Nonlinear Dynamics of Shoaling Gravity Waves.

Discussion of a one-point closure approximation.

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Outline

Shallow-water closure problem.

A one-point closure approximation.

Dissipation and nonlinearity.

Outlook towards operational wave models.

Take away.

$$\eta(x,t) = \sum_{\omega_1} \frac{A_1(x)}{\sqrt{C_{g,1}}} \exp\left[-i\omega_1 t\right]$$

$$\left[\frac{d}{dx} - ik_1\right] A_1(x) = i \sum_{2+3=1} \mathcal{W}_{23} A_2(x) A_3(x)$$

$$\frac{d}{dx}\langle AA\rangle = \langle AA\rangle + \langle AAA\rangle$$

$$\frac{d}{dx}\langle AAA\rangle = \langle AAA\rangle + \langle AA\rangle\langle AA\rangle + \langle AAAA\rangle^C$$

$$+\mu\langle AAA\rangle$$

$$\frac{d}{dx}\langle AAAA\rangle^C = \ldots$$

$$\eta(x,t) = \sum_{\omega_1} \frac{A_1(x)}{\sqrt{C_{g,1}}} \exp\left[-i\omega_1 t\right]$$

$$\left[\frac{d}{dx} - ik_1\right] A_1(x) = i \sum_{2+3=1} \mathcal{W}_{23} A_2(x) A_3(x)$$

$$\frac{d}{dx}\langle AA\rangle = \langle AA\rangle + \langle AAA\rangle$$

$$\frac{d}{dx}\langle AAA\rangle = \langle AAA\rangle + \langle AA\rangle\langle AA\rangle + \mu\langle AAA\rangle$$

See e.g. Holloway & Hendershot 1977; Salmon, 1998.

Nonlinear wave evolution

$$\frac{d\mathcal{E}_1}{dx} = -2\nu_1\mathcal{E}_1 - 4\sum_{2+3=1} \mathcal{W}_{23}\Re\left(\int_0^x \mathcal{H}_{23}(x,s)Q_{23}(s)\,ds\right)$$

$$\mathcal{E}_1(x) = \langle A_1(x)A_1^*(x)\rangle \qquad Q_{12} = \mathcal{E}_{(1+2)}\left(\mathcal{W}_{(1+2)(-2)}\mathcal{E}_2 + \mathcal{W}_{(1+2)(-1)}\mathcal{E}_1\right) + \mathcal{W}_{12}\mathcal{E}_1\mathcal{E}_2$$

resonant mismatch — dissipation — NL-wave damping (relaxation)

$$\mathcal{H}_{12}(x,s) = \exp\left[\int_s^x \left(i\Lambda_{12}(s') - \nu_{12}(s') - \mu_{12}(s')\right)\,ds'\right]$$

2-point QN2P

$$H_1(x,s) = \frac{\mathcal{E}_1(x,s)}{\mathcal{E}_1(s)}$$

1-point (QN)

$$\mu_{12} = 0$$

$$\nu_{12} = \nu_1 + \nu_2 + \nu_{1+2} \qquad \Lambda_{12} = \varkappa_1 + \varkappa_2 - \varkappa_{1+2}$$

Nonlinear wave evolution

$$\frac{d\mathcal{E}_1}{dx} = \underbrace{-2\nu_1\mathcal{E}_1}_{\mathrm{Dis}_1} \underbrace{- 4\sum_{2+3=1}\mathcal{W}_{23}\Re\left(\int_0^x \mathcal{H}_{23}(x,s)Q_{23}(s)\,ds\right)}_{\mathrm{Nl}_1}$$

$$\mathcal{E}_1(x) = \langle A_1(x)A_1^*(x)\rangle$$

$$Q_{12} = \mathcal{E}_{(1+2)}\left(\mathcal{W}_{(1+2)(-2)}\mathcal{E}_2 + \mathcal{W}_{(1+2)(-1)}\mathcal{E}_1\right) + \mathcal{W}_{12}\mathcal{E}_1\mathcal{E}_2$$

resonant mismatch — dissipation — NL-wave damping (relaxation)

$$\mathcal{H}_{12}(x,s) = \exp\left[\int_s^x \left(i\Lambda_{12}(s') - \nu_{12}(s') - \mu_{12}(s')\right) ds'\right]$$

2-point QN2P

$$H_1(x,s) = \frac{\mathcal{E}_1(x,s)}{\mathcal{E}_1(s)}$$

1-point (QNR)

$$\mu_{12} = \beta\frac{|\mathrm{Dis}_1 + \mathrm{Dis}_2 + \mathrm{Dis}_{(1+2)}| + |\mathrm{Nl}_1 + \mathrm{Nl}_2 + \mathrm{Nl}_{(1+2)}|}{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_{(1+2)}}$$

1-point (QN)

$$\mu_{12} = 0$$

$$\nu_{12} = \nu_1 + \nu_2 + \nu_{1+2} \qquad \Lambda_{12} = \varkappa_1 + \varkappa_2 - \varkappa_{1+2}$$

Nonlinear wave evolution

$$\frac{d\mathcal{E}_1}{dx} = -2\nu_1\mathcal{E}_1 - 4\sum_{2+3=1}\mathcal{W}_{23}\Re\left(\int_0^x \mathcal{H}_{23}(x,s)Q_{23}(s)\,ds\right)$$

$$\mathcal{H}_{12}(x,s) = \exp\left[\int_s^x \left(i\Lambda_{12}(s') - \nu_{12}(s') - \mu_{12}(s')\right)\,ds'\right]$$

2-point QN2P

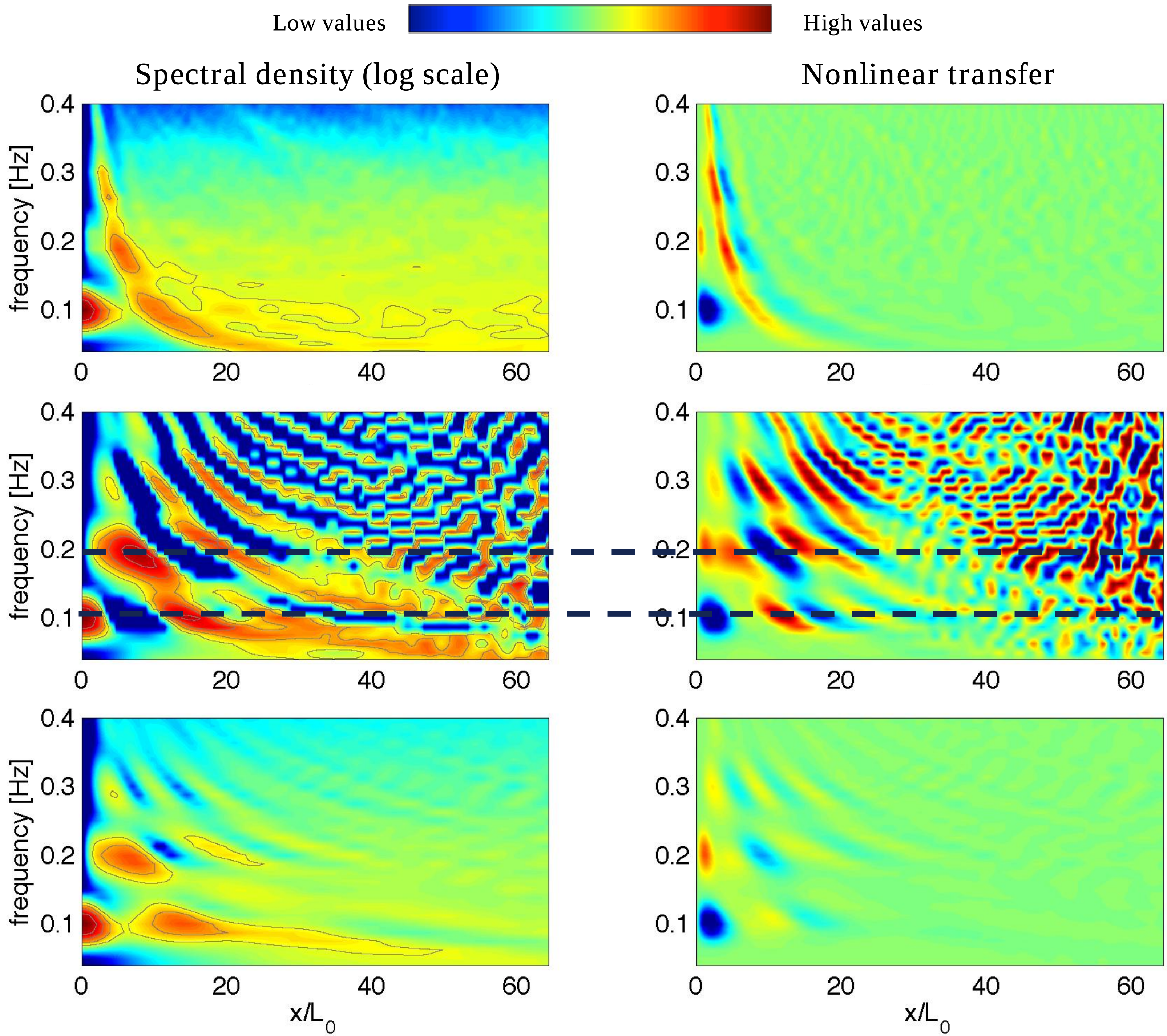
- transport equation cross-correlations
- numerically very intensive
- no empirical coefficients
- benchmark

1-point (QNR)

- parameterization nonlinear background wave field
- quasi-empirical
- efficient
- 2-equation model

1-point (QN)

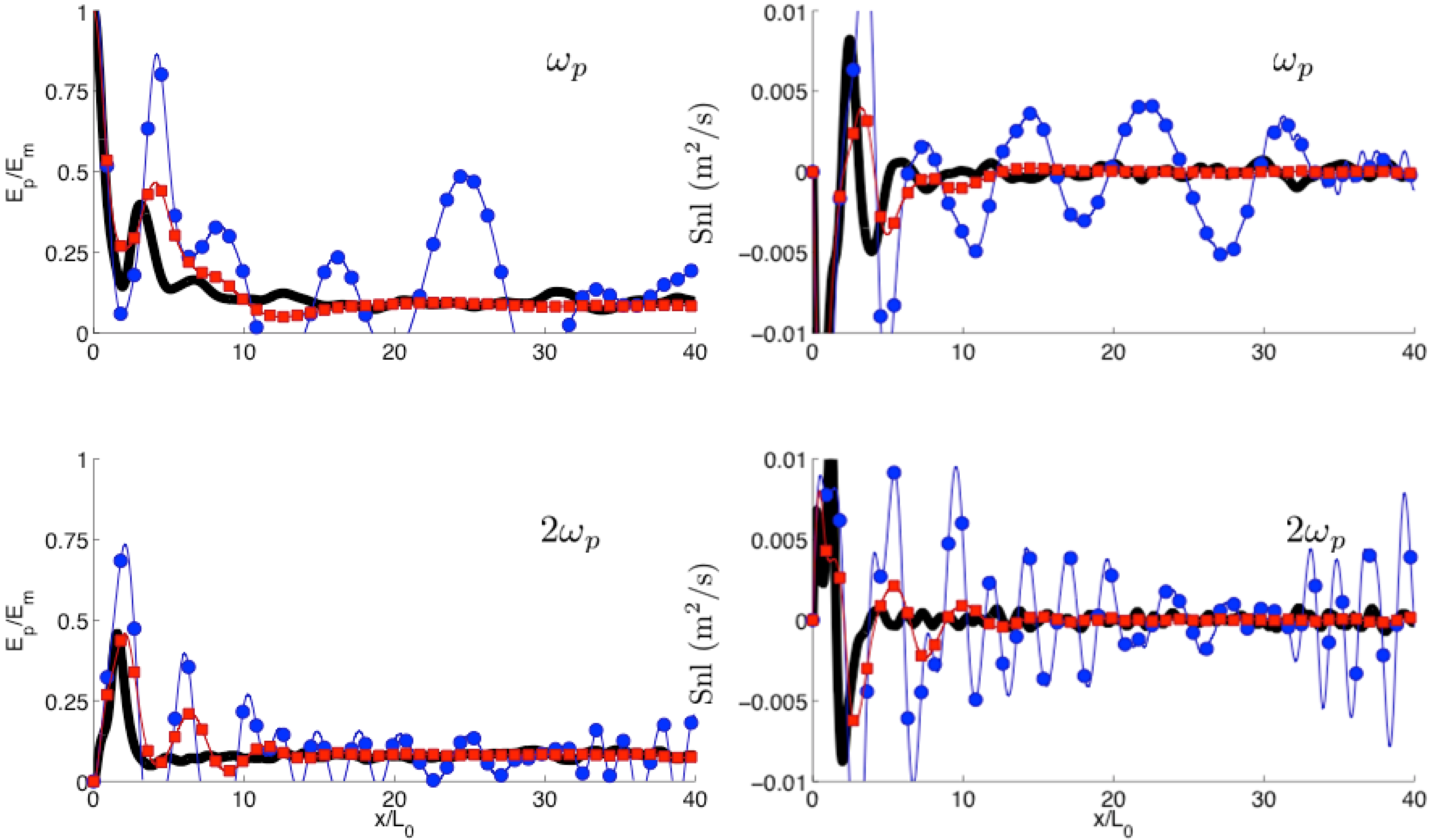
- breaks down in strongly nonlinear areas
- no empirical coefficients
- efficient
- 2-equation model



Low values
High values
Spectral density (log scale)
Nonlinear transfer
frequency [Hz]
0.4
0.3
0.2
0.1
0
20
40
60
x/L_0

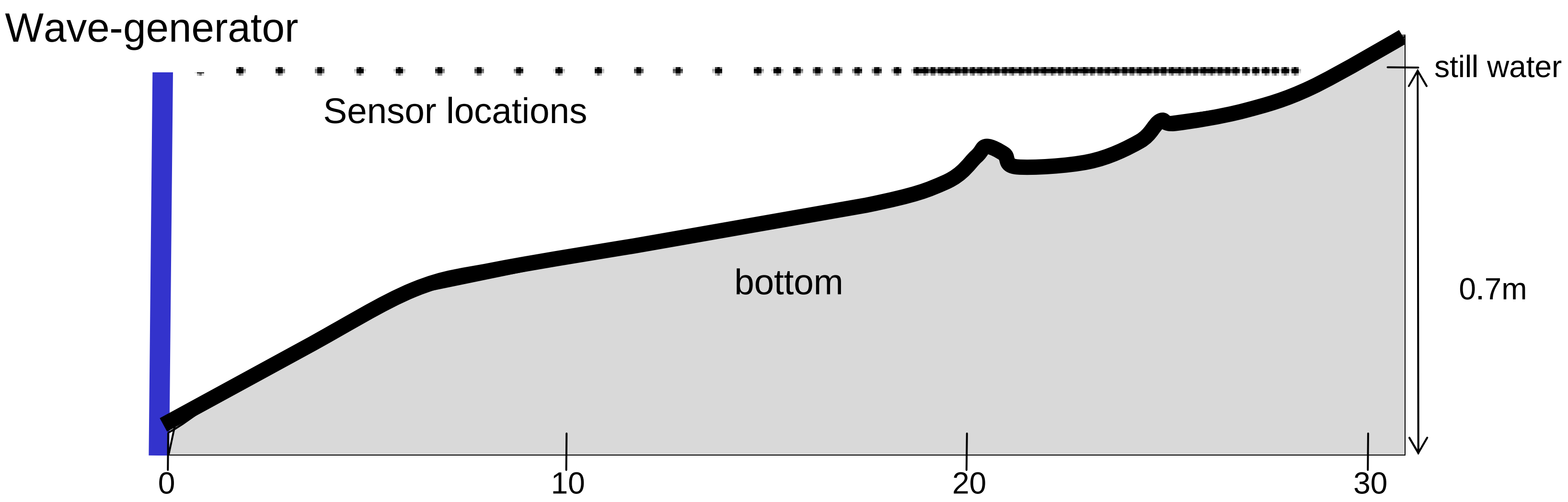
Normalized energies

Nonlinear energy transfer



Shallow-water wave evolution

Dissipation & Nonlinearity



Wave height = 0.10m

Period = 3.33 s

(weakly nonlinear and surf-zone breaking)

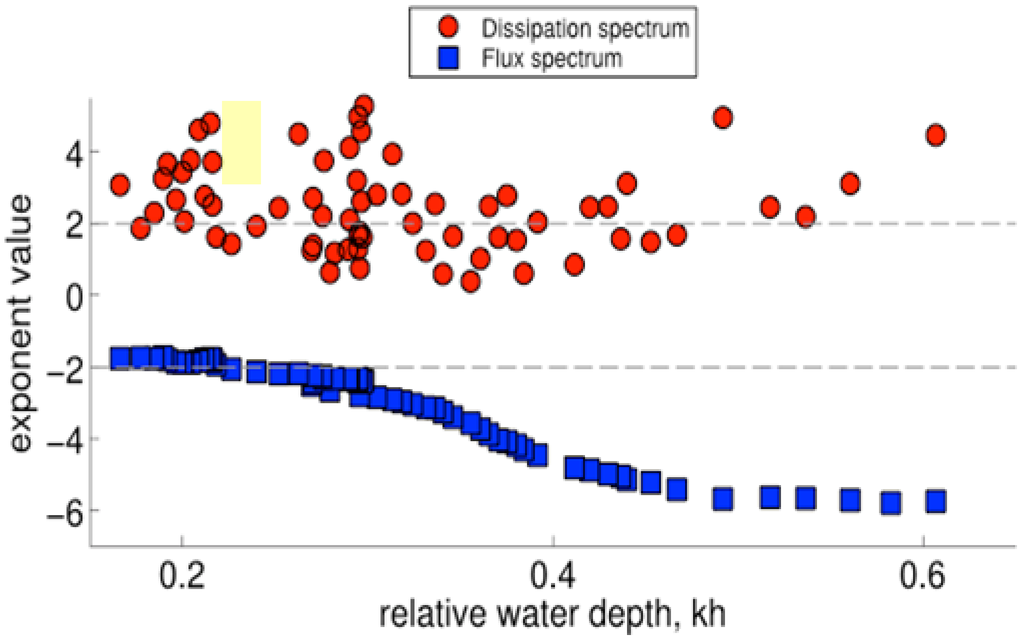
Boers, 1996, PhD thesis Delft University of Technology

Shallow-water wave evolution

Dissipation

$$\frac{d\mathcal{E}_1}{dx} = -2\nu_1\mathcal{E}_1 - 4\sum_{2+3=1}\mathcal{W}_{23}\Re\left(\int_0^x \mathcal{H}_{23}(x,s)Q_{23}(s)\,ds\right)$$

$$\overline{\nu}_1 = -\frac{1}{2\overline{\mathcal{E}}_1}\left[\frac{\Delta\mathcal{E}_1}{\Delta x} + 4\sum_{1+2=3}\overline{\mathcal{W}_{23}\Re\left(\int_0^x \mathcal{H}_{23}(x,s)Q_{23}(s)\,ds\right)}\right]$$



$$\nu(\omega) = \omega^n B$$

$$\mathcal{E}(\omega) = \omega^n \mathcal{E}_0$$

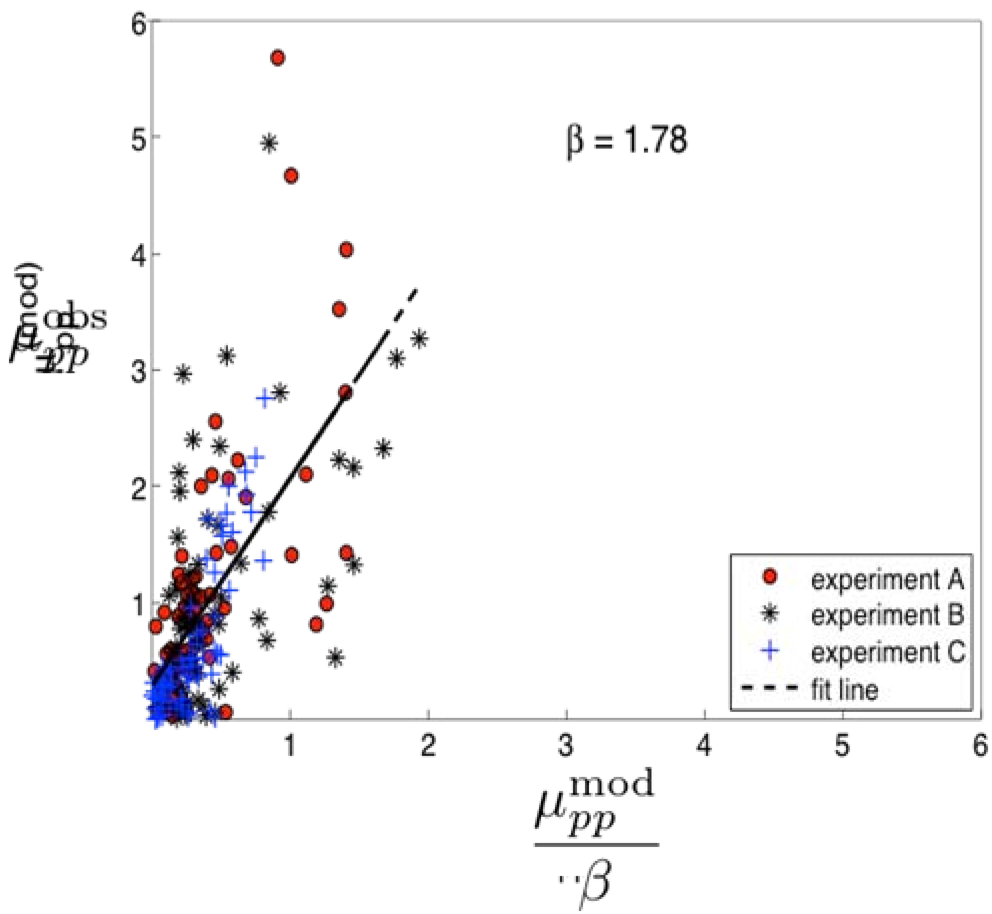
Nonlinearity

$$\frac{d}{dx}\mathcal{C}_{12} = (i\Lambda_{12} - \nu_{12} - \mu_{12})\,\mathcal{C}_{12} + 2i\mathcal{Q}_{12}$$

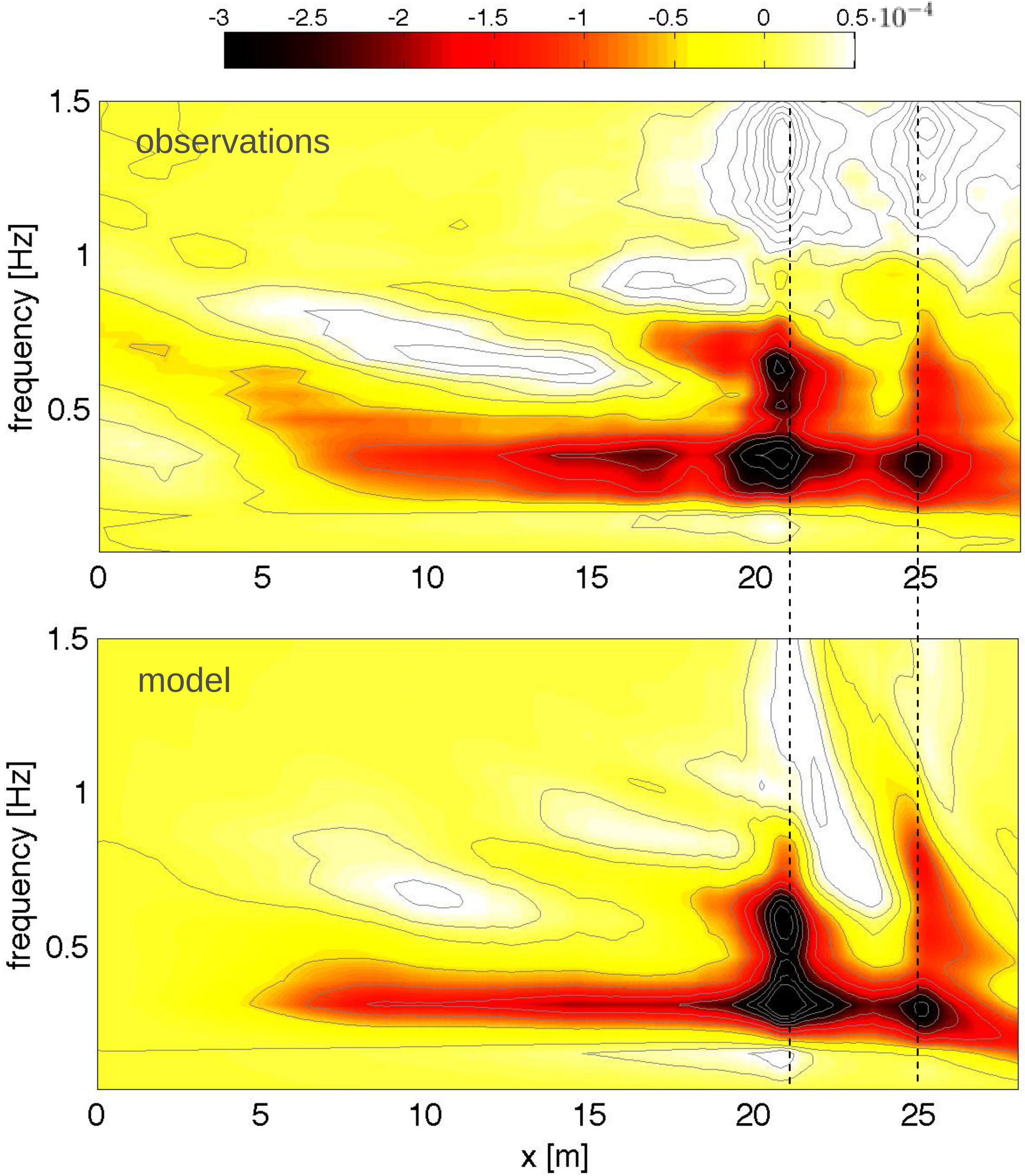
$$\int_0^x H_{12}(x,s)Q_{12}(s)\,ds = \frac{1}{2i}\mathcal{C}_{12}(x)$$

$$\overline{\mu}^{\mathrm{obs}}_{12} = -\frac{1}{\overline{\mathcal{C}}_{12}}\left[\frac{\Delta\mathcal{C}_{12}}{\Delta x} - 2i\overline{\mathcal{Q}}_{12}\right] + \left(i\overline{\Lambda}_{12} - \overline{\nu}_{12}\right)$$

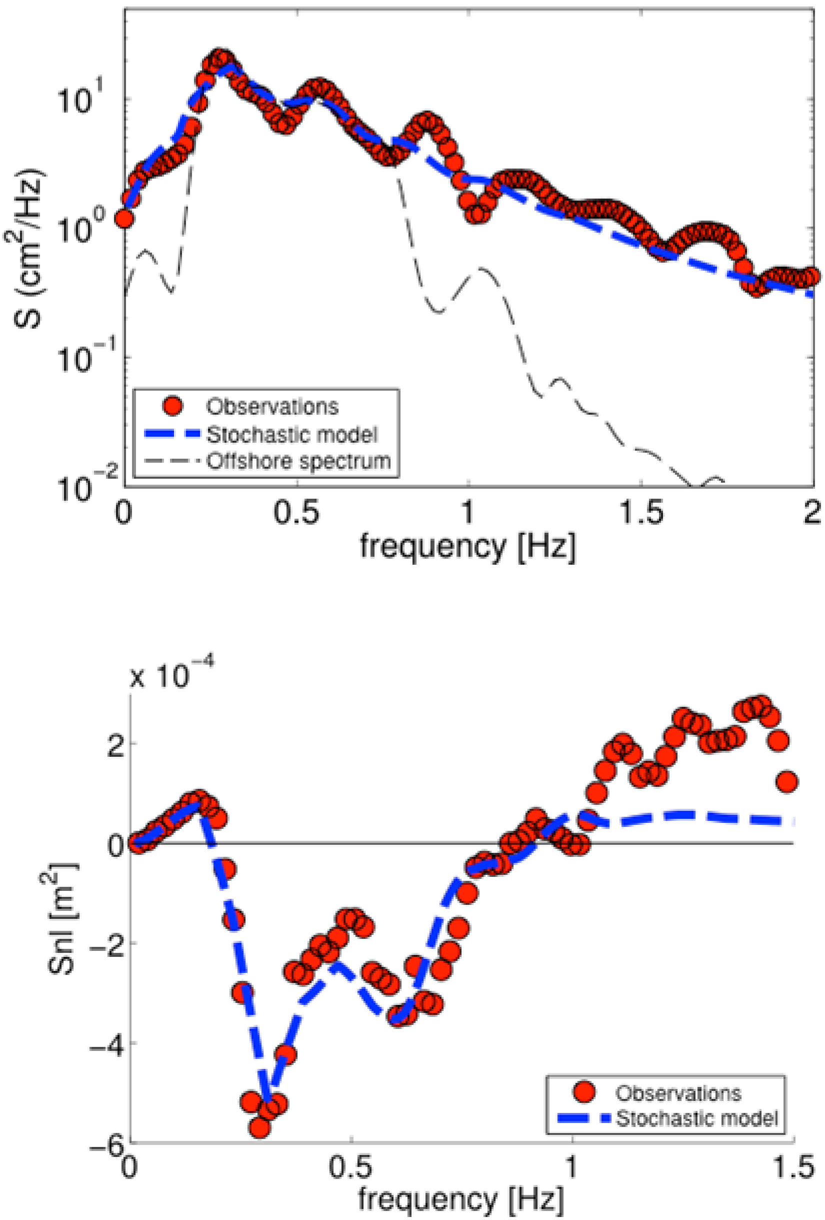
$$\mu^{\mathrm{mod}}_{12} = \beta\frac{|\mathrm{Dis}_1 + \mathrm{Dis}_2 + \mathrm{Dis}_{(1+2)}| + |\mathrm{Nl}_1 + \mathrm{Nl}_2 + \mathrm{Nl}_{(1+2)}|}{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_{(1+2)}}$$



Nonlinear Transfer

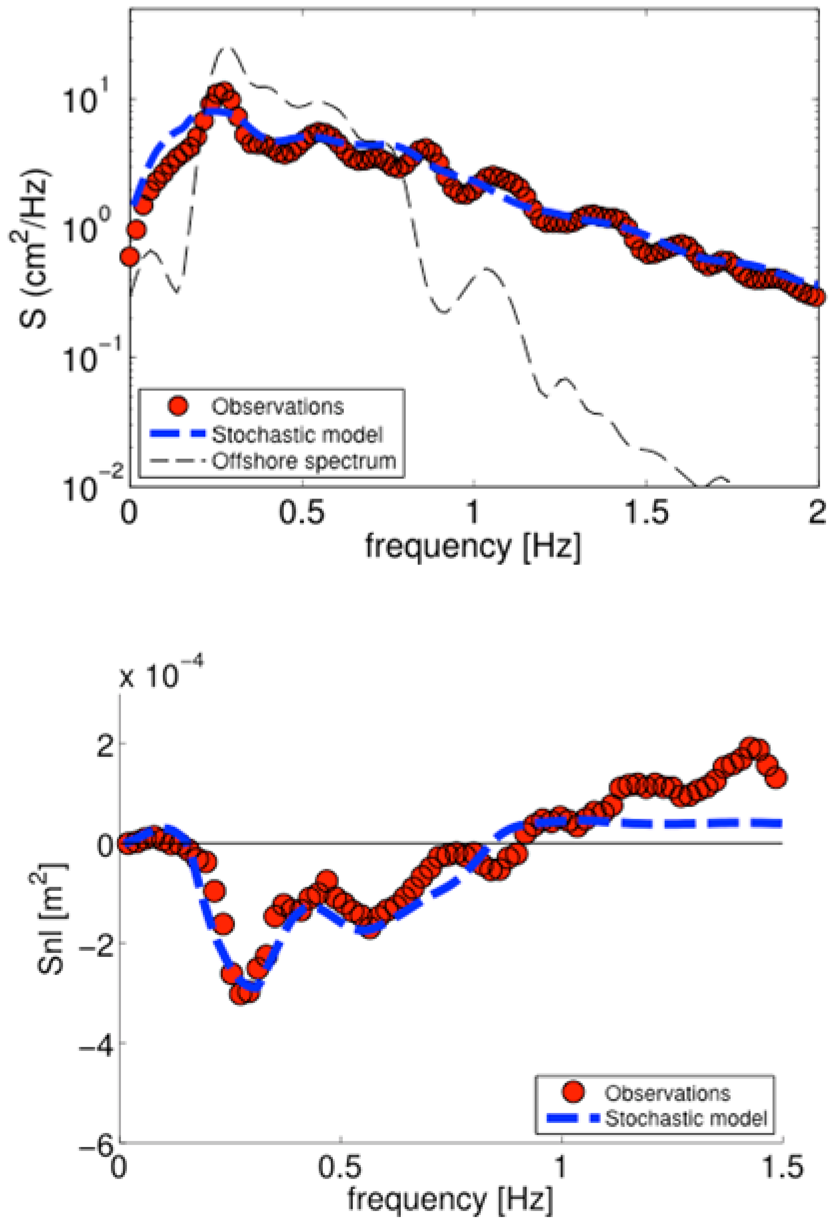


x = 21m

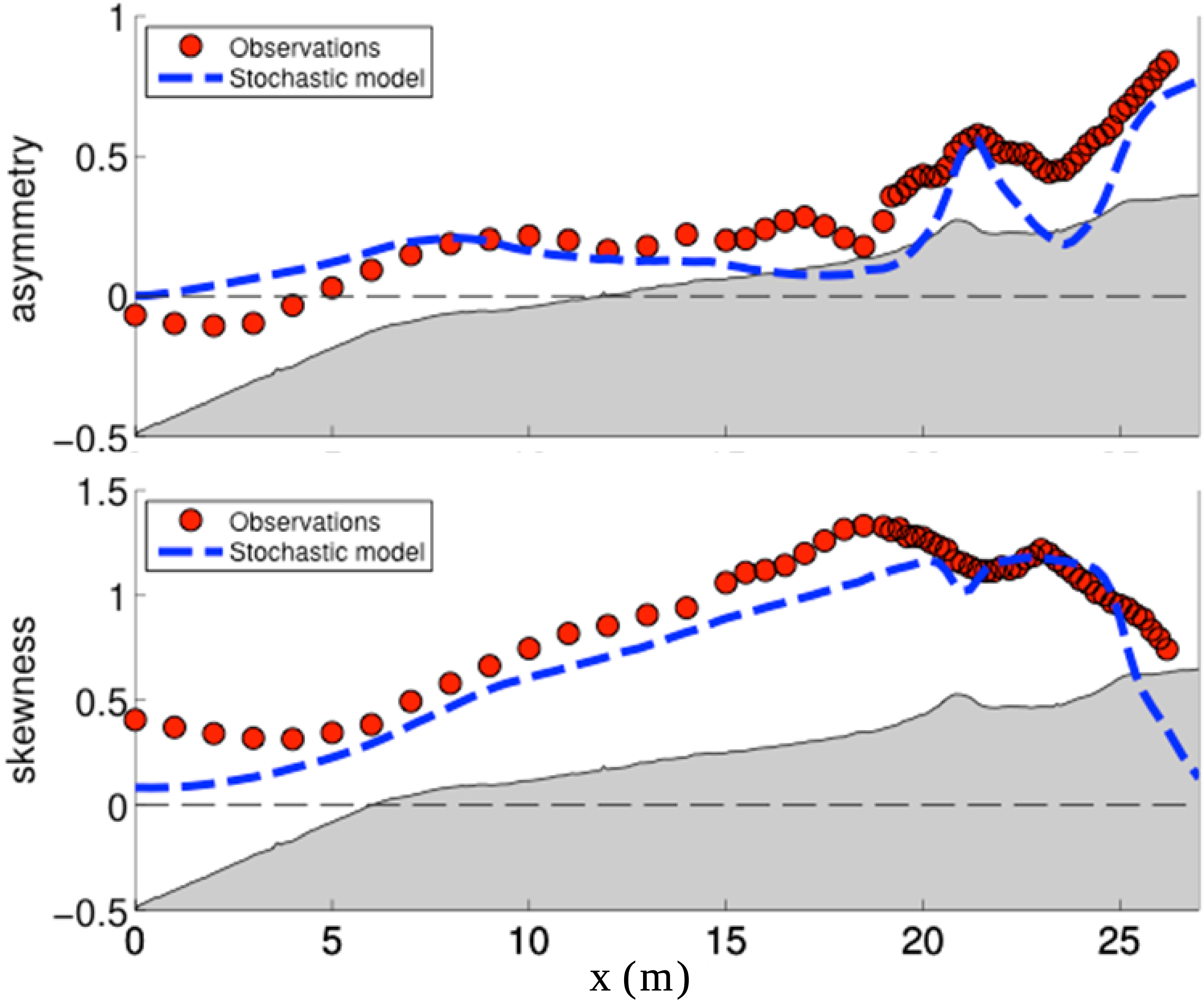


S (cm²/Hz)
frequency [Hz]
Observations
Stochastic model
Offshore spectrum
x 10⁻⁴
Snl [m²]
frequency [Hz]
Observations
Stochastic model

x = 25m



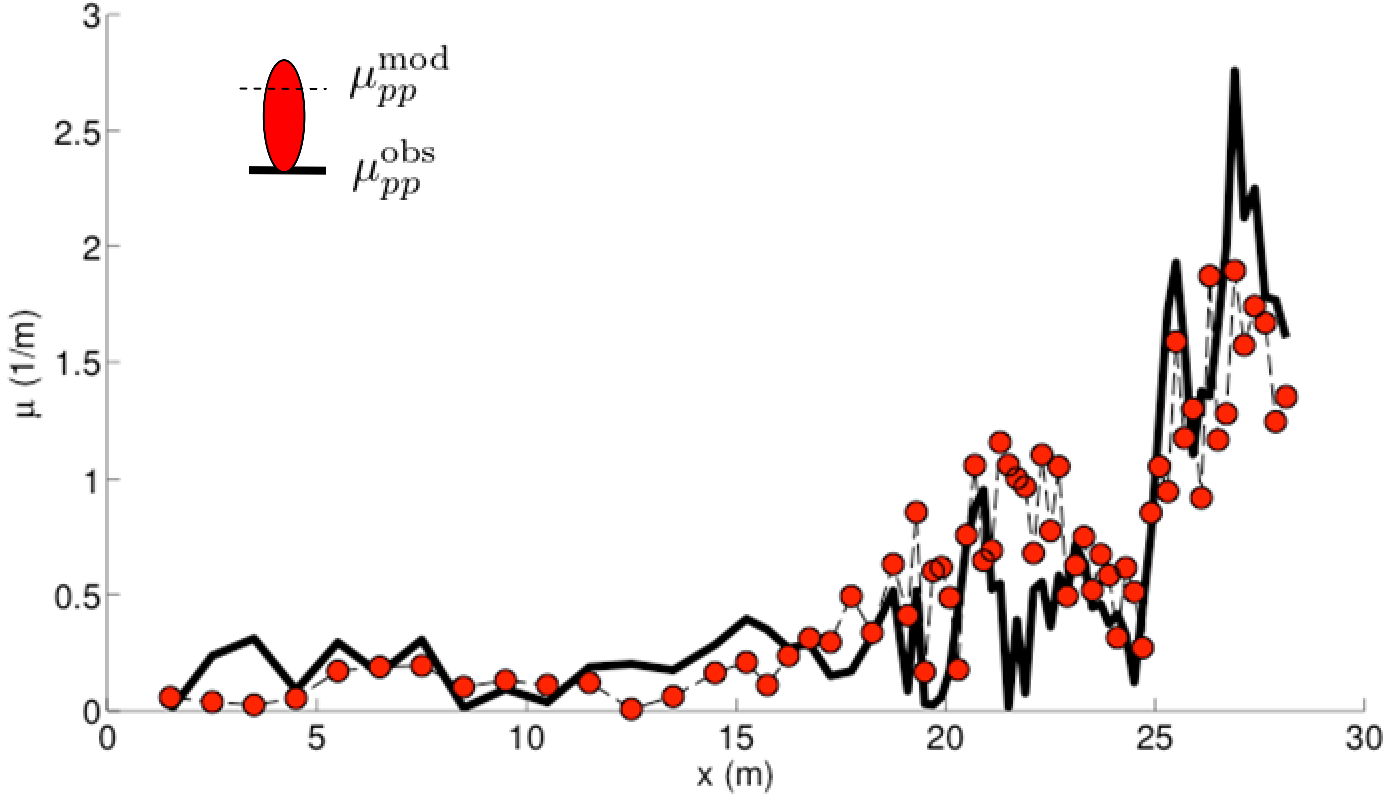
S (cm²/Hz)
frequency [Hz]
Observations
Stochastic model
Offshore spectrum
x 10⁻⁴
Snl [m²]
frequency [Hz]
Observations
Stochastic model



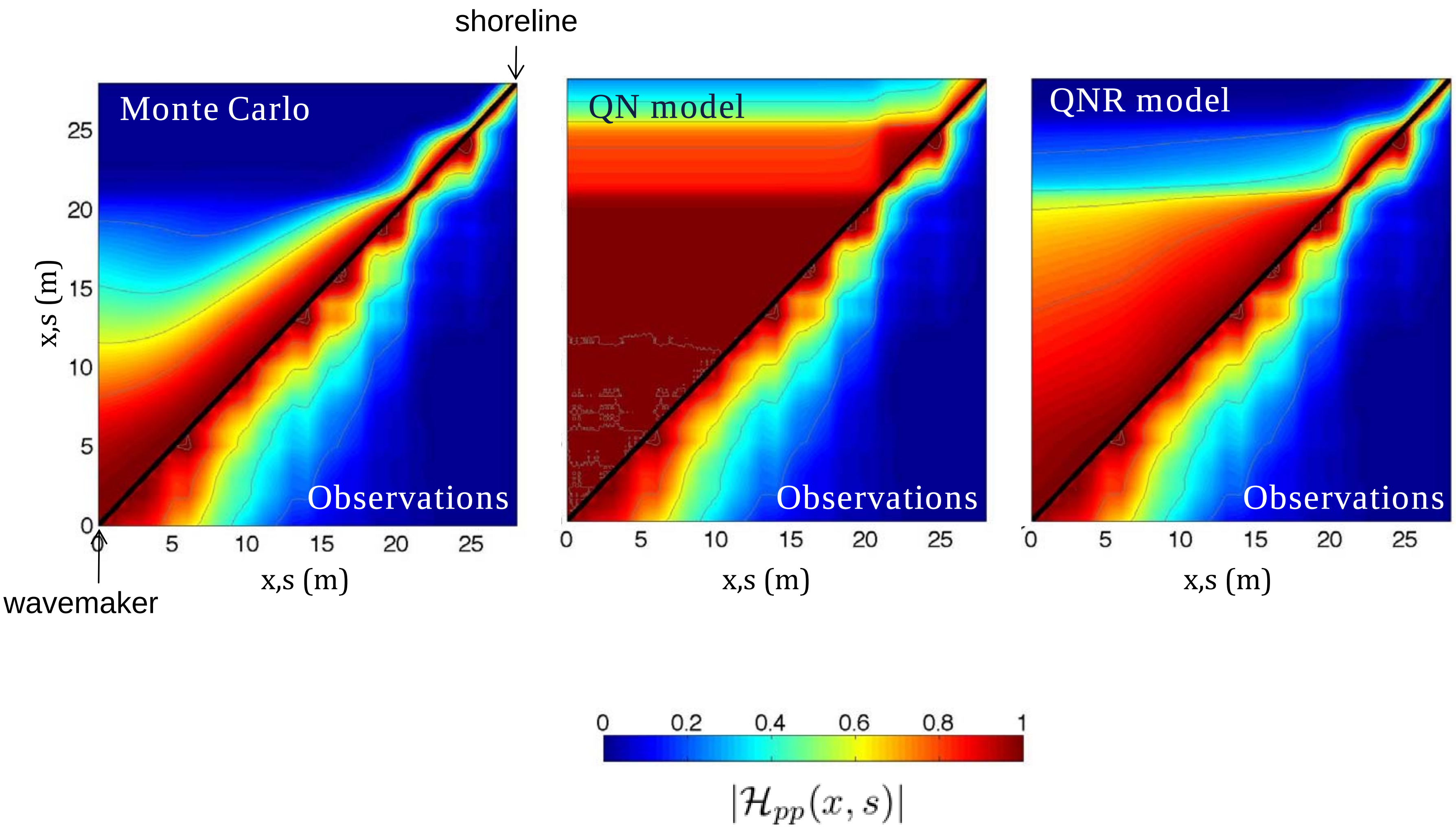
Observations
Stochastic model
asymmetry
1
0.5
0
−0.5
Observations
Stochastic model
skewness
1.5
1
0.5
0
−0.5
0
5
10
15
20
25
x (m)

Relaxation length scale

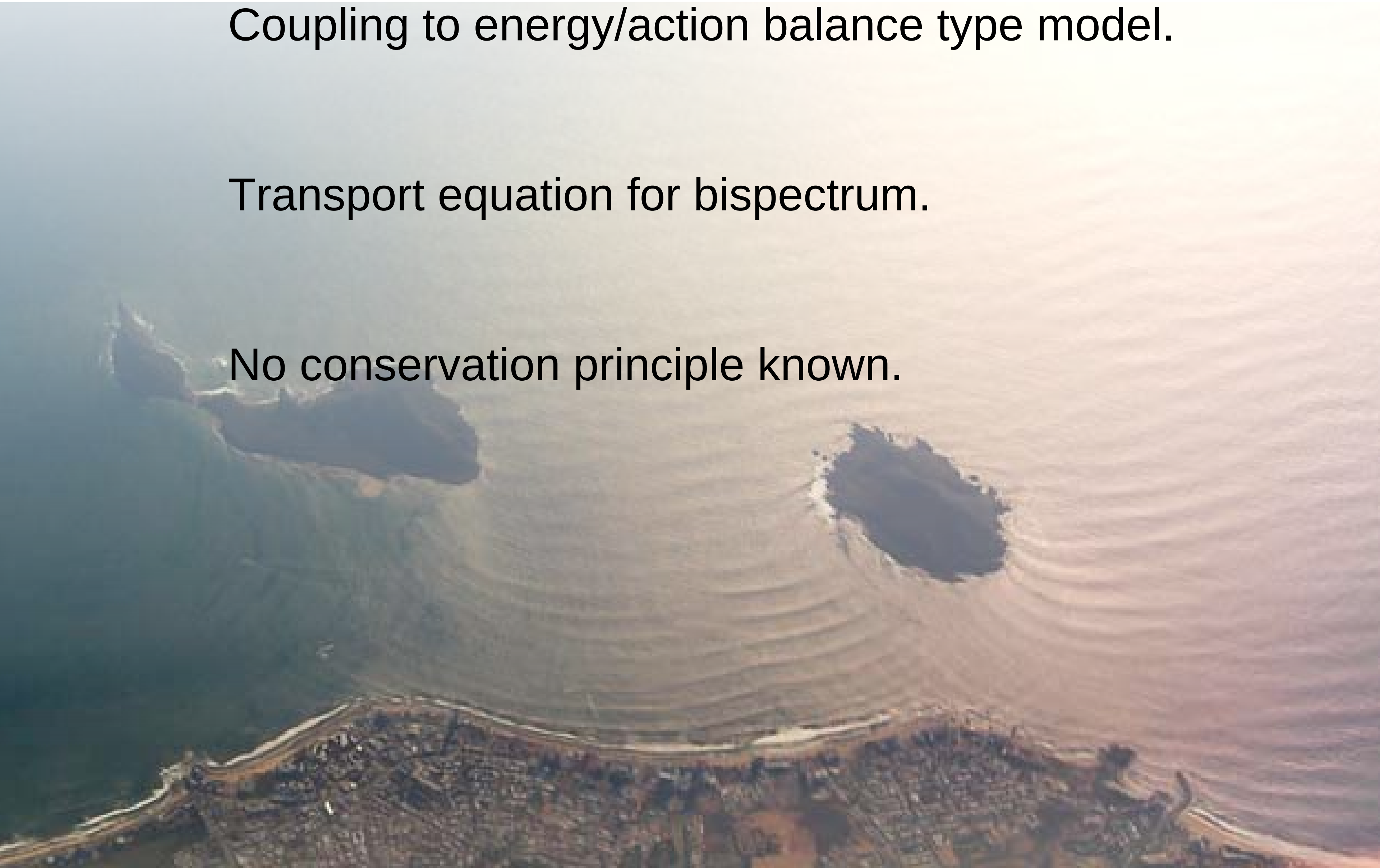
$$\overline{\mu}_{12}^{\text{obs}} = -\frac{1}{\overline{\mathcal{C}}_{12}}\left[\frac{\Delta\mathcal{C}_{12}}{\Delta x} - 2i\overline{\mathcal{Q}}_{12}\right] + \left(i\overline{\Lambda}_{12} - \overline{\nu}_{12}\right) \qquad \mu_{12}^{\text{mod}} = \beta\frac{|\text{Dis}_1 + \text{Dis}_2 + \text{Dis}_{(1+2)}| + |\text{Nl}_1 + \text{Nl}_2 + \text{Nl}_{(1+2)}|}{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_{(1+2)}}$$



Wave ‘memory’


shoreline
Monte Carlo
QN model
QNR model
Observations
Observations
Observations
x,s (m)
x,s (m)
x,s (m)
x,s (m)
wavemaker
0
5
10
15
20
25
0
0.2
0.4
0.6
0.8
1
$|\mathcal{H}_{pp}(x,s)|$

Outlook operational wave models

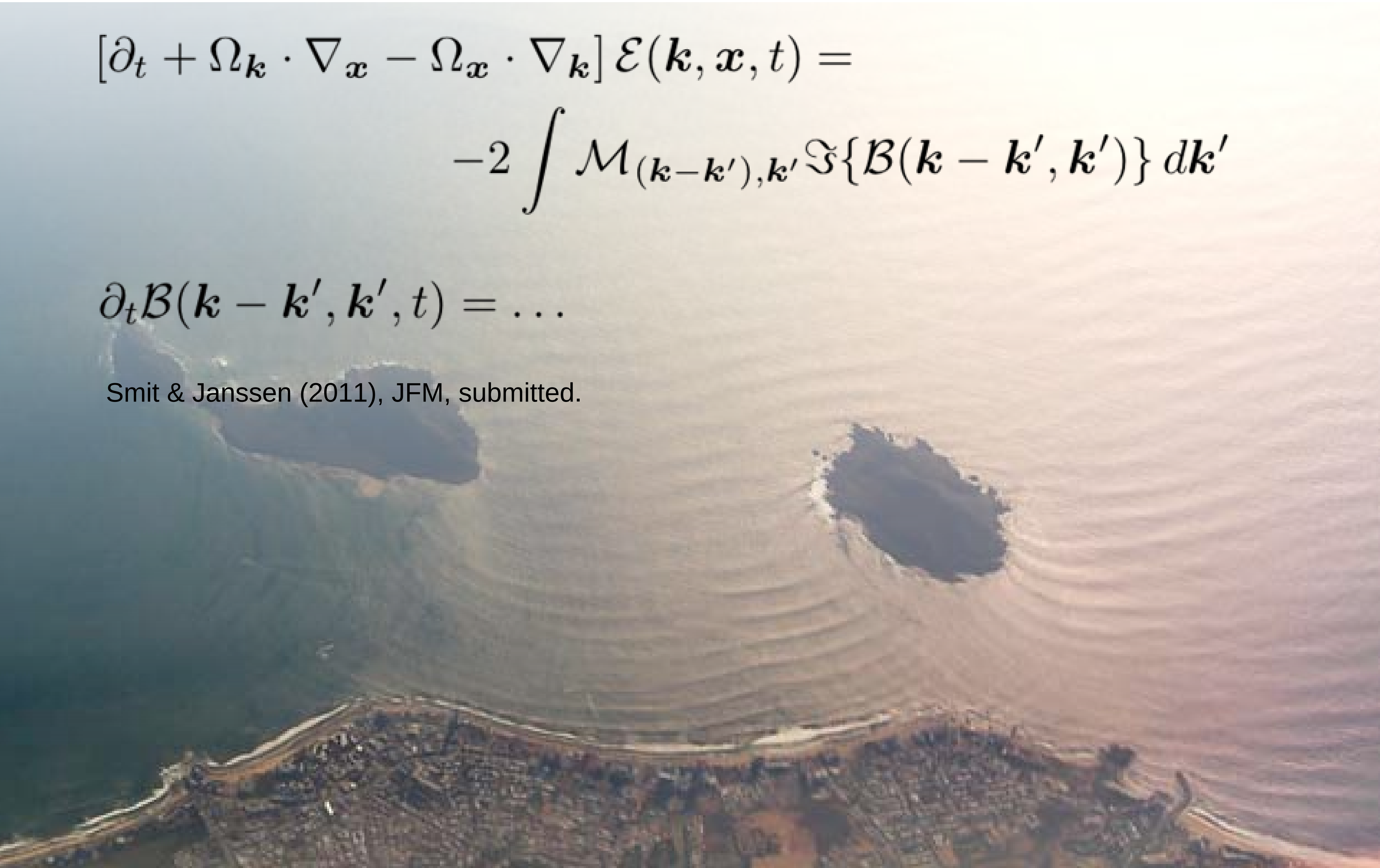


Coupling to energy/action balance type model.

Transport equation for bispectrum.

No conservation principle known.

Outlook operational wave models



$$[\partial_t + \Omega_{\boldsymbol{k}} \cdot \nabla_{\boldsymbol{x}} - \Omega_{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{k}}]\, \mathcal{E}(\boldsymbol{k}, \boldsymbol{x}, t) = -2 \int \mathcal{M}_{(\boldsymbol{k}-\boldsymbol{k}'), \boldsymbol{k}'} \Im\{\mathcal{B}(\boldsymbol{k}-\boldsymbol{k}', \boldsymbol{k}')\}\, d\boldsymbol{k}'$$

$$\partial_t \mathcal{B}(\boldsymbol{k}-\boldsymbol{k}', \boldsymbol{k}', t) = \ldots$$

Smit & Janssen (2011), JFM, submitted.

Conclusions

A one-point closure approximation can be effective and efficient (dissipation matters).

To incorporate history of evolution, a coupled spectrum/bispectrum model is needed (2-equation).

This requires transport equations for higher-order correlations.