A kurtosis-dependent GEV model for freak waves

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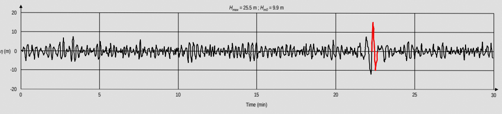
OBJECTIVE

OBJECTIVE: To develop an extreme value model to quantify the probability of occurrence of freak waves taking into account the kurtosis (κ_{40}) and the number of waves (n) in a sea state.
The model is fitted using wave gauge data.

Freak wave definition: $$\frac{H_{max}}{H_{m0}} > 2$$



BACKGROUND: Freak waves studies based on data measurements is used to predict the probability of occurrence of freak waves as function of κ_{40} and n (Mori et al. 2011).

DATA & METHODS

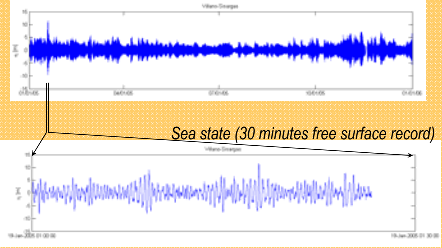
BUOY DATASET

15 buoy records all around Spain

“Puertos del Estado” quality control

Long-term free surface record

Sea state (30 minutes free surface record)



Statistical analysis

Spectral analysis

>300000 free surface sea states

$n \rightarrow$ number of individual waves

$H_{\max} \rightarrow$ maximum wave height $H_{m0} \rightarrow$ significant wave height

$\kappa_{40} \rightarrow$ *kurtosis*

STATISTICAL MODEL

Kurtosis-dependent Generalized Extreme Value distribution.
GEV as a function of κ_{40} (kurtosis) and n (sample size):

$$F(x;\theta) = \exp\left\{-\left[1+\xi^*\left(\frac{x-\mu^*}{\psi^*}\right)\right]^{-1/\xi^*}\right\}; \quad x = \frac{H_{\max}}{H_{m0}}$$

Izaguirre et al. (2010)

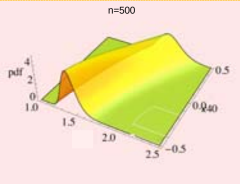
location $$\mu^*(n,\kappa_{40}) = \mu(\kappa_{40}) + \frac{\psi(\kappa_{40})}{\xi(\kappa_{40})}\left(s(n)^{\xi(\kappa_{40})}-1\right)$$

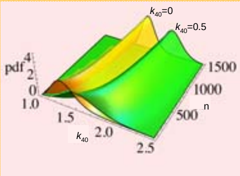
scale $$\psi^*(n,\kappa_{40}) = \psi(\kappa_{40})s(n)^{\xi(\kappa_{40})}$$

shape $$\xi^*(\kappa_{40}) = \xi(\kappa_{40})$$

$$s(n) = \frac{n}{N_{ref}}; \quad N_{ref} = 500$$

$$\theta : \begin{cases} \mu(\kappa_{40}) = \beta_0 + \beta_1\kappa_{40} + \beta_2{\kappa_{40}}^2 + \beta_3{\kappa_{40}}^3 \\ \psi(\kappa_{40}) = \alpha_0 + \alpha_1\kappa_{40} + \alpha_2{\kappa_{40}}^2 + \alpha_3{\kappa_{40}}^3 \\ \xi(\kappa_{40}) = \gamma_0 + \gamma_1\kappa_{40} + \gamma_2{\kappa_{40}}^2 + \gamma_3{\kappa_{40}}^3 \end{cases}$$





MAXIMUM LIKELIHOOD ESTIMATION

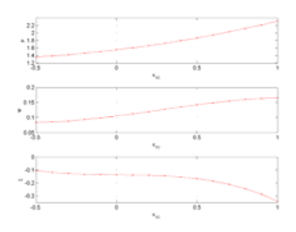
$$Data: \{x_i, n_i, \kappa_{40i}\}; \quad i = 1,\dots,m \quad m = 305592$$

$$\ell(\theta \mid n_i, \kappa_{40i}) = -\sum_{i=1}^{m}\left\{\log\psi^*(n_i,\kappa_{40i}) + (1+1/\xi^*(\kappa_{40i}))\log\left[1+\xi^*(\kappa_{40i})\left(\frac{x_i-\mu^*(n_i,\kappa_{40i})}{\psi^*(n_i,\kappa_{40i})}\right)\right]_+ + \left[1+\xi^*(\kappa_{40i})\left(\frac{x_i-\mu^*(n_i,\kappa_{40i})}{\psi^*(n_i,\kappa_{40i})}\right)\right]_+^{-1/\xi^*(\kappa_{40i})}\right\}$$

$$\theta = \{\beta_0,\beta_1,\beta_2,\beta_3,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\gamma_0,\gamma_1,\gamma_2,\gamma_3\}$$

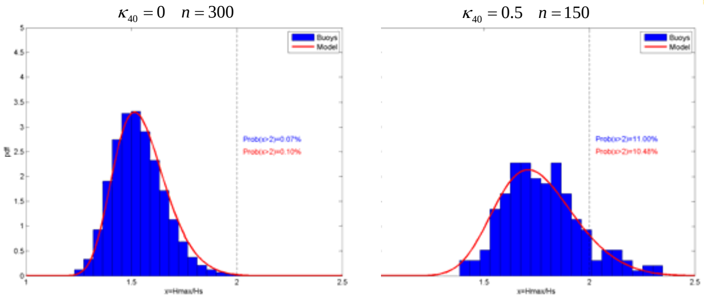
RESULTS

Fitted Model

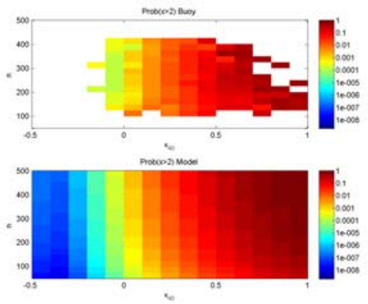


$$\begin{cases} \mu(\kappa_{40}) = 1.5538 + 0.5048\kappa_{40} + 0.2473{\kappa_{40}}^2 + 0.0065{\kappa_{40}}^3 \\ \psi(\kappa_{40}) = 0.1050 + 0.0696\kappa_{40} + 0.0323{\kappa_{40}}^2 - 0.0411{\kappa_{40}}^3 \\ \xi(\kappa_{40}) = -0.1363 - 0.0131\kappa_{40} + 0.0049{\kappa_{40}}^2 - 0.1946{\kappa_{40}}^3 \end{cases}$$

Model Verification: Buoys vs Model pdf


$\kappa_{40} = 0 \quad n = 300$ $\kappa_{40} = 0.5 \quad n = 150$

Freak waves Probability



CONCLUSIONS

- A statistical approach to estimate the probability of occurrence of freak waves in a sea state has been developed. The approach takes into account *n* (number of individual waves) and κ_{40} (kurtosis).
- The Generalized Extreme Value model has been applied to the Spanish deep-water buoy dataset.
- Results reveal an adequate fitness between the model and the data from buoys.
- A common parameterization has been found to different buoy data.

References:

Izaguirre C., Mendez F.J., Menendez M., Luceno, A, Losada I.J. (2010) Extreme wave climate variability in southern Europe using satellite data. Journal of Geophysical Research 115 C04009, doi:10.1029/2009JC0058028.
Mori, N., Onorato, M., Janssen, P.E.A.M. (2011) On the Estimation of the Kurtosis in Directional Sea States for Freak Wave Forecasting. Journal of Physical Oceanography 41:8, 1484-1497 doi: 10.1175/2011JPO4542.1

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