Non-Breaking Wave Turbulence and Swell Propagation

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Conclusions

- *the wave motion should be able to generate turbulence even in absence of wave breaking.*

potential significance of such turbulence source is due to the waves in the ocean being ever present and wave-caused speeds of water motion being at least an order of magnitude greater than those of shear currents and Langmuir circulations which are usually attributed with turbulence supply.

- *decoupling of the wave-induced non-breaking turbulence from analogies with the wall-layer law tradition*

the principal difference of the wave turbulence is the existence of the characteristic length scale (radius of the wave orbit) as opposed to the wall turbulence which does not have a characteristic length.

- *wave-induced turbulence would enhance the upper ocean mixing on behalf of the normal component of the wind stress*

wave motion can directly affect the upper-ocean mixing, and thus skipping the wave phase of momentum transformation undermines accuracy of approaches based on the assumption of direct mixing of the upper ocean due to the wind.

The wave-induced turbulence links together three ocean features which are routinely treated as separate properties: the wind waves, the near-surface turbulence and the upper ocean mixed layer

Publications on the wave turbulence

Swinburne

- Babanin et al., *Coasts and Ports*, 2005
- Babanin, *Geophys. Res. Lett.*, 2006
- Babanin and Haus, *J. Phys. Oceanogr.*, 2009
- Babanin et al., *Ocean Modelling*, 2009
- Dai et al., *J. Phys. Oceanogr.*, 2010
- Pleskachevski et al., *J. Phys. Oceanogr.*, 2011
- Toffoli et al., *EGU*, 2011
- Babanin and Chalikov, *EGU*, 2011
- Babanin, *Cambridge Uni. Press*, 2011

FIO, China, GKSS, Germany

- Qiao et al., 2004, 2008, 2010
- Gayer et al., 2006
- Dai et al., 2010

Earlier

- Cavaleri and Zecchetto, 1987
- Efimov and Khristoforov, 1969, 1971

How far back is the topic going?

Kinsman, 1965: Wind Waves

Navier-Stokes equation

linearised boundary conditions, with surface tension *T*

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w - g$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \eta}{\partial t} = w_{z=0}$$

$$p - 2\mu\frac{\partial w}{\partial z} = -\frac{\partial^2 \eta}{\partial x^2}\mathrm{T}_{z=\eta}$$

$$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0_{z=\eta}$$

Solutions

vorticity

$$\omega = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = \nabla^2\Psi$$

$$\omega = \beta\frac{i\sigma}{\nu}e^{mz}e^{i(kx+\sigma t)} =$$

$$= -2\gamma k\sigma \exp\left(\sqrt{\frac{\sigma_{real}}{2\nu}}z - \frac{2\sigma_{real}}{\mathrm{Re}_w}\right)\exp\left\{i\left(kx + \sqrt{\frac{\sigma_{real}}{2\nu}}z + \sigma_{real}t\right)\right\}$$

$$\frac{\delta_z}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{2\nu}{\sigma_{real}}} = \frac{1}{2\pi}\sqrt{\frac{2\nu k^2}{\sigma_{real}}} = \frac{\sqrt{2}}{2\pi}\frac{1}{\sqrt{\mathrm{Re}_w}}$$

- exponential decay in *z* and *t*
- oscillations in *x*, *z* and *t*
- ‘length’ of vertical vorticity oscillation is much smaller than λ

Wave-induced turbulence, however, is the topic of this talk

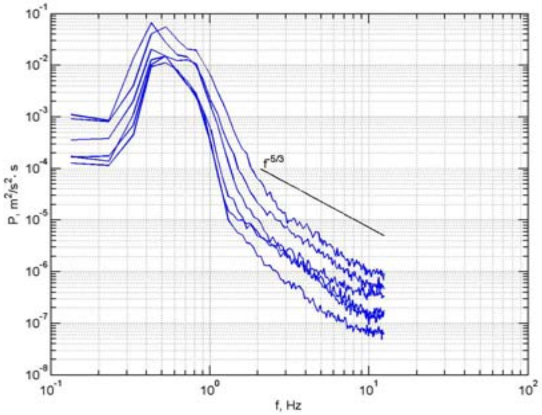
How presence of the turbulence and upper-ocean mixing is currently explained?

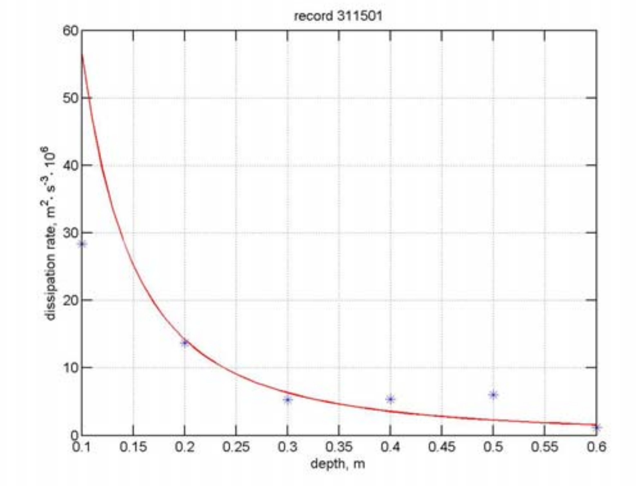
- wind stress (major factor)
- heating and cooling, advection, wave breaking, Langmuir circulation, internal waves etc

There is accumulating evidence that in absence of wave breaking, and even wind stress, turbulence still persists through the water column and not only the boundary layers

$$\varepsilon(z) = \begin{cases} const & z \le 0.4H_s & \\ z^{-1} & z > 0.4H_s & U < 7.5 \\ z^{-2} & z > 0.4H_s & U \ge 7.5 \end{cases}$$

Babanin et al., 2005



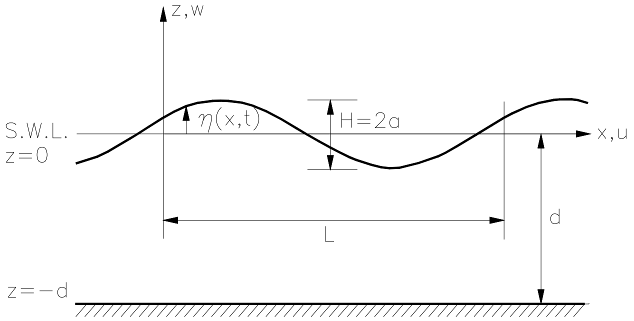


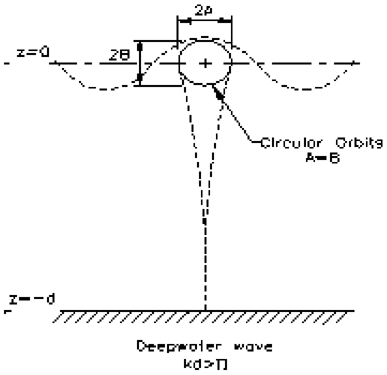
Babanin, 2006

Hypothesis of the Wave Reynolds Number

$\eta(x,t) = a_0 \cos(\omega t + kx)$

$a(z) = a_0 \exp(-kz)$





It is the hypothesis that the a-based Reynolds number

$$\mathrm{Re} = \frac{aV}{\nu} = \frac{a^2\omega}{\nu}$$

where V= ωa is orbital velocity, and ν is kinematic viscosity of the ocean water, indicates transition from laminar orbital motion to turbulent

Critical Reynolds Number for the Wave-Induced Motion, and Depth of the Mixed Layer

$$\mathrm{Re}(z) = \frac{\omega}{\nu} a_0^2 \exp(-2kz) = \frac{\omega}{\nu} a_0^2 \exp(-2\frac{\omega^2}{g} z)$$

$$z_{cr} = -\frac{1}{2k} \ln(\frac{\mathrm{Re}_{cr}\,\nu}{a_0^2\omega}) = \frac{g}{2\omega^2} \ln(\frac{a_0^2\omega}{\mathrm{Re}_{cr}\,\nu})$$

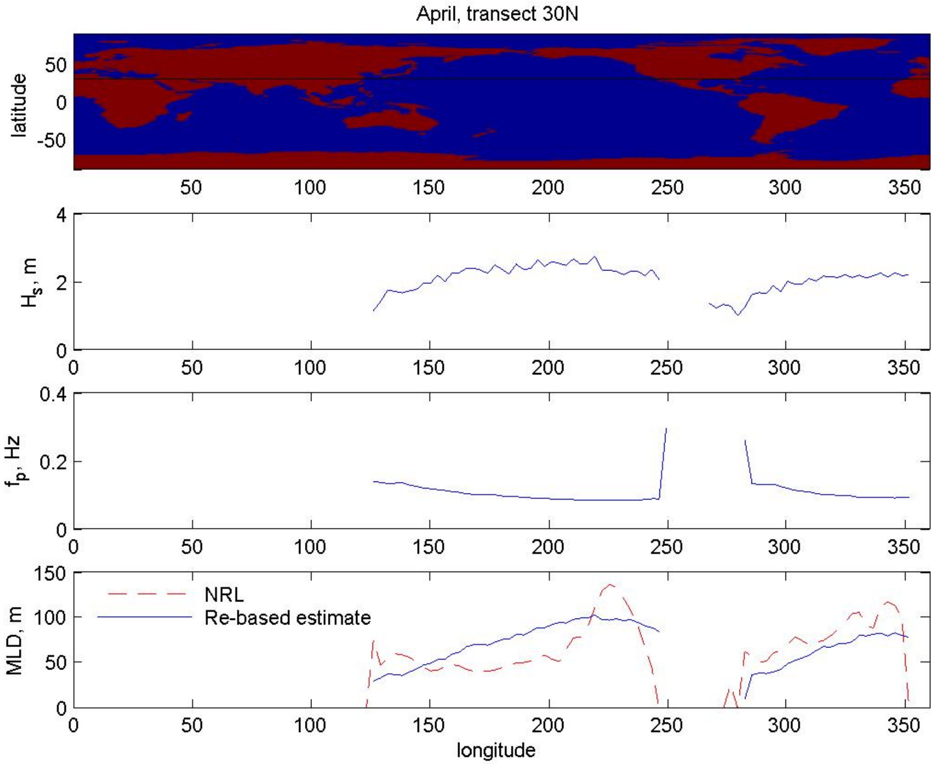
$$\mathrm{Re}_{cr} = \frac{\omega}{\nu} a_0^2 \exp(-2kz_{cr}) = \frac{\omega}{\nu} a_0^2 \exp(-2\frac{\omega^2}{g} z_{cr})$$

- Based on Black-Sea measurements it was found ***Re$_{cr}$=3000***

- Laboratory experiments with traces in mechanically-generated non-breaking waves confirmed the number

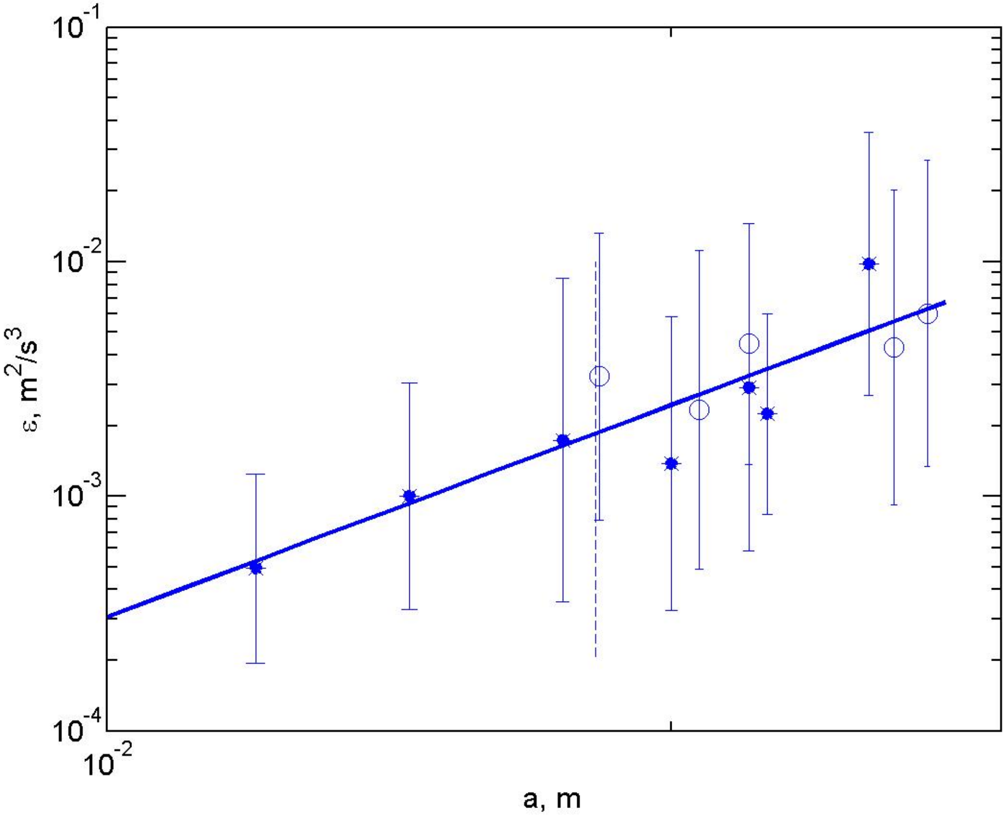
- estimates of ocean MLD compare with measurements within 10% uncertainty

- *Re*-based estimate of MLD (solid line) and NRL estimate of MLD (dashed line) compare well for a global latitude transect



Laboratory Experiment at ASIST, RSMAS, University of Miami

turbulence is highly intermittent, most frequent at rear wave face



$$\varepsilon = 300 \cdot a^{3.0\pm1.0}$$

This is close to the expectation: since the force due to the turbulent stresses is proportional to a^2, the energy dissipation rate should be $\sim a^3$.

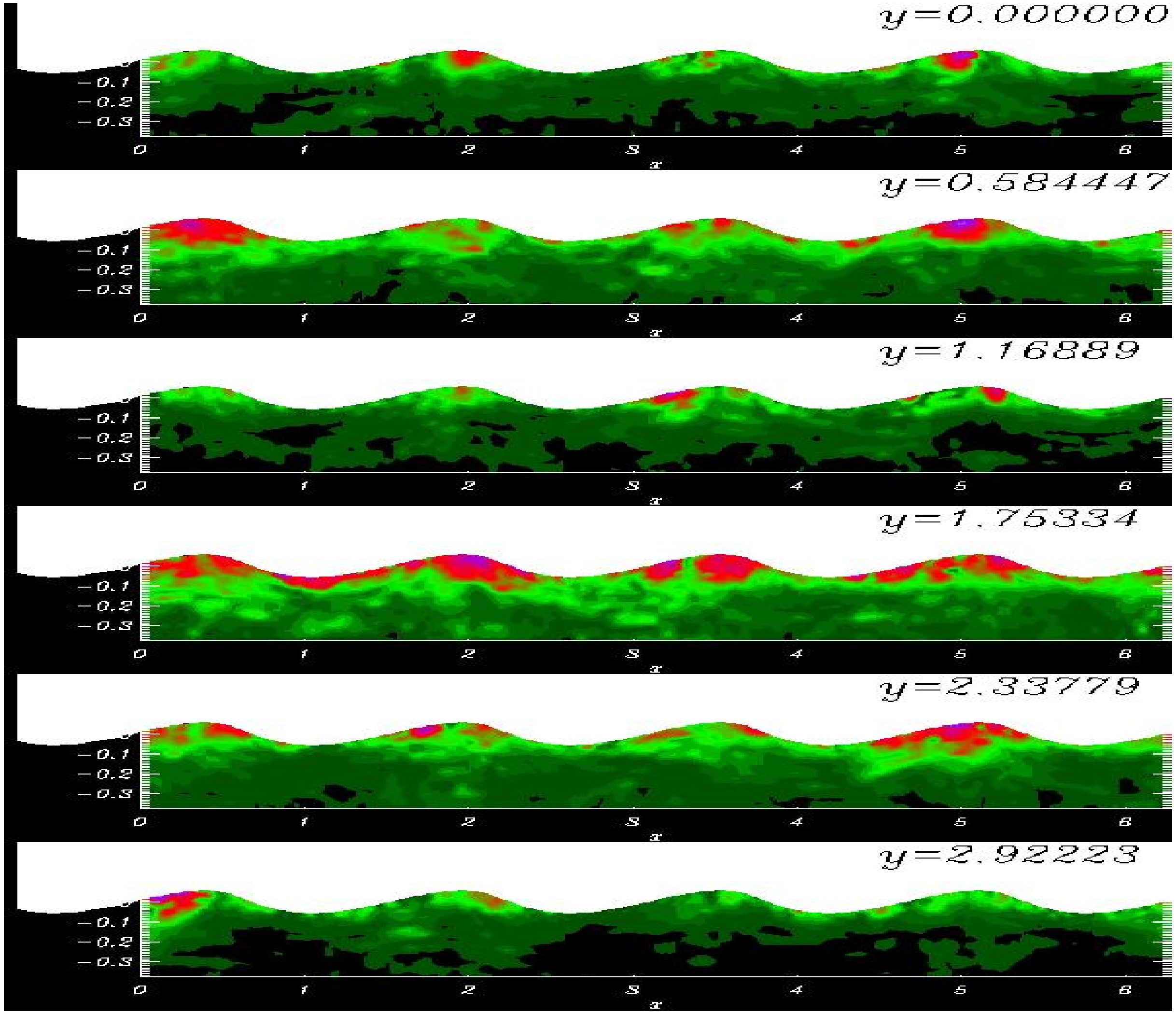
Babanin and Haus, 2009

Model of generation of turbulence in potential waves.

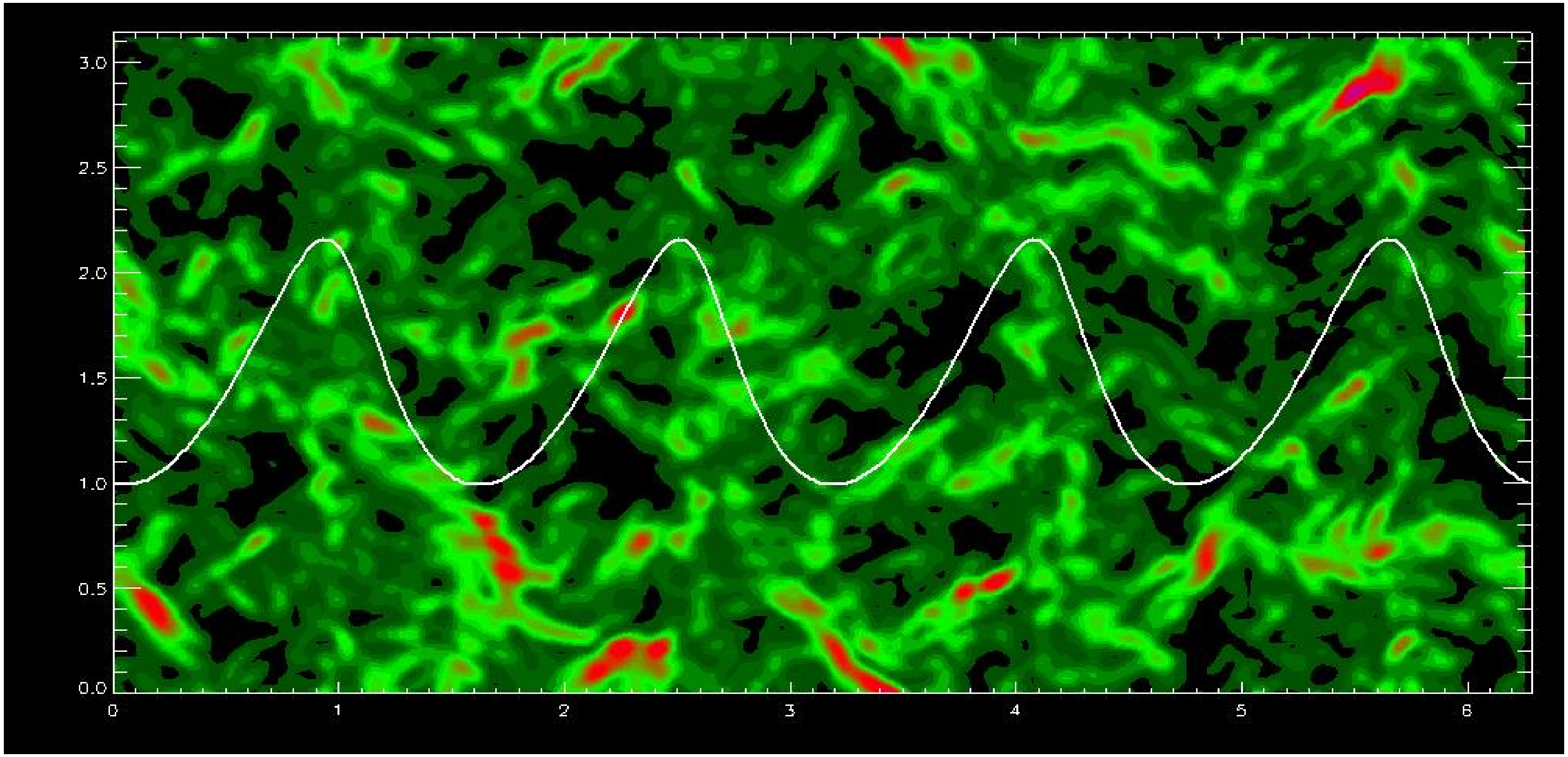
Model is based on exact 2-D (x-z) model of surface waves coupled with 3-D LES (x-y-z) model of vortical motion based on Reynolds equation with parameterized subgrid turbulence. Both system of Equations are written in conformal cylindrical surface-following coordinates.

The one-way coupling of models occurs through components of potential orbital velocity and vorticity components.

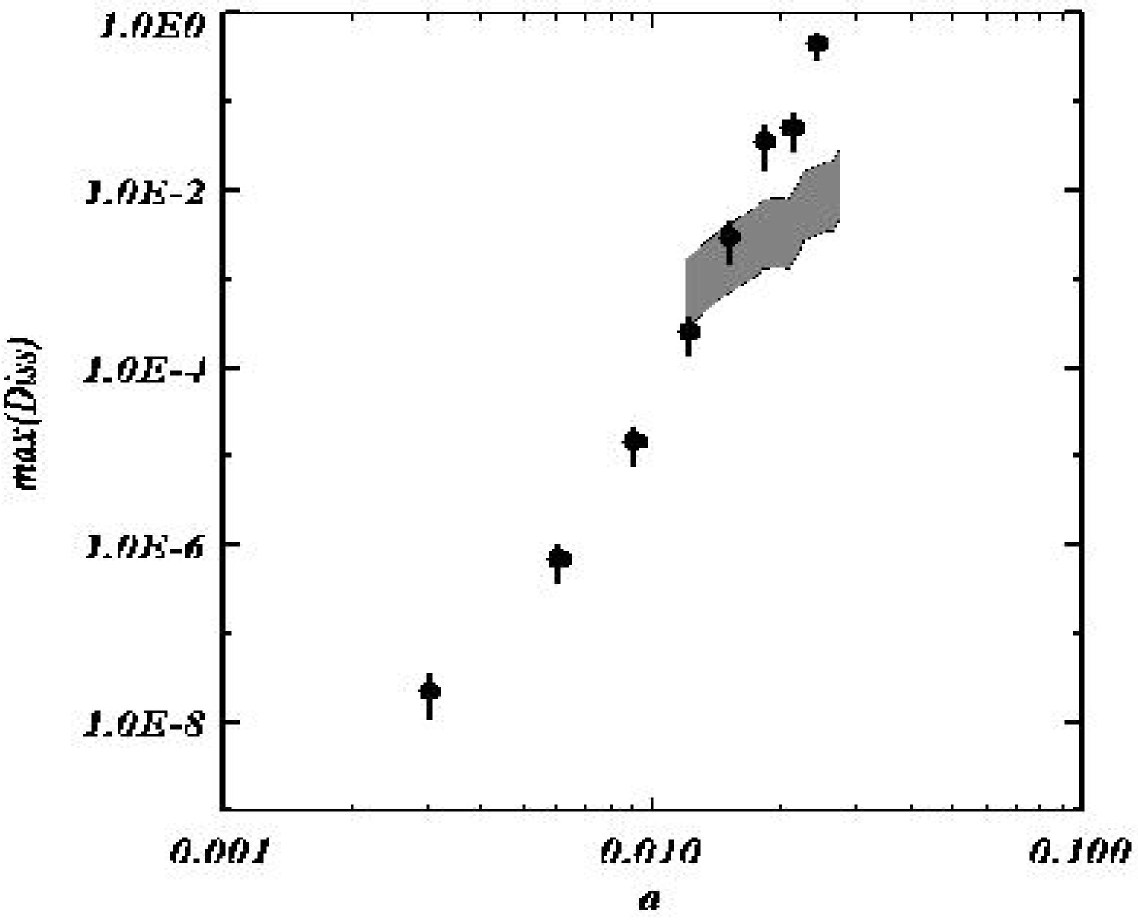
Total energy of non-potential motion at different crossection of 3-D flow



Distribution of kinetic energy of non-potential flow in top level (white curve indicates the shape of wave).

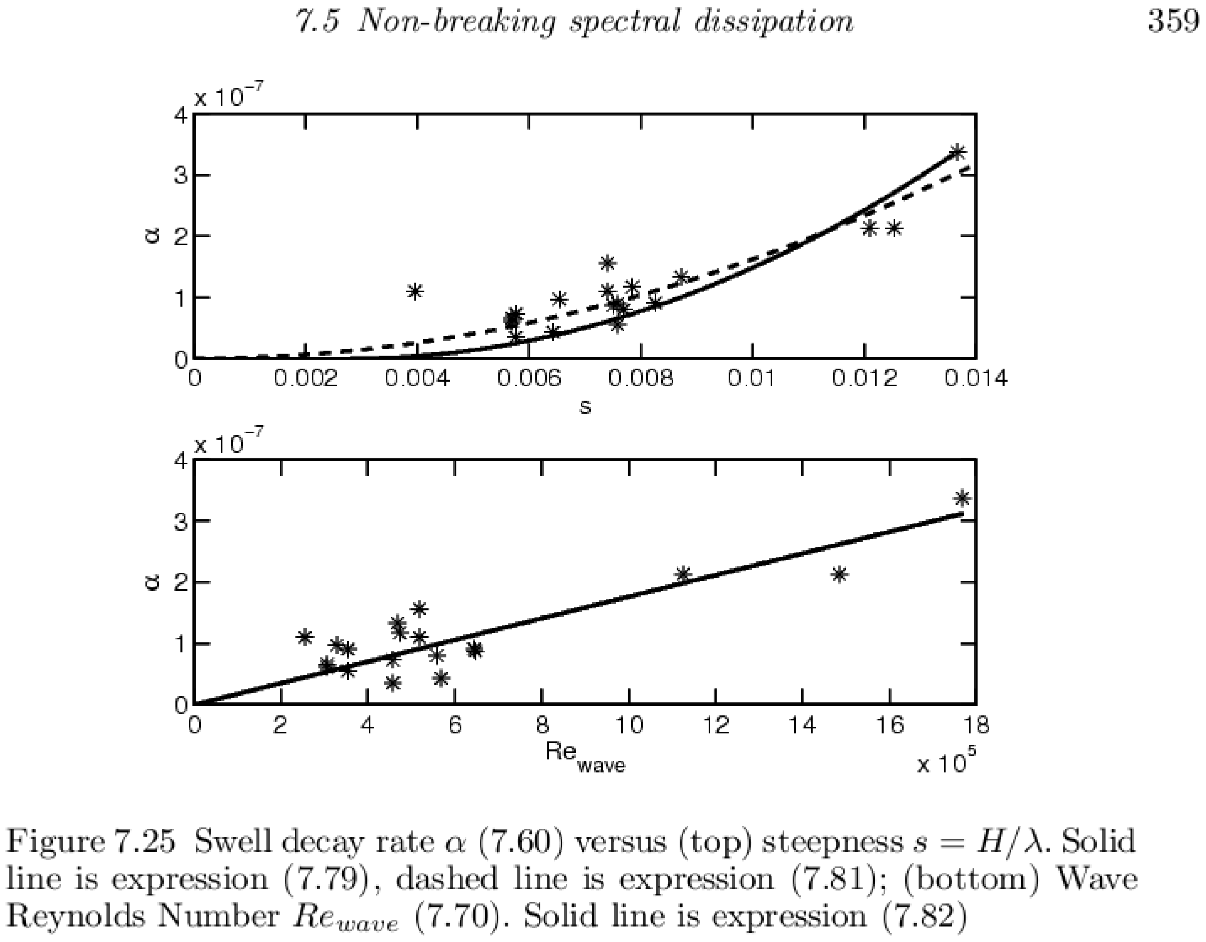


Maximum rate of dissipation vs wave amplitude (grey area – experimental data)



Swell attenuation

data of Ardhuin et al. (2009)



7.5 Non-breaking spectral dissipation 359

Figure 7.25 Swell decay rate α (7.60) versus (top) steepness $s = H/\lambda$. Solid line is expression (7.79), dashed line is expression (7.81); (bottom) Wave Reynolds Number Re_{wave} (7.70). Solid line is expression (7.82)

Swell attenuation

$\varepsilon = 300 \cdot a^{3.0\pm1.0}$ $b = b_1 k\omega^3 = 30.$ $b_1 = 0.004$

Dissipation

- volumetric

$$\epsilon_{dis} = b_1 k\omega^3 a_0^3 = 0.004 k u_{orb}^3.$$

- per unit of surface

$$D_a = b_1 k \int_0^\infty u(z)^3 dz = b_1 k u_0 \int_0^\infty \exp(-3kz) dz = \frac{b_1}{3} u_0^3.$$

- per unit of propagation distance

$$D_x = \frac{1}{c_g} D_a = \frac{b_1}{3} 2 \frac{k}{\omega} u_0^3 = \frac{2}{3} b_1 k \omega^2 a_0^3 = \frac{2}{3} b_1 g k^2 a_0^3.$$

$$\frac{g}{2} \frac{\partial (a_0(x)^2)}{\partial x} = \frac{2}{3} b_1 g k^2 a_0(x)^3,$$

$$a_0(x)^2 = \frac{4}{B^2} x^{-2} = \frac{9}{4 \cdot b_1^2 k^4} x^{-2} = \frac{9}{64} 10^6 k^{-4} x^{-2}.$$

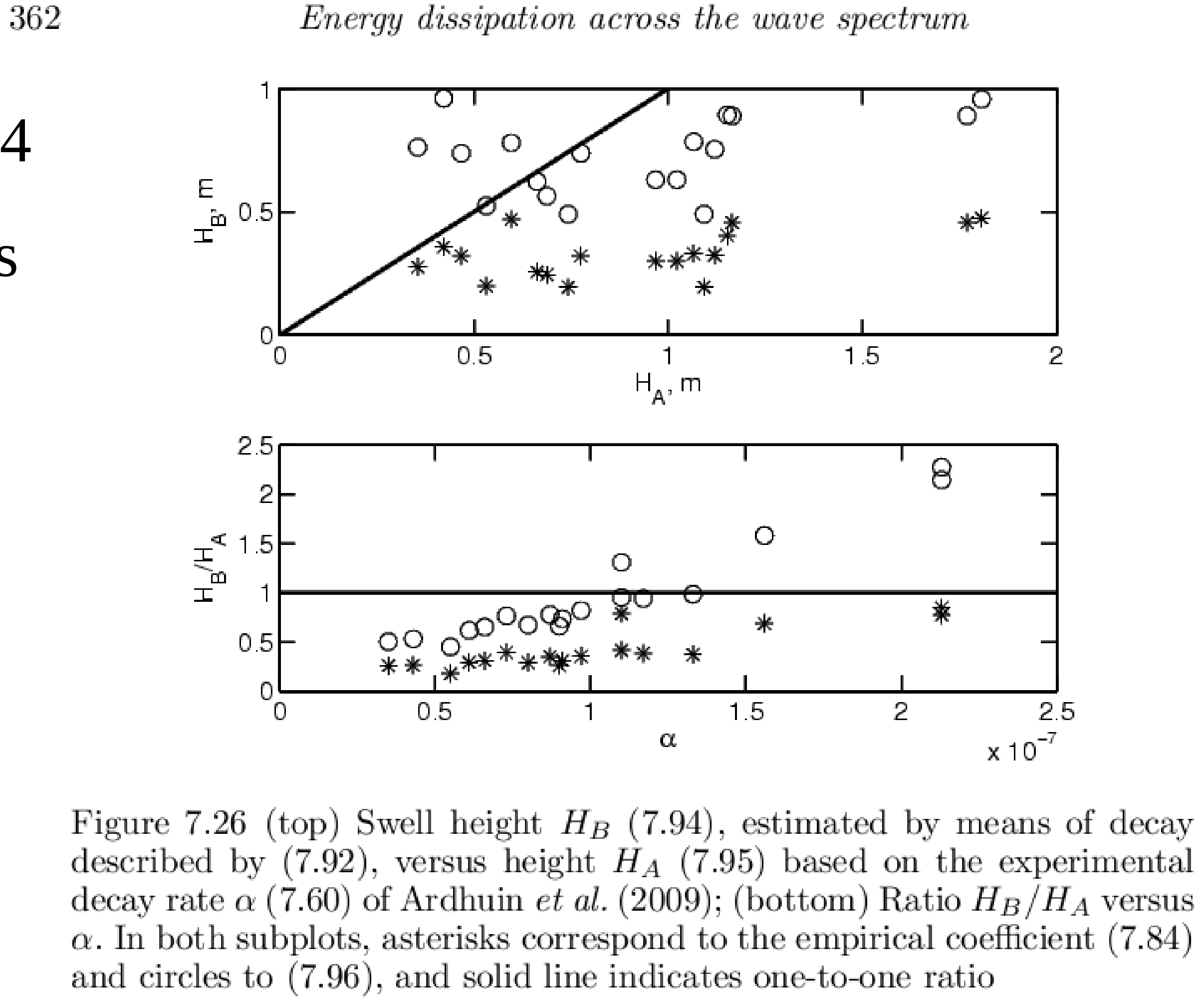
Swell attenuation

b_1=0.004

asterisks

b_1=0.002

circles



362 *Energy dissipation across the wave spectrum*

Figure 7.26 (top) Swell height H_B (7.94), estimated by means of decay described by (7.92), versus height H_A (7.95) based on the experimental decay rate α (7.60) of Ardhuin *et al.* (2009); (bottom) Ratio H_B/H_A versus α. In both subplots, asterisks correspond to the empirical coefficient (7.84) and circles to (7.96), and solid line indicates one-to-one ratio

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