SPECTRAL DESCRIPTION OF THE DISSIPATION MECHANISM FOR WIND WAVES. TURBULENT VISCOSITY MODEL

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ABSTRACT

The objective of this paper is parameterization of the dissipation function for wind waves in the spectral form. To solve the problem, the similarity statements are formulated; dimensionless characteristics of the system are considered, and general phenomenological representation of the dissipation function for wind waves *DIS(S)* is constructed, which is valid in a local approximation in the spectrum. Analysis of the representation parameters allows us to set the dissipation function in the form $DIS(S) \propto S^2$. The further systematic formulation of the final presentation of phenomenological function *DIS(S)* is performed by joining numerous results of the author. The physical meaning of the accepted parameterization parameters is analyzed, and a correspondence of the latter with the new experimental facts found in this field in the last 5-10 years is discussed. The facts are presented that illustrate a successful application of the constructed dissipation function. A theoretical justification of the revealed phenomenological representation of *DIS(S)* is based on the well-known Hasselmann's theory (Hasselmann, 1974). With this in mind, the theory was modified by the system of postulates allowing the existence of the dissipation mechanism due to the viscosity friction provided by the turbulence in the upper water layer. The final theoretical representation of the dissipation function in the form $DIS(S) \propto S^2$ does fully correspond to the initial phenomenological formulation. This fact provides the physical justification of the approach proposed.

Key words: wind-wave spectrum, numerical model, source function, evolution mechanism, wind-wave dissipation, turbulence.

1. Introduction and formulation of the problem

Wind sea is a stochastic hydrodynamic phenomenon at the air-sea interface, where shear currents, waves, and turbulent motions take place simultaneously. This multi-scale feature of the motions brings forward essential difficulties in solving the problem of wind-wave dynamics. Nevertheless, during the century and a half history of studying the physics of wind waves, many problems obtained their solutions.

In particular, it is well established (Komen et al., 1994; Cavaleri et al., 2007) that on the scales of hundreds of dominant wave lengths and periods, the description of stochastic and non-stationary wave field can be realized in the terms of two-dimensional energy spectrum $S(\sigma,\theta;\mathbf{x},t)\equiv S$, defined in the domain of space $\mathbf{x}$ and time t (here, σ and θ are the frequency and direction of wave component propagation, respectively, corresponding to the wave vector $\mathbf{k}(\sigma,\theta)$). In the case of deep water and negligible surface currents influence, the wave spectrum evolution is described by the following transport equation[1]

$$\frac{dS}{dt} = F \equiv NL + IN - DIS\ . \tag{1.1}$$

The left-hand side of equation (1.1) defines the mathematical part of the model, which is not discussed here. The right-hand side of (1.1), called the source function (SF) F, contains the physical properties of the model[2]. Usually three main constituents of SF are distinguished: three parts of the united evolution mechanism of wind waves. They are as follows

- The rate of conservative nonlinear energy transfer through a wave spectrum, NL, ("nonlinear-term");
- The rate of energy transfer from wind to waves, IN, ("input-term");
- The rate of wave energy loss, DIS, ("dissipation-term").

The physical sense of equation (1.1) is evident: it is the energy conservation law applied to each of the wave spectral components. Therefore, in principle, it can be postulated and written as an initial phenomenological equation. Nevertheless, a certain procedure exists, allowing the derivation of (1.1) from a system of basic hydrodynamic equations and resulting in specification of a spectral representation for each term in the right side of (1.1).

In the case of an ideal and incompressible fluid, and deep water approximation, the system mentioned before can be written as (Komen et al., 1994; Cavaleri et al., 2007)

$$\rho\frac{d\mathbf{u}}{dt} = -\vec{\nabla}_3 P - \rho\mathbf{g} + \mathbf{f}(\mathbf{x},t)\big|_{z=\eta(\mathbf{x},t)}\ , \tag{1.2}$$

$$\vec{\nabla}_3(\mathbf{u}) = 0\ , \tag{1.3}$$

$$u_z\big|_{z=\eta(\mathbf{x},t)} = \frac{\partial\eta}{\partial t} + (\mathbf{u}\vec{\nabla}_2\eta)\ , \tag{1.4}$$

$$u_z\big|_{z=-\infty} = 0\ . \tag{1.5}$$

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[1] A more general representation of evolution equation (1.1) is not discussed here due to the brevity.
[2] Hereafter, abbreviation SF is used instead of words "source function".

Here the following designations are used: ρ is the fluid density; $\mathbf{u}(\mathbf{x}, z, t) = (u_x, u_y, u_z)$ is the velocity field; $P(\mathbf{x}, z, t)$ is the atmospheric pressure; g is the acceleration due to gravity; $f(\mathbf{x}, z, t)$ is the external forcing (surface tension, wind stress and so on); $\eta(\mathbf{x}, t)$ is the surface elevation field; $\mathbf{x} = (x, y)$ is the horizontal coordinates vector; z is the vertical coordinate up-directed; $\vec{\nabla}_{\mathbf{2}} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ is the horizontal gradient vector; $\vec{\nabla}_{\mathbf{3}} = (\vec{\nabla}_{\mathbf{2}}, \frac{\partial}{\partial z})$ is the full gradient, and the full time-derivative operator is defined as $\frac{d}{dt}(...) = \left(\frac{\partial}{\partial t} + \mathbf{u}\vec{\nabla}_3\right)(...)$.

Due to nonlinearity of the problem, a numerical solution of system (1.2)-(1.5) is very difficult, to say nothing about an analytical solution. Nevertheless, by the present time, evolution mechanisms *NL* and *IN* have already obtained the widely recognized spectral representations, *NL(S)* and *IN(S)*, derived from equations (1.2)-(1.5) (for references, see Hasselmann, 1962; Miles, 1962; Komen et al., 1994; Polnikov, 2005; Cavaleri et al., 2007). However, such type of representation for the dissipation mechanism *DIS(S)* does not exist. This fact has several important causes, the main of which are the following.

In the theoretical aspect, the causes mentioned above are provided by the fact that numerous physical processes resulting in the wave energy dissipation are related to different kind of instabilities, including disruption of the air-sea interface surface (wave breaking, white capping, air-bubble intrusion, and others). All these processes form a specific multi-scale or, more exactly, cascade mechanism of wave dissipation. In this cascade, some “fast” events (mainly wave breaking) play a role of primary process generating different kinds of small-scale motions. The actual, “slow” dissipation of wave spectral components takes place on the background of these motions (so far as the spectrum is defined on a large time scale, only). In such a case, application of the standard equations, for example (1.2)-(1.5), requires their essential modifications corresponding to a proper mathematical method of their solution, and this causes the main theoretical problems. These problems are the basis that the widely recognized and physically justified model for wave energy dissipation mechanism does not exist.

Despite a presence of numerous theoretical papers, for example, statistical estimations (Longuet-Higgins, 1969; Battjes and Janssen, 1978), direct analytical derivations (Hasselmann, 1974), dimensional considerations (Phillips, 1985; Polnikov, 1994; Zaslavskii, 1999; Donelan, 2001), numerical simulations (Chalikov and Sheinin, 1998; Zakharov et al., 2007), and others, a sufficiently convincing solution of this problem was not found. Unfortunately, this issue is rarely discussed in scientific literature from a general and physical point of view. This fact slows down the construction of a justified formulation for a spectral representation of dissipation function *DIS(S)*. This paper is dedicated to accelerate the solution of the problem.

A more important and even the key factor complicating the solution of the problem is the fact that any experimental investigation of wave dissipation processes is principally restricted. This occurs due to the existence of numerous poorly observed and therefore hardly measurable processes taking place on the background of visible dissipation wave processes (mainly wave breaking). Non-controlled instability of shear and orbital currents (on the background of stochastic waves) and permanent generation of turbulence both in the upper water layer and in the air layer nearby the interface can be attributed to the class of poorly observed processes. Besides, the wave dissipation occurs on the background of permanent energy exchange between wind and waves (*IN*-mechanism) and immeasurable nonlinear energy transfer between waves (*NL*-mechanism). Therefore, the previous statement about the strong restriction of possibilities for experimental studying of the whole package of wind wave dissipation mechanisms becomes much more justified and, to our opinion, quite convincing.

Additional difficulty of measurements appear when it is necessary to get the information about the wave energy dissipation rate in the spectral representation, i.e. representation of the dissipation term in a form of its distribution through the wave spectrum components, $DIS(S(\sigma,\theta))$, rather than in the form of integral losses over the whole wave spectrum.

One can find a confirmation of the previous statement in the analysis of new and most advanced experimental data related to the wind wave dissipation properties, found in the last 5-10 years. These experimental results are widely represented in the papers by Australian scientists (Banner and Young, 1994; Banner and Tian, 1998; Babanin et al., 2001; Young and Babanin, 2006; Babanin and Westhuysen, 2008) and in a comprehensive overview by Babanin (2009). The results mentioned above are supplemented by the findings of American colleagues (for example, see references in Gemmrich et al., 2009, Gemmrich, 2010, Thomson et al, 2009).

First of all, it is seen from this literature that the overwhelming majority of experimental works, regarding to the wave dissipation, are mainly restricted by the research of wave breaking processes. Secondly, even the most important result in this field, reported in

(Young and Babanin, 2006) and presented in Fig. 1, does not answer the question about the nature and intensity of the wind-wave dissipation process.

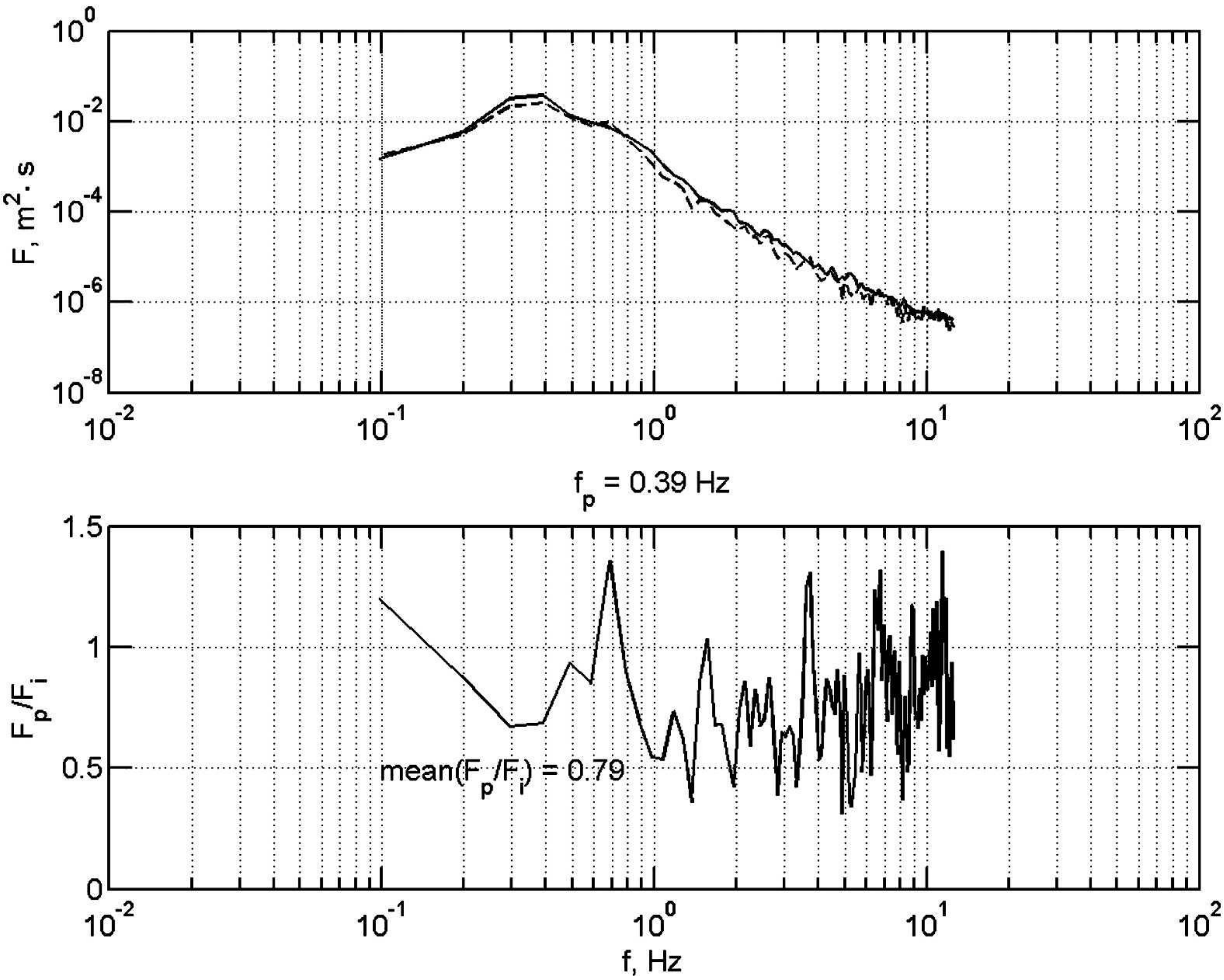

Fig. 1. Top panel: Mean power spectrum of incipient-breaking (solid line, F_i) and postbreaking (dashed line, F_p) waves. Bottom panel: Ratio of the spectra shown in the top panel (following to Young and Babanin, 2006) .

Indeed, the so called “difference” between the wave spectra measured “before” and “after” breaking, shown in Fig.1, provides evidence that the energy of wave component corresponding to the spectrum peak is not lost totally. It is most likely that it is randomly distributed among other wave components. We do not dwell on a detailed analysis of this data (which is described in Appendix A). We only note that the result shown here emphasizes the validity of the above statement about the restriction of the experimental research of the wave dissipation nature.

Nevertheless, despite the difficulties discussed above, there is a remarkable series of theoretical and experimental results in this field. Moreover, these results were for a long time used more or less successfully in numerous wind-wave numerical models of the form (1.1): WAM (The WAMDI Group, 1988), WAVEWATCH (WW) (Tolman and Chalikov, 1996) and others (Yongzeng et al, 2005; Ardhuin and Le Boyer, 2006; and so on). Though, unlike relatively well developed and widely recognized parameterizations of mechanisms *NL* and *IN*, the dissipation functions *DIS(S)* used in numerous numerical models are poorly justified physically. In particular, the parameterizations in the form of linear spectral function $DIS(S) \propto S$ used in such world-wide spread models as WAM and WW have a semi-phenomenological origin. The latter means that they are constructed on the basis of the general physical considerations, which are not directly and unambiguously related to the measurements and fundamental physical equations. Therefore, the parameterizations *DIS(S)* mentioned above are often and reasonably criticized from one or other viewpoints (Komen et al., 1994; Polnikov, 2005; Cavaleri et al., 2007).

According to the aforesaid, the necessity of more logical and theoretically justified description of wind-wave energy dissipation processes in the spectral representation is still needed.

In addition, we note that an idea exists already for a long time to construct the dissipation model, based on the statement that surface-wave energy losses can occur due to the turbulent viscosity provided by the interaction of waves with the turbulence in the upper water layer (Hasselmann, 1960). This idea was formally realized in several versions 25 years ago in (Efimov and Polnikov, 1986) where some essential modelling preferences of the proposed representations of *DIS(S)* were demonstrated. Later, this idea was developed by the author up to a semi-phenomenological theory, though its results were published only partially (Polnikov, 1994), whilst the main derivations were retained in a manuscript of the author's doctoral dissertation (Polnikov, 1995). Many years later, the author have sophisticated the final theoretical results (Polnikov, 2005) and successfully verified them by means of their implementation in the mathematical shells of the world-wide known models mentioned above: WAM(Polnikov et al., 2008) and WW(Polnikov and Innocentini, 2008). Nevertheless, the turbulent viscosity model is not widely recognized yet, as far as its full justification does not exist, though some experimental results in the field of wave-induced turbulence have appeared (Babanin and Haus, 2009; Gemmrich, 2010; Dai et al., 2010).

The issue of convincing justification of the spectral representation for wind-wave dissipation mechanism was many times discussed with A. Babanin, who is one of the leading investigators in this field. These discussions precisely stimulated the appearance of this paper. It is dedicated to gain the general understanding of the problem and definiteness in its solution. This is the main objective of the present work that has several constituents.

The first of the objectives is generalization of the point of view on the problem. With this in mind, the author performed a critical analysis of the most important approaches to the problem (theoretical works: Longuet-Higgins, 1969; Battjes and Janssen, 1978; Hasselmann, 1974; Phillips, 1985; Polnikov, 1994; Zaslavskii, 1999; Donelan, 2001; Chalikov and Sheinin, 1998; Zakharov et al., 2007; and experimental researches: Banner and Young, 1994; Banner and Tian, 1998; Babanin et al., 2001; Young and Babanin, 2006; Babanin and Westhuysen, 2008). The result of this analysis showed that clear and physically well justified models of wind wave dissipation mechanism do not exist yet. As an alternative to the present approaches, the full-scaled similarity method was applied in this paper for the first time.

The similarity approach, formulated in section 2, allows us to construct the most general spectral form for phenomenological function *DIS(S)* in the local representation in *S*, which, under some assumptions, is given as $DIS(S) \propto S^2$. As far as this particular form was repeatedly used by the author earlier (for references, see Polnikov, 2010), the corresponding numerical evidence is presented in section 2, demonstrating the preferences of the proposed representation for *DIS(S)* with respect to the versions of *DIS(S)* used in WAM and WW models (see Fig. 2 and Tab. 1 in section 2).

The second, and rather self-contained constituent of our objective is the construction of theoretical justification of the found phenomenological representation, $DIS(S) \propto S^2$ (section 3). The goal of this part of work is to find such physical mechanism that supports the mentioned dependence *DIS(S).* The mathematical approach, described in Hasselmann (1974), and the idea of turbulent viscosity, as the main mechanism of wave energy losses, were applied to solve the problem. Eventually, the physical model of wind wave energy losses corresponding to form $DIS(S) \propto S^2$ is constructed on the basis of equations system (1.2)-(1.5). It is our opinion that this part of the work gives a convincing proof that the turbulent viscosity can be considered as the most general dissipation mechanism for wind waves, which can be represented in the spectral form.

The final overview for application of the results obtained is given in conclusive section 4.

2. Construction of function $DIS(\sigma,\theta,W,S)$ by means of similarity method

Keeping in mind a multi-scale and stochastic feature of the processes taking place at the air-sea interface, we state that the most effective approach to the problem posed is the similarity method (or the method of dimension consideration), widely used in different fields of statistical hydromechanics (Monin and Yaglom, 1971).

In our case, the essence of similarity approach consists in the fact that dissipation rate $DIS(\sigma,\theta,W,S)$, realizing in the spectral domain (σ,θ), is provided by the local wind **W** and local wave spectrum intensity $S(\sigma,\theta)$. The following fact testifies to the benefit of this statement: in the case of stationary and homogeneous wind, the spectrum "tale" (i.e. the part of spectrum $S(\sigma,\theta)$ in the frequency domain $\sigma > 2\sigma_p$) has a fixed shape named the equilibrium spectrum $S_{eq}(\sigma,\theta)$ (Komen et al., 1994; Cavaleri et al., 2007). Moreover, it is widely recognized (see the same references) that formation of the equilibrium shape for a spectrum tale is stipulated by the balance between input and dissipation rates:

$$IN(\sigma,W,S_{eq}) - DIS(\sigma,W,S_{eq}) = 0 \quad . \qquad (2.1)$$

According to ratio (2.1), one should expect the local feature of the dissipation function in spectrum *S*, by analogy to feature of the input one. The latter means that there are no integrated (cumulative) spectral terms in *DIS(S)*.

All these facts serve as the basis for application of the dimension consideration method for postulating an initial kind of function $DIS(\sigma,\theta,W,S)$. Herewith, a physical nature of wave energy losses is quite inessential, as far as this approach has the phenomenological feature. The physical nature of the dissipation process could be revealed at the stage of theoretical justification of the phenomenological result found by the similarity method.

In conclusion we note that the proposed solution of the problem is represented here for the first time.

2.1. Phenomenological construction of the general kind of $DIS(\sigma,W,S)$

The principal parameters of the system considered are as follows: the wave spectrum $S(\sigma,\theta)$; the peak frequency of the spectrum σ_p; the steepness of waves at current frequency $\varepsilon(\sigma)$; the phase speed of wave crests, which is of the order of $c_p = g/\sigma_p$; the local wind speed W; and the gravity acceleration *g*. Thus, in the problem posed, one can form a long series of dimensionless parameters:

- The dimensionless spectrum,

$$\hat{S}(\sigma) = \sigma^5 S(\sigma,\theta) / g^2 \equiv \varepsilon^2(\sigma) \ , \tag{2.2a}$$

- The dimensionless current frequency,

$$\hat{\sigma} = \sigma / \sigma_p \, , \tag{2.2b}$$

- The dimensionless phase speed of crests (or the wave age A)

$$A = \hat{c}_p = c_p / W \ , \tag{2.2c}$$

- The dimensionless wind

$$\hat{W} = W\sigma / g \quad , \tag{2.2d}$$

and so on. To solve the problem, it needs to choose a combination of these parameters, providing for the general phenomenological representation of function $DIS(\sigma,W,S)$.

Let us write presentation for $DIS(\sigma,W,S)$ in the kind

$$DIS(\sigma,W,S) = const \cdot \sigma S(\sigma,\theta) \cdot \Phi(\hat{S},\hat{W},A,\varepsilon,..) \tag{2.3}$$

where $\Phi(\hat{S},\hat{W},A,\varepsilon,..)$ is the sought function of a whole set of the parameters mentioned above. With no account of the angular dependence of *DIS*, clarification of which is still not under consideration, expression (2.3) can be accepted as the most general representation of function $DIS(\sigma,W,S)$. With the aim of finding the most simple form of function $\Phi(...)$ among a multitude of its specifications, we will put some restrictions to its representation.

First of all, assuming the dissipation function is regular, growing, and local function of spectrum *S*, one can represent function $\Phi(...)$ as a series to powers in spectrum (with no restriction of generality of the spectral representation in local approximation)

$$\Phi(...) = \sum_n \alpha_n (A,\hat{\sigma},\hat{W},\varepsilon)\hat{S}^n \, . \tag{2.4}$$

Now, take into account that a value of dimensionless spectrum $\hat{S}$ given by ratio (2.2a) is a small parameter of the system, which is of the order of Phillips' constant α_p (Komen et al, 1994)

$$\hat{S} = \sigma^5 S(\sigma,\theta) / g^2 \cong \alpha_p \cong 0.01 << 1 \, . \tag{2.5}$$

In such case, it immediately follows from (2.4) that the representation for $\Phi(...)$ can be restricted by several first terms of series (2.4), which have a real physical sense.

Second, let us specify the meaning of the essential terms of series (2.4). To this end, it needs to attract the well known conception about equilibrium spectrum $S_{eq}(\sigma,\theta)$ realizing in a tail-domain of the spectrum, defined by the ratio

$$\hat{\sigma} > (2 \div 2.5)\,. \qquad (2.6)$$

The commonly recognized condition of the equilibrium spectrum existence is the balance between the input and dissipation mechanisms, given by equation (2.1). In turn, the commonly recognized representation for input function $IN(W,S)$ is the linear in spectrum function (Miles, 1962; Komen et al., 1994; Cavaleri et al., 2007)

$$IN(W,S) \;=\; \beta(\sigma,\theta,\mathbf{W})\,\sigma\,S\,(\sigma,\,\theta)\quad . \qquad (2.7)$$

Coefficient $\beta(...)$ in the right-hand side of (2.7) is the so called wave-growing increment depending on the dimensionless parameters mentioned above. Specification of function $\beta(...)$ is not principal here, it is sufficient to accept that the analytical representation of $\beta(...)$ is fully defined (Komen et al., 1994; Cavaleri et al., 2007).

In this case, it immediately follows from formula (1.1) that the mathematically similar, linear in spectrum terms of *IN* and *DIS* can be united to a single summand of SF, non-dimensional coefficient of which can be treated as the increment $\beta(...)$ commonly used in the input function *IN* (with no loss of physical sense). This circumstance allows us to exclude the linear in spectrum summand of series (2.3) for the representation of function *DIS*. Consequently, the only term of series (2.3), which has the physical sense, is the summand with the second power in spectrum *S*. As a result, with no loss for physical sense of SF, the general representation for dissipation function (2.3) is reduced to the form

$$DIS(\sigma,W,S) = \sigma S(\sigma,\theta)\cdot\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)\hat{S} = \alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)\,\frac{\sigma^6 S^2(\sigma,\theta)}{g^2}\,. \qquad (2.8)$$

Representation (2.8) specifies fully the spectral dependence for function *DIS(S)*, and the rest ambiguity in *DIS* is reduced to the new unknown function $\alpha_1(...)$ independent of *S*.

Thus, the problem of spectral representation of dissipation function is solved in the frame of the accepted phenomenological approach under assumption that this function is local in wave spectrum. As it was shown above, the final representation of *DIS(S)* has neither the linear in spectrum term (due to its mathematical similarity to the input term) nor the terms with powers in spectrum higher than the second one (due to their negligibility).

2.2 . Specification of function $DIS(\sigma,\mathbf{W},S)$.

Farther specification of the dissipation function is defined by the specification of the auxiliary function $\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)$. It is realized by the following way.

First of all, write the condition of the equilibrium spectrum existence in the form

$$\left[IN - DIS\right]\Big|_{S=S_{eq}} \approx 0 \qquad . \tag{2.9}$$

It is implied here that condition (2.9) is valid in domain (2.6), only. Substitution of (2.7) and (2.8) into (2.9) leads, in fact, to the equation for the spectrum shape, $S_{eq}(\sigma,\theta)$, solution of which has the kind [3]

$$S_{eq}(\sigma,\theta) = \frac{\beta(\sigma,\theta,\mathbf{W})}{\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)} g^2\sigma^{-5} \qquad (\text{for } \sigma > (2 \div 2.5)\sigma_p). \tag{2.10}$$

According to (2.10), the specification of $\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)$ is unequivocally defined by the setting the equilibrium spectrum shape, $S_{eq}(\sigma,\theta)$. So, this shape is the constructive element of the parameterization under derivations, together with the kind of representation for wave growth increment $\beta(...)$.

Combining the known models for the equilibrium spectrum shape, proposed by Phillips(1958) and Toba(1972) (also accepted in Phillips, 1995), one can get for it the following united form

$$S_{eq}(\sigma,\theta) = const(n)\cdot g^2\sigma^{-5}\cdot\hat{W}^n \qquad , \tag{2.11}$$

allowing to vary the power n of the dimensionless wind in a wide range. Herewith, value n = 1 corresponds to Toba's spectrum, whilst n = 0 does to Phillips' one. As a result, substitution of (2.11) into (2.10) gives

$$\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon) = C_{dis}(n)\phi(A,\hat{\sigma},\varepsilon)\beta_M(\sigma,\theta,\mathbf{W})/\hat{W}^n \tag{2.12}$$

where the fitting coefficient $C_{dis}(n)$, depending on the excepted form for $S_{eq}(\sigma,\theta)$, is shown explicitly, and the new dimensionless function $\phi(A,\hat{\sigma},\varepsilon)$ is extracted, which is not defined yet.

It should be mentioned here that due to a possible change the sign of increment $\beta(\sigma,\theta,\mathbf{W})$ (for swell components or low frequency components of spectrum, overtaking the local wind), the factor $\beta_M(\sigma,\theta,\mathbf{W})$ in (2.12) should be taken in the form of simple modification for increment $\beta(\sigma,\theta,\mathbf{W})$:

$$\beta_M(\sigma,\theta,\mathbf{W}) = \max[\beta_L, \beta(\sigma,\theta,\mathbf{W})] \qquad . \tag{2.13}$$

Here, the standard designation max[*a, b*] means a choice of maximum among values under the brackets. This modification is needed to secure a proper sign for the dissipation function.

[3] By the way, this is the key algebraic advantage of the quadratic form for *DIS(S)*.

The positive constant, β_L, is the so call "background" value of factor β_M, defined during the model fitting process (see subsection *2.3* below).

Expressions (2.12-2.13) finalize, in fact, the part of specification of dissipation function *DIS(...)* regarding to its dependences both on spectrum *S* and local wind **W.** This fact ensures independence of function $\phi(A,\hat{\sigma},\varepsilon)$ of *S* and **W**. Moreover, in the domain given by (2.5), it is quite acceptable asymptotic $\phi(A,\hat{\sigma},\varepsilon) \approx 1$, as far as the proper representation for *DIS(...)*describes fully the formation of known empirical tale of wave spectrum.

In the vicinity of the spectrum peak, i.e. in the domain given by the ratio $0.5 < \hat{\sigma} < 2$, the specification of function $\phi(A,\hat{\sigma},\varepsilon)$ needs attraction of new empirical information and phenomenological hypothesis regarding to behavior of function *DIS(...)* in this domain. It was proposed earlier (Polnikov, 1994, 2005) the following final form

$$\phi(A,\hat{\sigma},\varepsilon) = \phi(\theta,\theta_w,\hat{\sigma}) = \max\left[0,\ (1 - c_\sigma \hat{\sigma}^{-1}\right] T(\theta,\theta_w,\hat{\sigma}) \tag{2.14}$$

with the angular function given by purely phenomenological form

$$T(\theta,\theta_w,\hat{\sigma}) = \left\{1 + 4\sin^2\left(\frac{\theta-\theta_w}{2}\right)\hat{\sigma}\right\} \max\left[1,\ 1-\cos(\theta-\theta_w)\right] \tag{2.15}$$

where c_σ is the additional parameter of the dissipation model, θ_w is the local wind direction. It should be especially mentioned here that the angular representation (2.15) has the two-mode feature well corresponding to the recent empirical results (Young and Babanin, 2006) (for details, see Appendix A).

Thus, the formulas (2.8) and (2.12)-(2.15) finalize totally the proposed phenomenological parameterization of the dissipation function. As it has turned out later, this parameterization corresponds to the most reliable features of the modern empirical data (Young and Babanin, 2006), excluding the cumulative and threshold features of *DIS(S)* discussed in details in Appendix A.

2.3. Physical meaning of the parameters and the effectiveness of phenomenological function $DIS(\sigma,\mathbf{W},S)$

Moving the theoretical justification of the proposed spectral representation, *DIS(S)* $\propto$ S^2, to the afterfollowing separate consideration (section 3), give here the short description of meaning of the parameters introduced above, and show some evidence of new parameterization preference with respect to the analogues used in WAM and WW.

First of all, note that in the dissipation parameterization proposed above, there are only four fitting values: C_{dis}, β_L, c_σ, and n .

The meaning of the fitting coefficient C_{dis} is evident: it regulates the dissipation intensity. This parameter is inevitable in any model that has the SF represented in the additive form (1.1). Moreover, C_{dis} is the most actively varied in the course of fitting the model given by the wave spectrum evolution equation of the form (1.1).

Parameter n is the next in importance. Its meaning is evident also: it is responsible for the shape of an "expected" equilibrium spectrum, which is provided by the numerical model. Value n = 1 supports the Toba's equilibrium spectrum, $S_{eq}(\sigma) \propto Wg\sigma^{-4}$, whilst value $n = 0$ does the Phillips' spectrum, $S_{eq}(\sigma) \propto g^2\sigma^{-5}$. Consequently, by means of varying the value of n, users of the model are free to choose the expected shape of numerical equilibrium spectrum. Hereof, by the way, it follows that the falling law "-4" of the spectrum tale (i.e. $S(\sigma) \propto \sigma^{-4}$) is easily realized with no account of nonlinear term *NL* (as it is discussed in Phillips, 1985; Zakharov, 1974). So, the proposed approach gives an additional freedom for physical treating the Toba's spectrum formation.

Side by side with n, parameter c_σ is the element of fine fitting the calculating spectrum shape. It allows varying the dissipation intensity in the spectral peak domain and, at some extent, the dissipation intensity in the spectrum-tale domain. In fact, existence of this coefficient corresponds to the empirical data (Young and Babanin, 2006) that dissipation intensity at the spectrum tale is defined by the intensity of the dominant waves breaking. For example, our simulations show (Polnikov, Innocentini, 2008; Polnikov, 2010) that varying c_σ allows, at some extent, to change the mean frequency, σ_m, which is one of integrated characteristics of the spectrum, given by the ratio

$$\sigma_m = \iint \sigma S(\sigma,\theta) d\sigma d\theta \,/ \iint S(\sigma,\theta) d\sigma d\theta \quad . \qquad (2.16)$$

Relationship between σ_p and σ_m is important as one of the "checked" magnitudes analyzed in the course of numerical model verification (Polnikov, 2010). By the way, this relationship is defined by a choice of parameter *n*, as well.

Finally, several words about parameter β_L. In the light of the said above for explanation of formula (2.13), this parameter is used to regulate the dissipation intensity during the processes of rapid change of wind velocity vector **W** (turning or going down), corresponding to the transition of wind components to a swell. For such components, the value of increment $\beta(\sigma,\theta,\mathbf{W})$ undergoes the rapid decrease resulting in diminishing the intensity of wave breaking. Herewith, a certain extent of the background turbulence in the upper water layer is still retained what ensures a remarkable attenuation of the wind-wave components that became the swell. Together with the wave generation process, this attenuation permits waves to turn faster to the new wind direction (see detailed simulations in Polnikov, 2005). These situations are poorly described by the models with the traditional dissipation functions (WAM and WW), where the background dissipation is simply absent. The choice of the order of value β_L is defined by the empirical and numerical simulation results (Chalikov and Sheinin, 1998; Polnikov, 2005).

In conclusion we note that phenomenological function of dissipation angular distribution $T(\sigma,\theta,\theta_w)$ of kind (2.15) (which is totally made "by hands" still in Polnikov, 1994), corresponds rather well to the empirical data (Young and Babanin, 2006) described in Appendix A.

Dwell now on the preferences of new dissipation function, realized during numerical modelling of wind waves. First of all, we note that the only possibility to proof these preferences is the procedure of comparative verification. Such kind verification was performed in papers (Polnikov et al., 2008; Polnikov and Innocentini, 2008; Polnikov, 2010) where the models WAM and WW were accepted as the reference ones. The comparative verification procedure was performed for the integrated characteristics of wave field: the significant wave height H_S, and the mean period T_m, only. They were calculated both with the original version of the models mentioned and with their modifications made by replacing the corresponding SF in both models.

Note that in both cases, modification of SF is related to the change of all three terms. Herewith, the change of terms *NL* and *IN* does not practically change the physics of SF, whilst the change of *DIS*, in opposite, does it radically. Variation of the models modification, realized by means of a consecutive and separate change of the SF-terms, permitted us to find that all principal differences in accuracy of wave-field calculations are caused by the change of the *DIS*- term (Polnikov and Innocentini, 2008).

In all the mentioned papers, it was shown for a great data base that the change of *DIS*-term resulting in increasing the simulation accuracy for significant wave height H_s on 15-20% and for mean period T_m up to 50% (Tab. 1, following to Polnikov, 2010).

Table 1.

Verification errors of
original model (WAM) and modified model (NEW)
for 15 buoys in the North Atlantic in 2006y (following to Polnikov, 2010).

No of buoy/ Model type	Types of r.m.s. errors		Win of accuracy	
	δH_S, m	δT_m, s	$\frac{(\delta H_s)_{wam}}{(\delta H_s)_{new}}$	$\frac{(\delta T_m)_{wam}}{(\delta T_m)_{new}}$
41001/WAM	0.67	1.12	1.31	1.37
/NEW	0.51	0.82		
41002/ WAM	0.47	1.24	1.17	1.28
/NEW	0.40	0.97		
41010/ WAM	0.42	1.47	1.07	1.12
/NEW	0.39	1.31		
44004/ WAM	0.62	1.35	1.15	1.29
/NEW	0.54	1.05		
44008/ WAM	0.88	0.95	1.69	0.87
/NEW	0.52	1.09		
44011/ WAM	0.61	1.57	1.42	1.45
/NEW	0.43	1.08		
44137/ WAM	0.43	1.44	1.00	1.58
/NEW	0.43	0.91		
44138/ WAM	0.56	2.12	0.98	1.88
/NEW	0.57	1.13		
44139/ WAM	0.45	2.03	0.85	1.60
/NEW	0.53	1.27		
44141/ WAM	0.42	1.77	0.70	1.81
/NEW	0.60	0.98		
62029/WAM	0.54	2.44	1.02	1.71
/NEW	0.53	1.43		
62081/WAM	0.52	2.38	1.16	1.79
/NEW	0.45	1.33		
62105/WAM	0.50	2.34	1.00	1.77
/NEW	0.50	1.32		
62108/WAM	0.66	2.07	1.18	2.20
/NEW	0.56	0.94		
64046/WAM	0.54	2.09	0.96	1.77
/NEW	0.56	1.18		

Note. Shading marks the cases when the model NEW has less accuracy.

For more convincingness, we plot here the time history $H_s(t)$ (Fig. 3) which illustrates that the change of SF (the model NEW) results in better description of the extreme wave-heights and their sequential attenuation. Such attenuation is evidently related to the change of the local wind at the point under consideration, and these situations are better described by our *DIS*-term.

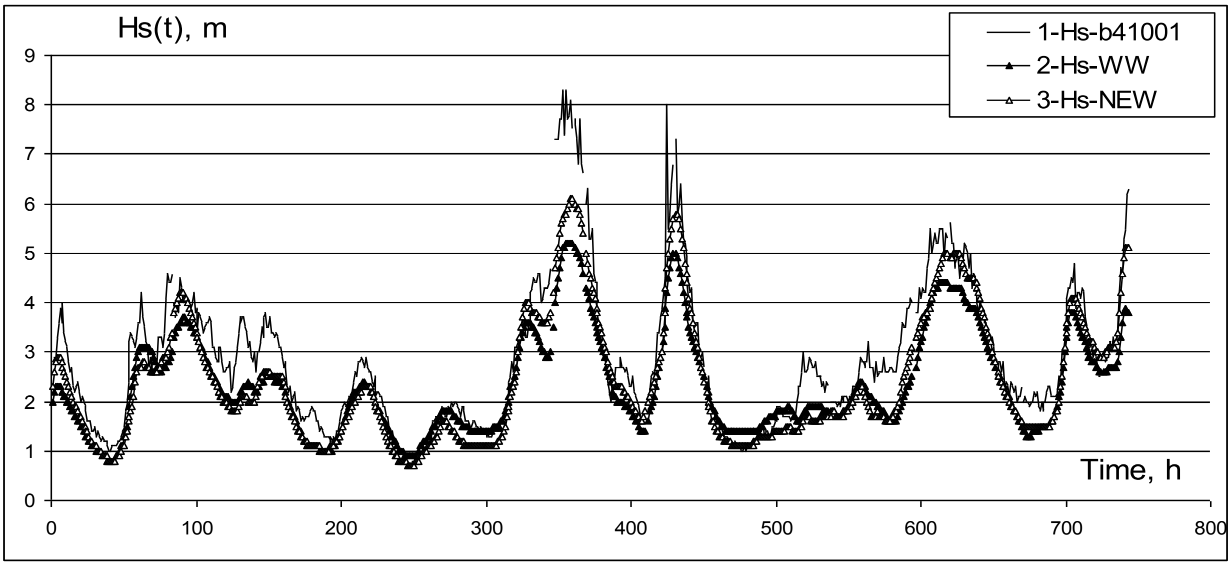

Fig. 2. The time history of significant wave height $H_s(t)$ at the location of buoy 41001 in the North Atlantic for the period January 2006: 1- buoy observations, 2 – results of modelling with the model WW, 3 – results of modelling with the modified model WW (following to Polnikov and Innocentini, 2008).

Here we have no place (and necessity) for a full analysis of the comparison methodic and for demonstrating more details, which can be found in (Polnikov et al., 2008; Polnikov and Innocentini, 2008; Polnikov, 2010). At the moment it is important to emphasize only the fact of remarkable increasing of simulations accuracy what is achieved simply by changing the physical content of the source functions used in WAM and WW. Herewith, the key item of this changing is the dissipation term, detailed study of which is the main aim of this work.

For completeness of the problem solution, it is left to consider the point of physical justification the quadratic dependence, $DIS(S) \propto S^2$, resulting in the mentioned positive results of verification.

3. Theoretical justification of form $DIS(S) \propto S^2$. The turbulent viscosity model

3.1. Reynolds stresses and main fundamentals of the model

The main fundamental of the proposed model states that, on the scales of validity the evolution equation for wind waves in the spectral form, the central and most general cause of the wind-wave energy dissipation is the turbulence of the upper water layer. Herewith, the specification of the processes producing the turbulence is insignificant.

It is evident that a reasonable part of the turbulence intensity is provided by the wave breaking processes. Though, it is also clear that a multitude of accompanying processes (mentioned earlier in section 1) generates a cascade of chaotic motions with no determined scales, i.e. the turbulence motions. In our mind, these motions make the main contribution to the wind-wave dissipation. In other words, the proposed theoretical approach is based on the accounting all dissipative processes resulting in the turbulence production in the upper water layer.

According to the said, without any restriction of generality, the field of currents $\mathbf{u}(\mathbf{x},t)$ can be written in a wavy water layer in the form of two constituents

$$\mathbf{u}(\mathbf{x},t) = \mathbf{u}_w(\mathbf{x},t) + \mathbf{u}^{'}(\mathbf{x},t) \quad . \tag{3.1}$$

The first summand in the right-hand side (farther, the r.h.s.) of (3.1), $\mathbf{u}_w$, we treat as the potential motion attributed to the wind waves, whilst the second summand, $\mathbf{u}^{'}$, is treated as the turbulent constituent of full currents field, totally uncorrelated with $\mathbf{u}_w$ in the statistical sense. Herewith, it is important to note that a corresponding representation for the elevation field $\eta(\mathbf{x},t)$ in not necessary, if one accepts the Hasselmann's statement about "weakness in mean" for processes of the water surface ruptures (Hasselmann, 1974).

Now, substituting representation (3.1) into initial equation (1.2) (with no external forces) and into (1.4), by means of averaging over turbulent scales we get the following system (subindex *w* is omitted)

$$\frac{\partial \overline{u}_i}{\partial t} + \sum_j \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -g\delta_{i,3} - \sum_j \frac{\partial < u_i^{'} u_j^{'} >}{\partial x_j} \tag{3.2}$$

$$\frac{\partial \overline{\eta}}{\partial t} + \sum_{i=1,2} \overline{u}_i \frac{\partial \overline{\eta}}{\partial x_i} = \overline{u}_3 \quad . \tag{3.3}$$

Here, the tensor form of equations is used; the mean wave variables are denoted by the bar in above (farther, the bar will be omitted for simplicity); and brackets <...> are the symbol of averaging over the turbulent scales. Due to nonlinearity of the system, a new term is appearing in the r.h.s. of (3.2):

$$P_i \equiv \sum_j \frac{\partial < u_i^{'} u_j^{'} >}{\partial x_j} \quad . \tag{3.4}$$

Physical meaning of the term **P** is the forcing which results in the wave motion dissipation (Hasselmann, 1974).

Numerator $< u_i^{'} u_j^{'} > \equiv \tau_{ij}$ in expression (3.4) is the well known (in the turbulence theory) magnitude called the Reynolds stress tensor (Monin and Yaglom, 1971) written here in the normalization to the water density. Methods of parameterization for τ_{ij} are also rather well developed. Therefore, to construct the theory, it needs to specify τ_{ij} in terms of the wave variables, η and **u,** and to ascribe to this specification a certain physical content. After this, with the aim of getting the final result, one can use the mathematics of (Hasselmann, 1974).

To reach the aim posed, let us accept a series of additional fundamentals. Firstly, following to (Hasselmann, 1974), we accept the fundamental about "weakness of distortions in mean". It allows us to retain the common meaning for the wave profile, $\eta(\mathbf{x},t)$, and potential wave motion, **u(x,***t*), and to introduce any derivatives for these variables. Secondly, we suppose that the magnitude of nonlinear stresses τ_{ij} does essentially exceed the weak dynamic nonlinearity of the system, described by the second terms in the l.h.s. of (3.2) and (3.3). Hereby, we postulate the fundamental of the "strong" turbulence. Thus, system (3.2), (3.3) takes the following kind

$$\frac{\partial u_i}{\partial t} + g\delta_{i,3} = -P_i(\mathbf{u},\eta) \quad , \tag{3.5}$$

$$\frac{\partial \eta}{\partial t} = u_3 \quad . \tag{3.6}$$

3.2. Phenomenological closure of Reynolds stress

Now, we formulate the main grounds of our concept for the procedure of the Reynolds stress closure, the aim of which is to express the turbulent characteristic τ_{ij} via wave variables η and **u**.

To do this, we remind the version of such closure, consisting in using the well known Prandtl's conception of random mixing length λ' (Monin and Yaglom, 1971). In the simplest case of near-wall turbulence, the Prandtl's approach results in the Reynolds stress closure in the form of quadratic function in the velocity field

$$\tau_{ij} = < \lambda_i' \, \lambda_j' (\partial u_i / \partial x_3)(\partial u_j / \partial x_3) > . \tag{3.7}$$

In our case, besides of the velocity field instability, the turbulence is provided by the variability of the interface surface. Therefore, representation (3.7) should be properly generalized. As a certain result of this generalization, the distortion forcing $-P_i(\mathbf{u},\eta)$ can be represented, for example, by the following quadratic function of wave variables $\mathbf{u}$ and η

$$-P_i(\mathbf{u},\eta) = < \sum_j \frac{\partial}{\partial x_j}\{\left[\lambda_i'(\partial u_i/\partial x_3)+\upsilon_i'(\partial\eta/\partial x_i)\right]\left[\lambda_j'(\partial u_j/\partial x_3)+\upsilon_j'(\partial\eta/\partial x_i)\right]\} > \qquad (3.8)$$

where the random values υ_i' have the meaning of mixing velocities.

Here it is important to note that we do not know real processes of the turbulence formation in the upper water layer, therefore there is no sense to construct any more complicated and specified approximation for distortion **P** via physical variables **u** and η, as it was done in the previous papers by the author (Polnikov, 1994, 1995). It is essential only that representation (3.8) retains the following main features of the problem: nonlinearity of the distortion forcing **P** in the wave variables, and dependence **P** on the gradients on both velocity field $\mathbf{u}(\mathbf{x},z,t)$ and elevation field $\eta(\mathbf{x},t)$. As it will be shown below, the nonlinear feature of the distortion forcing gives sufficient grounds for finding the general kind of spectral representation for *DIS(S)*.

3.3. General kind of the problem solution in the spectral representation

In the case of deep water, the full system of equations to be solved includes dynamic equations (3.5), (3.6), written at the interface surface $\eta(\mathbf{x},t)$, and equations (1.3), (1.5) valid in a whole volume of fluid. The proper system of equations, written in the potential motion approximation, has the kind

$$\dot{\Phi} + g\eta = -\hat{P}(\eta,\Phi) \quad , \qquad (3.9)$$

$$\dot{\eta} = \frac{\partial\Phi}{\partial z} \quad , \qquad (3.10)$$

$$\sum_i \frac{\partial^2\varphi}{\partial x_i^2} = 0 \quad \text{и} \quad \frac{\partial\varphi}{\partial z}\Big|_{z=-\infty} = 0 \quad . \qquad (3.11)$$

Here, the distortion function $\hat{P}(\eta,\Phi) = (\vec{\nabla}_3)^{-1}[P(\eta,\mathbf{u})]$ represents the certain modification of function (3.8), induced by the procedure of transition to the scalar potential of velocity field $\varphi(\mathbf{x},z,t)$, whilst $\Phi \equiv \varphi(\mathbf{x},\eta(\mathbf{x}),t)$ is the potential at the elevation surface.

Farther, the variables are written in the form of the Fourier-Stiltjes decomposition in the wave vector space (see details in Hasselmann, 1974; Zakharov, 1974)

$$\eta(\mathbf{x},t) = const \cdot \int_{\mathbf{k}} \exp[i(\mathbf{kx})]\eta_{\mathbf{k}}(t)d\mathbf{k}\,;\, \varphi(\mathbf{x},z,t) = const \cdot \int_{\mathbf{k}} \exp[i(\mathbf{kx})]\, f(z)\varphi_{\mathbf{k}}(t)d\mathbf{k} \quad (3.12)$$

where **k** is the wave vector, and *f(z)* is the vertical structure function of the potential, to be found from two equation (3.11). In our case, *f(z)*=exp(-*kz*). After substitution of representations (3.12) into (3.9)-(3.11), equation (3.11) result in the mentioned structure function *f(z)*, and two other equations get the kind

$$\dot{\Phi}_{\mathbf{k}} + g\eta_{\mathbf{k}} = -P_{\mathbf{k}}(\eta_{\mathbf{k}}, \Phi_{\mathbf{k}}) \quad , \qquad (3.13)$$

$$\dot{\eta}_{\mathbf{k}} = k\Phi_{\mathbf{k}} \quad . \qquad (3.14)$$

Here, $P_{\mathbf{k}}(\eta_{\mathbf{k}}, \Phi_{\mathbf{k}}) \equiv F^{-1}[\hat{P}(\eta, \Phi)]$, and the operator F^{-1} means the inverse Fourier transform applied to function $\hat{P}(\eta, \Phi)$. The latter procedure is commonly used in the course of transition to the final equations written for the Fourier components, $\eta_{\mathbf{k}}$ and $\Phi_{\mathbf{k}}$ (for details, see Zakharov, 1974; Polnikov, 2007).

Here we should especially note that the inverse Fourier transform procedure needs a calculation of integrals of the kind

$$\int_{\mathbf{k}} d\mathbf{x} \left[\exp(-i\mathbf{kx}) < \lambda'_i \lambda'_j \int_{\mathbf{k}_1} \int_{\mathbf{k}_2} \phi(\mathbf{k}_1, \mathbf{k}_2) \exp(i\mathbf{k}_1\mathbf{x}) \exp(i\mathbf{k}_2\mathbf{x}) \eta_{\mathbf{k}_1} \eta_{\mathbf{k}_2} d\mathbf{k}_1 d\mathbf{k}_2 > \right], \qquad (3.15)$$

which are typical in the nonlinear theory for waves (Zakharov, 1974; Polnikov, 2007). In the traditional theory, the form (3.15) gets the final expressions with the three-wave resonances. These resonances are defined by the factor $\delta(\mathbf{k}_1 \pm \mathbf{k}_2 \pm \mathbf{k})$ under the sign of the final integral, which is provided by the analytical integration in (3.15) on *d***x.** But in our case, such resonances can result in the cumulative terms in the dissipation function, which are not desirable (see discussion in section 4).

To avoid appearing the cumulative terms in the final result, we use the principal difference between our expression (3.15) and the analogous one appearing in the traditional (conservative) nonlinear theory. In our case, this difference consists in the fact that the turbulent-scale averaging operator, given by the brackets <…>, takes place under the integral in (3.15). This fact allows us to accept the following hypothesis: *the random multipliers of the closure (3.8), standing under the averaging brackets in (3.15) (alike* $\lambda'_i\lambda'_j$ *and analogous ones), due to their random feature, provide with a radical rearrangement of the phase (exponential) multipliers in the integrand of (3.15), resulting in the non-resonance* *feature of interaction between wave and turbulence*. This hypothesis simply means a postulating the following rule for statistical averaging in the integrals of kind (3.15):

$$\int_{\mathbf{k}} d\mathbf{x}\left[\exp(-i\mathbf{kx}) < \lambda_i'\lambda_j' \int_{\mathbf{k}_1}\int_{\mathbf{k}_2} \phi(\mathbf{k}_1,\mathbf{k}_2)\exp(i\mathbf{k}_1\mathbf{x})\exp(i\mathbf{k}_2\mathbf{x})\eta_{\mathbf{k}_1}\eta_{\mathbf{k}_2} d\mathbf{k}_1 d\mathbf{k}_2 >\right] = \\ = \int_{\mathbf{k}_1}\int_{\mathbf{k}_2} L(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\eta_{\mathbf{k}_1}\eta_{\mathbf{k}_2} d\mathbf{k}_1 d\mathbf{k}_2 \tag{3.16}$$

where $L(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$ is the regular function of its arguments.

Acceptance of this hypothesis allows avoiding the resonance (cumulative) forms in the final expression for nonlinear terms (3.15) and consider distortion forcing $P_{\mathbf{k}}(\eta_{\mathbf{k}},\Phi_{\mathbf{k}})$ in equation (3.13) as a regular quadratic form of Fourier variables $\eta_{\mathbf{k}}$ and $\dot{\eta}_{\mathbf{k}}$, alike in (3.16). In such case, the system (3.13), (3.14) is reduced to the standard equation for an oscillator with a weak and regular disturbance (see analogues in Hasselmann, 1974; Zakharov, 1974)

$$\ddot{\eta}_{\mathbf{k}} + gk\eta_{\mathbf{k}} = -kP_{\mathbf{k}}(\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}}) \quad . \tag{3.17}$$

The solution of equation (3.17) can be found in the spectral form by means of the method described in (Hasselmann, 1974), which is very close to the standard method of the weak turbulence, presented for the first time in (Zakharov, 1974) and detailed in(Polnikov, 2007).

Moving the explanation algebra to Appendix B, here we only note that the quadratic form of distortion function $P_{\mathbf{k}}(\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})$ leads to the solution of (3.17) of the kind

$$DIS(S,\mathbf{k},\sigma) = \sum_{n=2}^{N} \alpha_n(\mathbf{k},\sigma) F_n[S(\mathbf{k})] \quad , \tag{3.18}$$

where $F_n[S(\mathbf{k})]$ is the functional to *n*-th power in the wave spectrum. As seen, expression (3.18) is fully equivalent in the functional representation of initial formulas (2.3)-(2.4) of the phenomenological similarity method. This equivalence of the spectral representations for dissipation function $DIS(S)$ in both approaches finalizes the sought theoretical justification of the similarity method used (for details, see Appendix B).

In conclusion of this section, we should especially note that the main fundamental of the turbulent viscosity model, resulting in (3.18), is nothing else the nonlinear closure for Reynolds stress of kind (3.8) (or any other quadratic form), accepted above. Consequently, both result (3.18) and the results of section 2 have the proper physical justification. These results make a basis of the proposed model of the wave-energy losses due to the turbulent viscosity of the upper water layer.

4. Conclusion

In conclusion of the paper, we resume shortly the main fundamentals of the proposed model for wind-wave dissipation, the range its applicability, and prospective for its further elaboration.

Firstly, about fundamentals. An explicit functional form for the dissipation term of SF can be found in the frame of the phenomenological similarity method (section 2). But the physical justification of the initial formulas of the similarity method, (2.3)- (2.4), heeds attraction of the basic hydrodynamic equations. This aim is reached in the frame of the proposed turbulent viscosity model.

The main fundamental of the model consists in the assumption that on the scales of wave-spectrum evolution description, i.e. hundreds of the dominant periods, the main physical cause of the wave-spectrum component dissipation is the turbulence of the upper water layer, induced by the whole package of dissipative processes taking place at the air-sea interface. It is natural that laws of the turbulence formation are hardly known for us. Therefore, in the course of the model construction, the following two important assumptions are postulated: a) the nonlinear feature of the forcing resulting in the wave dissipation; and b) the specific modification of the phase structure for those Fourier-components of wave elevation and velocity, which are involved in the Reynolds stress closure. Acceptance of these presumptions is stipulated by the aim of obtaining observable final formulas. The point of their practical applicability is to be justified *a posteriori* by means of the verification procedure. Successful results of such procedure are presented in section 2 (Tab.1, Fig. 2) what, in fact, justifies all the assumptions accepted.

Secondly, about the scales of applicability of the approach proposed and treating the role of breaking events. This issue can be considered on the basis of the fact that functions *IN(S)*, *DIS(S)*, and *NL(S)* are linear, quadratic, and cubic in the spectrum, respectively[4]. It is known that each power of spectrum in the SF-terms of equation (1.1) increases the temporal scales for the proper mechanism in ε^{-2} times. Therefore, the input mechanism is the most fast one. This mechanism is balanced by the more slow dissipation mechanism what result in the equilibrium spectrum-tale formation in the high frequencies domain. But in the peak domain where *IN(S)* and *DIS(S)* have the cutting factors, the most slow and energy-conservative nonlinear mechanism, *NL(S)*, plays the most principal role (for details, see Polnikov, 2005,

[4] More details about roles of SF-terms and proper references can be found in (Polnikov, 2005, 2009)

2009). Thus, the issue of treating the role of very fast and strongly nonlinear breaking processes finds its solution based on the mentioned spectral consideration.

Really, the only logically justified matching the fast wave breaking processes with the rather slow wave-energy dissipation rate consists in the conclusion that the breaking means mainly the chaotic redistribution of the wave energy through the wave frequency band but do not the fact of wave-energy dissipation. Figure 1, being the direct measurement result of the wave-energy losses due to breaking, is the fine illustration to the said (see also text in section 1 and detailed comments in Appendix A). In this aspect, the known Hasselmann's hypothesis about "weakness of breaking in mean" (Hasselmann, 1974) means, in fact, that the fast, non-conservative and nonlinear distortion during breaking is not directly related to a slow process of wave-energy losses, realized in the spectral representation. One can say about losses only on the scales of the spectrum existence, i.e. hundreds of dominant periods.

Herewith, it becomes clear that in the spectral representation for the wave dissipation mechanism, the threshold feature of the dissipation function, which is often fixed in the field and numerical experiments related to the study of separate breaking events (Young and Babanin, 2006; Zakharov et al., 2007), should be totally smoothed due to statistical distribution of these event in time and space.

The said defines an applicability range for the dissipation model discussed here: it does not describe fast (bur rare at the fixed point of observation) separate breaking events; rather it is intended to description of the wave spectrum evolution realized on the scales of spatial-temporal variability of the whole statistical ensemble for a random wave field.

The last point, about prospective of farther elaborating the theoretical justification of the model. Such a prospective is seen in the direction of specification the hypothesis about a phase-structure modification for the Fourier-components involved into the Reynolds stresses closure procedure (formula (3.16) in subsection *3.3*). It is quite feasible that, in truth, some kind of the phase structure is conserved for the wave variables being under the turbulent-scales averaging sign in the integral of kind (3.15). Then, in the course of derivation the final expression for *DIS(S)*of kind (3.18), functional $F_n[S(\mathbf{k})]$ could have the integral expressions of the convolution type. This change of functional kind for *DIS(S)* is equivalent to appearing the cumulative dissipation term analogous to one proposed in (Young and Babanin, 2006)(see Appendix A). But in such case, according to ratios (2.1) and (2.9), the functional form for input function *IN(S)* should be properly modified, to save the widely accepted phenomenon of the equilibrium spectrum existence. Apparently, investigation of this point is a matter of a far future what is stipulated by an extraordinary complexity of verification the SF-terms of the cumulative type.

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Appendix A

For completeness of the picture, describing the most important experimental results dealing with the study of breaking processes in waves, we give here some extractions from paper (Young and Babanin, 2006) and our comments to their results.

Basing on the result of data processing, represented in Fig. 1, and attracting other experimental facts, the motioned authors have recommended the following parameterization of *DIS(S)*, provided by the breaking of the dominant wave only[5]

$$DIS(S)= a\sigma H[S(\sigma)-S_{th}(\sigma)]An(\sigma) + b\int_{\sigma_p}^{\sigma}\big(H[S(q)-S_{th}(q)]An(q)\big)dq\,. \qquad \text{(A.1)}$$

Here, a and b are the fitting constants; $H[\dots]$ is the step-function given by the ratio

$$H[x]=\begin{cases} x & if \;\; x>0 \\ 0 & if \;\; x\le 0 \end{cases}; \qquad \text{(A.2)}$$

$S_{th}(\sigma)$ is the threshold spectrum level chosen especially; $An(\sigma)$ is the dimensionless function of frequency, depending on the angular spreading of the wave spectrum. The integrated summand in the r.h.s. of (A.1) has the meaning of the "cumulative" impact of the low-frequency breaking on intensity of the dissipation for higher frequency components.

Results presented above can be characterized by the following way.

1) Dissipation function has the threshold feature;
2) Empirical function *DIS(S)* is linear in spectrum;

[5] Formula (A.1) is given with minor editing changes which are not principle in the physical sense.

3) The rate of losses in the frequency domain above the peak frequency σ_p has the cumulative feature, i.e. it is growing with the growth of losses at lower frequencies[6].

Among the properties enumerated above, the linear dependence *DIS(S)* is the most essential empirical fact related to our aim. The threshold and cumulative features of empirical function *DIS(S)* are interesting also, and they will be discussed later.

In a whole, it needs to acknowledge the importance and usefulness of the found empirical behavior of function *DIS(S)*, provided by the breaking only. Herewith, in the frame of our statement about incompleteness of describing the whole package of the wave dissipation processes by the breaking only, the detailed assessment of the results represented is the following.

Firstly, the result shown in Fig. 1 does hardly allow getting the reliable and quantitative dependences *DIS(S)*, for the reason of inevitable statistical noise. Here we have two remarks.

Thus, even accepting, as the analysis basis, the upper plot in Fig. 1, it evidently follows from the lower plot that the energy of breaking wave-components located at the spectral peak is not fully lost, it is rather chaotically spread among the others spectrum components.

Moreover, insufficient justification of the breaking-events separation in the wave-frequency band (see discussion of this point in Banner and Young (1994); Banner and Tian (1998); Babanin, Young, and Banner (2001); Young and Babanin (2006); Babanin and Westhuysen (2008)) leads automatically to a strong noise in the empirical information of the dissipation function dependence on frequency, $DIS(\sigma,W,S)$. Therefore, it is not following from Fig. 1, by no way, the conclusion about the cumulative feature of the dissipation processes, and the form given by the second term in (A.1) is not evident.

Secondly. From the point of view of the statistical ensemble meaning, which is very important for the spectrum definition, it seems that one may not separate the unique wave ensemble in two ensembles: "before breaking" and "after breaking". Such separation is the methodical arbitrariness.

Really, per se of the wave-field ensemble definition, *in any realization of the stochastic field (at any certain time moment), there is always a certain percent of the field square where the breaking takes place.* Therefore, one may not separate "by hand" the realization

[6] It is worth while to mention here one more empirical fact which does not influence on the dependence *DIS(S)* but relates to the angular dependence of dissipation intensity. This is the fact of the two-lobe profile for angular function $DIS(\sigma,\theta,\theta_w)$: the dissipation intensity is growing with initial growth of difference between direction of wave propagation θ and wind direction θ_w, followed by the intensity going down with the farther growth of (θ - θ_w).

with the breaking from the realization without the breaking, despite of the fact that there is no breaking at the point of measurement at the moment.

By the way, thereby the issue of the threshold feature of *DIS(S)* is closed. There is the threshold feature for a separate breaking, but the threshold feature of the breaking process is absent in the wave ensemble (due to a statistical structure of the field ensemble).

Thirdly. The breaking events are always too fast, to get reliable estimations of dynamics for the spectra “before breaking” and “after breaking”. Such estimations can be realized for the frequencies far exceeding peak frequency σ_p. But these frequencies can not be separated from wave elevation records having their duration smaller several dominant periods: for this rather long time, the amplitudes of high frequency components “forget” about very fast breaking event (if any). It is truer due to the stochastic feature of the wave field itself.

Fourthly. For each, very short period of the wave breaking at some space point, there are much more long periods of previous and past state of the wave field without breaking. Here the Hasselmann’s postulate about “weakness of breaking impact” on geometrical features of the interface surface becomes evident, as far as the wavy surface has the stochastic feature. By the way, just this postulate allows using the spectrum description for the stochastic elevations of the water surface.

The last two items say about the same: there is incomparability of the temporal scales for events of breaking (taking a small part of dominant period $T_p = 2\pi/\sigma_p$) and the scales of applicability of the spectral representation (hundreds of the dominant periods).

Fifth, let us say several words about representation (A.1), as it is. Omitting the threshold feature discussed earlier, we note that formula (A.1) has no explicit factor ensuring “autonomic” dependence of function *DIS* on local wind *W*. In principal, such factor could be included into coefficients *a* and *b*. Herewith, due to the similarity consideration, the wind value could appear in *DIS* via the proper dimensionless parameters of the system. It means that values of *a* and *b* should depend on the spectrum parameters. The absence of such dependence in (A.1) leads to the loss of information with respect to one, already known from the representations for function *DIS(S, W)* already used (in WAM or WW).

Herewith, the new information is given by the cumulative summand in (A.1). Though, due to importance of condition (2.9) for treating the equilibrium spectrum formation, existence of the cumulative summand in *DIS(S)* demands a reconsideration of the spectral representation for input term *IN(S)* which has not such summand (yet). This inconsistence of the representations for *DIS(S)* and *IN(S)* is hardly acceptable, as far as it essentially complicates the treatment of the phenomenon of equilibrium spectral shape existence, the fact of which is empirically proved and widely recognized (see references in Komen et al., 1994; Cavalery et al., 2007).

Thus, interesting experimental representation (A.1), in our mind, does not practically result in any advancement in the problem of construction a realistic parameterization of function *DIS(σ,W,S)* (but makes the latter even more difficult to treat). Hereby, present analysis of the results obtained on the basis of wave-breaking processes investigations (Young and Babanin, 2006), supports our earlier statement about impossibility to study the whole package of real dissipative processes in waves by means of up-to-date experimental methods.

Appendix B

A short version of equation (3.17) solution in the spectral representation is the following. Following to (Hasselmann, 1974), let us introduce the so called generalized variables

$$a^s_{\mathbf{k}} = 0.5(\eta_{\mathbf{k}} + s\frac{i}{\sigma(k)}\dot{\eta}_{\mathbf{k}}), \qquad \text{(where} \quad s = \pm \text{ and } \sigma(k) = (gk)^{1/2}\text{).} \qquad \text{(B.1)}$$

Then, equation (3.17) can be rewritten in the well-known form with the time-derivative of the first order (Hasselmann, 1974; Zakharov, 1974: Krasitskii, 1994):

$$\dot{a}^s_{\mathbf{k}} + is\sigma(k)a^s_{\mathbf{k}} = -is\sigma(k)P_{\mathbf{k}}(\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})/2g\,. \qquad \text{(B.2)}$$

Now, let us multiply equation (B.2) by the complex conjugated component $a^{-s}_{\mathbf{k}}$, make the sum of the resulting equation with the initial one, and make averaging the equation obtained over the wave ensemble. If we use the following definition of the wave spectrum from (Hasselmann, 1974)

$$2 << a^s_{\mathbf{k}} a^{s'}_{\mathbf{k}} >> = S(\mathbf{k})\delta(s+s') \qquad \text{(B.3)}$$

where the double brackets <<…>> means the statistical averaging over wave ensemble, and $S(\mathbf{k})$ is the wave spectrum, we can get the following evolution equation for a wave spectrum

$$\dot{S}(\mathbf{k},t) = \frac{2\sigma_k}{g}\,\mathrm{Im} << P_{\mathbf{k}}(\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})a^-_{\mathbf{k}} >> \,. \qquad \text{(B.4)}$$

Now, making a certain specification of function $P_{\mathbf{k}}(\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})$ on the basis of closure (3.8), one can get the general kind of dissipation function *DIS*(*S*).

For the reason of the qualitative representation of closure (3.8), there is no necessity to reproduce exactly all algebraic computations. Nevertheless, it is important to enumerate here for more convincingness the following circumstances enabling to get the final result:

a) The structure of the generalized variables $a^s_{\mathbf{k}}$ in (B.1) has the form of the Fourier-components sums of $\eta_{\mathbf{k}}$ and $\dot{\eta}_{\mathbf{k}}$. Initial form of distortion (3.8) is the products of the same sums what allows us to express the distortion via the generalized variables ;

b) Hypothesis (3.16) of the non-resonant feature for the wave-turbulence interactions allows the distortion forcing to be represented by a regular function of variables $a^s_{\mathbf{k}}$;

c) Hypothesis (3.16) can be used for any transformations of the integrands in the r.h.s. of equations (B.2) and (B.4) in the course of solution getting.

Items b) and c) allow to perform all actions with nonlinear summands of distortion function $P_{\mathbf{k}}(\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})$ without appearance any integrated convolution terms, in contrast to the resonant integrated terms typical in the conservative nonlinear theory (numerous details of such kind mathematics one can find in Zakharov, 1974; Krasitskii, 1994, Polnikov, 2007).

On the basis of items a)-c), for the aim of getting the general kind of solution, it is acceptable to write the final expression for $P_{\mathbf{k}}(\eta_{\mathbf{k}},\Phi_{\mathbf{k}})$ in the following simple form

$$P_{\mathbf{k}}(\eta_{\mathbf{k}},\Phi_{\mathbf{k}}) = \sum_{\mathbf{k}_1,\mathbf{k}_2} T(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)[a^+_{\mathbf{k}1}a^-_{\mathbf{k}2} + a^-_{\mathbf{k}1}a^+_{\mathbf{k}2}] . \qquad \text{(B.5)}$$

Herewith, both an explicit form of factor $T(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$ and certain specification of quadratic function in the r.h.s. of (B.5) are not significant. From the mathematical point of view, the main feature of expression (B.5) is its nonlinear representation in amplitudes $a^s_{\mathbf{k}}$.

Accepting (B.5), one can easily get the general kind of the r.h.s. of evolution equation (B.4) in the spectral form. Really, substitution of (B.5) into the r.h.s. of (B.4) results in the sum of the third statistical moments in amplitudes $a^s_{\mathbf{k}}$ of the kind $<< a^{s1}_{\mathbf{k}1}a^{s2}_{\mathbf{k}2}a^{s3}_{\mathbf{k}3} >>$. No of these moments can be expressed directly via spectral function $S(\mathbf{k})$ which is the even function to powers of the wave amplitudes. In such case, it needs to write and solve a proper equation for each the third moments by means of using basic equation (B.2) (for details, see Zakharov, 1974; Krasitskii, 1994).

The third moment will be expressed via a set of the fourth moments of the kind $<< a^{s1}_{\mathbf{k}1}a^{s2}_{\mathbf{k}2}a^{s3}_{\mathbf{k}3}a^{s4}_{\mathbf{k}4} >>$ with various sets of superscripts. A part of these moments, for which the condition *s1+s2+s3+s4* $\neq 0$ is met, can be put to zero, in accordance with definition (B.3).

The rest forth moments are to be decoupled to the products of the second moments corresponding to the spectrum definition. By this way the first nonvanishing summands appear in the r.h.s. of the wave-spectrum evolution equation (B.4), and they will be proportional to the second power in $S(\mathbf{k})$.

The described procedure can be continued by means of writing the proper equations for some of the uncoupled fourth moments, leading to appearance the fifth moments. After this, the procedure can be repeated for the fifth moments, and so on. Finally, this continuation ensure in the r.h.s. of the evolution equation (B.4) the fast converging series to powers in spectrum $S(\mathbf{k})$ (the reason of convergence is stated in section 2, ratio (2.5)). As far as the whole r.h.s. of the evolution equation (B.4) has the meaning of the dissipation mechanism (by origin of equation (3.17)), the solution described ensure the dissipation function *DIS(S)* in the following general kind

$$DIS(S,\mathbf{k},\sigma) = \sum_{n=2}^{N} \alpha_n(\mathbf{k},\sigma) F_n[S(\mathbf{k})]. \qquad \text{(B.6)}$$

Here, $F_n[S(\mathbf{k})]$ is the functional on the *n*-th power in the wave spectrum, and functions $\alpha_n(...)$ are the implicit factors of dissipation intensity. Both factors under sign of sum in expression (B.6) are written in the most general kind. Their specification is to be done on principals not related to the point of equation (3.17) solution (alike the similarity method).

The only specification can be done here, consisting in the local-type representation of functional $F_n[S(\mathbf{k})]$ via powers of $S(\mathbf{k})$. That is provided by the hypothesis (3.16) of non-resonant feature for the interactions considered. After this specification we have fully the same form of function *DIS(S)*, which was used as the initial representation for *DIS(S)* in the similarity method (formulas (2.3), (2.4)). Thus, the theoretical justification of the latter is done.

References

Ardhuin, F., Le Boyer, A., 2006. Numerical modelling of sea states: validation of spectral shapes(in French). Memoires presentes en 2006. A. T. M. A. 2006 [http://www.atma asso.fr].

Babanin, A.V., 2009. Breaking of ocean surface waves. Acta Physica Slovaca. 59, 305-535.

Babanin, V.A., Young, I.R., Banner M.L., 2001. Breaking probabilities for dominant surface waves on water of finite constant depth. J. Geophys. Res. 106C**,** 659–11 676.

Babanin, A.V., Van der Westhuysen A.J., 2008. Physics of "saturation-based" dissipation functions proposed for wave forecast models. J. Phys. Oceanogr. 38, 1831-1841

Babanin, A.V., Haus, B.K., 2009. On the Existence of Water Turbulence Induced by Nonbreaking Surface Waves. J. Phys. Oceanogr. 39, 2675–2679.

Banner, M.L., Young, I.R., 1994. Modeling spectral dissipation in the evolution of wind waves. Part I: Assessment of existing model performance. J. Phys. Oceanogr. 24**,** 1550 – 1571.

Banner, M.L., Tian, X., 1998. On the determination of the onset of wave breaking for modulating surface gravity water waves. J. Fluid Mech. 367, 107-137.

Battjes, J.A., Janssen, J.P.F.M., 1978. Energy Loss and Set-up due to Breaking of Random Waves. Proc. Coastal Eng. Conf., 16th, 569-587.

Cavaleri, L., et al (The WISE group), 2007. Wave modelling - the state of the art. Progress in oceanography.75, 603-674.

Chalikov, D., Sheinin, D., 1998. Direct Modeling of One-dimensional Nonlinear Potential Waves, in: Perrie, W. (ed.), Advances in Fluid Mechanics. 17, pp. 207 – 222.

Dai, D., Qiao, F., Sulisz, W., Han, L., Babanin, A., 2010. An Experiment on the Nonbreaking Surface-Wave-Induced Vertical Mixing. J. Phys. Oceanogr. 40, 2180-2188

Donelan, M.A., 2001. A nonlinear dissipation function due to wave breaking. Proc. Workshop on ocean wave forecasting, Reading, United Kingdom, ECMWF. 87–94.

Efimov, V.V., Polnikov, V.G., 1986. Numerical experiments on the basis of wind wave model with turbulent dissipation(in Russian). Marine Hydrophysical Journal. 17-25.

Gemmrich,J.R., Banner, M.L, Garret, C., 2009. Spectrally Resolved Energy Dissipation Rate and Momentum Flux of Breaking Waves. J. Phys. Oceanogr. 38, 1296-1312.

Gemmrich, J,R., 2010. Strong Turbulence in the Wave Crest Region. J. Phys. Oceanogr. 40, 583–595.

Hasselmann, K., 1960. Grundglichungen der Seegangsvoruassage (in German). Shifttechnik. 7, 191-195.

Hasselmann, K., 1962. On the non-linear energy transfer in a gravity wave spectrum. Pt.1. General theory. J. Fluid Mech. 12, 481-500.

Hasselmann, K., 1974. On the spectral dissipation of ocean wave due white capping. Boundary Layer Meteorology, 6, 107-127.

Komen, G. L., Cavaleri L., Donelan M. et al., 1994. Dynamics and Modelling of Ocean Waves. Cambridge University Press.

Krasitskii, V.P., 1994. On reduced equations in the Hamiltonian theory of weakly nonlinear surface waves. J. Fluid Mech. 272, 1-20.

Longuet-Higgins, M.S., 1969. On wave breaking and the equilibrium spectrum of wind generated waves. Proc. Roy. Soc. A310**,** 151-159.

Miles, J.W. 1962. On the generation of surface waves by shear flows. Pt. 4. J. Fluid Mech. 13, 433-448.

Monin, A.S., Yaglom, A.M., 1971. Statistical Fluid Mechanics: Mechanics of Turbulence, v. 1. The MIT Press, Cambridge, Massachusetts, and London, England.

Phillips, O.M. 1958. The equilibrium range in the spectrum of wind-generated waves. J. Fluid Mech. 4**,** 231-245.

Phillips, O.M., 1985. Spectral and statistical properties of the equilibrium range in wind-generated gravity waves. J. Fluid Mech. 156, 505-631.

Polnikov, V.G., 1994. On a description of a wind-wave energy dissipation function, in: Donelan, M.A., Hui, W.H., Plant, W.J. (Eds.). The Air–Sea Interface: Radio and Acoustic Sensing, Turbulence and Wave Dynamics. Rosenstiel School of Marine and Atmospheric Science, University of Miami, pp. 277–282.

Polnikov, V.G., 1995. The study of nonlinear interactions in wind wave spectrum (in Russian). Doctor of Science dissertation. Marine Hydrophysical Institute of NASU. Sebastopol.

Polnikov, V.G., 2005. Wind-wave model with an optimized source function (English transl.). Izvestiya, Atmospheric and Oceanic Physic. 41, 594–610.

Polnikov, V.G., 2007. Nonlinear theory for a stochastic wave field in water (in Russian). LENAND publishing house. Moscow.

Polnikov, V.G., 2009. The Role of Wind Waves in Dynamics of the Air-Sea Interface (English transl.). Izvestiya, Atmospheric and Oceanic Physics. 45, 346–356.

Polnikov, V.G., 2010. An Extended Verification Technique for Solving Problems of Numerical Modeling of Wind Waves (English transl.). Izvestiya, Atmospheric and Oceanic Physics. 46, 551–564.

Polnikov, V.G., Farina, L., 2002. On the problem of optimal approximation of the four-wave kinetic integral. Nonlinear Processes in Geophysics. 9, 497-512.

Polnikov, V.G., Dymov, V.I., Pasechnik, T.A., et al., 2008. Testing and verifying the wind wave model with an optimized source function (English transl.). Oceanology. 48, 7–14.

Polnikov, V.G., Innocentini, V., 2008. Comparative study performance of wind wave model: WAVEWATCH- modified by the new source function. Engineering Applications of Computational Fluid Mechanics. 2, 466-481.

The WAMDI Group, 1988. The WAM – a third generation ocean wave prediction model. J. Phys. Oceanogr. 18, 1775-1810.

Thomson, J., Gemmrich, J.R., Jessup, A.T., 2009. Energy dissipation and the spectral distribution of whitecaps. Geophysical Research Letters 36, L11601.

Toba, Y., 1972. Local balance in the air-sea boundary processes. Pt. 1: On the growth process of wind waves. J. Oceanogr. Soc. Japan. 28, 109-121.

Tolman, H.L., Chalikov, D.V., 1996. Source terms in a third generation wind wave model. J. Phys. Oceanogr. 26, 2497-2518.

Yongzeng, Y., Qiao, F., Zhao, W., Teng, Y., and Yuan, Y., 2005. MASNUM ocean wave numerical model in spherical coordinates and its application. Acta Oceanologica Sinica. 27, 1-7 (In Chinese).

Young, I.R., Babanin, A.V., 2006. Spectral distribution of energy dissipation of wind-generated waves due to dominant wave breaking. J. Phys. Oceanogr. 36, 376–394.

Zakharov, V.E., 1974. Hamiltonian formalism in the theory of waves in nonlinear media with dispersion (in Russian). Izv. VUZov, Radiofizika. 17, 431-453.

Zakharov, V.E., Korotkevich, A.O., Pushkarev, A., and Resio, D., 2007. Coexistence of Weak and Strong Wave Turbulence in a Swell Propagation. Phys. Rev. Lett. 99, 16-21.

Zaslavskii, M.M., 1999. The blocking spectra for wind waves (in Russian). Izvestiya, Atmospheric and Oceanic Phys. 35, 269-277.