The modeling of quadruplets

The LQA method for the computation of non-linear four-wave interactions and shallow water effects

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Scope and results

- Progress in the development of efficient methods for the computation of non-linear four-wave interactions in discrete spectral wind-wave models

- Improved efficiency of WRT method, including LQA
- Resemblance of stripped-down WRT method and mDIA
- Inclusion of improved shallow water physics in coupling coefficient slows down down-shifting of spectral peak

Contents

- Non-linear four-wave interactions in wind-wave models
- Computational methods
- Discrete Interaction Approximations
- Reducing the work load in the WRT method
- Equivalence of WRT and DIA's
- Effect of modified shallow water physics

Non-linear four-wave interactions in wind-wave models

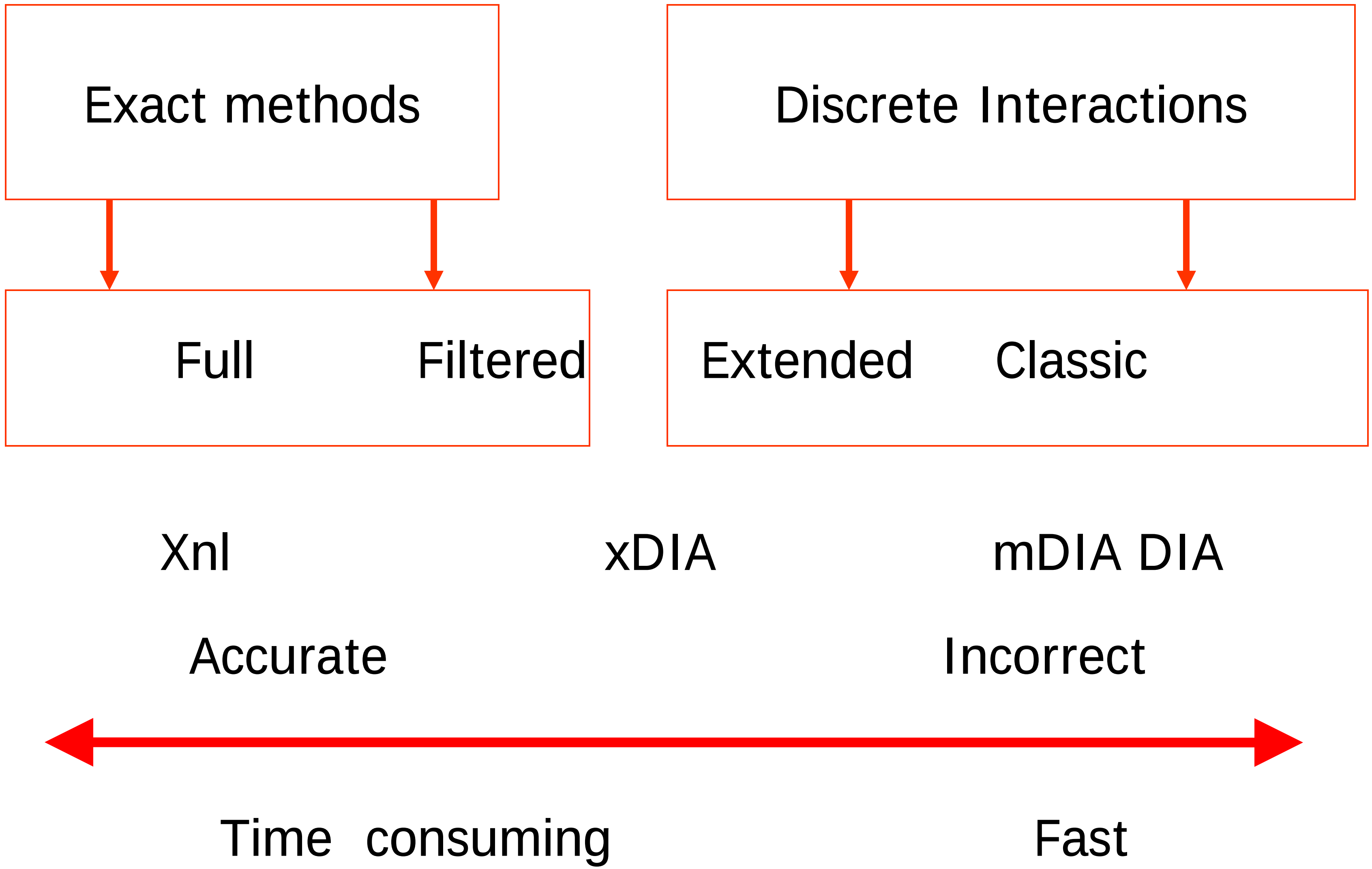
- Non-linear four-wave interactions essential for wind-wave evolution
- Source term fully known, but too time-consuming for practical use
- Operational models like WW III, SWAN, WAM use crude but fast DIA
- Error's in DIA's are compensated by tuning other source terms
- Need for more accurate and 'fast' parameterisations of non-linear four-wave interactions

Computational methods

- Discrete Interaction Approximation (Hasselmann et al., 1985)
- Exact reformulations of Boltzmann integral (Webb, 1978; Masuda, 1980; Polnikov, 1997; Lavrenov, 2001;...)
- Practical exact solution methods (Tracy and Resio, 1982; Van Vledder, 2005; Komatsu and Masuda, 2001; Gagnaire-Renou et al., 2010; ...)
- Two Scale Approximation (Resio and Perrie, 2009, 2010)
- Dominant transfer (Perrie et al., 2010)

Challenge: bridging the gap

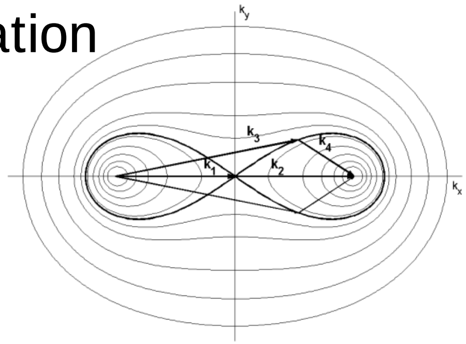
- Exact methods (accurate and time consuming)
- Discrete Interaction Approximation (fast and inaccurate)
- Speeding up exact solution method (filtering; courser interpolation; higher-order quadrature methods; smarter choice of integration space, ...). Reduced Integration Methods
- Extending the DIA with more wave number configurations
- Will both methods meet somewhere ?


Exact methods
Discrete Interactions
Full
Filtered
Extended
Classic
Xnl
xDIA
mDIA DIA
Accurate
Incorrect
Time consuming
Fast

Classic DIA and it extension

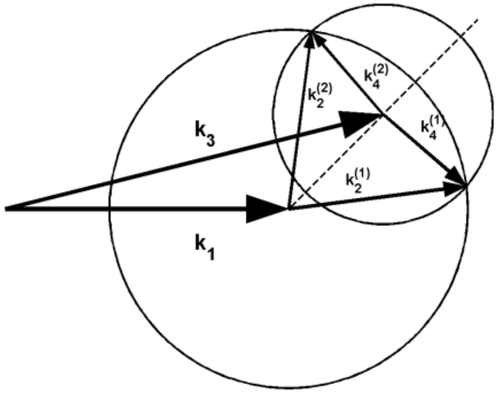
- DIA with one type of configuration

$$\boldsymbol{k}_1 = \boldsymbol{k}_2$$
$$\omega_1 = \omega_2 = \omega$$
$$\omega_3 = (1+\lambda)\omega$$
$$\omega_4 = (1-\lambda)\omega$$



- Generalized DIA with arbitrary configuration (Van Vledder, 2001; symmetric form proposed by Tolman 2004).

$$\boldsymbol{k}_1 \neq \boldsymbol{k}_2$$
$$\theta_2 = \theta_1 + \Delta\theta$$
$$\omega_1 = \omega$$
$$\omega_2 = (1+\mu)\omega$$
$$\omega_3 = (1+\lambda)\omega$$
$$\omega_4 = (1-\mu-\lambda)\omega$$



Discrete Interaction Approximations

- Classic DIA, λ=0.25, Hasselmann et al., 1985
- Multiple DIA, λ_i, i=1,2 (Van Vledder et al., 2000)
- Hashimoto and Kawaguchi (2001), Tolman (2004)

- DIA limited to one type wave number configuration
- Adding of this type of configuration no solution

- Generalized Multiple DIA with λ_i, μ_i, $\Delta\theta_i$ (Van Vledder, 2001, Tolman, 2004, SRIAM, Komatsu and Masuda, 2001)

Coefficients of mDIA

- Each wave number configuration has 3 shape factors (λ, μ, $\Delta\theta$) and a coefficient of proportionality C_{nl4}
- How to choose these coefficients?

- Least square analysis against limited set of (often academic) spectra
- Holistic approach, growth curve analysis (Hasselmann et al., 1985; Tolman, 2010)

- What is the next best wave number configuration?
- What is the best combination of 2, 3, 4, ..., configurations?

- Start from the other end: mathematically consistent stripping down of WRT method to end up with discrete interaction configurations

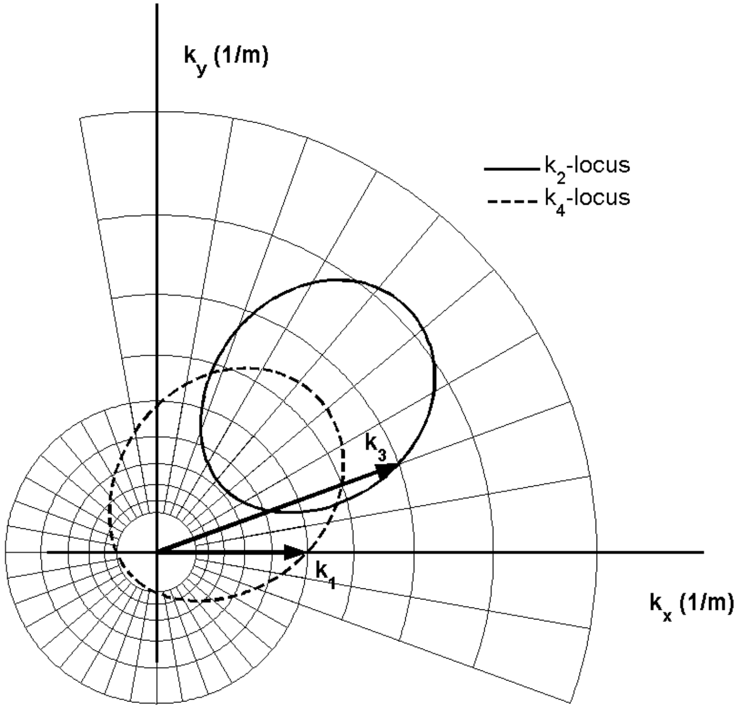
Strip down exact method to mimic a Discrete Interaction

- Workhorse is the WRT method of Resio and Perrie (1992), Van Vledder (2006)

$$\frac{\partial \boldsymbol{n}_1}{\partial t} = \int d\boldsymbol{k}_3 T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right)$$

$$T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) = \int_s ds G(s) J(s) N(s)$$

The T-function in the WRT method

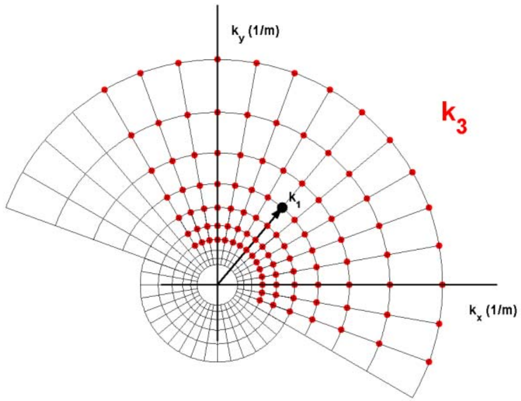


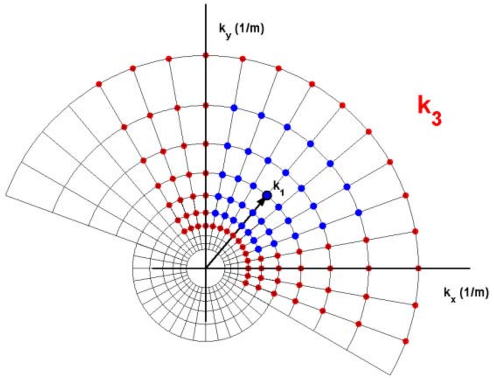
$\mathbf{k}_1$ and $\mathbf{k}_3$ loop over all discrete wave numbers of a spectrum

For each $\mathbf{k}_1$,$\mathbf{k}_3$ combination the resonant $\mathbf{k}_2$ and $\mathbf{k}_4$ wave numbers form closed path (locus)

T($\mathbf{k}_1$,$\mathbf{k}_3$) integrates product of functions (coupling coefficient, Jacobian term, wave number product) along locus

Range of $\mathbf{k}_3$ in discrete wave number grid

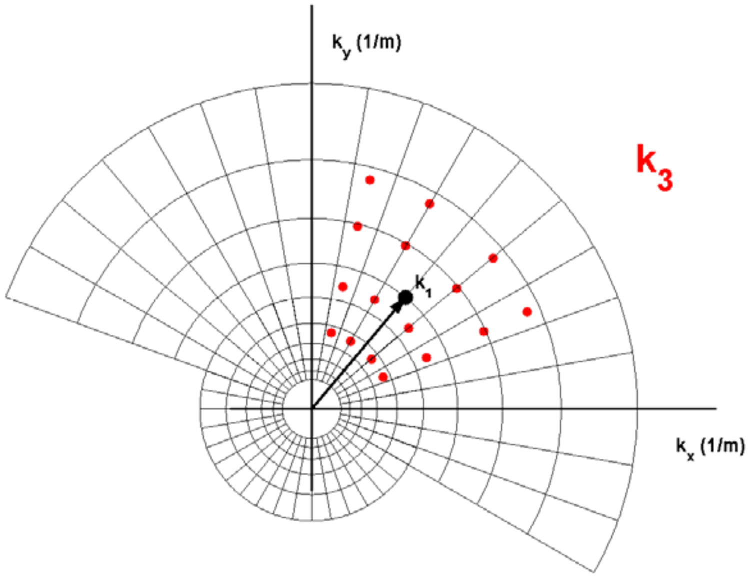




Speed up by choosing only $\mathbf{k}_1$ - $\mathbf{k}_3$ combinations that are not too far separated in wave number space.

Effective method to reduce workload, examples in Van Vledder (2006)

Modifying outer $\mathbf{k}_3$ integration loop in WRT method

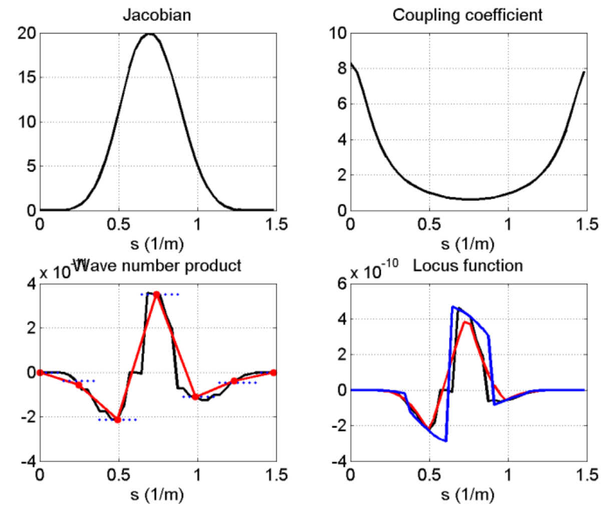


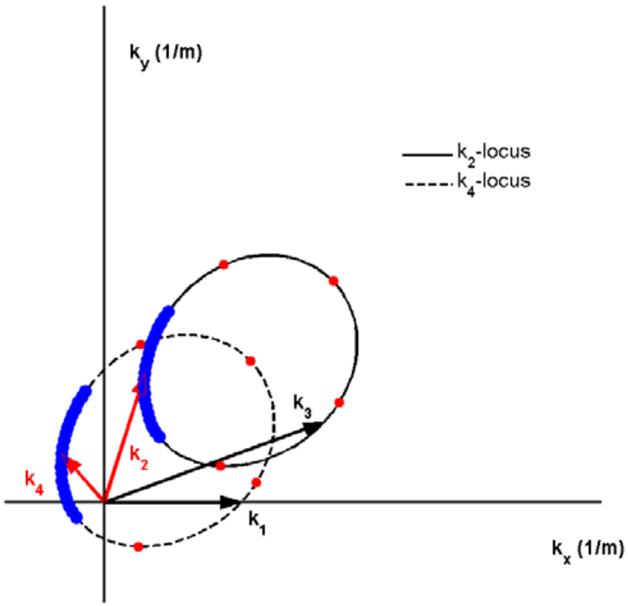
Disadvantage of present implementation of WRT method: $\mathbf{k}_1$ and $\mathbf{k}_3$ fixed to discrete wave number grid.

Distribute $\mathbf{k}_3$ around $\mathbf{k}_1$ according to e.g. Gauss-Legendre quadrature including proper weights

Integration along locus, LQA

Pick a few points on locus, but keep all information of G and J

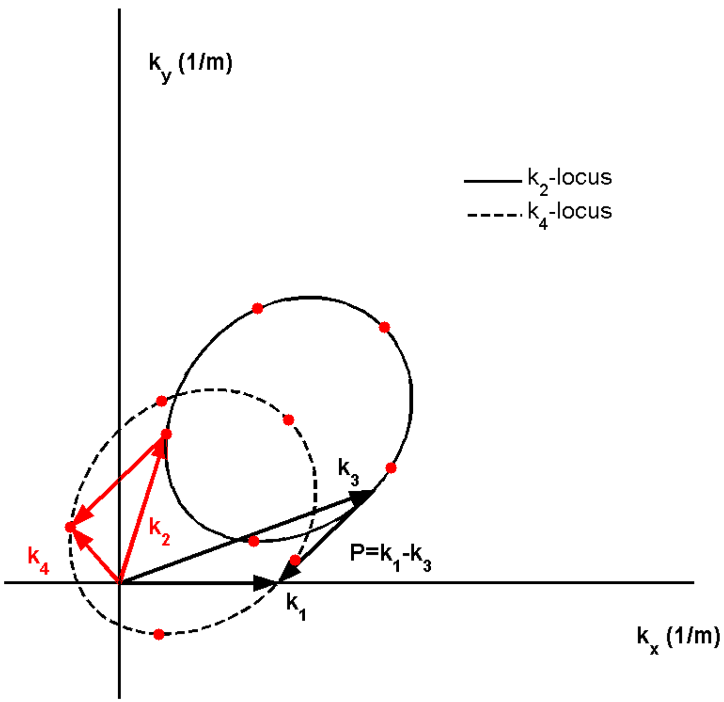




Piece wise integration along locus, lump contribution of coupling coefficient G and Jacobian J, which can be precomputed

$$T = \sum_{i=1}^{N} N\left(s_i\right) \int_{s_i - 0.5\Delta s_i}^{s_i + 0.5\Delta s_i} G(s) J(s) ds$$

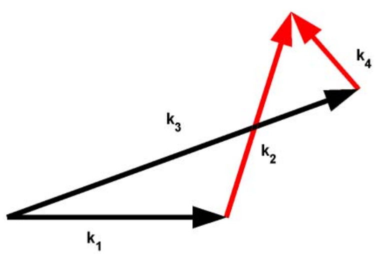
Incremental integration along locus



Dual points on locus form a quadruplet

Identify individual wave number configuration on locus

Determine shape factors λ, μ, $\Delta\theta$



Equivalence of stripped WRT and Discrete Interaction

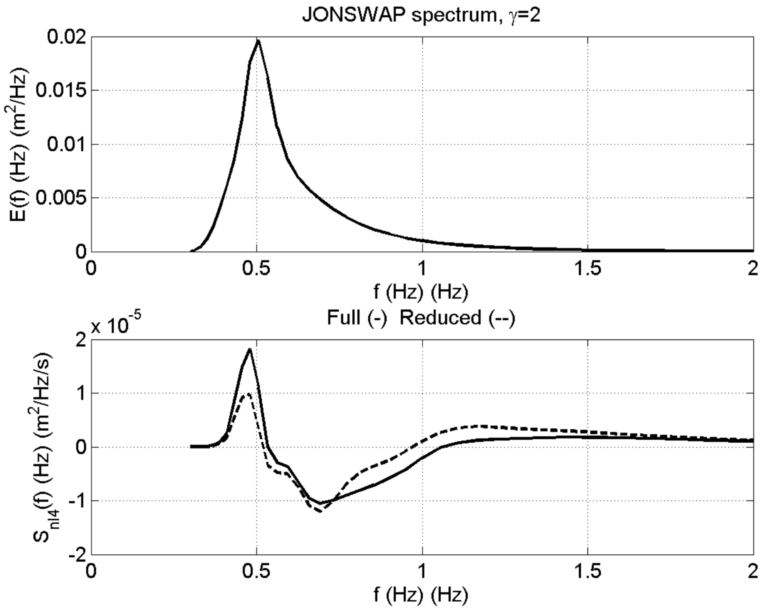
- In WRT changes are made **only** to each pair of discrete $n(\mathbf{k}_1)$ and $n(\mathbf{k}_3)$, while using information from loci of $\mathbf{k}_2$ and $\mathbf{k}_4$. Action densities at the latter wave numbers are affected further on in the looping process.

- In DIA changes are made simultaneously to all four wave numbers in a configuration of $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$ and $\mathbf{k}_4$

- Principle of detailed balance $\Delta n_1 = \Delta n_2 = -\Delta n_3 = -\Delta n_4$

- Strength of individual T-contributions determine weight of quadruplets. Account for scaling with wave number.

Testing new approximations



- Example of comparison
- Renormalization needed
- First check on individual spectra
- Stability analysis, growth curves
- Field cases

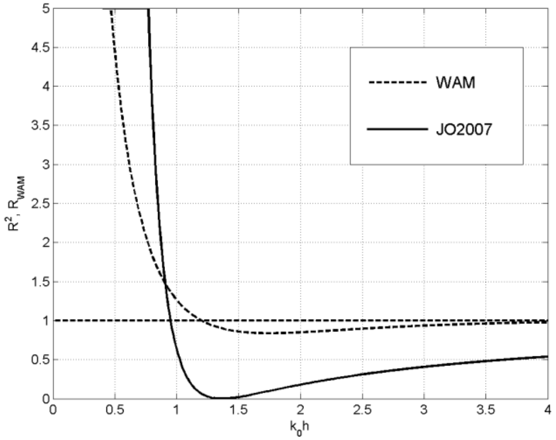
Shallow water effects

- WRT method also suitable for shallow water

- Extension of (mG)DIA to shallow water
- WAM: Overall scaling factor R(kh), based on narrow peak approximation Herterich and Hasselmann (1980)
- Shape remains constant, whereas it will change !
- Shallow water DIA (Van Vledder and Bottema, 2002), depth included in dispersion relation and coupling coefficient
- More advanced msDIA developed by Tolman (2010)

Modulational instabilities in shallow water

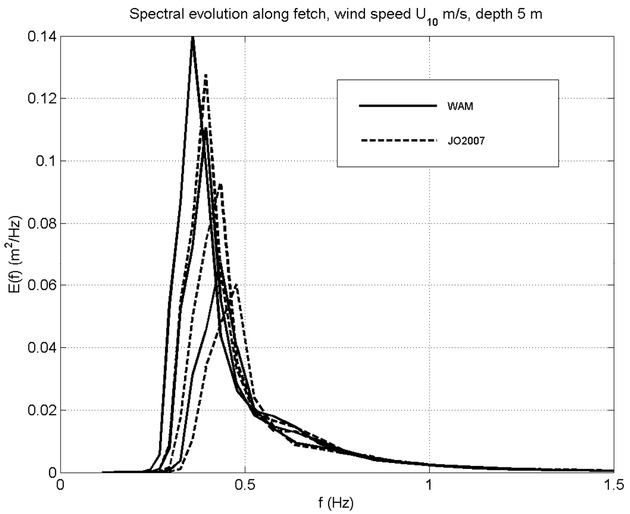
- (Peter) Janssen and Onorato (2007) investigated effects of modulational instabilities and wave induced currents in shallow water on non-linear transfer rate
- For narrow peak approximation they show that S_{nl4} vanishes for kh = 1.363
- They suggest alternative scaling of deep water transfer rate
- Full coupling coefficient and narrow band approximation



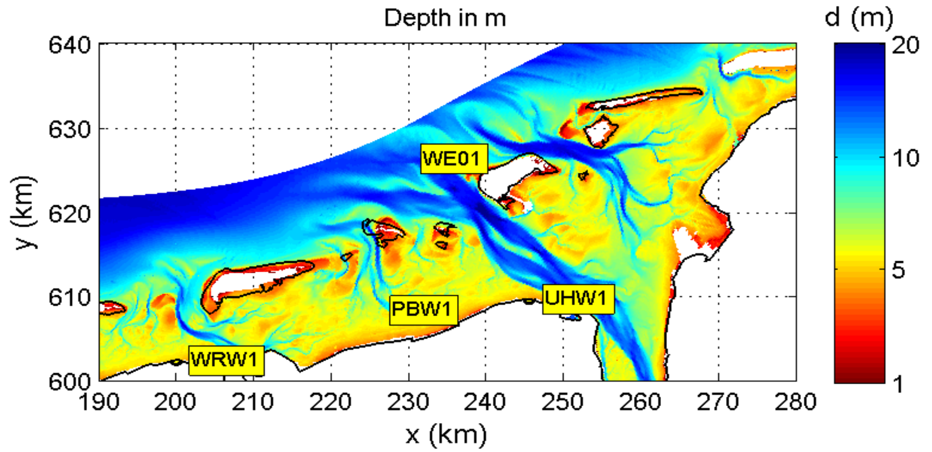
- Narrow-band scaling implemented in SWAN model
- Fetch-limited wave growth
- Storm condition in Dutch Wadden Sea

Fetch-limited wave growth

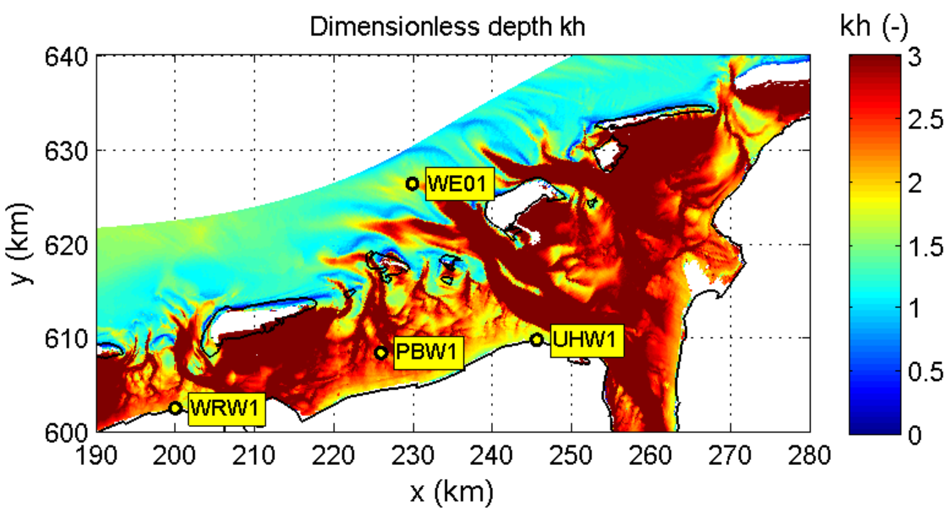
- U_{10} = 10 m/s, depth = 5 m, fetch = 10 km
- Slower downshifting of spectral peak
- Spectra shown at 5, 7.5 and 10 km
- Wave heights and periods 10% smaller



Storm of 9 November 2007

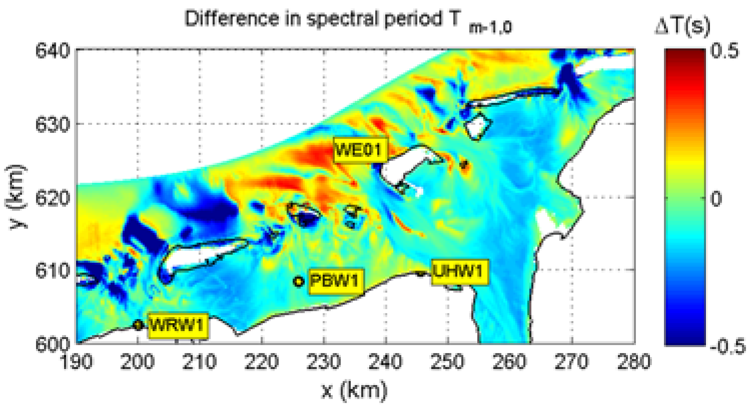


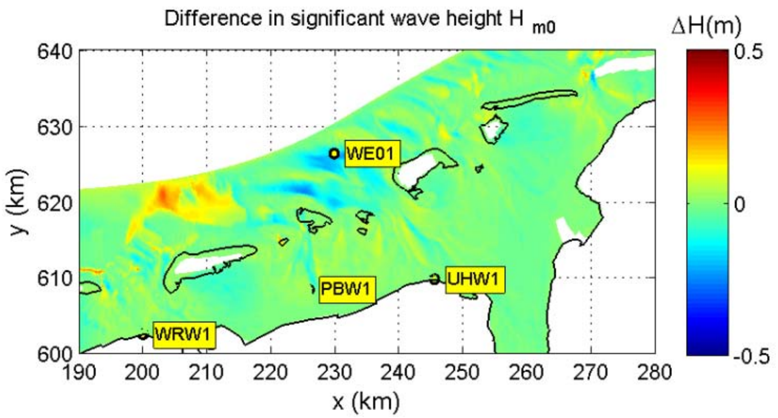
Eastern Wadden Sea and buoy location

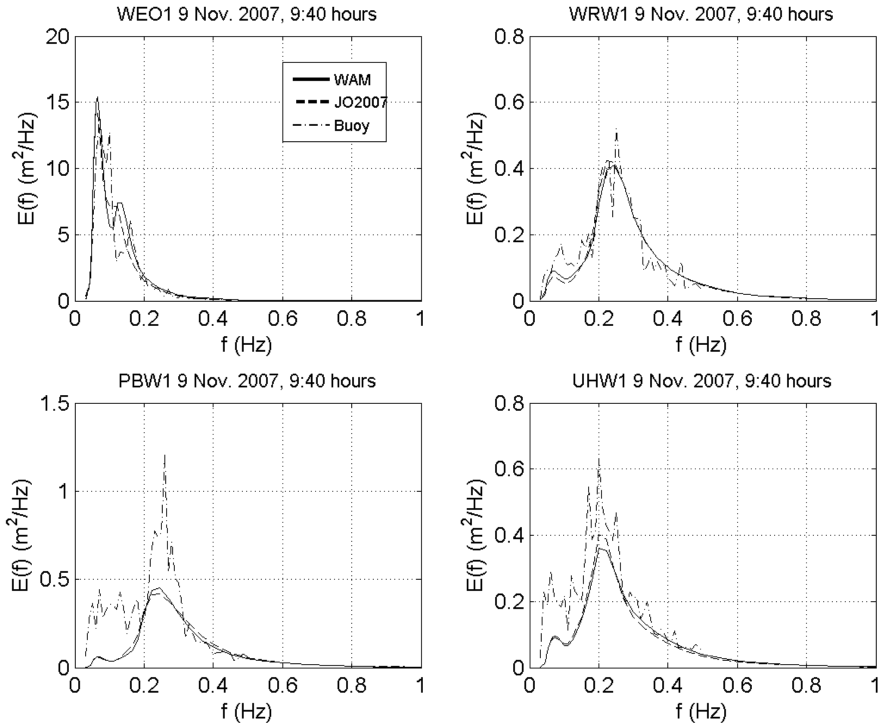


Moment of high water,
Wind speed 20 m/s
Offshore H_{m0} = 8 m, T_p=12 s

Effect on periods and heights (5%) and spectra at buoy positions







Summary and conclusions

- Efficiency of WRT improved (no claims yet about performance)
- Stripped down WRT resembles set of discrete interactions
- Further testing and choice of settings in progress

- Narrow band depth scaling of Janssen and Onorato (2007) slows down downshifting of spectral peak
- Consequences in Wadden Sea small and local
- Implementation and testing of full modified coupling coefficients in progress