SPECTRAL THEORY OF WIND WAVE DISSIPATION

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Introduction

WIND WAVE is a complicated <u>physical phenomenon</u>
at the air-sea interface, <u>having stochastic features</u>.
Thus, it should be described in a spectral representation:

$$\frac{\partial S}{\partial t} + C_{gx}\frac{\partial S}{\partial x} + C_{gy}\frac{\partial S}{\partial y} = F \equiv IN - DIS + NL \qquad (1)$$

Here, $S(\mathbf{k},\mathbf{x},t) \equiv S$ **is the 2D spectrum in the space, x, and time, *t*;**
the l.h.s. is the full derivative - <u>math. part of model</u>
(not discussed below)**;**
the r.h.s the source function, *F*, <u>phys. part of model</u> ,
describing <u>mechanisms of wind wave evolution</u>.
It is used to represent *F* as a sum of 3 (separate) mechanisms:
energy input, *IN*, **energy lost,** *DIS*, **nonlin en. conserv transf,** *NL*.

Initial equations

Principally, all terms of S.F. must be derived from the basic eqs.

$$\rho\frac{d\mathbf{u}}{dt}=-\vec{\nabla}_{\mathbf{3}}P-\rho\mathbf{g}+\mathbf{f}(\mathbf{x},t);\Big|_{z=\eta(\mathbf{x},t)} \qquad \mathbf{(2)}$$

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}_{\mathbf{3}}(\rho\mathbf{u})=0 \qquad \mathbf{(3)}$$

$$u_z\Big|_{z=\eta(\mathbf{x},t)}=\frac{\partial\eta}{\partial t}+(\mathbf{u}\vec{\nabla}_{\mathbf{2}}\eta) \qquad \mathbf{(4)}$$

$$u_z\Big|_{z=-\infty}=0 \qquad \mathbf{(5)}$$

There is a special procedure to do it (Miles 1957, Hass 1962, 1974, Zakharov 1974).But usually, each term is derived separately, due to complexity the of system and nonlin feature of (2)-(5).

Finally, the resulting eq. (1) should be valid on the scales of hundreds of dominant wave period $T_p=2\pi/\sigma_p$ **and wave length** $L_p=2\pi\sigma_p^2/g$ **, where** σ_p **is the sp. peak frequency(>10^{3}s, 10^{3}m).**

Nl-term

NL*-term *(energy transfer mechanism due to nonlinearity)
is the best investigated one.

Under some approximation, it is governed by the integral

$$NL(\mathbf{k}_4) = 4\pi \iiint d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 M^2_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} \cdot F3(S_{\mathbf{k}_1}, S_{\mathbf{k}_2}, S_{\mathbf{k}_3}, S_{\mathbf{k}_4}) \times \qquad \textbf{(6)}$$

$$\times \delta(\sigma_{\mathbf{k1}} + \sigma_{\mathbf{k2}} - \sigma_{\mathbf{k3}} - \sigma_{\mathbf{k4}}) \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)$$

called the Hasselmann's kinetic integral (KI) (Hasselmann, 1962).

This point is out of our present consideration.

For completeness, we should say that
the point of *NL*-term representation,
optimally corresponding for application in numerical models,
is exhaustively studied in Polnikov(Nonlin. Proc. in Geophys. 2002, 2003).
The FastDIA was proposed and successively verified. The NL problem was closed

IN-term

IN*-term *(energy pumping mechanism)

is <u>rather well studied theoretically and experimentally</u>.

The main theoretical result is the Miles' theory (1960). It reads

$$In = C_{in}\beta(\sigma,\ \theta, \mathbf{W})\,\sigma S(\sigma,\ \theta) \quad . \qquad \textbf{(7)}$$

where C_{in} is the main fitting parameter for *In-term*.

<u>Problem is to find the form of increment function $\beta(\ldots)$</u>.

There are two ways: num. simulations, or experim. measurements.

At present, <u>the best way is to combine the main theoretical and experimental features of $\beta(\ldots)$.</u>

Parameterization of <u>$\beta(\ldots)$</u> plays remarkable role in the point of parameterization for the DIS-term.

<u>Here we do need no more details about *IN*-term besides form (7)!</u>

DIS-term

***DIS*-term (*wave energy dissipation mechanism*)**
is the least studied one!!!

There were a lot of attempts to derive a kind of function *DIS(S):*

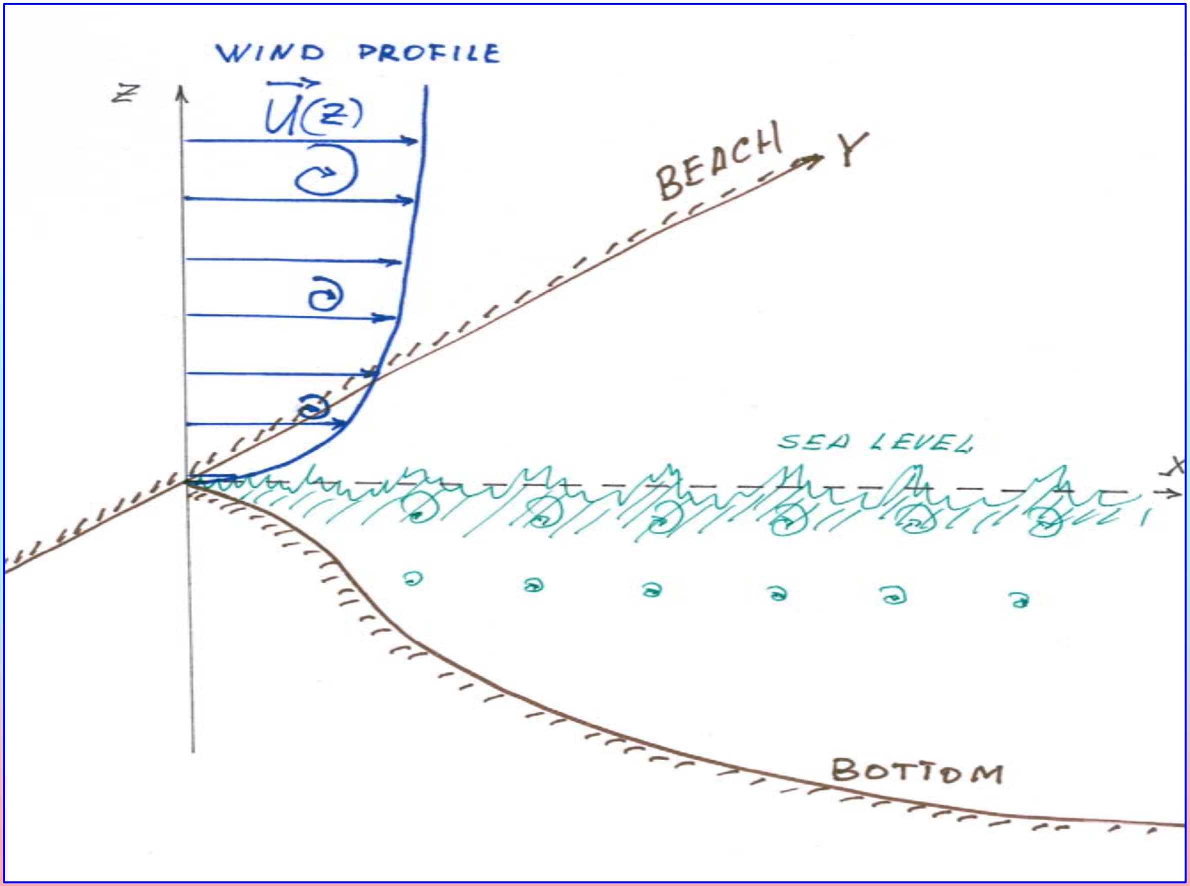
***analytical theory* (Hasselmann, 1974: *Dis ~ S*),**
***dimensional* considerations(Donelan(2001): *Dis ~* $S^{2,5}$, Phillips(1985): *Dis ~* S^3 ,**
***semi phenomenological theory* (Tolman&Chalikov, 1996): *Dis ~ S*,**
***numerical simulations* (Chalikov,2010; Zakharov et al, 2007+…)**
+
***a lot of empirical investigations which dealt with the breaking processes*, only(!) (see the last review by Babanin, 2009).**

Herewith, there are not clear the following items:

(1) general represent. of *DIS(S): first, second, or higher* power in *S*;
(2) any (widely) recognized general theory for DIS,
(3) any empirical result (in the spectral form) related to the measurements of the whole aggregate of dissipative processes, including turbulence of the water upper layer (WUL).

Remarks of measurements for DIS(S)

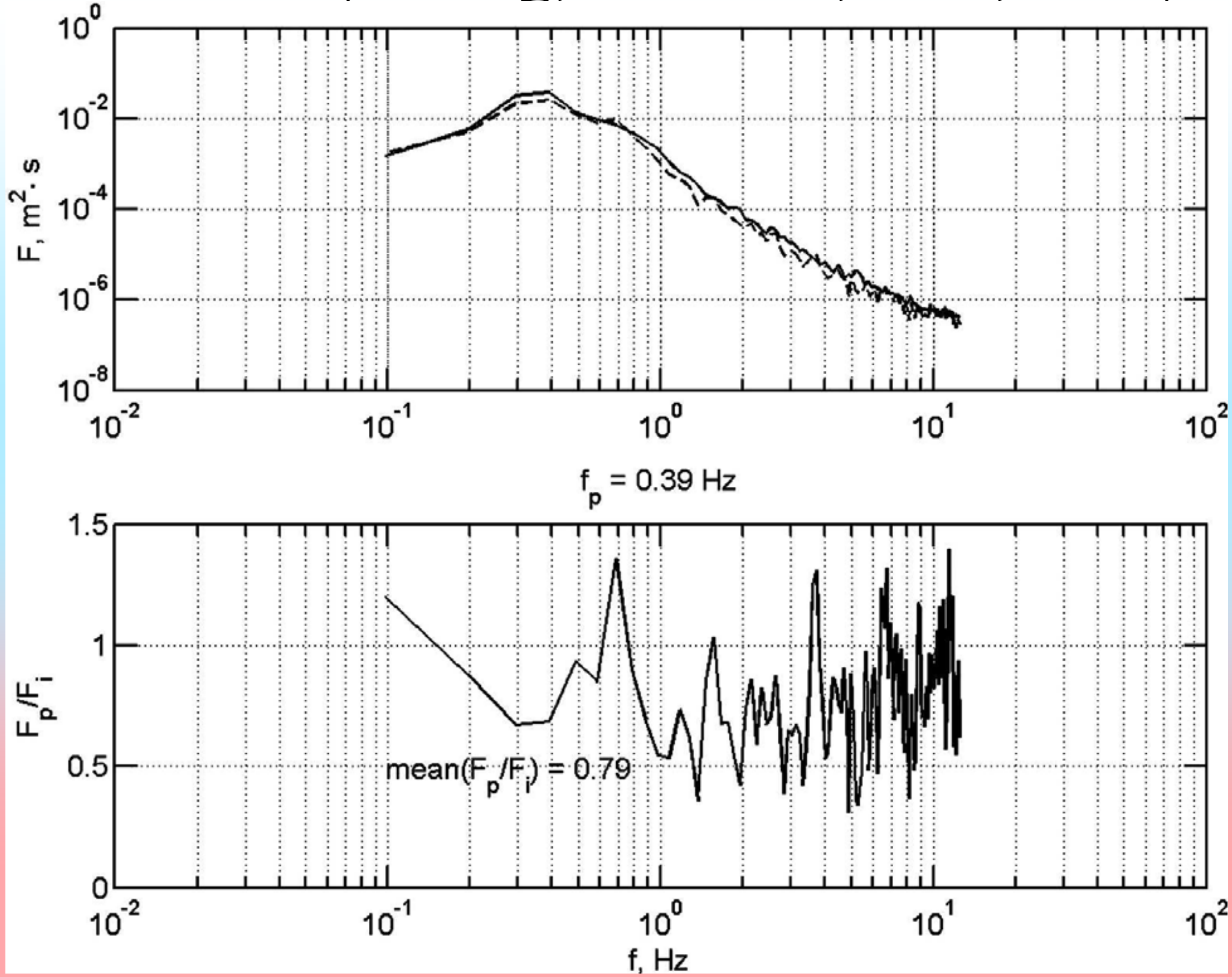
At the air sea interface
shear currents,
wave,
and turbulent motions
present simultaneously.
One cannot separate
them by equipment!!!



Thus, one cannot
separate *IN*put,
***NL*in transfer**
and *DIS*sipation
processes
present simultaneously!!!!!
Nevertheless, some optimists try to do it??
Let us see several of the results.

Example of empirics for *DIS*-term

Measurement results (Young, Babanin, JPO,2006)



From the picture we state:

Breaking is not directly a dissipation, it is a spreading of energy!!!

Empirical features of breaking

Despite of numerous empirical uncertainties,
there are several empirical effects useful to our aim.

Following to Young&Babanin (2006), they are as follows :

E1) Threshold feature of the wave breaking process;

E2) Influence of the long waves breaking on the intensity of breaking for shorter waves;

E3) Different feature of the dissipation rate for dominant waves and for waves in the tail part of wave spectrum;

E4) More intensive breaking of waves running at some angle to the mean wind direction
(i.e. *the two-lobe feature of the angular function for breaking intensity*)

The true theory should have some of these features.

Nevertheless, on the basis of the analysis of present measurements, we state that function DIS(S) cannot be measures, principally!
There is not a proper tool!!

Initial theor. ideas about *DIS*-term

Due to <u>impossibility to measure the spectral form of</u> *DIS*-term (!!),
<u>the solution of the problem is to derive a general theory</u>,
which could give a basis for *DIS*-term parameterization.

In 1986 the author has proposed, and in Polnikov (1994, 1995)
has justified a representation of *DIS*-term in the form

$$Dis = \nu_T(\sigma, \theta, \mathbf{W}, S) * k^2 * S(\sigma, \theta) \sim S^2(\sigma, \theta), \qquad \textbf{(8)}$$

where $\nu_T(\ldots)$ <u>has a meaning of an effective turbulent viscosity</u>
<u>provided by the aggregate of all dissipative processes in WUL</u>
(including: *breaking, sprinkling, capping, shear current instability, and so on*).

<u>The problem is to find</u> $\nu_T(\ldots)$ <u>theoretically</u>!!!

Similarity consideration

From dimensional consideration we can accept

$$\boxed{DIS(\sigma, W, S) = const \cdot \sigma S(\sigma, \theta) \cdot \Phi(\hat{S}, \hat{W}, A, \varepsilon, ..)} \qquad \textbf{(8a)}$$

where $\boxed{\Phi(\hat{S}, \hat{W}, A, \varepsilon, ..)}$ **is the function of dim-less parameters of the syst:**

spectrum $\hat{S}(\sigma) = \sigma^5 S(\sigma, \theta) / g^2 \equiv \varepsilon^2(\sigma)$ **;**

current frequency $\hat{\sigma} = \sigma / \sigma_p$

wind $\hat{W} = W\sigma / g$

age $A = \hat{c}_p = c_p / W$

steepness $\varepsilon = ka(k) = \sqrt{\hat{S}}.$

The main problem is to define the power of *S*(…) for func *Dis(S)*.

To do this we apply

the spectral representation of $\boxed{\Phi(\hat{S}, \hat{W}, A, \varepsilon, ..)}$ **in a local approximation**

$$\Phi(...) = \sum_n \alpha_n(A, \hat{\sigma}, \hat{W}, \varepsilon)\hat{S}^n \qquad \textbf{(8b)}$$

Similarity consideration (con'd)

Then, using the existence of small parameter

$$\boxed{\hat{S} = \sigma^5 S(\sigma,\theta)/g^2 \cong \alpha_p \cong 0.01 << 1} \quad , \qquad \textbf{(8c)}$$

the balance at high frequencies of the form

$$\boxed{IN(\sigma,W,S_{eq}) - DIS(\sigma,W,S_{eq}) = 0} \quad , \qquad \textbf{(8d)}$$

and the linear function of input term

$$IN(W,S) = \beta(\sigma,\theta,\mathbf{W})\,\sigma S(\sigma,\theta) \quad , \qquad \textbf{(8e)}$$

accumulating all linear terms of SF as the corrected *IN-term*, we get

$$\boxed{DIS(\sigma,W,S) = \sigma S(\sigma,\theta)\cdot\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)\hat{S} = \alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)\frac{\sigma^6 S^2(\sigma,\theta)}{g^2}} \qquad \textbf{(8f)}$$

The form of $\alpha_1(A,\hat{\sigma},\hat{W},\varepsilon)$ **is following from eq. (8d),**
<u>used as eq. for equilibrium spectrum shape</u> $S_{eq}(\sigma,\theta)$

<u>The principal result is the quadratic form: *DIS(S)* $\propto$ *S*^{2} (as in (8f))!</u>

A <u>more general</u> theoretical consideration is given below.

The main theory (turb. viscosity model).
1. *Basic statements*

1. The main fundamental of the theory states that on the scales of Eq. (1) validity, the most general reason for wind wave dissipation is a turbulence of the water upper layer.

Herewith, specification of processes producing the turbulence is quite unprincipled.

2) The velocity field in a wavy water layer can be written in the form of two constituents

$$\boxed{\mathbf{u}(\mathbf{x},z,t)=\mathbf{u}_w(\mathbf{x},z,t)+\mathbf{u}'(\mathbf{x},z,t)} \quad , \qquad (9)$$

where

$\mathbf{u}_w$ is the potential wave velocity field,

$\mathbf{u}'$ is the turbulent one, uncorrelated with $\mathbf{u}_w$.

3) The elevation field, $\eta(\mathbf{x},t)$, has a meaning of continuous field, permitting introduction of partial derivatives

(Hasselmann's hypotheses of "a disturbance weak in mean").

2. Reynolds stress

Substitution of (9) into (2) and (4) after averaging over turb. scales gives <u>for wave motions</u> the following eqs: (written below in the tensor kind)

$$\boxed{\frac{\partial \overline{u}_i}{\partial t} + \sum_j \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -g\delta_{i,3} - \sum_j \frac{\partial < u_i^{'} u_j^{'} >}{\partial x_j}\bigg|_{z=\eta(\mathbf{x},t)}} \qquad \textbf{(10)}$$

$$\boxed{\frac{\partial \overline{\eta}}{\partial t} = \overline{u}_3 - \sum_{i=1,2} \overline{u}_i \frac{\partial \overline{\eta}}{\partial x_i}} \quad . \qquad \textbf{(11)}$$

Here, <u>the new term</u> in the r.h.s. of (10),

$$\boxed{\sum_j \frac{\partial < u_i^{'} u_j^{'} >}{\partial x_j} \equiv P_i} \quad , \qquad \textbf{(12)}$$

<u>is the "forcing", resulting in the dissipation of a wind-wave energy</u>

(details can be found in my full paper text).

Terms $< u_i^{'} u_j^{'} > \equiv \tau_{ij}$ **are the well known <u>Reynolds stresses</u> (R.S.).**

Our <u>task is to close R.S. via wave motion variables</u> $\overline{\eta}$ **<u>and</u>** $\overline{\mathbf{u}}$ **.**

3. R.S. features and closure

To continue the consideration, we get the foll. assumptions:

(1)The relative value of Reynolds stress term, P_i, is much greater of the dynamical nonlinear term, $\boxed{\sum_j u_j \frac{\partial u_i}{\partial x_j}}$.

Thus, we get the linearized eqs. (but with nl-forcing term!!)

$$\boxed{\frac{\partial u_i}{\partial t} + g\delta_{i,3} = -P_i(\mathbf{u},\eta)} \quad \text{(13a)}$$

and

$$\boxed{\frac{\partial \eta}{\partial t} = u_3} \quad \text{(13b)}$$

(they are written at the surface)

+ two eqs. in whole water layer (used for description of the vertical structure).

(2) The R.S. closure has the complicated *nonlinear* (!) form

$$\boxed{-P_i(\mathbf{u},\eta) = <\sum_j \frac{\partial}{\partial x_j}\{[L_i(\partial u_i/\partial x_i) + C_i(\partial\eta/\partial x_i)][L_j(\partial u_j/\partial x_j) + C_j(\partial\eta/\partial x_j)]\}>} \quad \text{(14)}$$

(Prandtle-like closure, using gradients of velocity and elevation fields !!!)

4. Reduced equations

Introducing definitions for velocity potential variables

$$\boxed{\mathbf{u}_w(\mathbf{x},z,t)=\vec{\nabla}_3\varphi(\mathbf{x},z,t)} \quad \boxed{\Phi(\mathbf{x},t)\equiv\varphi(\mathbf{x},t)\big|_{z=\eta(\mathbf{x})}} \quad , \qquad \textbf{(15)}$$

rewrite the governing eqs in the form

$$\boxed{\frac{\partial\Phi}{\partial t}=-g\eta+\hat{P}(\eta,\Phi)} \quad , \quad \boxed{\frac{\partial\eta}{\partial t}=\frac{\partial\Phi}{\partial z}} \qquad \textbf{(principle eqs)} \qquad \textbf{(16)}$$

Other two eqs ($\Delta\varphi=0$ **and** $\frac{\partial\varphi}{\partial z}\big|_{z=-\infty}=0$) **are used for vert. structure definition. (17)**

Here $\boxed{\hat{P}(\eta,\Phi)=(\vec{\nabla}_3)^{-1}[\mathbf{P}(\eta,\mathbf{u})]}$ **is the <u>new form of the forcing term</u>.**

After using the Fourier-decompositions (F.T.)

$$\eta(\mathbf{x},t)=const\cdot\int_{\mathbf{k}}\exp[i(\mathbf{kx})]\eta_{\mathbf{k}}(t)d\mathbf{k} \qquad \varphi(\mathbf{x},z,t)=const\cdot\int_{\mathbf{k}}\exp[i(\mathbf{kx})]f(z)\varphi_{\mathbf{k}}(t)d\mathbf{k} \qquad \textbf{(18)}$$

we find the follow. <u>final reduced system of governing eqs</u>

$$\boxed{\dot{\Phi}_{\mathbf{k}}+g\eta_{\mathbf{k}}=\Pi(\mathbf{k},\eta_{\mathbf{k}},\Phi_{\mathbf{k}})} \qquad \textbf{(19)}$$

$$\boxed{\dot{\eta}_{\mathbf{k}}=k\Phi_{\mathbf{k}}} \qquad \textbf{(20)}$$

Where $\boxed{\Pi(\mathbf{k},\eta_{\mathbf{k}},\Phi_{\mathbf{k}})\equiv F^{-1}[\hat{P}(\eta,\Phi)]}$ **is <u>the next form of the 'forcing'.</u>**

5. Method of solution (following Hass, 1974)

First step: reducing system (19)-(20) to 1 equation

$$\boxed{\ddot{\eta}_{\mathbf{k}} + gk\eta_{\mathbf{k}} = -k\Pi(\mathbf{k},\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})} \quad . \qquad \mathbf{(21)}$$

Second step: introducing the generalized variables (Hass,1974)

$$\boxed{a_{\mathbf{k}}^{s} = 0.5(\eta_{\mathbf{k}} + s\frac{i}{\sigma(k)}\dot{\eta}_{\mathbf{k}})} \quad (s = \pm \text{ , } \sigma(k) = (gk)^{1/2}) \ . \qquad \mathbf{(22)}$$

That gives to Eq. (21) the form

$$\boxed{\dot{a}_{\mathbf{k}}^{s} + is\sigma(k)a_{\mathbf{k}}^{s} = -is\sigma(k)\Pi(\mathbf{k},\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})/2g} \quad . \qquad \mathbf{(23)}$$

Third step: defining the spectrum (by the wave ensemble averaging)

$$\boxed{<< a_{\mathbf{k}}^{s}\, a_{\mathbf{k}'}^{s'} >>= S(\mathbf{k})\delta(s+s')\delta(\mathbf{k}-\mathbf{k}')} \qquad \mathbf{(24)}$$

and getting the final eq for wave spectrum evolution(Hass,1974)

$$\boxed{\dot{S}(\mathbf{k},t) = \frac{2\sigma_k}{g}\operatorname{Im} << \Pi(\mathbf{k},\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})a_{\mathbf{k}}^{-} >> \equiv -DIS(S)} \qquad \mathbf{(25)}$$

Solution of (25) needs specification of $\boxed{\Pi(\mathbf{k},\eta_{\mathbf{k}},\dot{\eta}_{\mathbf{k}})}$ **!!!**

6. Specification of forcing

Take into account the following points:

(a) The generalized variables a_k^s **have a linear form of** $\eta_{\mathbf{k}}, \dot{\eta}_{\mathbf{k}}$ **;**

(b) The structure of R.S. $(\eta, \mathbf{u})$ **permits to express it via** a_k^s **;**

(c) The turb.scale-averaging (brackets <…>) leads to destruction of the wave-like phase structure under <..> in F.T. For example,

$$\boxed{\int_{\mathbf{k}_3} d\mathbf{x}\left[\exp(-i\mathbf{k}_3\mathbf{x}) < \lambda_i'\lambda_j' \int_{\mathbf{k}_1}\int_{\mathbf{k}_2} T(\mathbf{k}_1,\mathbf{k}_2)\exp(i\mathbf{k}_1\mathbf{x})\exp(i\mathbf{k}_2\mathbf{x})\eta_{\mathbf{k}_1}\eta_{\mathbf{k}_2} d\mathbf{k}_1 d\mathbf{k}_2 >\right] \Rightarrow \sum_{\mathbf{k}_1,\mathbf{k}_2} T(\mathbf{k}_1,\mathbf{k}_2)\eta_{\mathbf{k}_1}\eta_{\mathbf{k}_2}}$$

The said allows to write the final expression for $\Pi(\mathbf{k}, \eta_{\mathbf{k}}, \dot{\eta}_{\mathbf{k}})$ **in the most simple nonlinear kind (no convolution forms)**

$$\boxed{\Pi(\mathbf{k}, \eta_{\mathbf{k}}, \dot{\eta}_{\mathbf{k}}) = \sum_{s_i, s_j, \mathbf{k}'} T_{i,j}(\mathbf{k}, \mathbf{k}') a_{\mathbf{k}}^{s_i} a_{\mathbf{k}'}^{s_j}} \quad . \qquad \mathbf{(26)}$$

An explicit kind of multipliers *T*$_{ij}$, and a certain representation of the quadratic form in the r. h. s. of (26) is not principled, as far as the main physical feature: the *nonlinearity of forcing*, is conserved.

7. Traditional solution of nonlin. Eq.

To solve typical nonlinear eq. (23), we use traditional method of eqs. chain for statistical moments of the generalized variables.

It has 3 steps.

(1)To multiply amplitude Eq. (23) by the proper c. conjugated generalized component, and to make the ensemble averaging.

After some algebra, we have evolution eq. of the form

$$\dot{S}(\mathbf{k}) = -i \sum_{s1,s2,s3} f_1(\mathbf{k}, s1, s2, s3) T^{s1,s2,s3}_{k,k,k} L^{s1,s2,s3}_{k,k,k} \qquad (27)$$

where $L^{s1,s2,s3}_{k,k,k} \equiv << a^{s1}_{\mathbf{k}} a^{s2}_{\mathbf{k}} a^{s3}_{\mathbf{k}} >>$ **is the third order moment.**

Solution (continued)

(2) The evol. eq. for the 3rd order stat. moment, $\boxed{<< a_{\mathbf{k}}^{s1} a_{\mathbf{k}}^{s2} a_{\mathbf{k}}^{s3} >>}$, should be written from init. Eq. <u>by the same manner</u> via the fourth moment,

$$\boxed{<< a_{\mathbf{k}}^{s1} a_{\mathbf{k}}^{s2} a_{\mathbf{k}}^{s3} a_{\mathbf{k}}^{s4} >>},$$

which could be split into a set of products of the second moments expressed via the spectrum $S_{\mathbf{k}}$ (chaotic phase hypothesis)

By this way <u>the 2nd order term appears in the r.h.s. for the intermittent evolution Eq. (23) for the spectrum $S_{\mathbf{k}}$</u> (!!):

$$\boxed{\dot{S}(\mathbf{k}) = \mathrm{Re}\left\{ \sum_{s1,s2,s3} f_2(\mathbf{k}, s1, s2, s3) \left| T_{k,k,k}^{s1,s2,s3} \right|^2 FN_2[S(\mathbf{k})] \right\} + res} \quad \textbf{(28)}$$

Where FN_2 is the second power functional in $S_{\mathbf{k}}$.

Solution (continued)

(3) A part of the 4th moments could be specified further by the same manner.

Finally, we get the most general kind of *DIS(S)* of the form

$$\boxed{DIS(S,\mathbf{k},\mathbf{W}) = \sum_{n=2}^{N} c_n(\mathbf{k},\mathbf{W}) FN_n[S(\mathbf{k})]} \quad . \qquad \mathbf{(29)}$$

Thus, general form of *DIS* is a series of functional of S_k, starting from the second power term.

Due to assumption of wave-phase structure destruction (in the forcing term), in further consideration we can use the simplest local form: $FN_n(S_k) \sim (S_k)^n$.

Farther specification of *DIS(S) has a phenomenological feature.*

Parameterization of *DIS*

For the presence of a small parameter defined as

$$\alpha = \max[S(\sigma,\theta)\sigma^5 / g^2] \approx \alpha_{Ph} \cong 0.01 << 1 \qquad \textbf{(30)}$$

Due to this, without a loss of accuracy, in the lowest order of a **the phen. series (8a) for *DIS(S)* can be restricted by the first term**

$$\boxed{DIS(\ldots) \cong c_2(\ldots)S^2(\mathbf{k}) = \gamma(\sigma,\theta,\mathbf{W})\frac{\sigma^6}{g^2}S^2(\sigma,\theta)} \qquad \textbf{(31)}$$

The dimensionless factor γ(…) follows from the balance

$$\boxed{[IN - DIS]\Big|_{S=S_{eq}} \approx 0} \quad , \qquad \textbf{(32)}$$

valid at the equilibrium tail part of spectrum

$$\sigma > 2.5\sigma_p \qquad \textbf{(33)}$$

with the fixed shape $S_{eq}(\sigma,\theta)$.

(**In (32), it is taken into account a relatively small contr. of *NL(S)* in domain (33)).**

Parameterization of *DIS* (features)

Final theoretical result has the form (Polnikov, 2005, 2010)

$$Dis(\sigma,\theta,S,\mathbf{W}) = C_{dis} c(\sigma,\theta,\sigma_p)\max\left[\beta_{dis},\beta(\sigma,\theta,\mathbf{W})\right]\frac{\sigma^6}{g^2}S^2(\sigma,\theta). \quad \textbf{(34)}$$

A. The <u>special phenomenological function</u> *c(…)* of the kind

$$c(\sigma,\theta,\sigma_p) = 32\max\left[0,\ (\sigma - a_p\sigma_p)/\sigma\right]T(\sigma,\theta,\sigma_p) \quad \textbf{(35)}$$

describes losses <u>in the domain of peak of spectrum (empir. effects E2,E3)</u>

***c(…)* regulates different features of *Dis-term* in two parts of spectral domain (peak domain and tail part).**

B. The <u>angular function *T(.)*</u> has a two-lobe feature (empir.effect E4)

$$T(\sigma,\theta,\theta_w,\sigma_p) = \left\{1 + 4\frac{\sigma}{\sigma_p}\sin^2\left(\frac{\theta-\theta_w}{2}\right)\right\}\max\left[1,\ 1-\cos(\theta-\theta_w)\right] \quad \textbf{(36)}$$

C. Constants C_{dis}, β_{dis}, and a_p are the main fitting par-s in *Dis-term*.

All of them has a certain physical meaning!

At present,

<u>parameterization (34)-(36) is the best justified among others</u>!!!

Turb. Viscosity estimations

Comparing

$$Dis = \nu_T(\sigma, \theta, \mathbf{W}, S) * k^2 * S$$

with

$$Dis(\sigma,\theta,S,\mathbf{W}) = C_{dis} c(\sigma,\theta,\sigma_p) \max[\beta_{dis}, \beta(\sigma,\theta,\mathbf{W})] \frac{\sigma^6}{g^2} S^2(\sigma,\theta)$$,

one gets

$$\nu_T = C_{dis} c(\sigma_p) \beta[W, c_{ph}(\sigma)] \cdot \sigma^2 S \quad . \qquad \textbf{(37)}$$

With account of the widely accepted presentation

$$\beta[W, c_{ph}(\sigma)] \approx 30 \cdot (\rho_a / \rho_w)[2 \cdot 10^{-3} W^2 / c_{ph}^2(\sigma)] \quad , \qquad \textbf{(38)}$$

that leads to estimation

$$\nu_T \approx \max\{10^{-3}, 4 \cdot 10^{-3} \frac{W^2}{\sigma}\} \cdot \left(\frac{\sigma^5 S}{g^2} \right) \Bigg|_{\substack{W=10 \\ \sigma=1}} \cong 4 \cdot 10^{-3} m^2 s^{-1} \quad . \qquad \textbf{(39)}$$

Estimation (39) is 2 orders greater that kinematics viscosity of water!!!

Procedure of validation

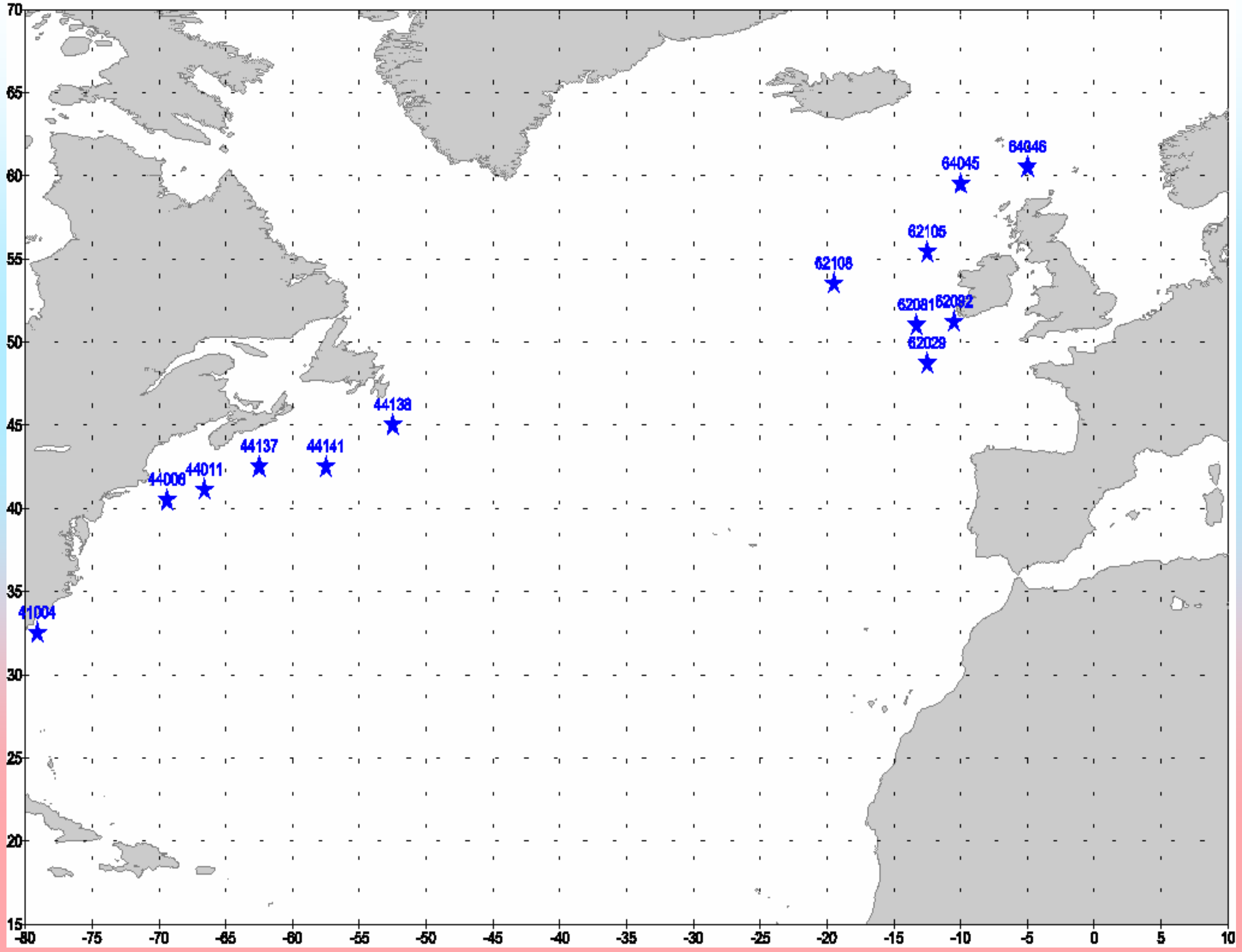
<u>Regulations</u> of comparative verification procedure demand a <u>fulfillment of the following series of conditions</u>:

- Reasonable data base of reliable observations for wind waves;
- Reliable wind field given on a rather thick space-time grid for the whole period of observations;
- Well designed math part for numerical model of the kind (1);
- Choice of a widely recognized wind wave model as an etalon for comparison.

These conditions were met by using buoy data of NBDC, wind field of NCEP and two models WAM and WW, published in (Polnikov et al, 2008) and (Polnikov&Innocentini, 2008).

Results for WW are presented below.

Simulating region in the North Atlantic and some buoys locations



Verification results

The best results are gained for the following values of fitting coeffs:

$$C_{nl} = 9 \cdot 10^3, \quad C_{in} = 0.4, \quad C_{dis} = 70, \quad \text{and} \quad a_p = 0.7 \qquad \textbf{(40)}$$

with the default values of the other fitting parameters (Pol., 2008).

A typical time history of significant wave height, $H_s(t)$, obtained in these simulations is shown in Fig. 1.

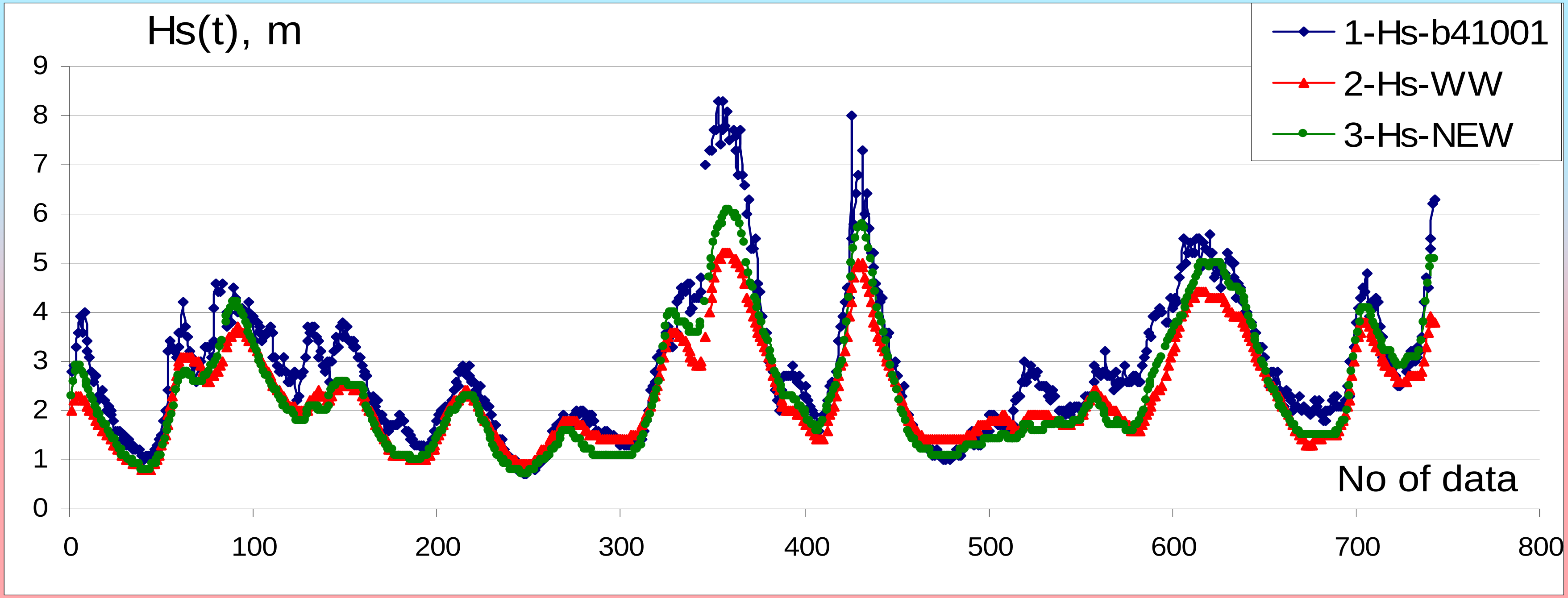


Fig. 1. Time history of the observed and simulated wave heights on buoy 41001 for January 2006.

Results (continue)

Tab. 1. R.m.s. errors for buoys in the Eastern P. of NA

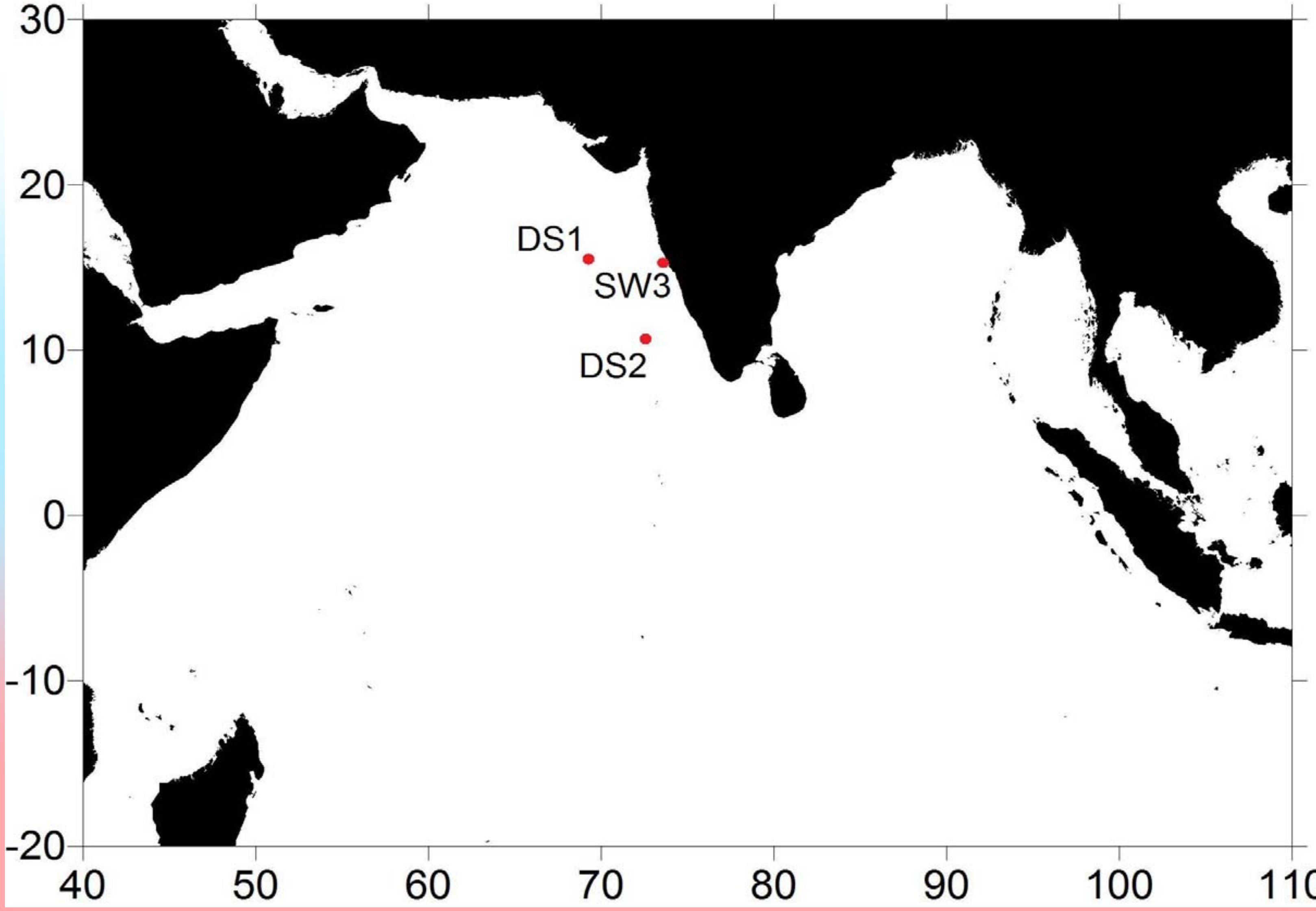
Eastern NA, No of buoy	Model WW		Model NEW		$\frac{(\delta H_s)_{WW}}{(\delta H_s)_{NEW}}$
	δH_s, m	ρH_s, %	δH_s, m	ρH_s, %	
62029	0.57	14	0.54	13	1.05
62081	0.67	15	0.56	13	1.20
62090	0.66	14	0.57	14	1.16
62092	0.58	14	0.53	14	1.09
62105	0.79	18	0.68	15	1.16
62108	0.99	15	0.84	13	1.18
64045	0.71	12	0.61	12	1.16
64046	0.72	15	0.76	15	0.95

Results (continue)

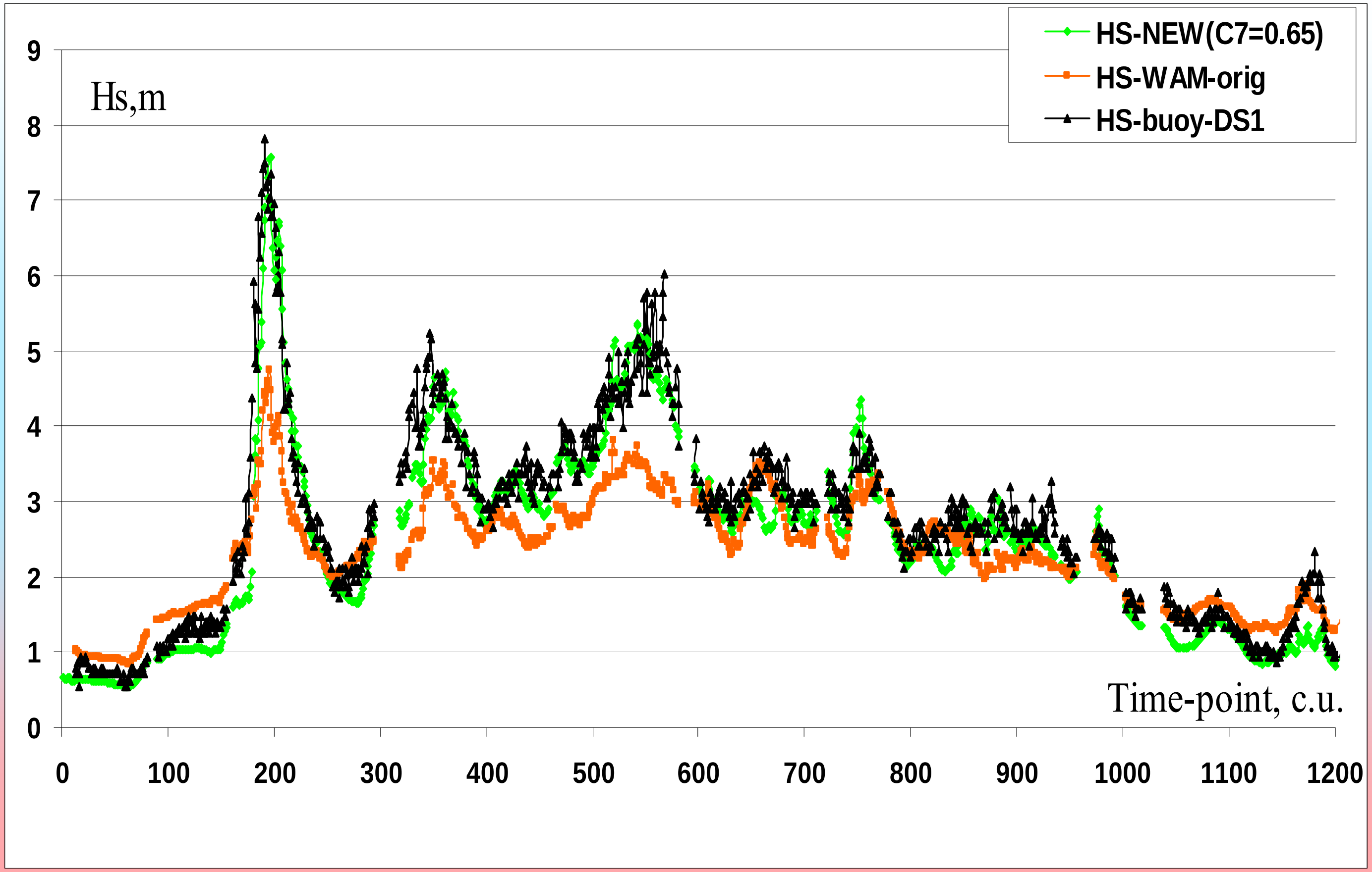
Tab. 2. R.m.s. errors for buoys in the Western P. of NA

Western NA	Model WW		Model NEW		$\frac{(\delta H_s)_{WW}}{(\delta H_s)_{NEW}}$
	δH_s,m	ρH_s,%	δH_s,m	ρH_s,%	
41001	0.81	22	0.66	20	1.23
41002	0.52	18	0.47	18	1.11
44004	0.82	25	0.68	26	1.21
44008	0.83	27	0.61	24	1.36
44011	0.82	23	0.55	18	1.49
44137	0.58	19	0.51	17	1.14
44138	0.70	19	0.74	19	0.95
44139	0.63	19	0.69	20	0.91
44140	0.78	19	0.80	19	0.97
44141	0.64	20	0.68	20	0.94
44142	0.81	27	0.48	18	1.69

Location buoys in IO, used for verification of model WAM



Results of the modified WAM verification



The time history for *Hs(t)* on buoy DS1:
black – buoy measurements; red- WAM original, green –WAM mod.

Verification in IO (continued)

After several attempts, we have found the table of r.m.s. errors :

The buoy index [coordinates]	r.m.s. err. , m WAM-orig	r.m.s. err., m WAM- modif
DS1 [15.5N, 69.3E]	0.75	0.47
DS2 [10.7N, 72.5E]	0.40	0.29
SW3 [15.4N, 73.7E]	0.45	0.28
Mean r.m.s. error	0.533	0.347

All other simulations of wave-fields were executed for the optimal fit of NEW model (i.e. the modified WAM).

(The detailed discussion of the point is not the subject of the work, it is under submission to publication).

Summary of validation

Analysis of these results shows the following.

1. The accuracy of the model NEW is regularly better with respect to one of the original WW.
This result is revealed for more than 70% of buoys considered.

2. Winning of accuracy for the model NEW is of the order of 15-20%, but sometime it can reach 70% (buoy 44142).

3. The relative error, ρH_s, calculated by taking into account each point of observations, is not so small (15-27%). It has a tendency of reducing for the model NEW, but it is not so well expressed.

Concluding remarks and future

1. The nonlinear feature of function *DIS*(k, *S*) (31) is robust to any assumptions of equilibrium spectrum shape and the specification of nonlinear R.S. closure,

   just due to nonlinearity of *the dissipative forcing term (*R.S.*)*.

2. Domain of validity of the result is hundreds of peak periods and wave lengths.

   For this reason, the threshold feature of breaking is smoothed due to wave statistics,

   and empirical effect E1 is not manifested in *DIS(S)*.

3. The most radical assumption of destruction for wave-like phase feature in the R.S. forcing, provided by the averaging over turbulent scale could be elaborated in further (if needed).

   This could change the local feature of *DIS*(k, *S*) in the k-space to the one of convolution form, $\int S(k-k')S(k')dk'$

   (corresponding to cumulative effect of *DIS(S*), phenomenological proposed but not be proved in Young&Babanin, 2006).

Some useful references

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Finishing remarks

The presentation and full text paper
are available from session laptop, 12thworkshop web-site,
and author.

If you have questions,
please be aloud , slow, and short.
Otherwise, please contact directly to the author.

Thank you for understanding