

Table 1 Percentiles found to match results from long term analyses for a number of North Sea/ Norwegian Sea response cases.

Case	Response	Percentiles fitted to long term extremes (%)		Response properties	Sea state properties	Location
		10^{-2} per year	10^{-4} per year			
Jacket	Deck displacement	85	90	Non-linear	Gaussian sea	Northern North Sea. Depth: 190m
	Base shear	85	90			
	Overturning moment	85	90			
TLP	Maximum tether force	90	95	Non-linear, several component processes	Gaussian sea	Northern North Sea. Depth: 310m
Wave elevation	Crest height	85	90	Linear	Gaussian sea	Central North Sea. Depth: 110m
Large semi-submersible	Surge	95	>95	Wave induced, No current, no wind	Gaussian sea	Northern North Sea
Steel Catenary Riser (TLP)	Tension and motion at hang – off, Tension and curvature at touch-down.	Ca. 90 Conservative except for curvature.	-	Wave induced response, No wind and no current.	Gaussian Sea	Northern North Sea weather. Depth: 600m
Flexible riser (Semi –Sub., lazy wave	Tension and curvature at top, sag – and hog-area	70-90	-	Lazy wave configuration, Hanging from semi-submersible	Gaussian Sea	Northern North Sea,
Gravity based concrete structure	Overturning moment	90		Response mainly linear, but effected by drag loading and ringing	Gaussian sea	Northern North Sea. Depth: 300m

4.2 CONTOURS FOR HURRICANE GOVERNED WAVE CLIMATE

4.2.1 STORM CHARACTERISTICS AND THEIR JOINT PROBABILISTIC MODEL

The question we will briefly discuss here is: Will the environmental contour method be useful in areas where extreme response is governed by the rare occurrences of severe hurricane events?

From the long term analysis we saw that the interesting variables were the hurricane maximum response and the most probable largest hurricane response. In order to obtain an environmental contour being decoupled from the response problem, we will assume that the hurricanes for long term predictions by the contour method are characterized with sufficient accuracy by the hurricane peak environmental characteristics. Assuming the response problem to be governed by the wave induced loading, a hurricane will then be characterized by the significant wave height, h_{sp}, and the spectral peak period, t_{pp}, at the storm peak.

Since hurricane maximum response is expected to be realized during the most severe part of the hurricane (say during sea states higher than 85-90% of storm peak significant wave height) and that the spectral peak period is not expected to change very much during the most severe part of the hurricane, the only main information not carried by these two characteristics is the duration of the most severe part of the hurricane (say the duration of time where the significant wave height exceeds 90% of storm maximum significant wave height).This means that we have neglected a source of long term, variability. As a consequence, the equivalent percentile can be expected to be slightly larger than what we would find if duration of severe part of storm were included (slowly varying variability is shifted to extreme response variability).

Since the number of hurricanes is rather low and our experience of using the environmental contour method in hurricane governed areas is rather limited, the following discussion should merely be considered as an illustration of a possible approach. Further work is necessary before the approach can be used for final design, but maybe it can represent a possible approach for early phase screening of concepts.

In order to prepare the contour line for h_{sp} and t_{pp}, we need the joint distribution of these two variables. The joint probability density function is given by:

$$f_{H_{sp}T_{pp}}(h,t) = f_{H_{sp}}(h) f_{T_{pp}|H_{sp}}(t\,|h) \tag{15}$$

The hurricane data sample considered here is from GOMOS08, Oceanweather Inc. (2010). We have selected an area of size 1^o x 1^o and identified all hurricanes above some threshold within this area. An artificial point series of hurricanes is constructed by assuming that the worst part of each hurricane is moving through the platform site inside the area. This introduces an element of conservatisms into the analysis, but the conservatism is small compared to the statistical uncertainty introduced by a rather low number of hurricanes. This and other approaches for predicting extremes in Gulf of Mexico are discussed *e.g.* by Heideman and Mitchell (2009).

A 3-parameter Weibull distribution is fitted to the observed hurricane peak values using method of moments. The fitted model is compared to the sample distribution in Fig. 9.

The conditional distribution for spectral peak period given hurricane peak significant wave height is modeled by a log-normal distribution. Due to a limited amount of data, rather large uncertainties are associated with this modeling. The conditional mean spectral peak period and the conditional 90% band for observed spectral peak periods are shown in Fig. 10. Large uncertainties will be associated with the parameters in the extrapolated range. In particular, we have hesitated in reducing the width of the 90% band as sea state severity increases. Further work on joint modeling of H_{sp} and T_{pp} is recommended.

As the model for the joint probability distribution of H_{sp} and T_{pp} is established, the contour lines are obtained using the Rosenblatt transformation, Eq. (9). Contour lines corresponding to various annual exceedance probabilities are shown in Fig. 11.

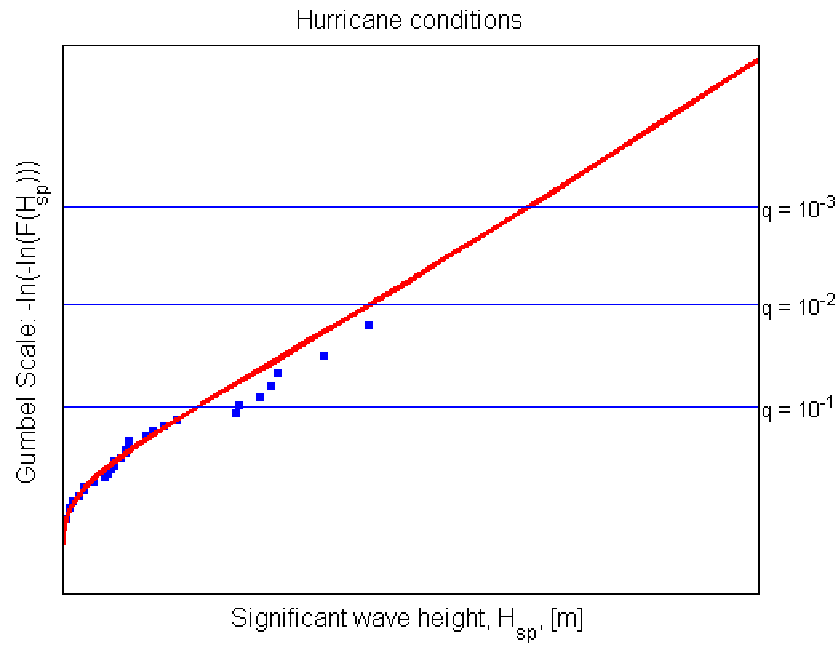

Fig. 9 Distribution function for hurricane peak significant wave height, H_{sp} .

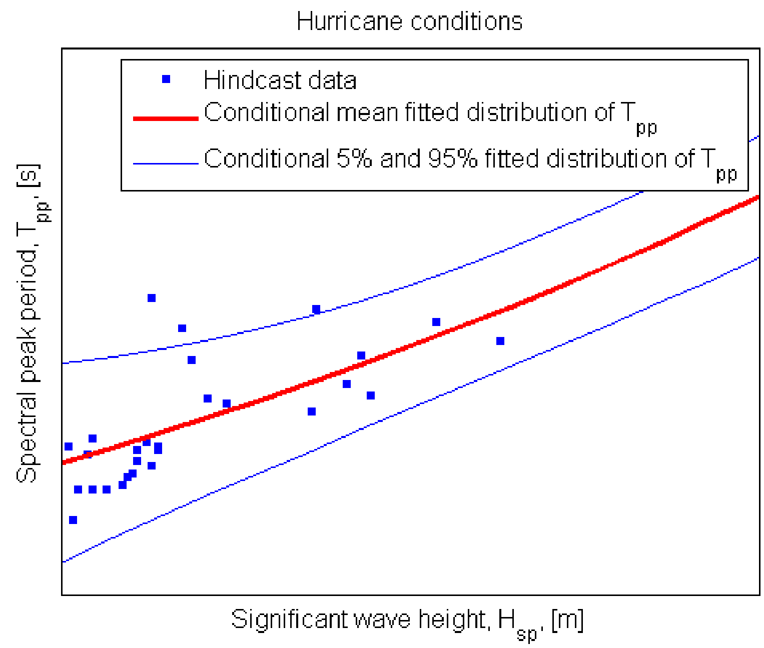

Fig. 10 Conditional mean of T_{pp} and 90% band of T_{pp} given peak significant wave height as function of hurricane peak significant wave height.

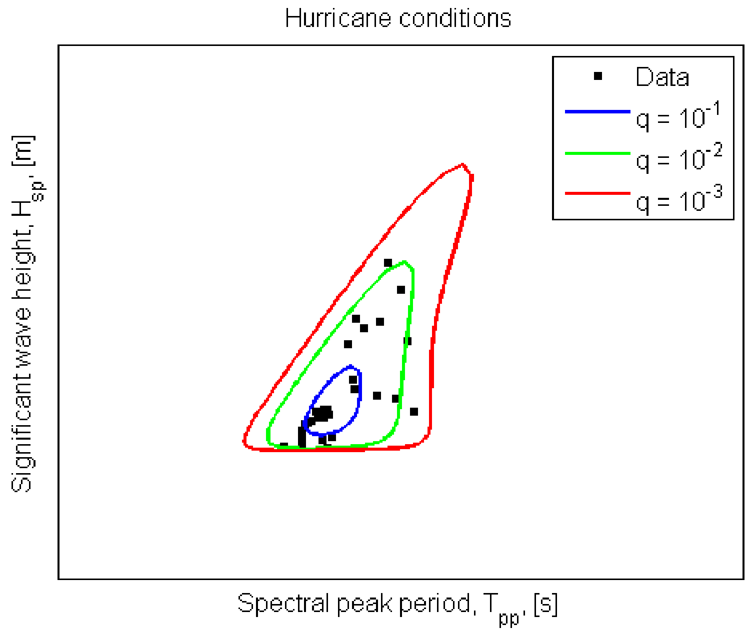

Fig. 11 Contour lines for hurricane peak significant wave height, H_{sp}, and associated spectral peak period, T_{pp}.

Each point along contour, in Fig. 11, is herein assumed to represent the whole hurricane event, *i.e*. there is a sequence of sea states represented by the hurricane peak event. The hurricane peak events along the contour are assumed to correspond to 30-minute duration. By the contour line method it is assumed that the worst hurricane peak sea state (in view of the response under consideration) along the q-probability contour line can be used as a proper design sea state. The location of the worst hurricane peak sea state will be response dependent, but it will in most cases be close to top of the contour or somewhat to the left of the of the top (steep sea part of contour). If this sea state had been the only sea state that could contribute to the exceedance of the q-probability responses, the q-probability response could be estimated by the median of the 30-minute extreme value distribution. In practice this is not the case. The median 30-minute maximum may be exceeded for another 30-min. episode of the worst hurricane along the contour. Additionally, it can also be exceeded in other hurricanes. A consequence of this is that selecting the median 30-minute maximum of the worst hurricane peak event will be considerably on the low side.

If we would like to use the worst sea state along the contour as the design sea state, we must therefore select a higher percentile in the 30-minute extreme value distribution. Alternatively, we may artificially increase the duration of the worst peak hurricane event.

The percentile will of course depend on the response under consideration. It may possibly also depend somewhat on the variability of the storm profile. If the method shall be useful for design applications, there must be a sufficient similarity over a range of practical response problems such that we can adopt a standard recommendation for the percentile levels. The similarity thought of here is with respect to the amount of variability carried by the response extremes of the 30-minute hurricane peak event versus the variability carried by the adopted (slowly varying) hurricane characteristics.

The variability of the target percentile level can be investigated if we can perform a full long term analysis and then see what percentile we have to use in combination with contour approach. If we do this for a broad range of response problems and find that the target percentile is not varying too much from case to case, then the method can possibly be useful for assessment of complex response problems.

We have not investigated possible application of the environmental contour line for hurricane governed areas sufficiently for concluding on its applicability. However, we have applied the method for some few cases. The results for these examples will briefly be shown below.

At first a long term analysis is carried out as indicated previously. All weather is assumed to come from the same direction and the sea surface is assumed to be long crested. We will adopt the q-probability response obtained from the long term analysis (see Eqs. (5 and 6)) as a good estimate for the true underling value and we will denote this value by $x_{LT,q}$.

When it comes to the contour method, it consists of the following steps:

- Identify the most unfavorable hurricane peak event in view of the response under consideration. For a complex response problem, this is done by doing some few time domain simulations or model tests of 30 minutes duration for some few sea states along the critical part of the contour. A measure of the 30-minute extreme levels for each hurricane is obtained by:

  $$x_{30-min,ext} = \bar{x} + k \cdot s_{X_{30-min,ext}}$$

  $\bar{x}$ is the mean and $s_{X_{30-min,ext}}$ is the standard deviation of the 30-minute extreme value for the various selected hurricane peak events along the contour. k is factor used for pointing to a proper extreme value level. It will typically be a value well in the excess of 1 – say 1.3-1.5 depending on the coefficient of variation of $X_{30-min,ext}$.

- As the worst hurricane peak sea state is selected, a large number of additional 30-minutes time domain simulations or tests are carried out. The number chosen for the design sea state should be so large that we can expect to see at least a couple of observations exceeding the target percentile level. If target percentile is 0.90, we should at least include 20. If target is 0.98, we should have about 100. (Note that the duration of each simulation/test is 30-minutes.)

- As the large sample of 30-minute extremes is available, a Gumbel extreme value distribution is fitted to these data. If data suggests that a Gumbel model is not adequate, another probabilistic model must be selected. Let us denote the fitted extreme value distribution by $F_{X_{30-min,ext}}(x)$.

- The target percentile, α_q is now estimated by: $\alpha_q = F_{X_{30-min,ext}}(x_{LT,q})$

4.2.2 EXAMPLE STUDIES GULF OF MEXICO

The cases considered below are North Sea platform cases. The reason for selecting these is that response surfaces for the parameters of the extreme value distributions are available (*i.e.* Gumbel parameters are available as functions of significant wave height and spectral peak period). These platforms will not necessarily be typical for Gulf of Mexico installations, but they will serve the purpose as example cases for demonstrating contour method. Since the validity of the available response surfaces are of questionable validity for very extreme sea states (*i.e.* sea states approaching and exceeding 10^{-3} – annual significant wave height), we will here limit the consideration to extremes corresponding to annual exceedance probabilities of 10^{-1} and 10^{-2}, respectively. More results for all cases are presented in Baarholm (2012).

Case 1: q-probability tether loads of TLP

First case is a dynamic wave induced response of a TLP tether. We will limit consideration to diagonal long crested sea propagating from southwest to northeast. The response quantities considered are tether tension in southwest and northeast corner. The percentiles required when using the environmental contour in order to match the long term result are shown in Table 2.

Case 2: q – probability response of flexible riser connected to a semi-submersible platform.

The second case is a flexible riser hanging form a semi-submersible. Several response quantities are discussed in Baarholm (2012). Here we will show results for tension and curvature at top of riser. We will again consider one wave direction of long crested sea. Direction is almost parallel to the direction of the flexible riser. Results regarding required percentiles are shown in Table 2.

Case 3: q- probability base shear, deck displacements and overturning moment of a gravity based concrete structure fixed structure

The last platform case is global loads of a gravity based structure (GBS), Troll A. The platform is installed in the Northern North Sea in about 300m water depth. The structural response is significantly affected by dynamics. Mud line forces, base shear and overturning moment and deck displacement are included. More details regarding the structural response for this case can be found in Baarholm et al (2010). The required percentiles are given in Table 2.

Summing up Table 2: For a 30-minutes duration of contour sea states (which is in agreement with the selected weather resolution) rather high percentiles are observed. For 0.1-annual probability, we see required percentiles are typically from 0.93 – 0.98. The variation range are more or less the same for $q = 0.01$, *i.e.* from 0.93 – 0.98. For North Sea conditions there is a slight tendency of increasing percentiles when exceedance probability is reduced. That is not observed here. The results for $q = 10^{-3}$ (not included here) suggest in fact a reduction in percentile. However, there is quite some scatter in these results and since large uncertainties are associated with our long term prediction of $q = 10^{-3}$ – annual probability results, further work are required before we can conclude on the tendency of target percentile versus target exceedance probability.

If – for some reason – one prefers to work with an artificial 3-hour duration for the peak hurricane sea states, the required percentile is considerably reduced. The target percentiles are then found by raising the target 30-minute percentiles to the power of 6. If we - as a first approximation - assume that the 0.95 percentile is valid for 30-minute sea state duration, the corresponding target percentile for a 3-hour sea state is close to 0.75. This is different from typical North Sea cases where a percentile of 0.90 seems fine for estimating 0.01 – annual probability response. Transforming the North Sea number to 30-minute duration, target percentile would be 0.9826. What we can say from this is that the short term variability (distribution of short term extreme response) is relatively more important for North Sea conditions than for Gulf of Mexico cases. This is equivalent to say the long term variability of the sea state characteristics is relatively more important for the Gulf of Mexico type of climate. This is not surprising since the coefficient of variation of annual maximum significant wave height is considerably larger for the Gulf of Mexico than for the North Sea/Norwegian Sea.

If we select 0.95 for the 30-minute duration case, there is a chance that we may be somewhat on the low side (non-conservative) or somewhat on the high side (conservative). Let us assume that the lowest likely “true” percentile is 0.9 and the highest “true” percentile 0.98. The error is dependent of the coefficient of variation of the 30-minute extreme value distribution. The results are shown for 3 levels of scatter in Fig. 12. For most practical problems the ratio of Scale Parameter to Location parameter will be between 0.08 and 0.2. For complicated response cases *e.g*. impact loads due to breaking waves, the coefficient may be considerably larger. Here a case with a coefficient of variation of 0.4 is included.

It is seen from Fig. 12 that for typical cases the error seems to be less than +/- 10%. This may be sufficient accuracy for early phase assessments. However, the robustness of the results shown herein needs to be further investigated.

If one should do model test planning based on these results, one should for the critical hurricane peak perform about 40 different 30-minutes test realizations or – more efficient – 7 different 3-hours tests (each 3-hour test gives 6 realizations of the 30-minute extreme value). Since the short term variable is less important for Gulf of Mexico applications than for North Sea cases, we can do with fewer repeats of the governing sea states.

Table 2 Target percentiles for the considered response cases, Baarholm (2012).

Platform concept	Response quantity	Target percentile contour method, 30 minute duration	
		$q = 0.1$/year	$q = 0.01$/year
TLP, diagonal sea from SW to NE (Case 1)	Max. dynamic tension in SW tether	0.93	0.94
	Min. dynamic tension in SW tether	0.98	0.97
	Max. dynamic tension in NE tether	0.93	0.88
	Min. dynamic tension in NE tether	0.84	0.93
Semi- sub., flexible riser (Case 2)	Maximal tension at hang-off	0.95	0.95
	Maximum curvature at hang-off	0.98	0.97
GBS, all weather form one direction (Case 3)	Maximum base shear	0.97	0.93
	Maximum deck displacement	0.97	0.93
	Maximum overturning moment	0.97	0.93
Average	Mean percentile	0.947	0.937
Scatter	Standard deviation of percentile	0.044	0.027

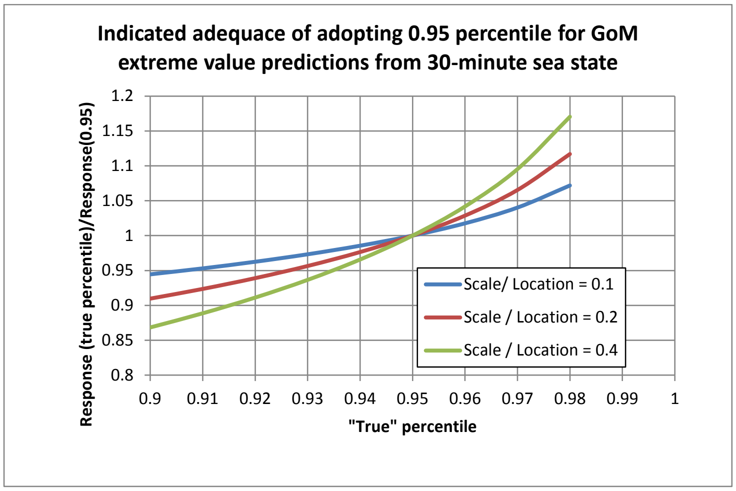

Fig. 12 Adequacy of 95-percentile for 30-minute sea state duration

We have also indicated the adequacy of the contour approach for predicting wave crest height. This is just for illustrating the approach. The short term crest height distribution is modeled by the Forristall crest height distribution for long crested sea, Forristall (2000). At first a long term analysis was done using the same approach as outlined for the response analysis except that the most probable response (here crest height), $\tilde{y}$, is replaced by the peak hurricane significant wave height. This means that the effect of T_p (and also the duration of the most severe part of hurricane) on extreme crest heights are neglected. Assuming 30-minute duration for short term sea state for the contour method, the required percentiles for matching the long term results are found to be 0.95 and 0.97, for $q = 0.1$ and 0.01, respectively (see Table 2). Increasing duration artificial to 3-hours percentiles decrease to 0.74 and 0.82, for $q = 0.1$ and $q = 0.01$, respectively. In contrast to the response cases, percentiles seem to increase slightly when q is reduced.

5. SOME CONDITIONS FOR MAKING CONTOUR METHOD CONVENIEN

A condition for the contour method to work is that the response severity is worsened as sea state severity measured by simultaneous occurrence of significant wave height and spectral peak period is worsened. If we consider the problem in u-space it means that response is monotonically worsened as we move out from origin in the most unfavorable direction.

It is also important to point out that all slowly varying variability (*i.e.* variability of environmental characteristics should be included when preparing the contour. If more than 2 variables are important a higher dimensional contour should in principle be used. This is manageable if the number of important slowly varying parameters is 3, for higher dimension the approach is not attractive. The consequence of of neglecting slowly varying variability is that the adequate equivalent percentile level will increase.

It should also be mentioned that most experience with the environmental contour method is for problems where the coefficient of variation of the extreme value distribution describing the short term variability is in $5 - 20\%$, i.e. a typical range of variability for wave induced response of offshore structures. If the short term variability is much higher than this, further verification is recommended since default recommendations may be un-conservative.

For cases deviating considerably from the typical level of short term variability, extra care should be shown when modeling the short term variability.

6. OTHER POSSIBLE FUTURE APPLICATIONS OF ENVIRONMENTAL CONTOUR METHOD

An alternative application of environmental contour method is to establish consistent associated environmental characteristics. At present two approaches are frequently used. According to Norwegian practice, the ULS (10^{-2}– annual probability) combinations is defined as 10^{-2}– annual probability sea state (defined as the worst sea state along the 10^{-2}– annual contour for h_s and t_p), 10^{-2}– annual probability mean 1-hour wind speed and 10^{-1} – annual probability 10-minute mean current speed. Extreme "average" wave and wind conditions are taken to be more or less fully correlated, while a certain lack of full correlation is accounted for regarding current by reducing the extreme current to be combined with 10^{-2} - annual probability waves and wind conditions.

An alternative approach could – for a wave governed problem - be to use the conditional mean value of other characteristics given 10^{-2}– annual probability wave conditions. However, this means that we have neglected variability and as a consequence our extreme values may be on the low side.

A third possible approach could be to use contours. This means that we determine q-probability contours for the involved slowly varying environmental characteristics. For a particular response problem one could then search for the worst combination on the contour surface and adopt this as an adequate design combination. In this connection one may well establish higher dimensional contours. The challenge with this approach is to establish a joint probabilistic description of the environmental characteristics. Some examples of contours are for illustrative purposes shown in Fig. 13. These contours are not of a sufficient accuracy to be used for design purpose.

Provided one has an accurate joint probabilistic model for the involved weather characteristics, we can determine contours with reasonable accuracy. We could also prepare 3-dimensjonal contours including both wind speed, significant wave height and surface current. When preparing the joint model it is important modeling the variables in agreement their importance for the problem under consideration. If – say – wind speed was the most important parameter, this quantity should be modeled by a marginal distribution. The second most characteristic (*e.g.* significant wave height) should then be modeled conditionally with respect to the wind speed. If a 3-dimensonal contour is to be prepared, the least important of the 3 included characteristics should be modeled conditionally on the two other characteristics. Since statistical uncertainties are involved, the contour will most likely not be independent of the ordering of the variables.

It is seen from previous sections that if an artificial duration of 3 hours are used the target percentile for the contour method will be around 75%. In these cases we assumed that the problem was defined by 2 slowly varying characteristics, significant wave height and spectral peak period. If 3 or more slowly varying weather characteristics are of importance, the target percentile becomes even lower. This means that for such a problem one could obtain a rather good estimate of the q-probability response by the mean 3-hour maximum response of the worst weather condition. But – and there is always a but – much more data are need to establish robust multidimensional contours, i.e. we need to be able to simulate realistic hurricane characteristics.

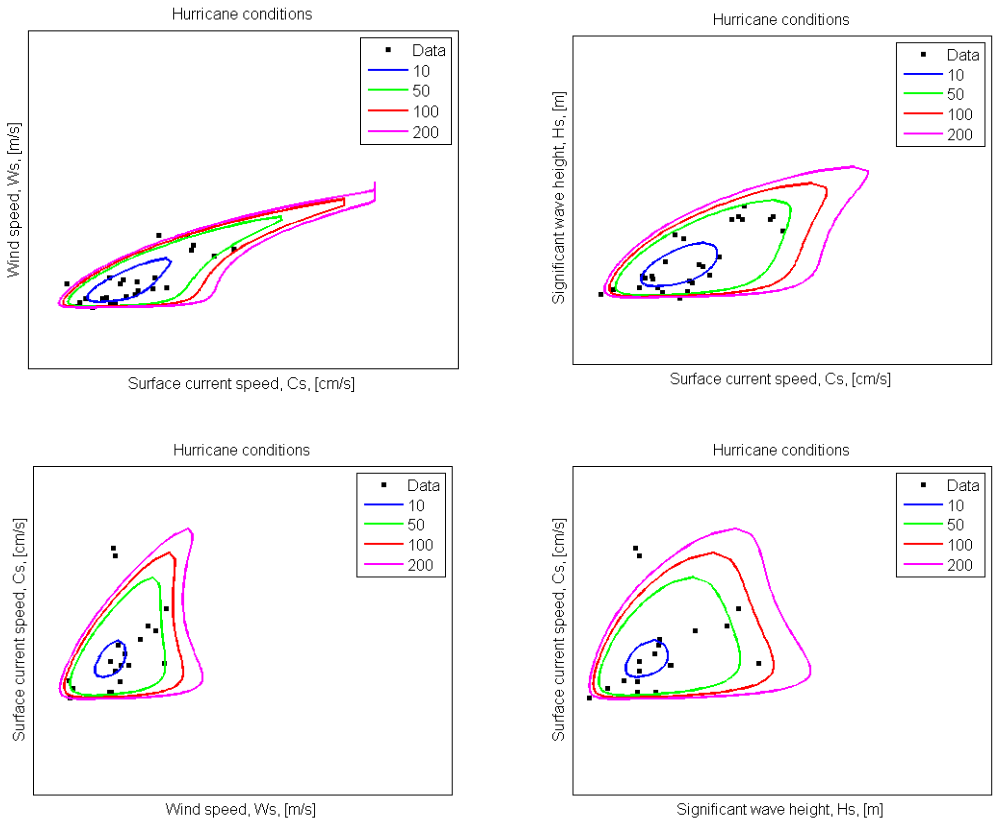

Fig. 13 Contour lines for surface current, significant wave height and mean wind speed

7. CONCLUDING REMARKS

The environmental contour approach has been presented North Sea applications and for Gulf of Mexico applications.

For the North Sea duration of sea states along the contour is taken to be 3 hours. For a broad range of problems, long term q-probability extremes can be estimated by finding the worst sea state along the q-probability contour in view of the response under consideration, and then by estimating 0.9-percentile of the 3-hour extreme value for this sea state. It is important to point out that the contour method is an approximate method. In the North Sea/Norwegian Sea, the choice of 0.9-percentile will often give a reasonable – but not necessarily perfect – estimate.

For the Gulf of Mexico we have here selected the hurricane peak characteristics as the major slowly varying variables. Contours are determined for these characteristics. The duration of sea states along the contour is taken to be 30 –minutes. Q-probability extremes are estimated by identifying the most unfavorable combination of hurricane peak characteristics along the q-probability contour and then finding the 0.95-percentile of the 30-minute extreme value for this hurricane peak event. We have hardly any experience with applying the method to hurricane governed areas, the Gulf of Mexico results should therefore be consider as examples. Further work is recommended if robustness for this type of climate shall be demonstrated for a broad range of applications.

If we are looking at a response problem depending only on 2 slowly varying characteristics, 0.95 seems to be realistic percentile level. If one more slowly varying characteristic is important, the percentile should most probably be somewhat reduced.

Before a more general application of contours can be applied for selecting adequate consistent combinations of hurricane characteristics, much more data are required.

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