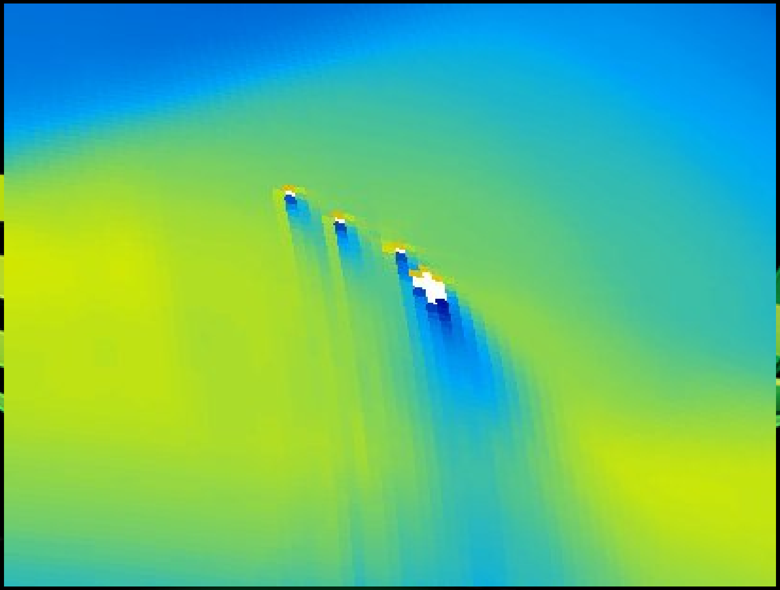


Implementation of the Spherical Multiple-Cell Grid in the WAVEWATCH III Model

Jian-Guo Li 1 November 2011

Contents

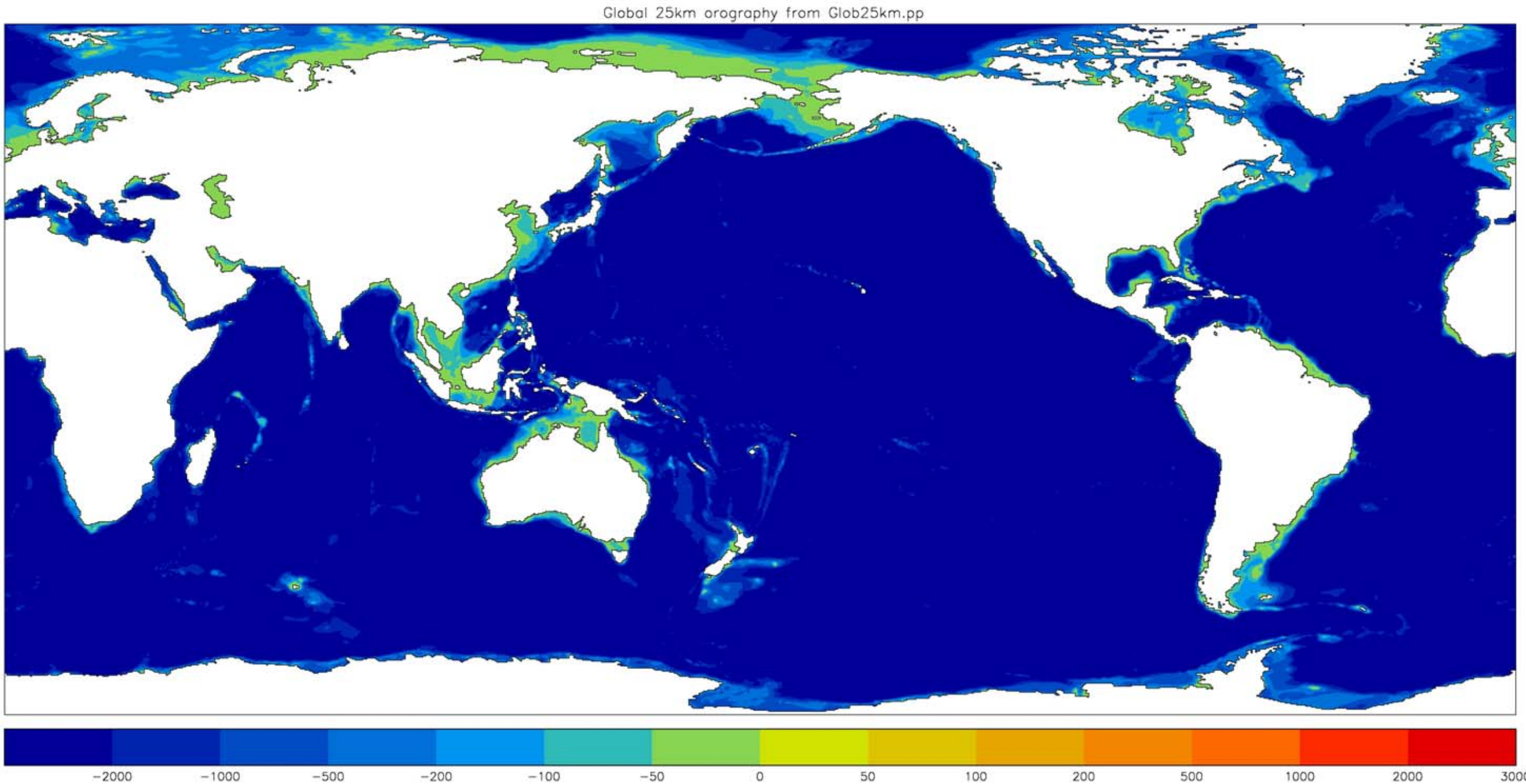
This presentation covers the following areas

- The global 25km and SMC grid wave models
- Comparison of two models via altimeter and spectral buoy data
- Wave spectral transport on 6-25km SMC grid
- Summary and conclusions

The 25km lat-lon grid model

Met Office

Resolves small islands and fine coastlines, required to improve commercial and defence applications in international waters.

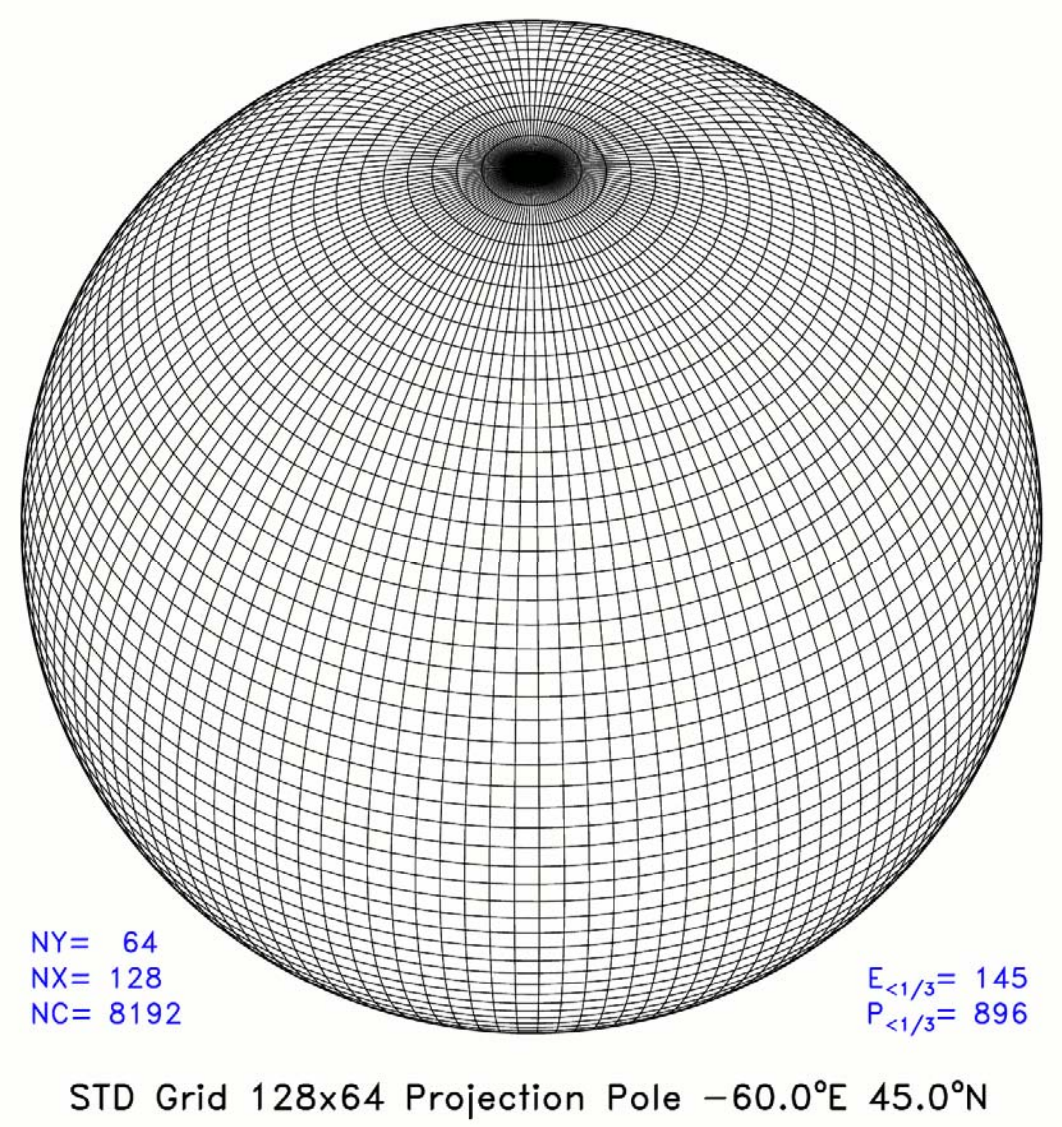


Computing cost problem

The 25km global model is much more expensive than the 60km model.

- 12hr hindcast of 60km ~ 140 s/task (8 pes on IBM)
- 12hr hindcast of 25km ~ 2000 s/task (14 times)
- Number of grids (1024x688 : 432x288 ~ 5.7)
- Number of directions (36 : 24 = 1.5)
- Reduction of time step (360 : 600 s = 0.6 Dec08 180 : 600 s = 0.3 Sept10)
- 5.7x1.5/0.6 ~ 14 (28).

CFL restriction in standard grid



Transportation equation in spherical system, 1 of 900:

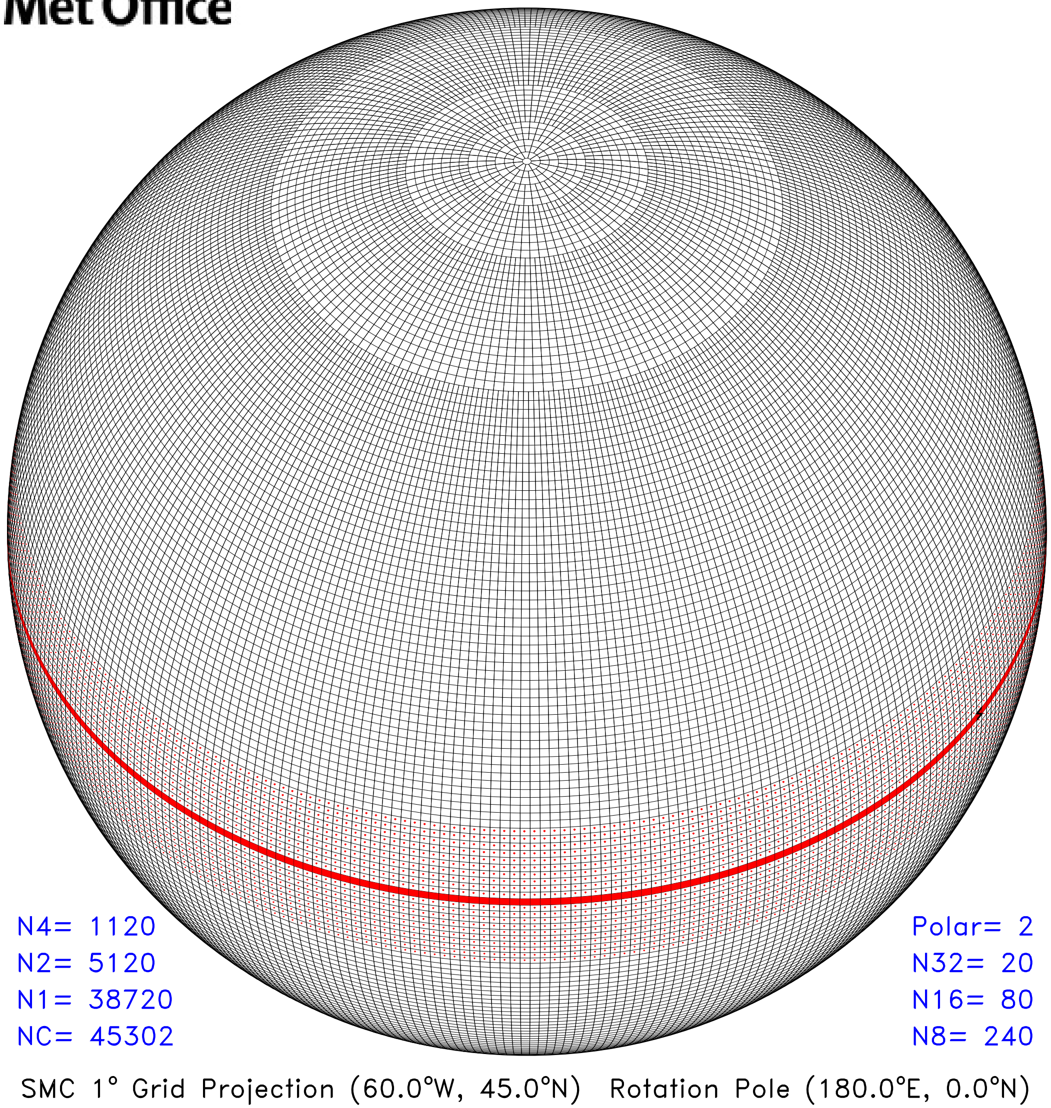
$$\frac{\partial \psi}{\partial t} + \frac{\partial (u\psi)}{\partial x} + \frac{\partial (\upsilon \psi \cos\varphi)}{\cos\varphi \partial y} = 0$$

Severe CFL restriction on Eulerian advection time step at high latitudes. Hence wave models stop at ~ 82º N.

The Pole is a singular point Flow has to go around it, not crossing it.

Spherical Multiple-Cell grid

Met Office



- Merged cells at high latitudes to relax CFL limit on time step.
- Introduce round polar cells with integral equation to avoid polar blocking and singularity.

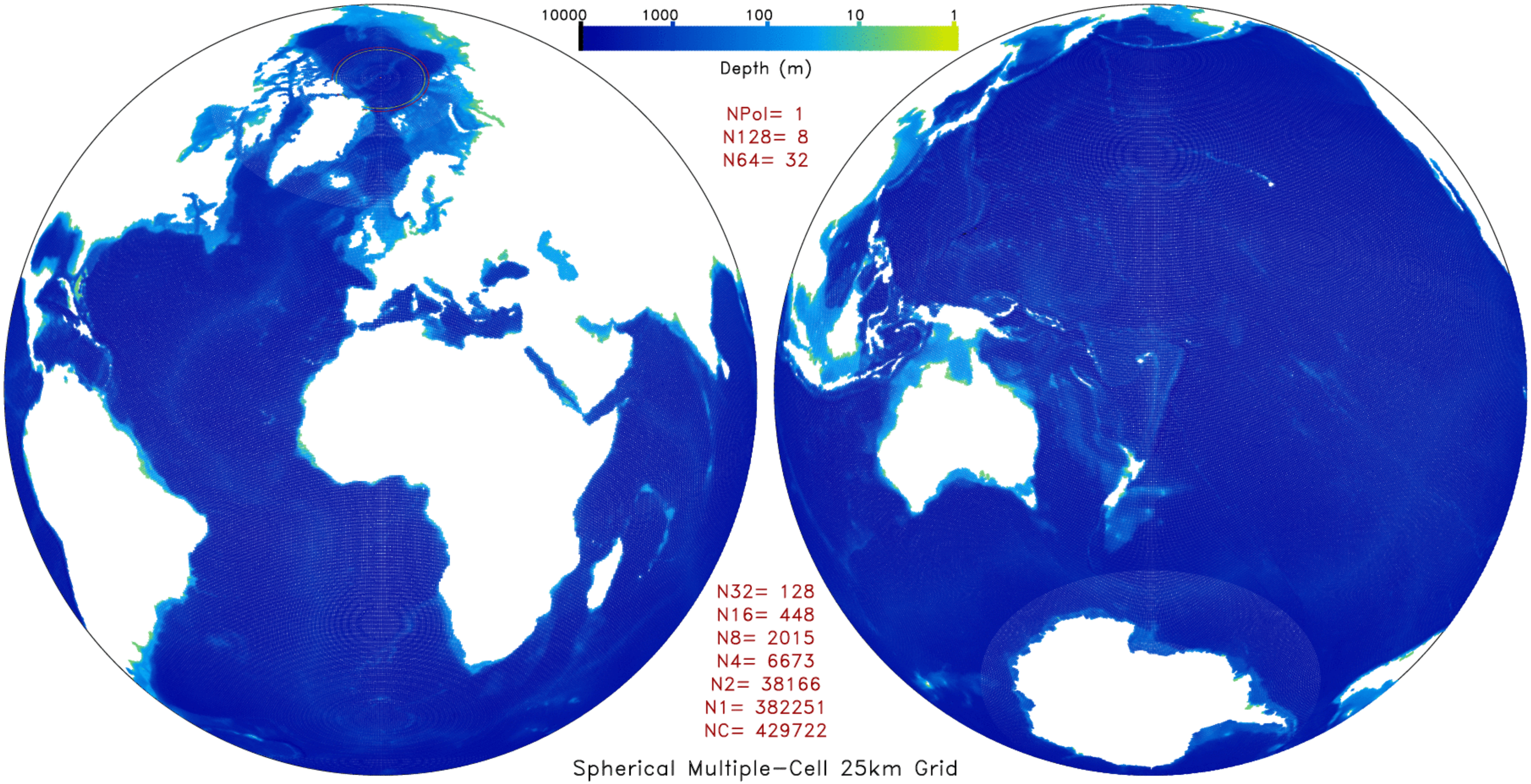
$$\frac{\partial}{\partial t}\iint_A \psi\, dA = -\oint_{C_A} \psi \mathbf{v}\cdot d\mathbf{s}$$

$$\psi_P^{n+1} - \psi_P^{n} = \pm\frac{\Delta t}{A_P}\sum_{i=1}^{m}\psi_i^{*}\upsilon_i \Delta s_i$$

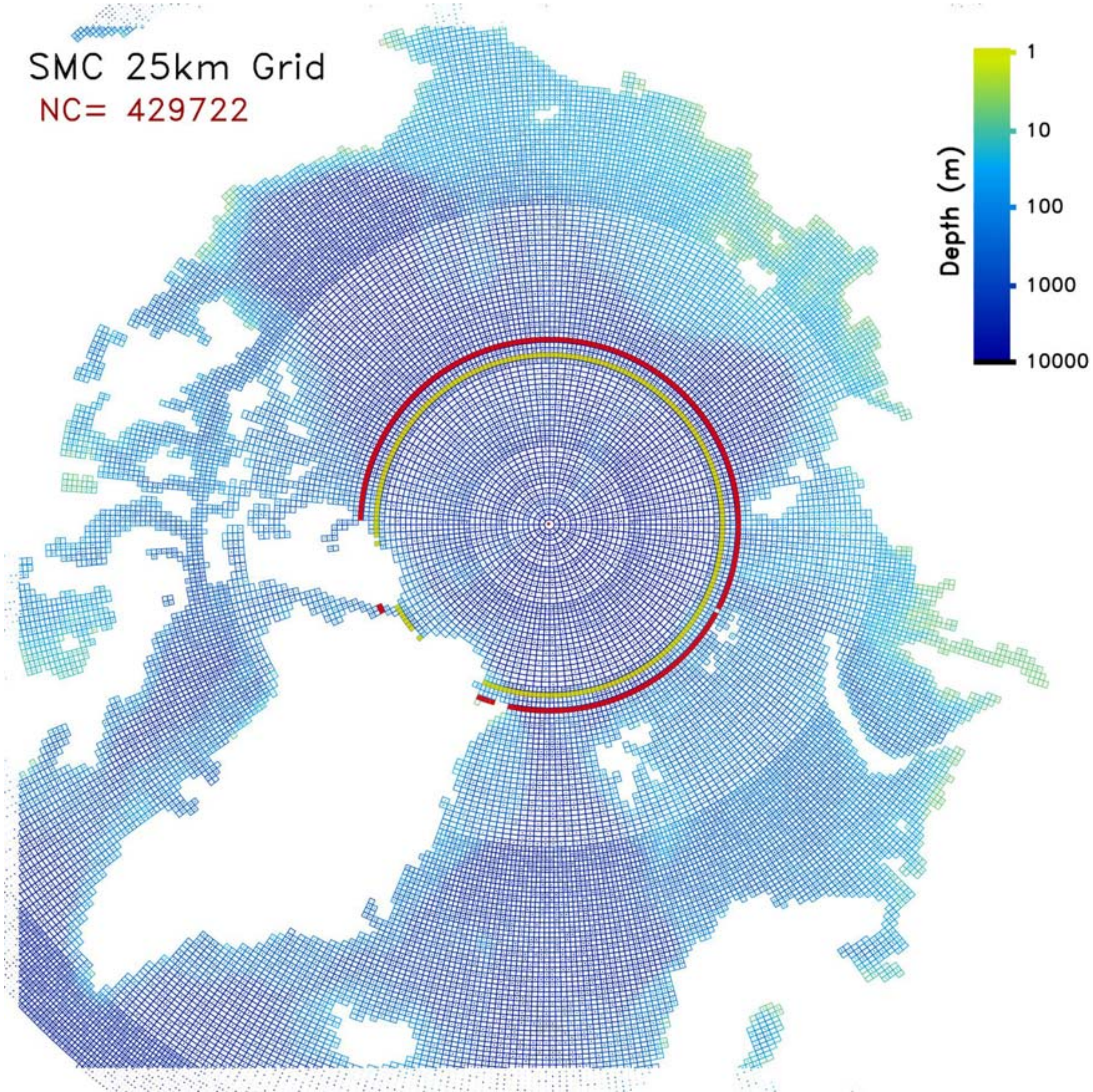
•More details please see: Li, J.G. 2011: *Mon. Wea. Rev.*, **139**, 1536-1555.

The 25km SMC grid

Total cell 429,722 ~ 55% of the lat-lon grid 1024x768



The 25km SMC grid - Arctic



Conforms to lat-lon grid but merges at high latitudes to relax CFL restriction ~4x.

Finite-difference on SMC grid as on lat-lon grid.

Polar cell to avoid singularity.

Arctic map-east zone for vector definition.

Surface wave energy balance equation on SMC grid

- Ocean surface wave spectral energy balance equation in spherical coordinate system:

$$\frac{\partial \psi}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial (F_y \cos\varphi)}{\cos\varphi \partial y} + \frac{\partial (\dot{k}\psi)}{\partial k} + \frac{\partial (\dot{\theta}\psi)}{\partial \theta} = S$$

$$F_x \equiv (u + U)\psi - D_x \partial\psi / \partial x$$

$$F_y \equiv (\upsilon + V)\psi - D_y \partial\psi / \partial y$$

Advection and diffusion terms are merged.

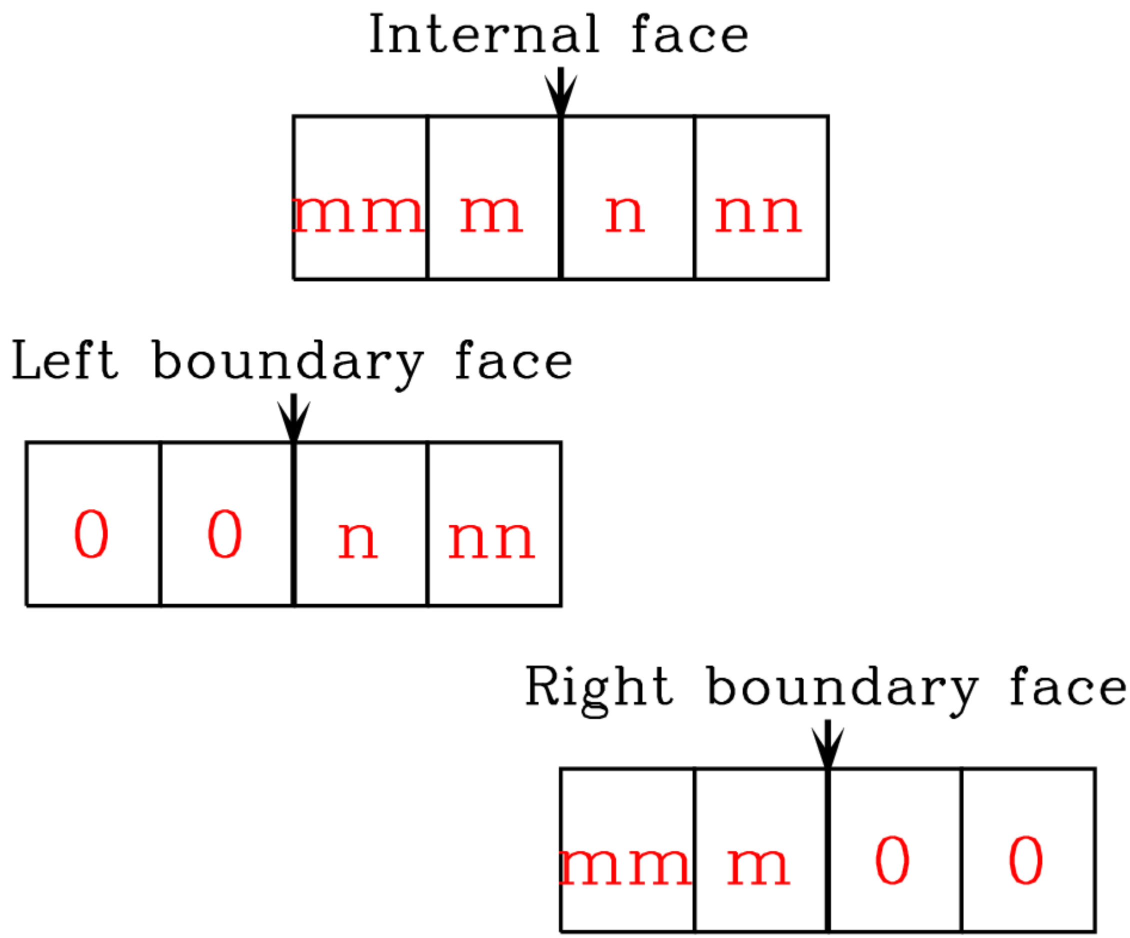
Great-circle turning is added to refraction.

Unstructured SMC grid flux face pointer array

- Internal and boundary faces are treated alike in 1-D array. No boundary for global model.

- Single point island is extended by 0-cells, allowing single-island blocking.

- Two-D spherical surface advection is done by 4 loops: u- and v-face flux loops and 2 cell update loops.



Upstream Non-Oscillatory 2nd Order (UNO2) Advection Scheme

$$\psi_j^{n+1} = \psi_j^n + \left(u_{j-1/2}\psi_{j-1/2}^{MF} - u_{j+1/2}\psi_{j+1/2}^{MF}\right)\Delta t/\Delta x_j$$

$$\psi_{j+1/2}^{MF} = \psi_C^n + \left(x_{MF} - x_C\right)G_C$$

$$x_{MF} - x_C = 0.5 sign\left(u_{j+1/2}\right)\left(\Delta x_C - \left|u_{j+1/2}\right|\Delta t\right)$$

$$G_C = Sign\left(G_{DC}\right)\min\left(\left|G_{DC}\right|, \left|G_{CU}\right|\right)$$

$$G_{AB} \equiv \left(\psi_A - \psi_B\right)/\left(x_A - x_B\right)$$

UNO2

You Know Too

Details see:

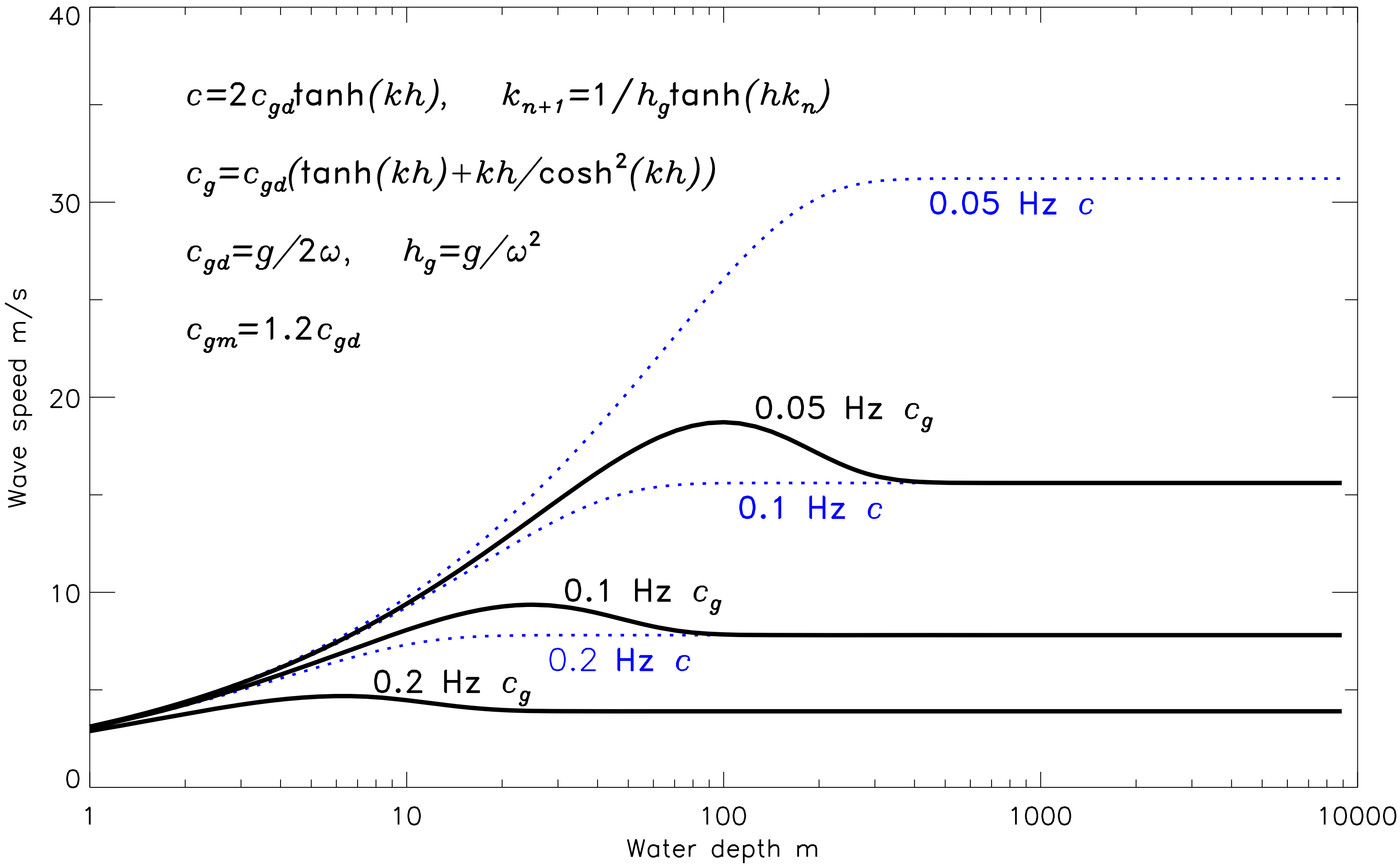
Li, J.G. (2008) *Mon. Wea. Rev.*, **136**, 4709-4729.

Upstream, Central and Downstream cells relative to velocity *u*.

$U_{j+1/2}$ →

U	C	D
j -1	j	j +1

Surface wave speed changes with depth, near shore refraction

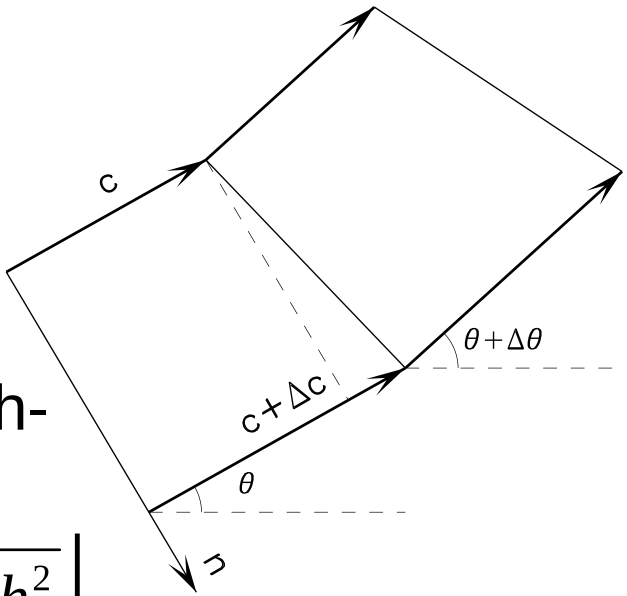


Refraction by rotation and depth gradient limiter

Ocean surface wave energy travels at depth dependent group speed and refraction occurs in shallow waters.

$$\dot{\theta}_{rfr} = -\xi \mathbf{n} \cdot \nabla h - \mathbf{n} \cdot \nabla U_k$$

$$\xi = \omega / \sinh(2kh)$$



Refraction by rotation and limited to h-gradient direction.

$$\alpha = \cos^{-1}\left[-\left(h_x \cos\theta + h_y \sin\theta\right) \Big/ \sqrt{h_x^2 + h_y^2} \right]$$

$$\Delta\theta_{mxrfr} = \eta \min(\alpha, \pi - \alpha)$$

Wave travels along great circle rather than fixed local direction

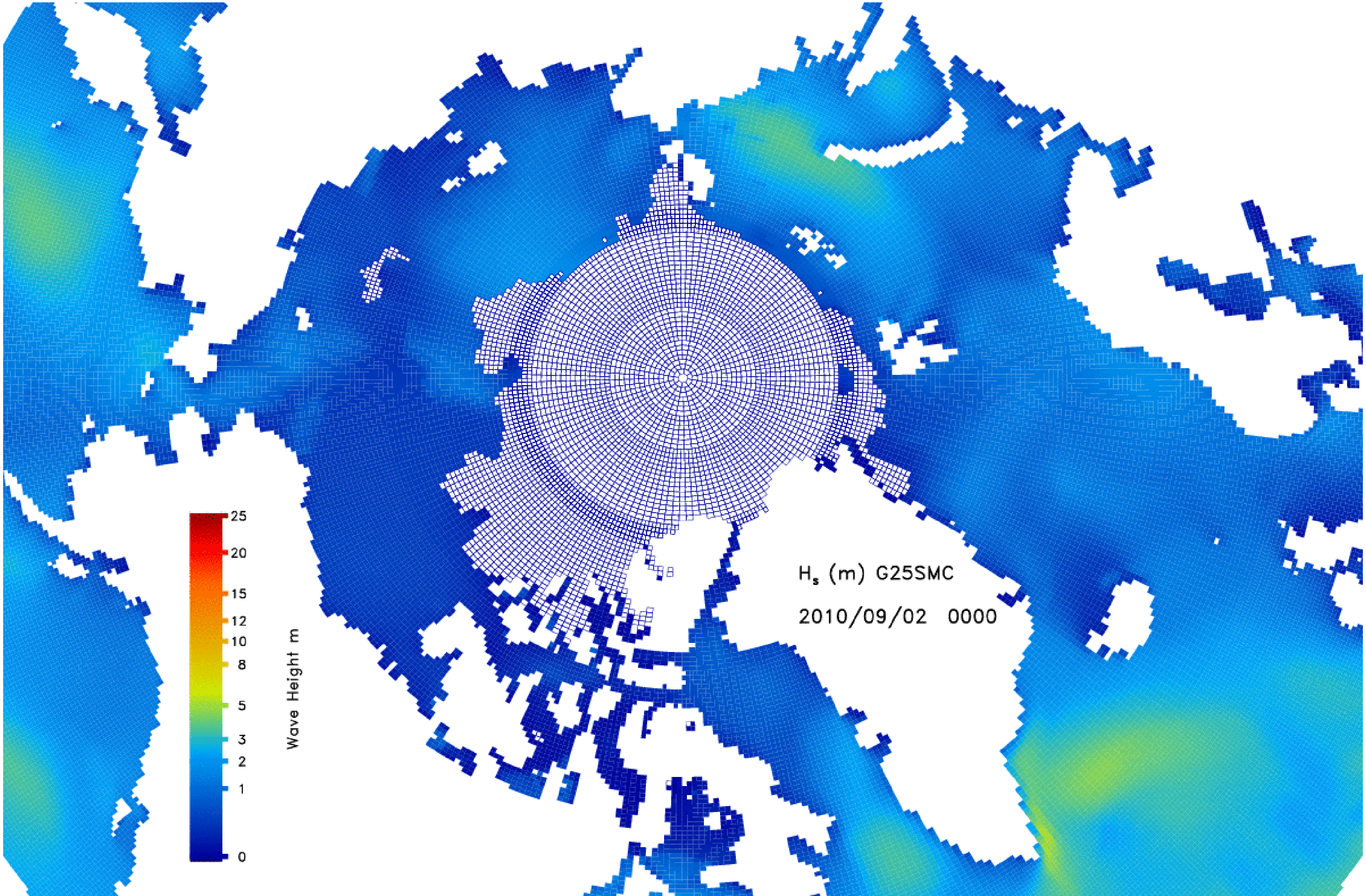
- Great circle distance is the shortest distance on a sphere, hence the nature likes it.
- Wave spectral component is defined in relative to its local east so it deviates from its direction in transport. The direction changing rate is

$$\dot{\theta}_{gct} = -\dot{\gamma}\cos\theta\tan\varphi, \qquad \dot{\gamma} \equiv c_g / r_{Earth}$$

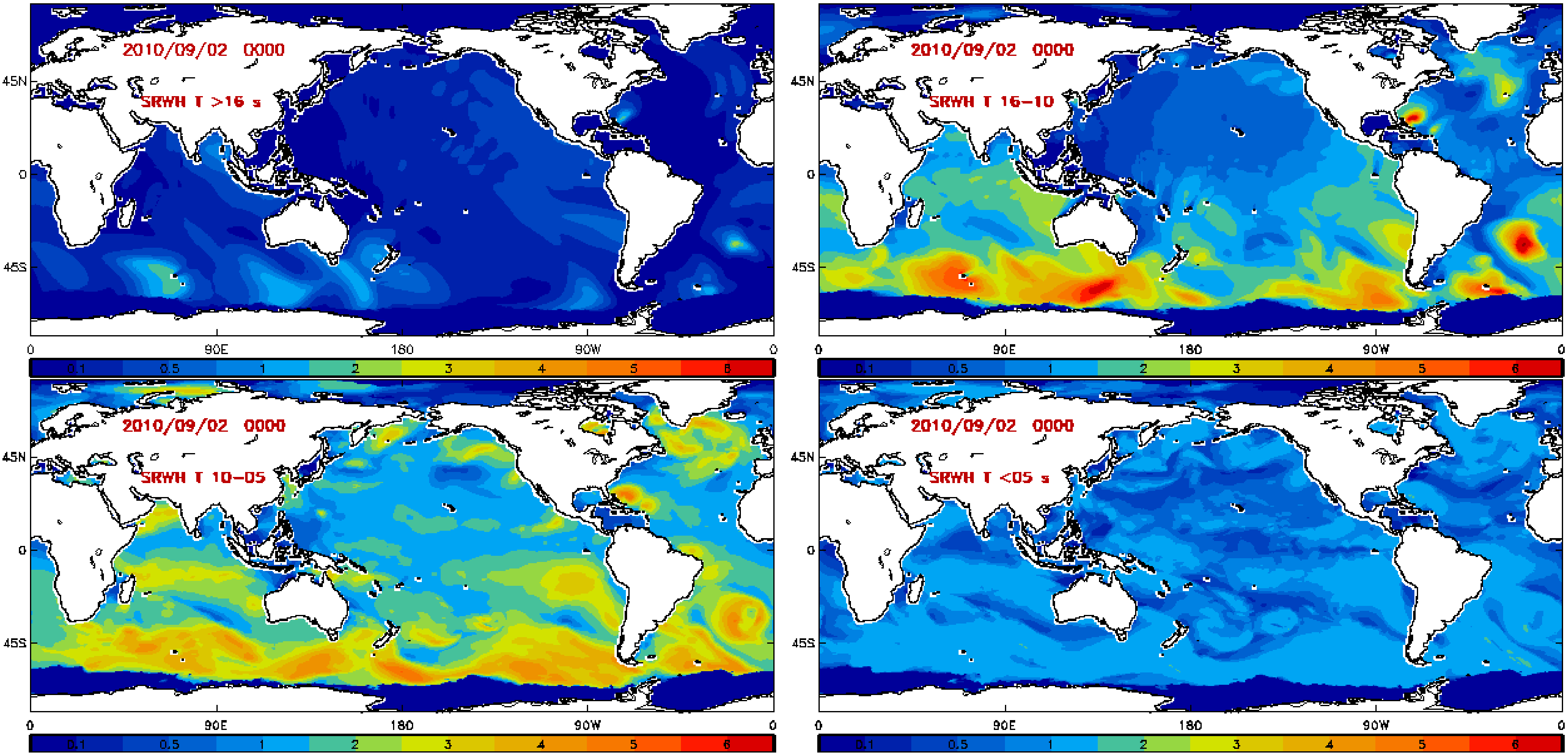
Refraction and GC turning may be treated as a single rotation, rather than directional advection.

$$\Delta\theta = \left(\dot{\theta}_{gct} + \dot{\theta}_{refr}\right)\Delta t$$

SWH output in Arctic region

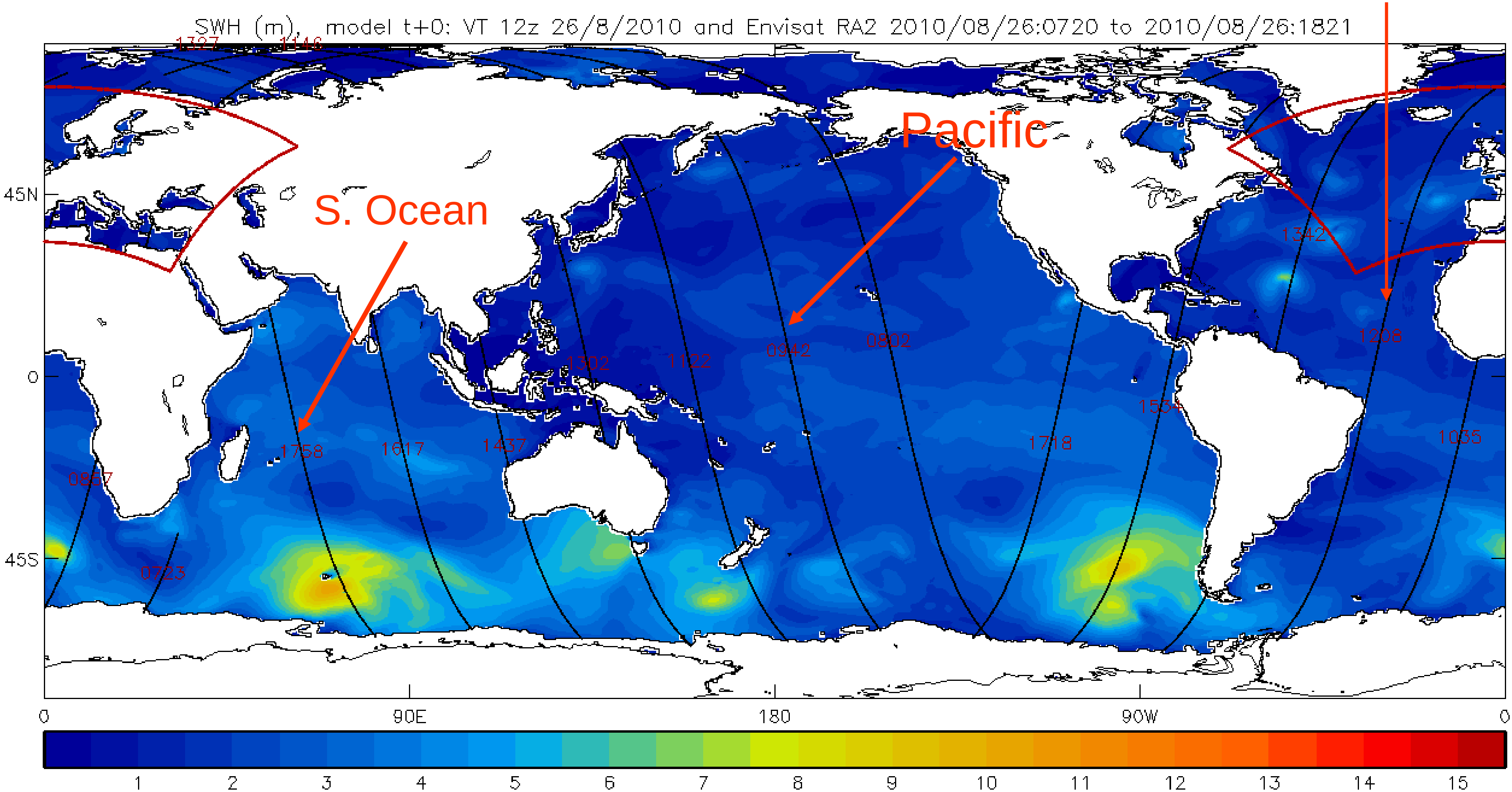


SRWH 4-bin output G25SMC

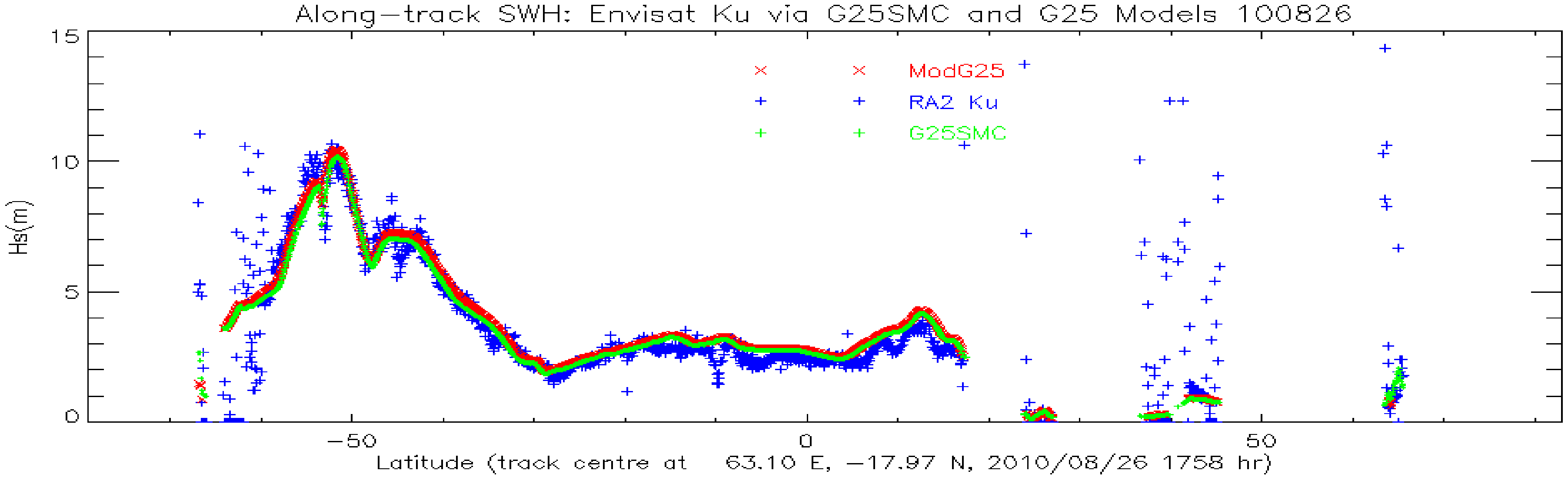


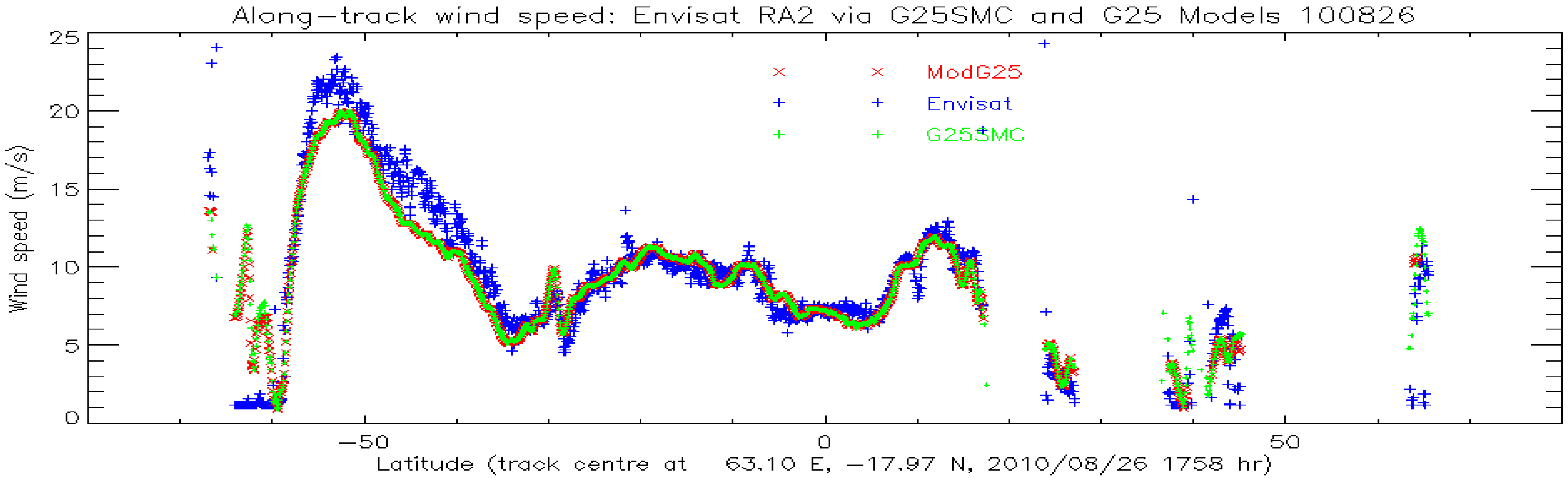
Validation with Envsiat RA2 SWH

- SWH along satellite track

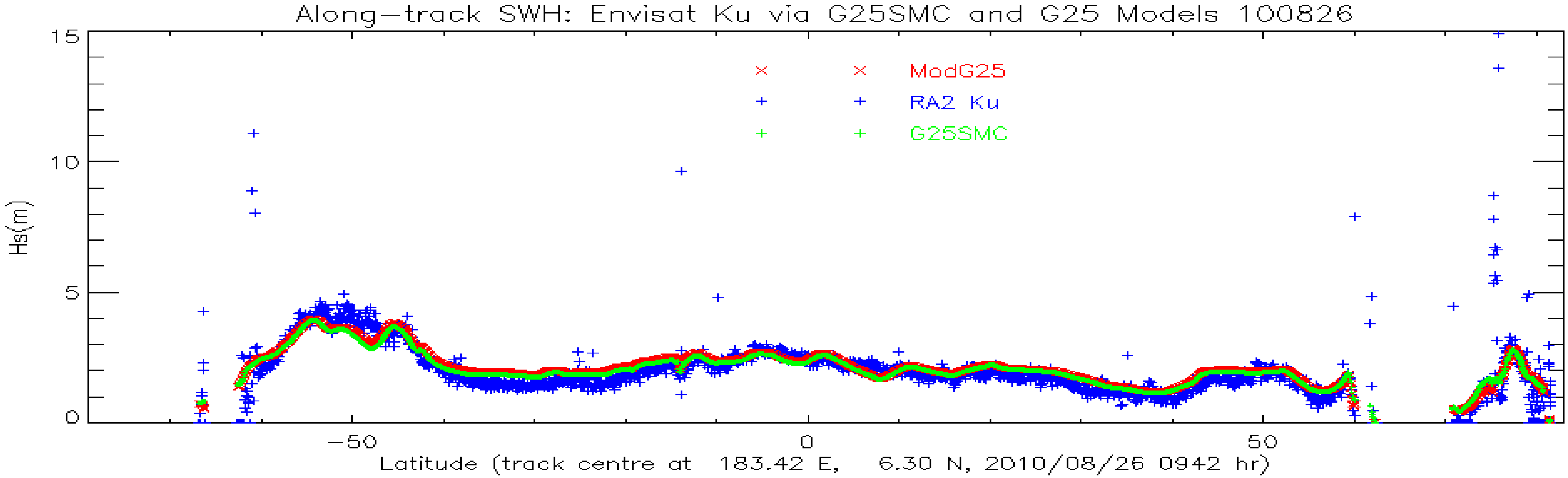


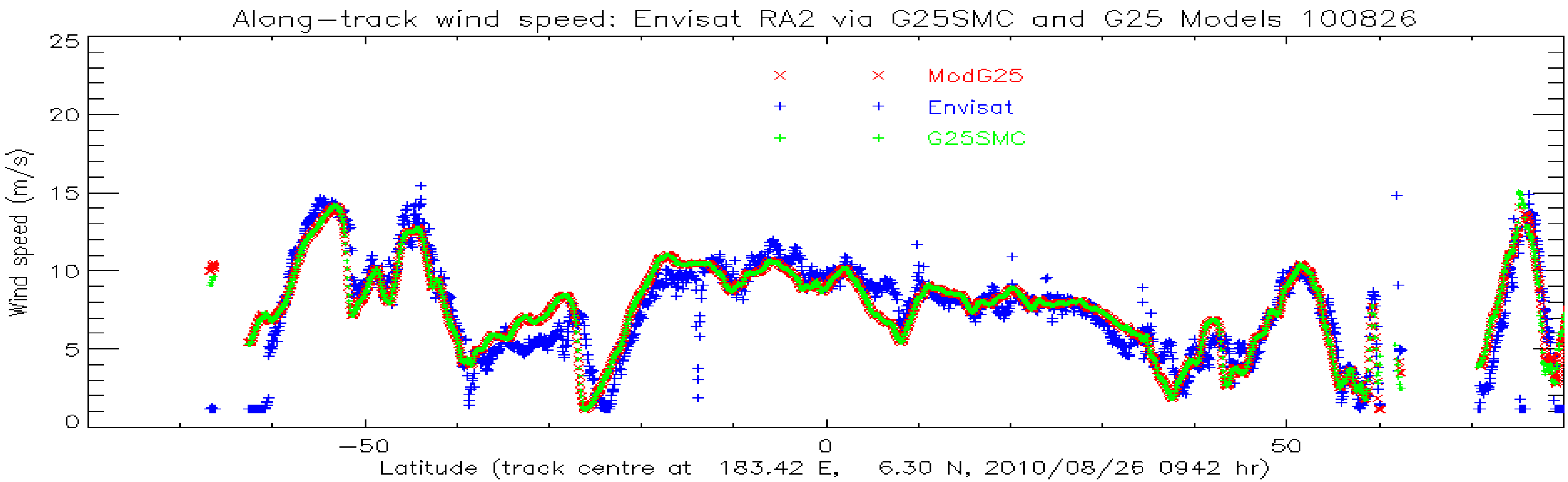
SWH along S Ocean track



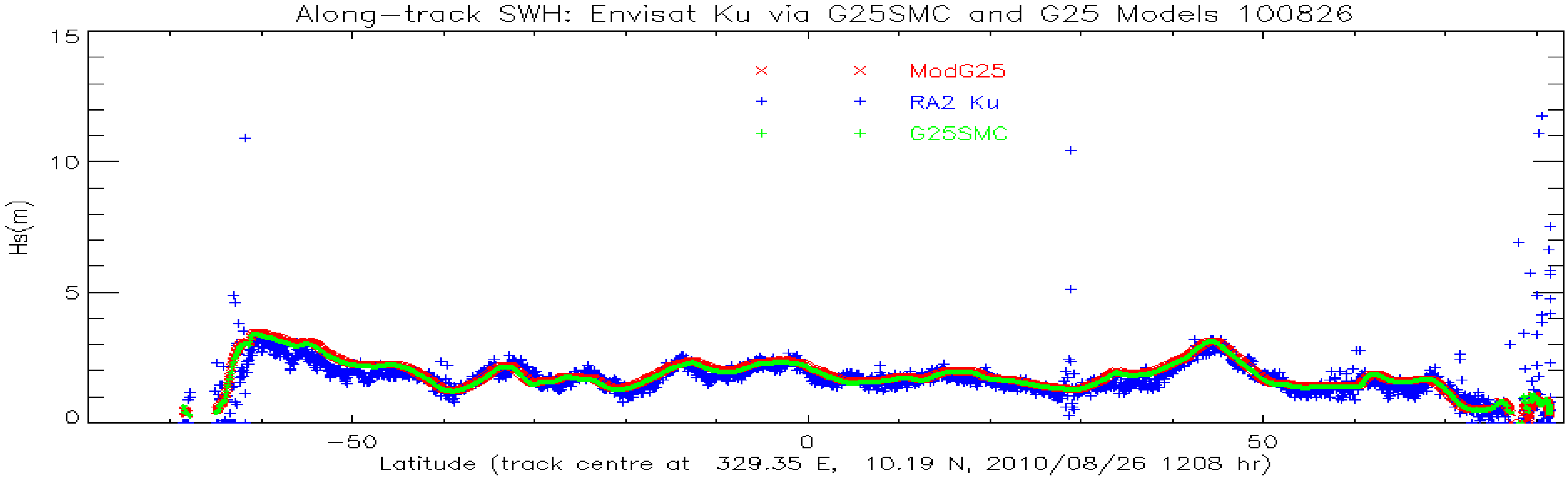


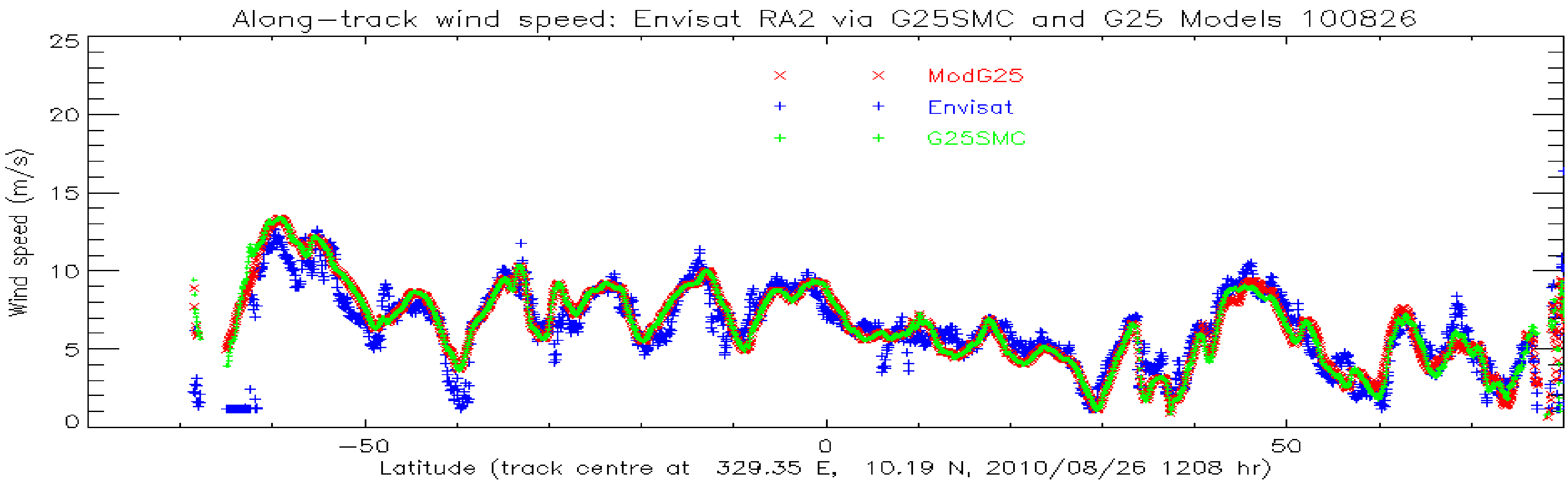
SWH along the Pacific track



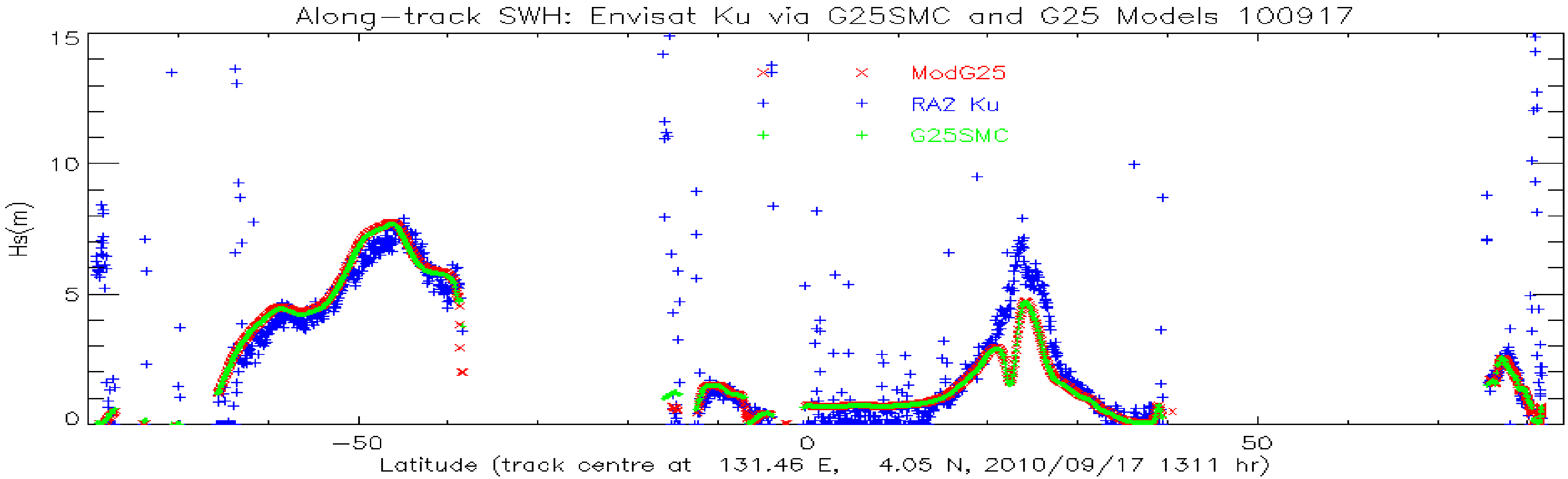


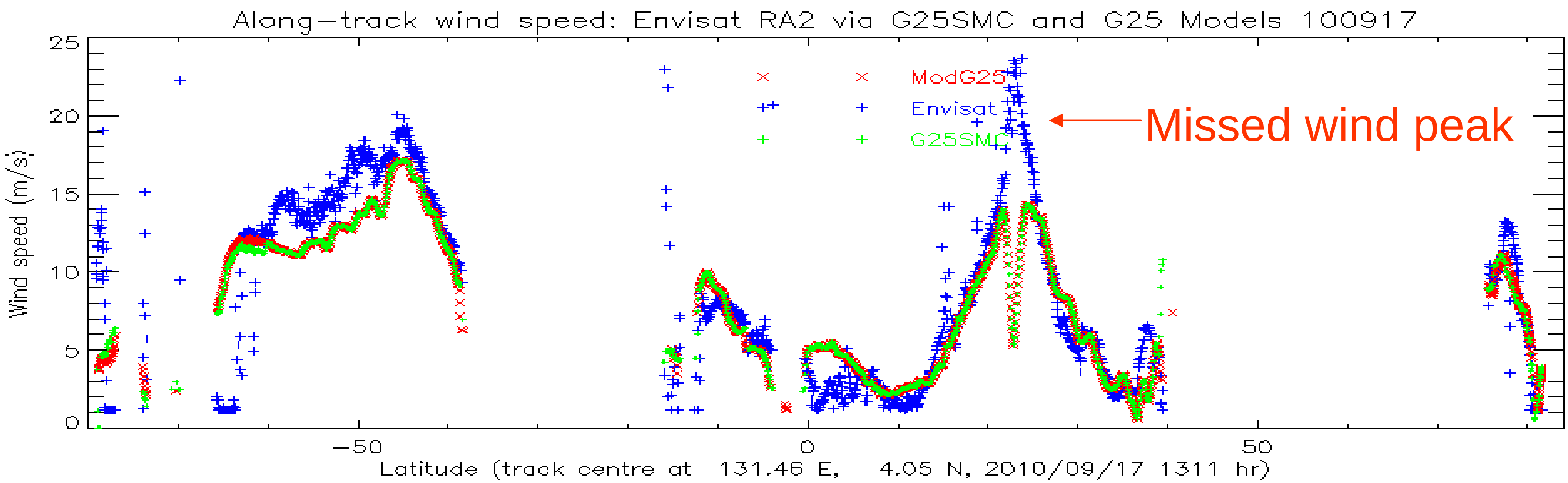
SWH along the Atlantic track





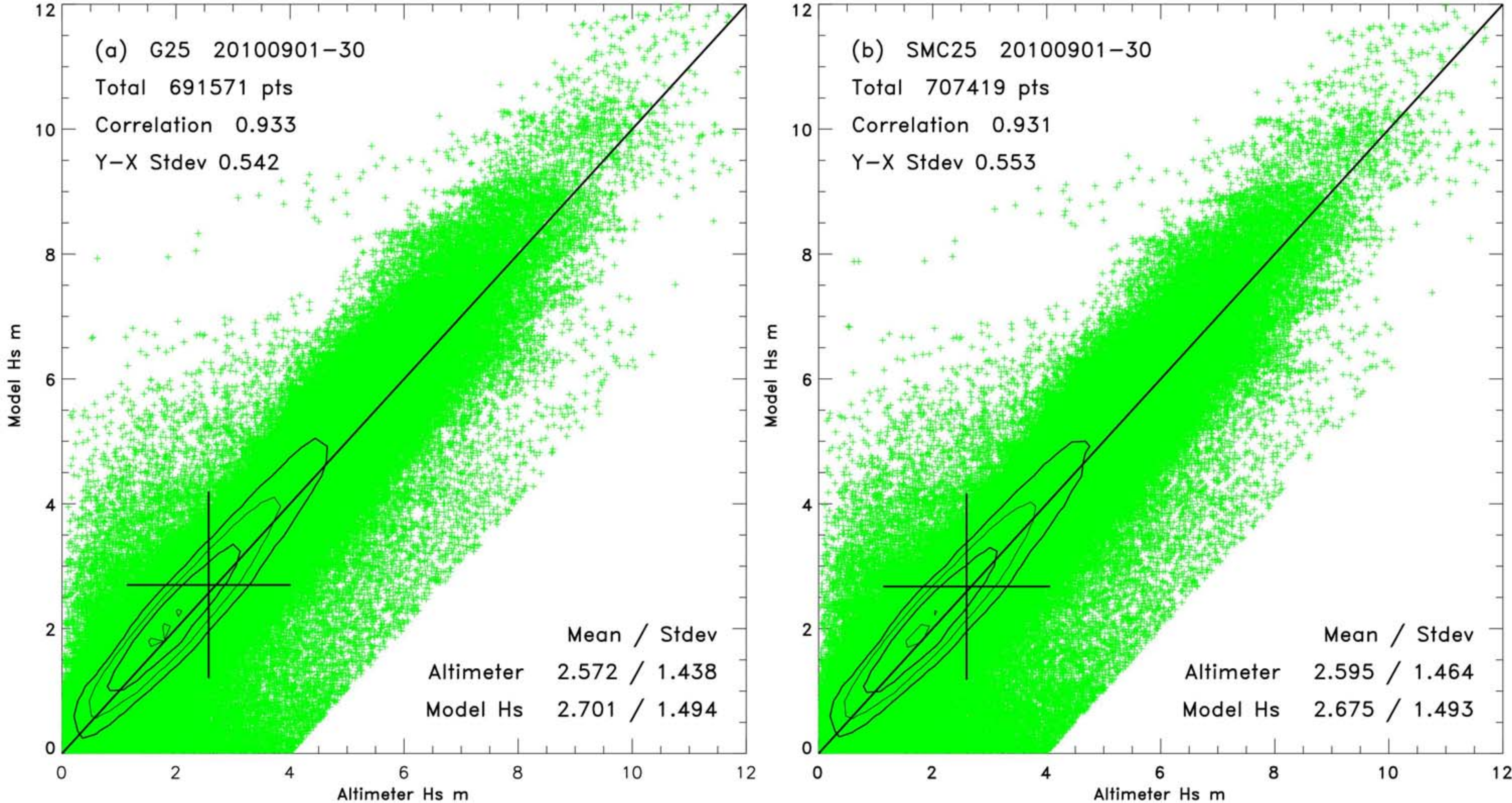
Wind influence on SWH output





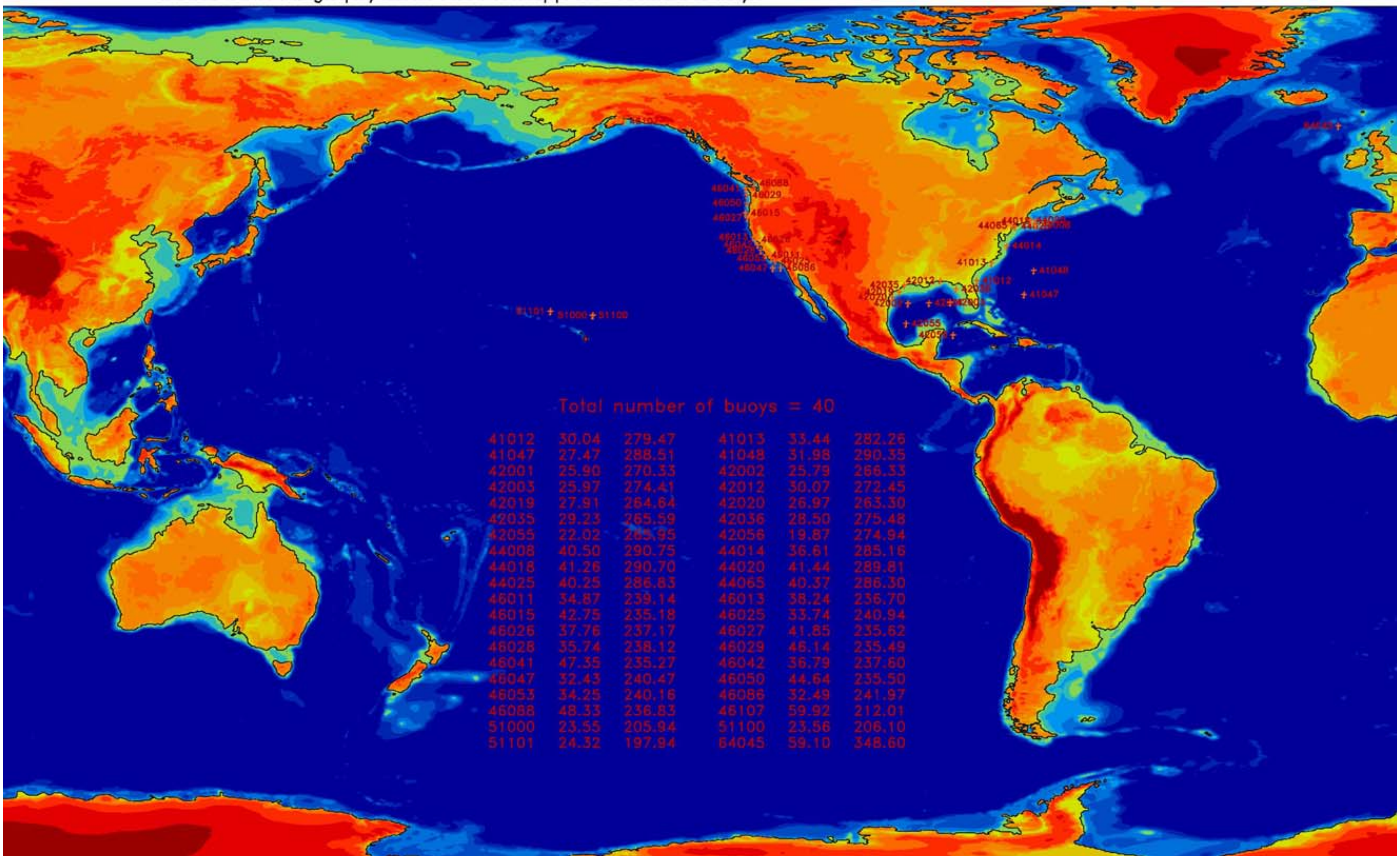
SWH scatter plot global

Extra 15848 coastal points in SMC25

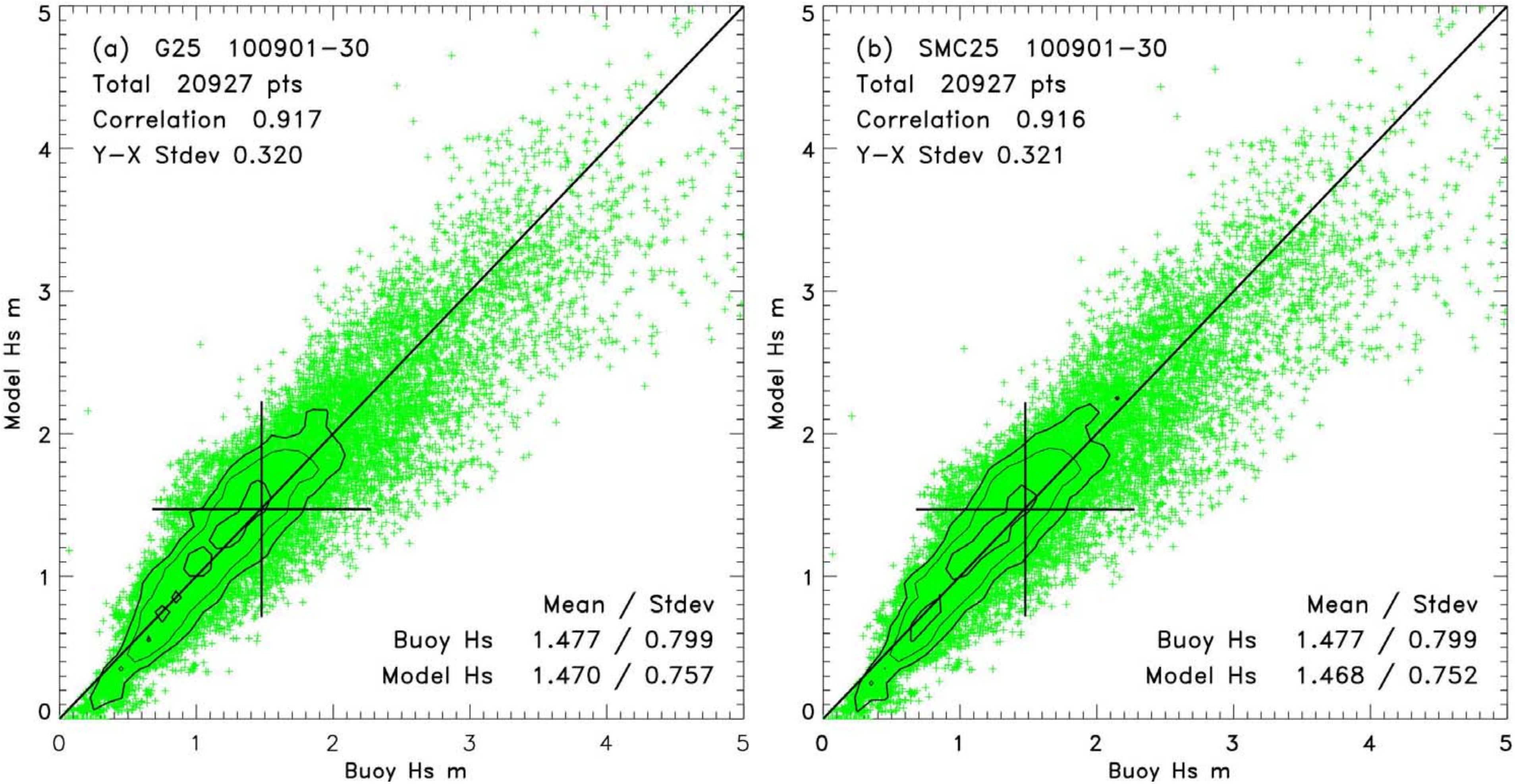


Comparison with spectral buoys

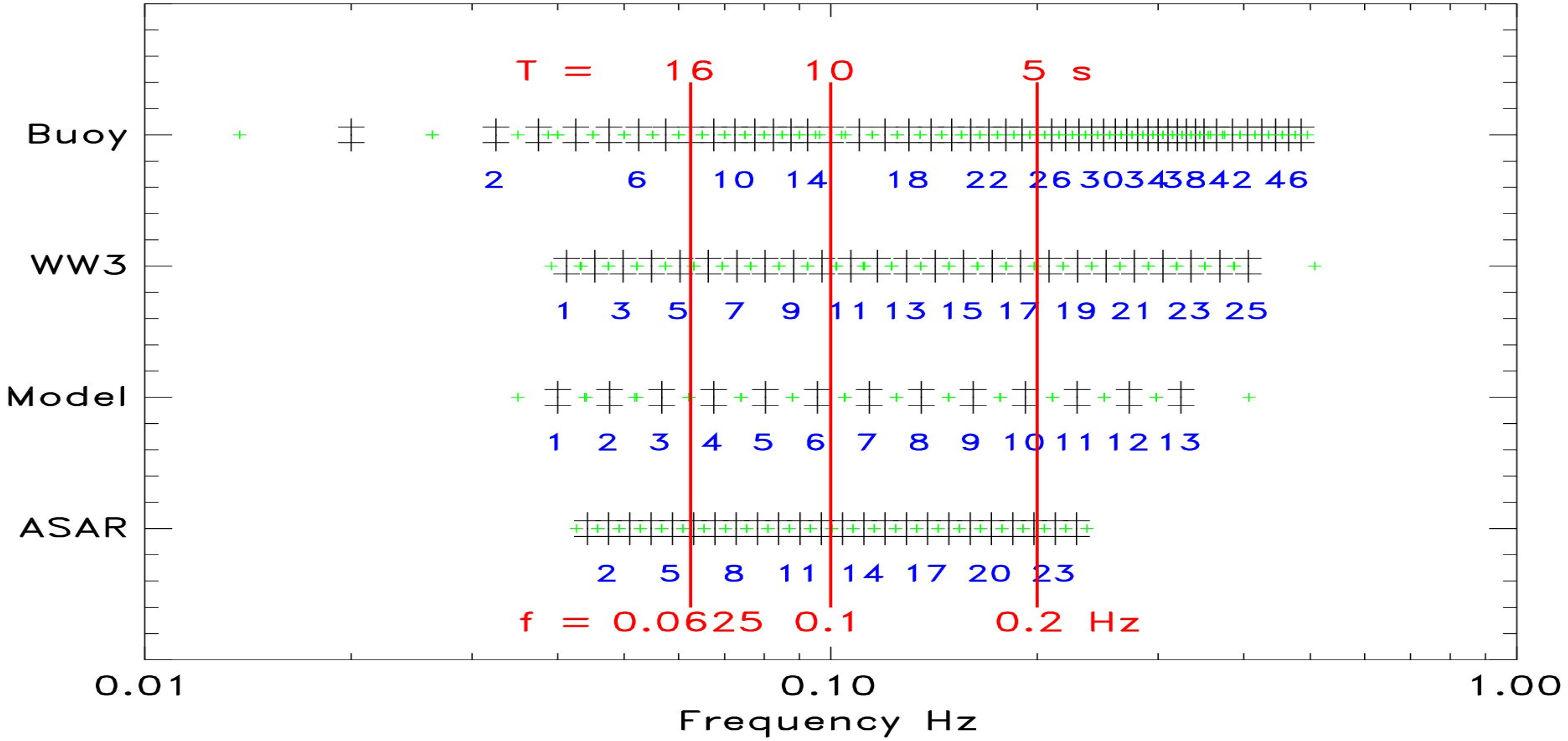
Global 25km orography from Glob25km.pp and selected buoys



Comparison of SWH of g25 and SMC models with 31 buoys

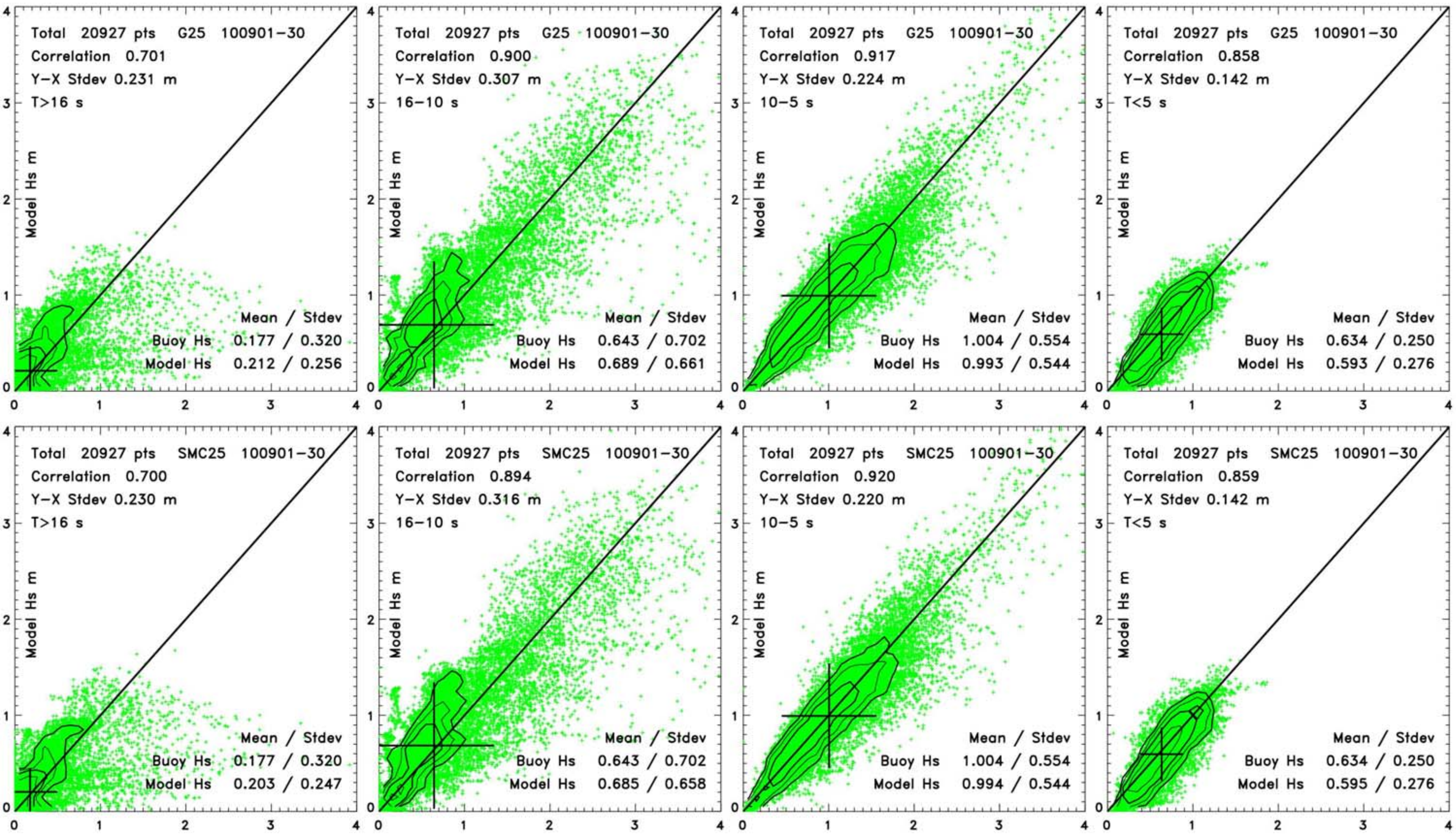


Frequency bins and 4-bin SRWH



SRWH defined as $$H_s(f_2 - f_1) = 4\left(\int_{f_1}^{f_2} df \int_0^{2\pi} d\theta \cdot E(f,\theta)\right)^{1/2}$$

Comparison of 4-bin SRWH of g25 and SMC models with 31 buoys



Comparison of G25 and SMC models

	G25 Lat-lon	SMC25 grid
Wave model grids	1024x688	429722 (61%)
Advection time step	180 s	600 s (333%)
One-day hindcast on 1-node 32 PEs (64 co)	330 s/task	210 s/task (64%)
5.5-day forecast on 4-nodes 128 PEs	1800 s/task	1240 s/task (69%)
RA2 rms / correlation	0.542 m/ 0.933	0.553 m/ 0.931
Buoy rms / correlation	0.320 m/ 0.917	0.321 m/ 0.916

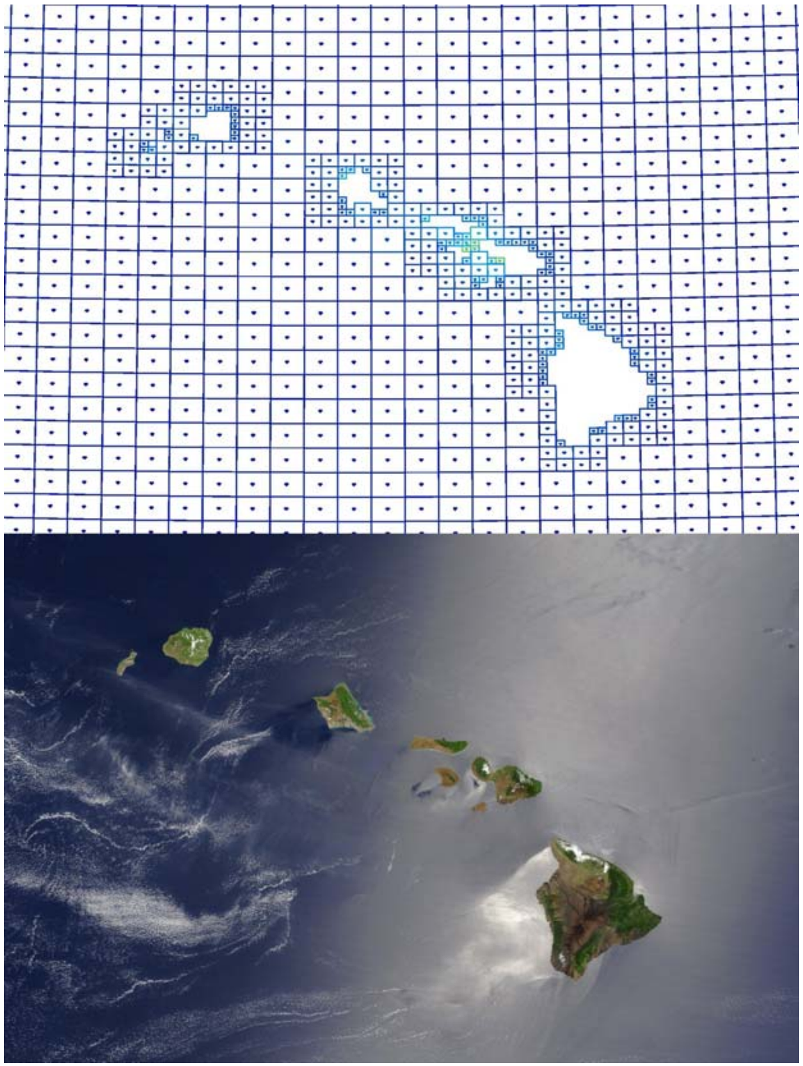
Total CPU time reduction is ~ 1/3

Met Office

Summary of SMC grid model

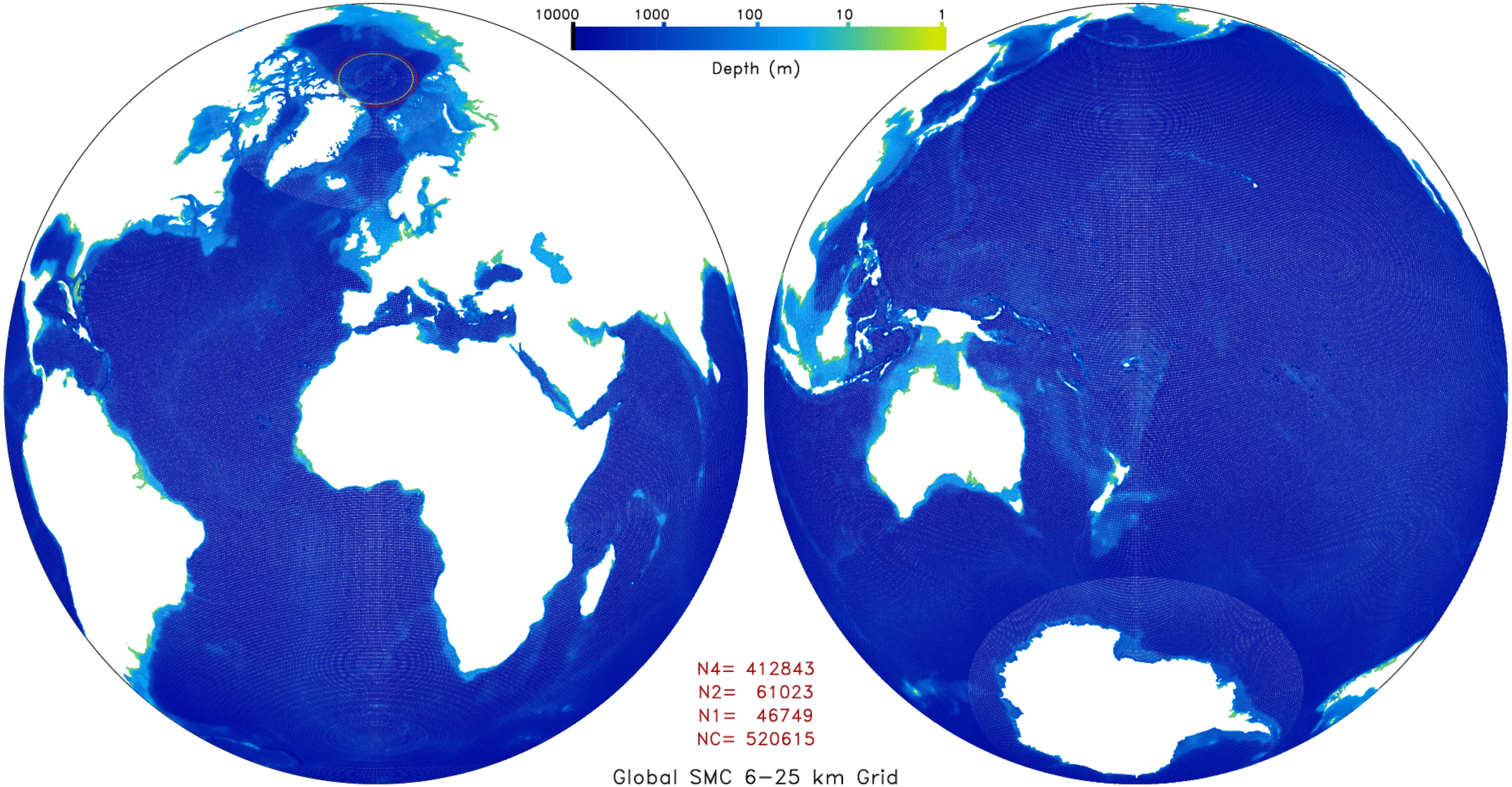
- SMC grid is successfully implemented in a global wave model, WAVEWATCH III.
- Reducing 45% grid points and increasing time step by 4 times, the SMC grid wave model saves ~1/3 time and maintain the accuracy as compared with lat-lon grid.
- Planning to use the SMC25 grid wave for our operational global wave model.
- A lower resolution (50km) SMC grid global wave model has been prepared for long-term hind-cast (1 yr 1 node 1 night) and global ensemble.
- Unstructured SMC grid has other useful features, like mesh refinement for detailed coastlines and small islands.

Refined 6-25km SMC grid

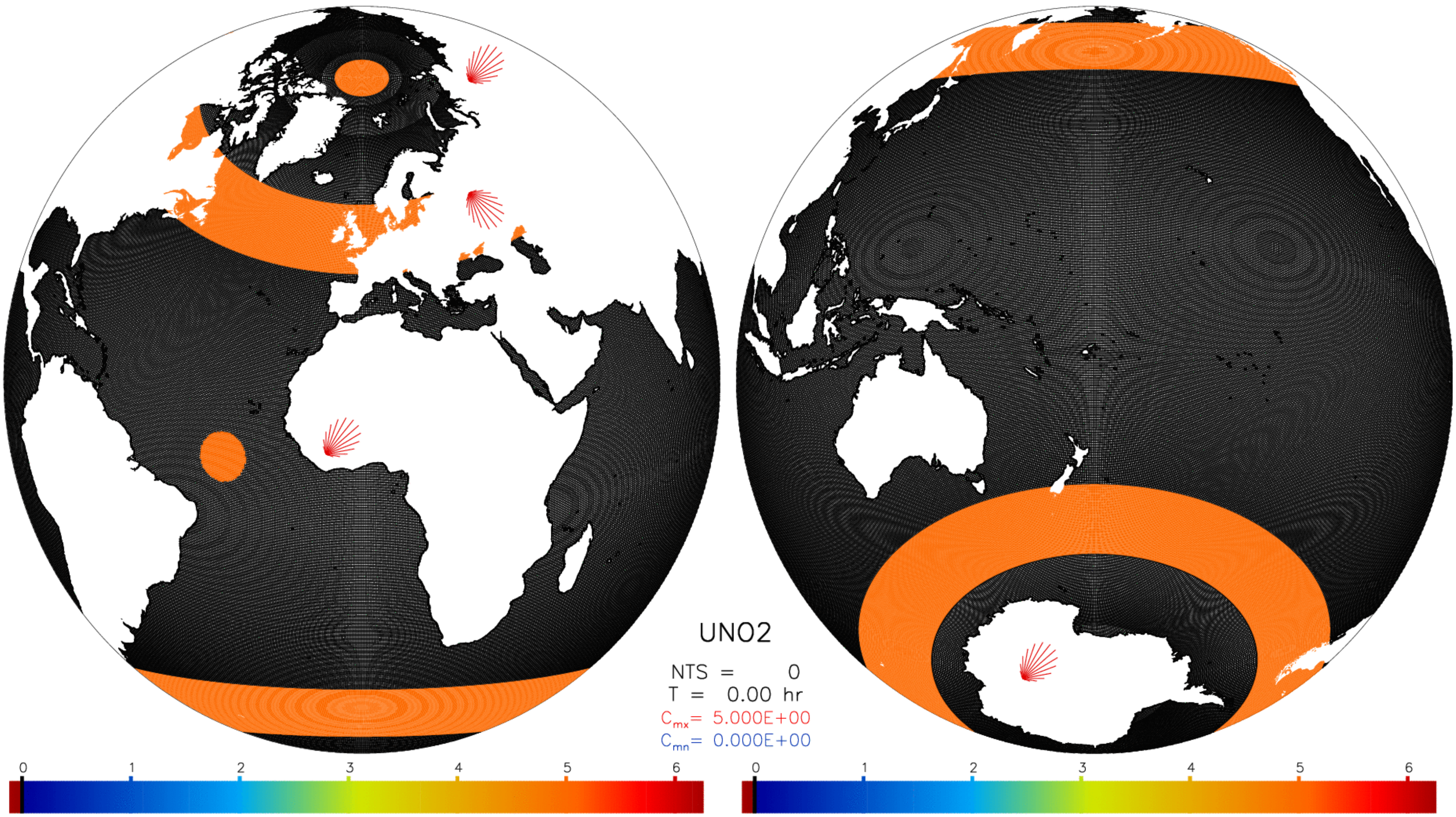


- Refined resolution up to 6km near coastlines.
- Number of cells (520615) still smaller than lat-lon grid (1024x768=786 432) 66%.
- Could make regional (NAEW) models redundant.
- Multi-time-steps to handle refined cells.
- Future improvement in coastal zone.

Refined 6-25km SMC grid



Wave spectra (36) transport on 6-25 SMC grid



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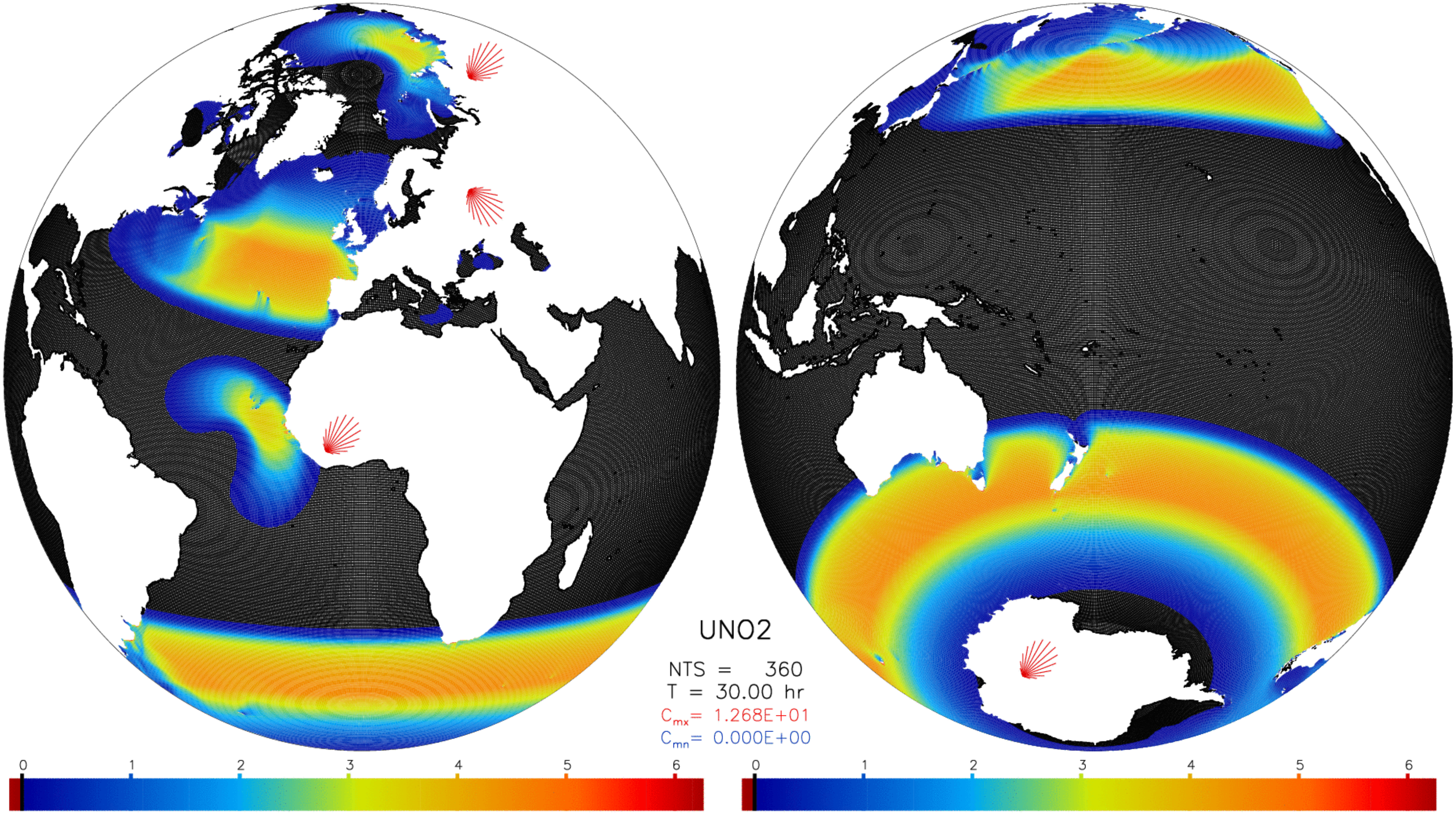

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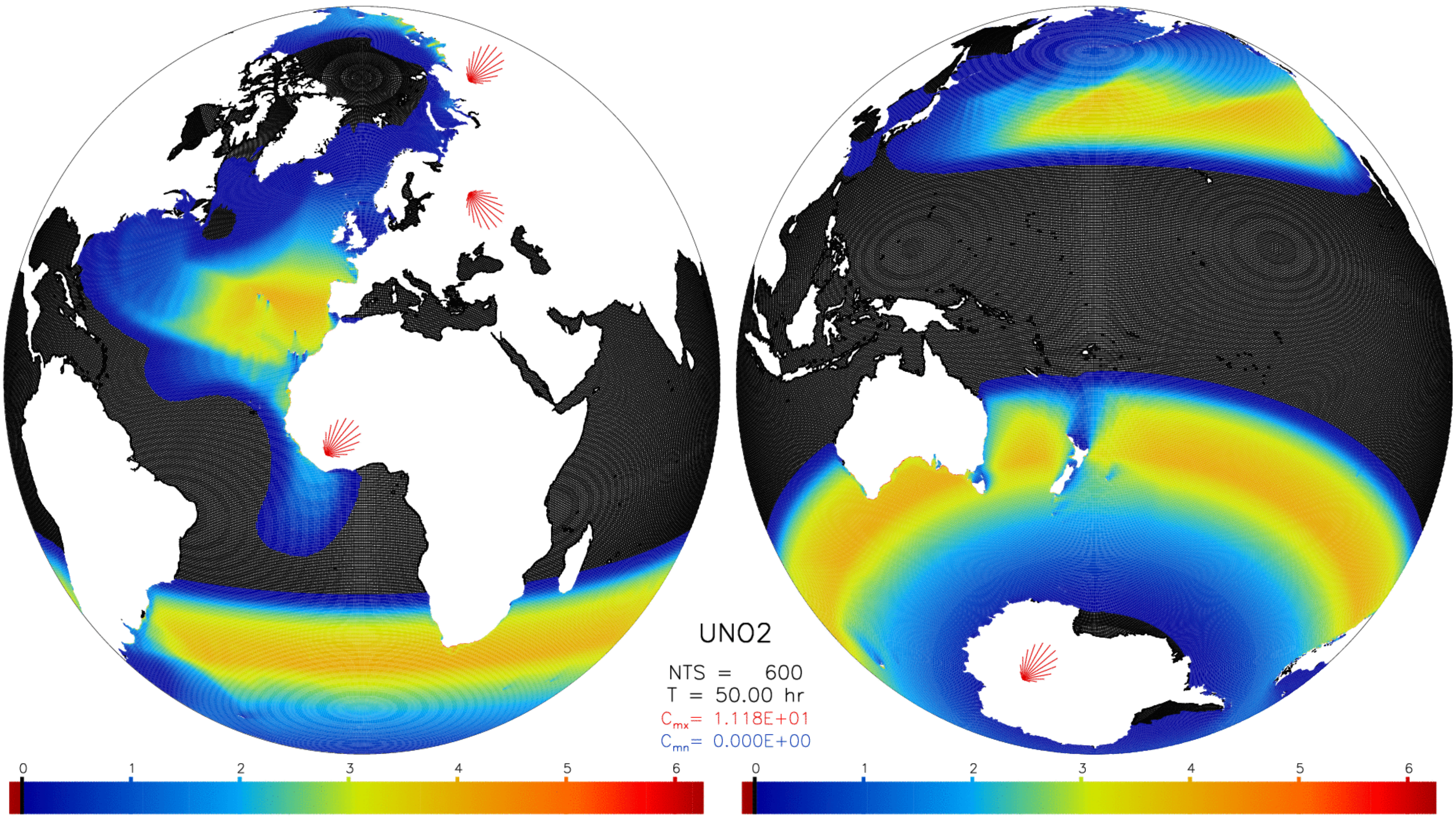
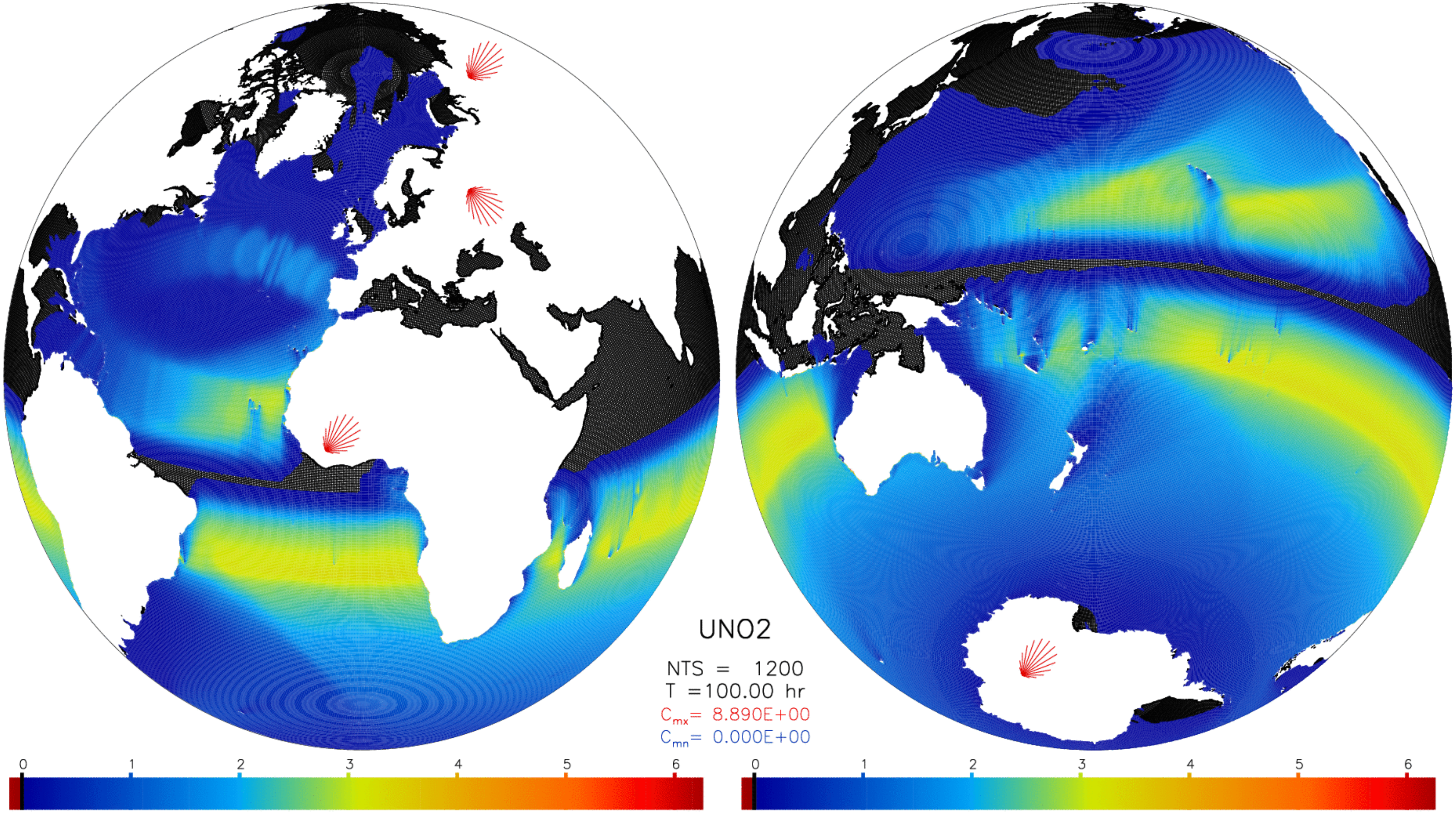


Wave spectra (36) transport on 6-25 SMC grid



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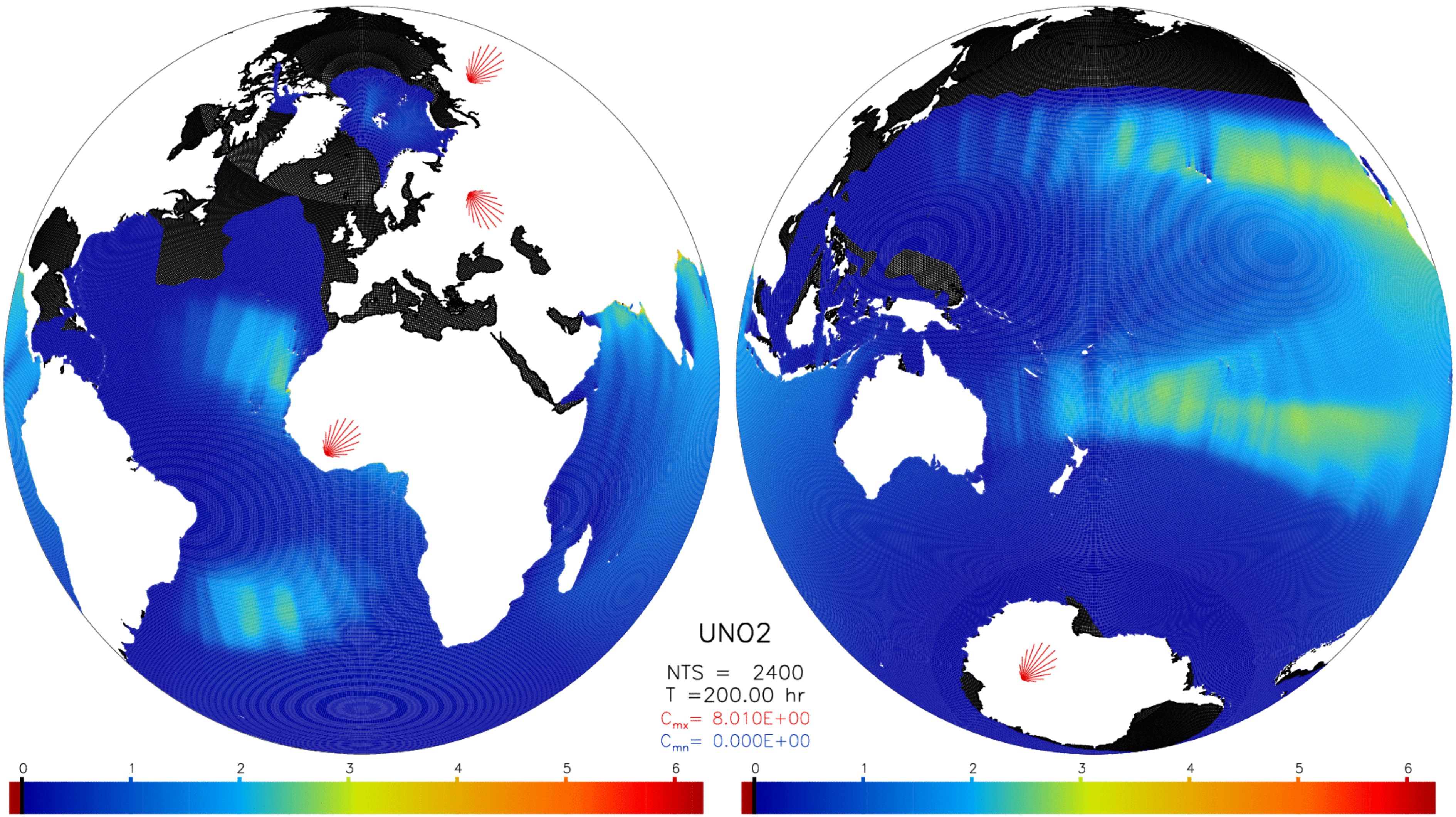

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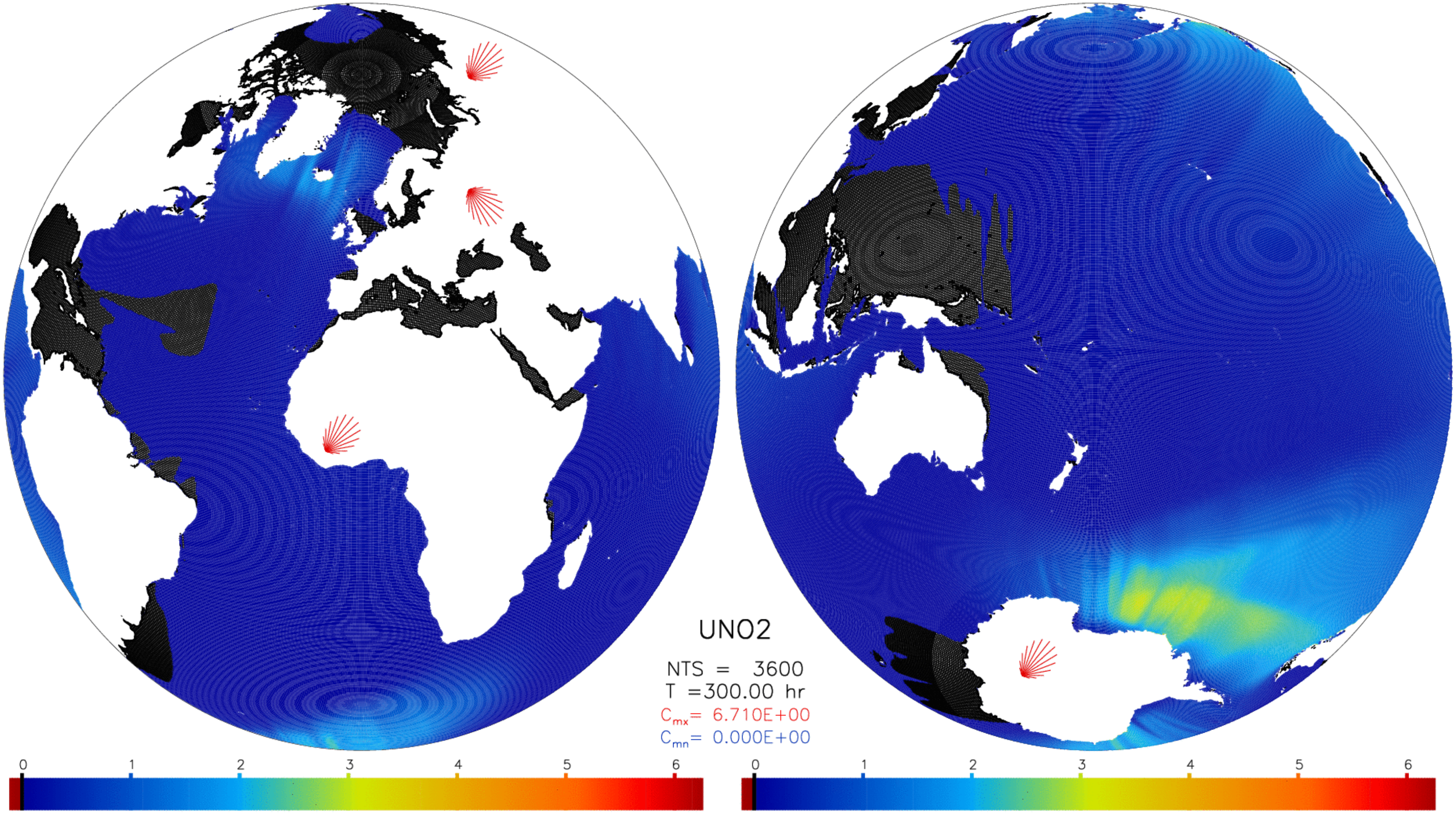
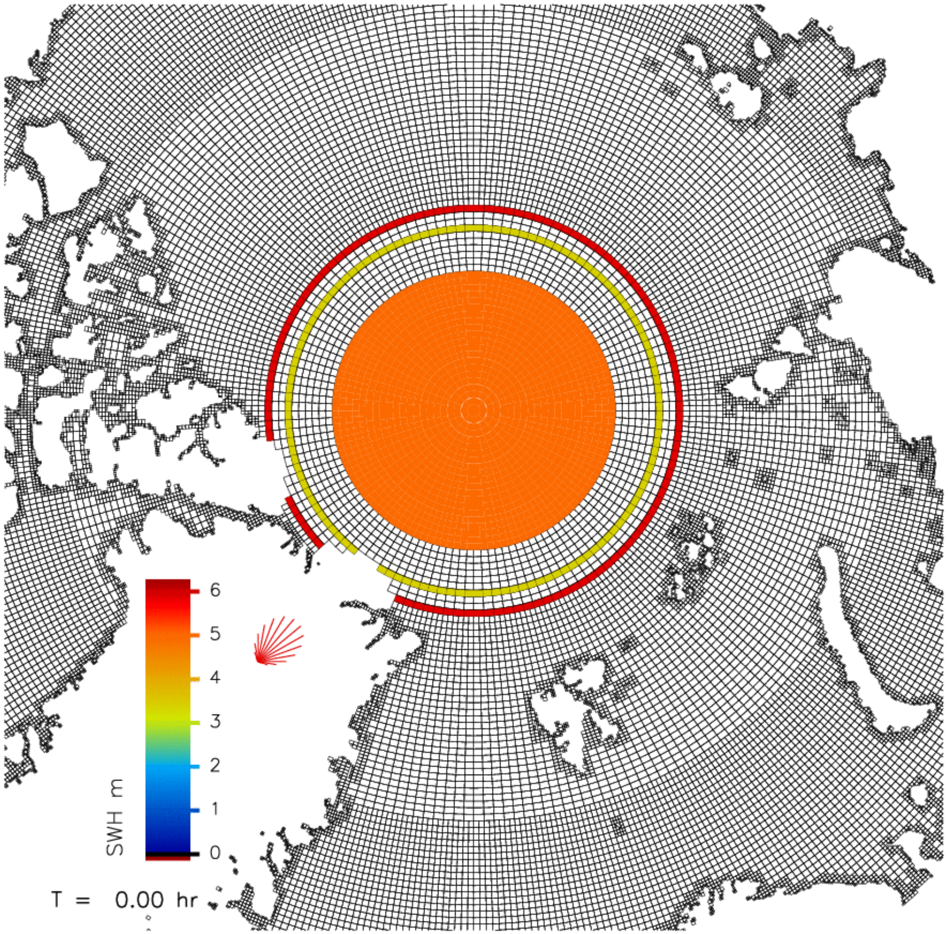
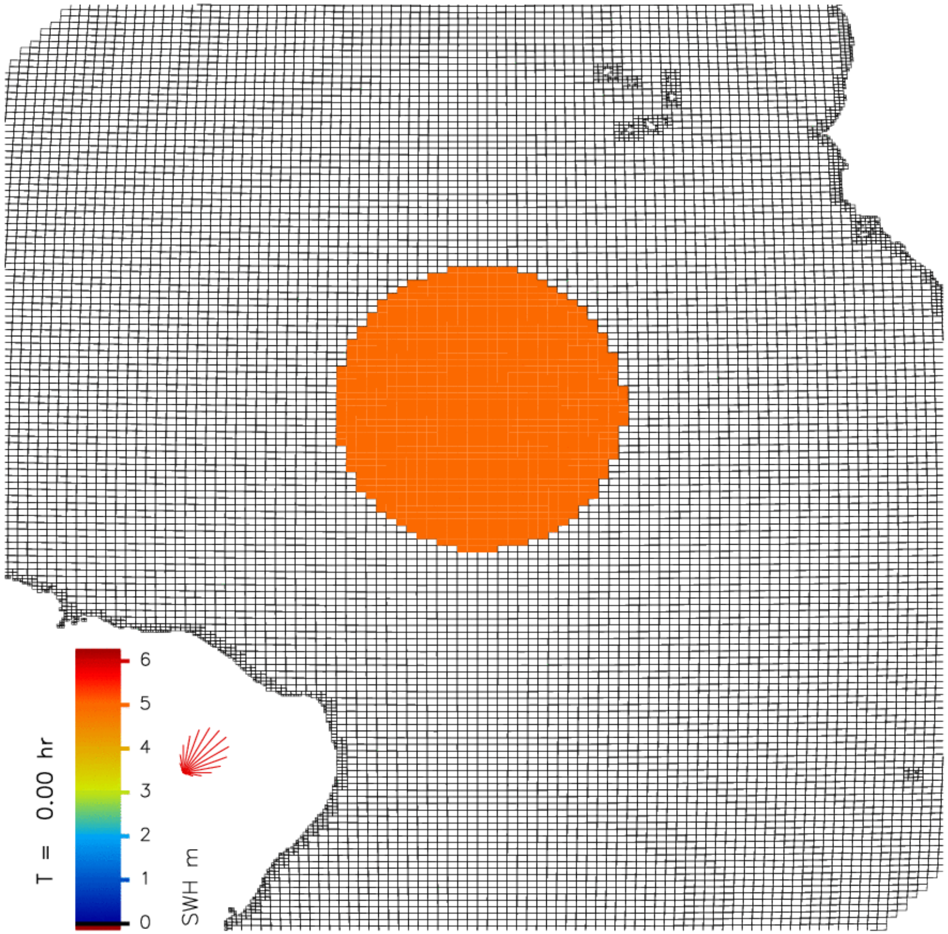
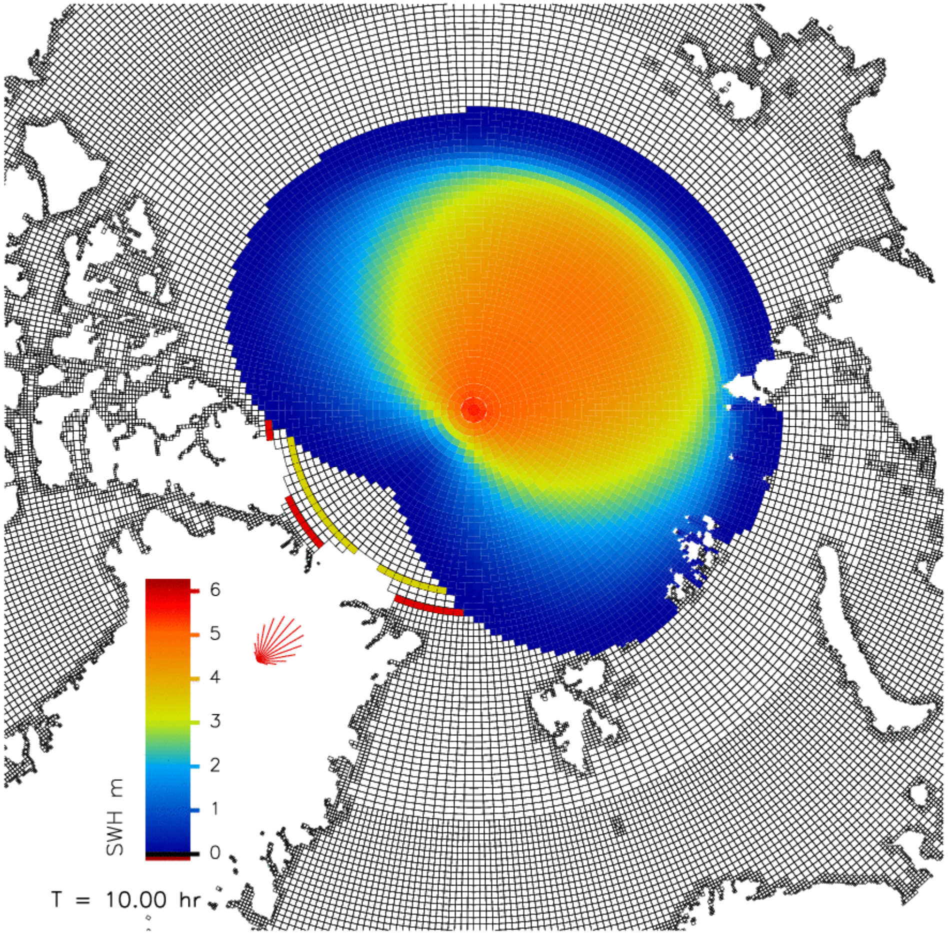


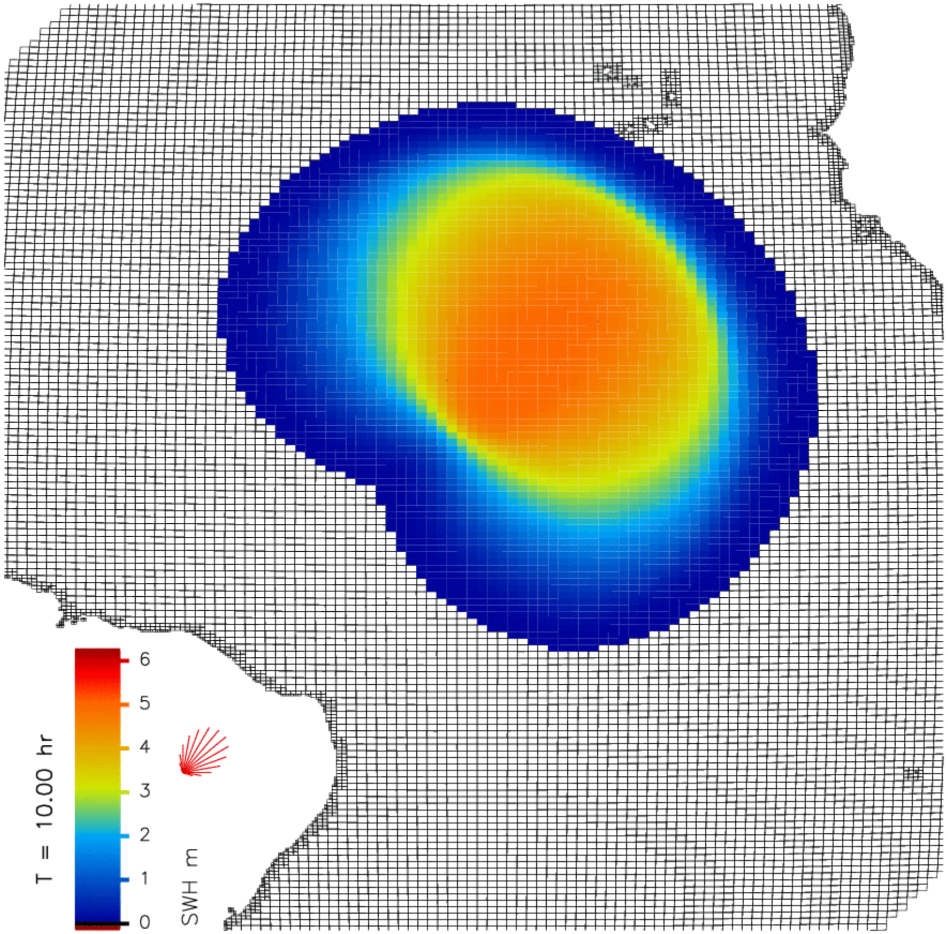
Wave transport on 6-25 SMC grid --- Arctic via Atlantic



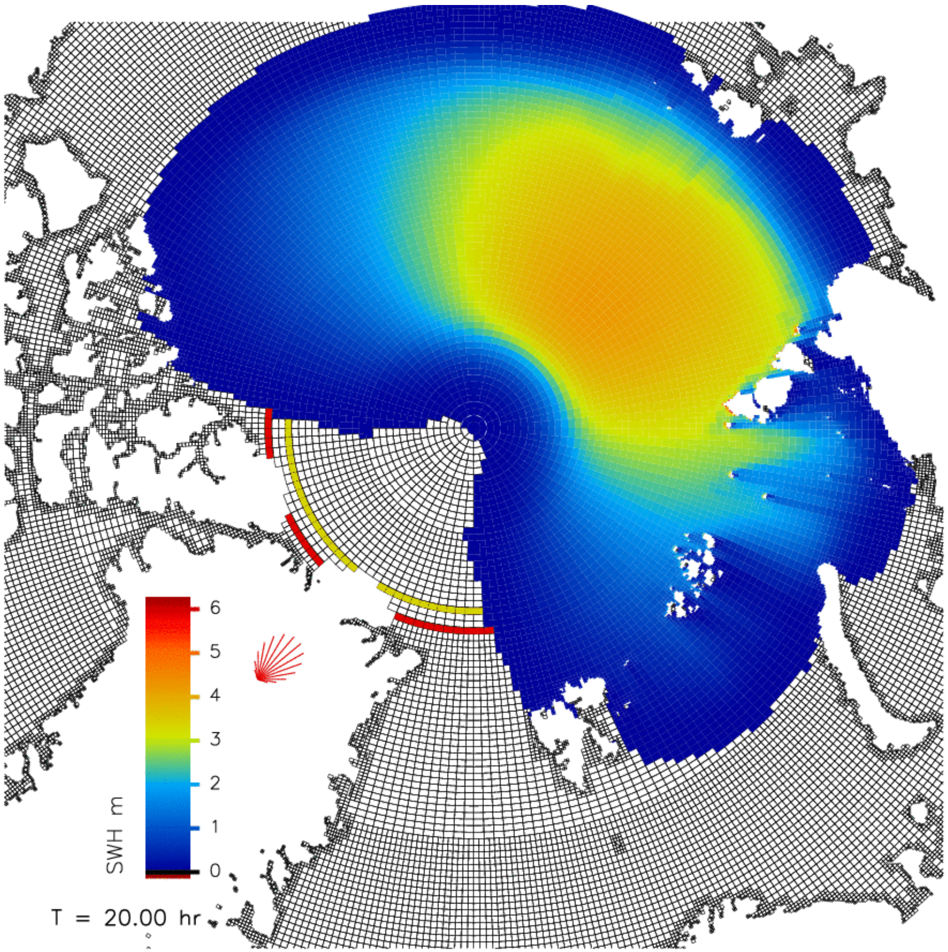


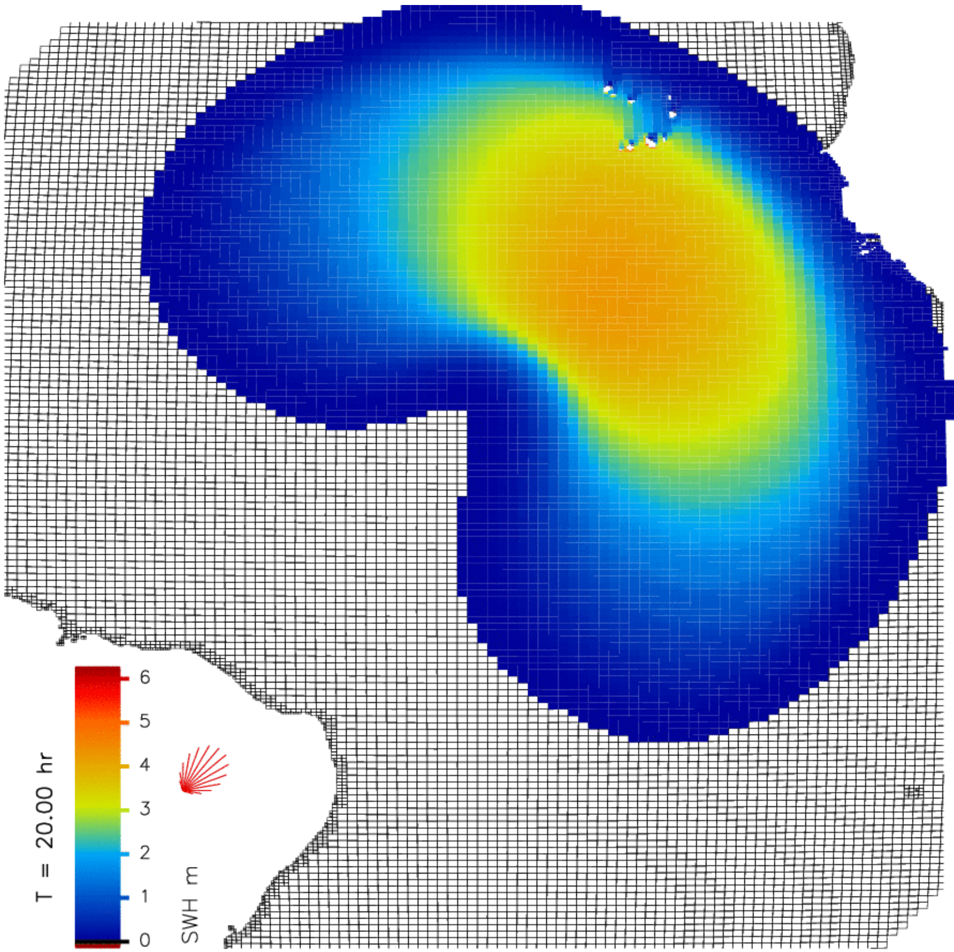
Wave transport on 6-25 SMC grid --- Arctic via Atlantic



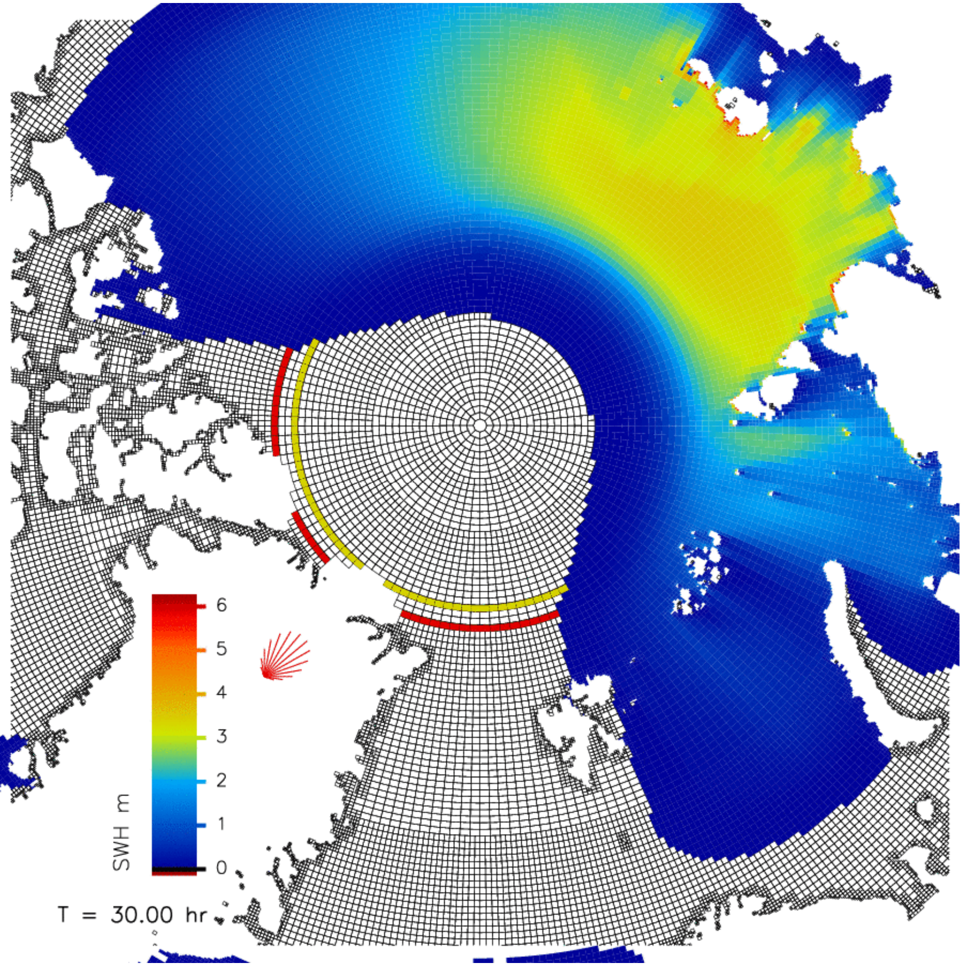


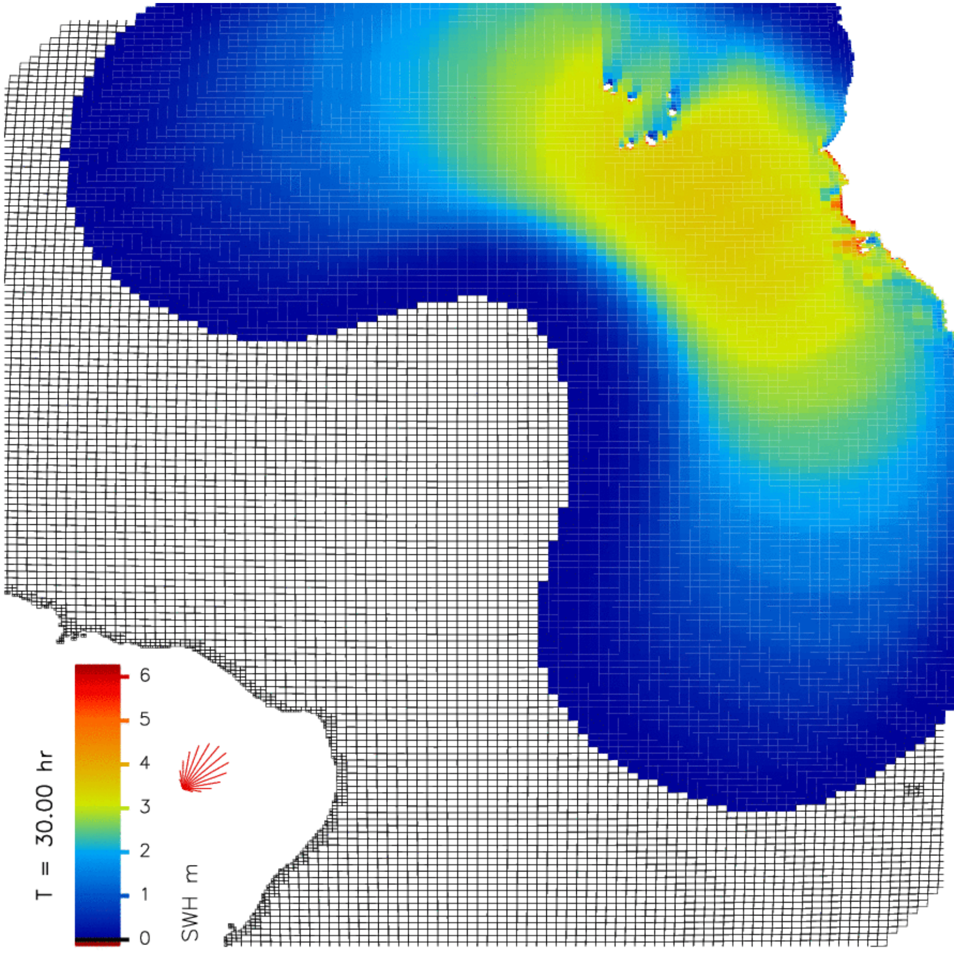
Wave transport on 6-25 SMC grid --- Arctic via Atlantic





Wave transport on 6-25 SMC grid --- Arctic via Atlantic





Summary and conclusions

- Retreating Arctic sea ice in recent summers has called wave modellers to expand their models to high latitudes. The polar problems in wave models are tackled with a SMC grid technique.

- SMC grid relaxes CFL limit on time step by merging cells at high latitudes and reduces computation cost by removing land points out of transportation.

- Multi-resolution is achieved for coastal regions with a 6-25km SMC grid and sub-time-steps are used to handle the refined cells. This makes it possible to merge global and regional wave models and include the Arctic if necessary.

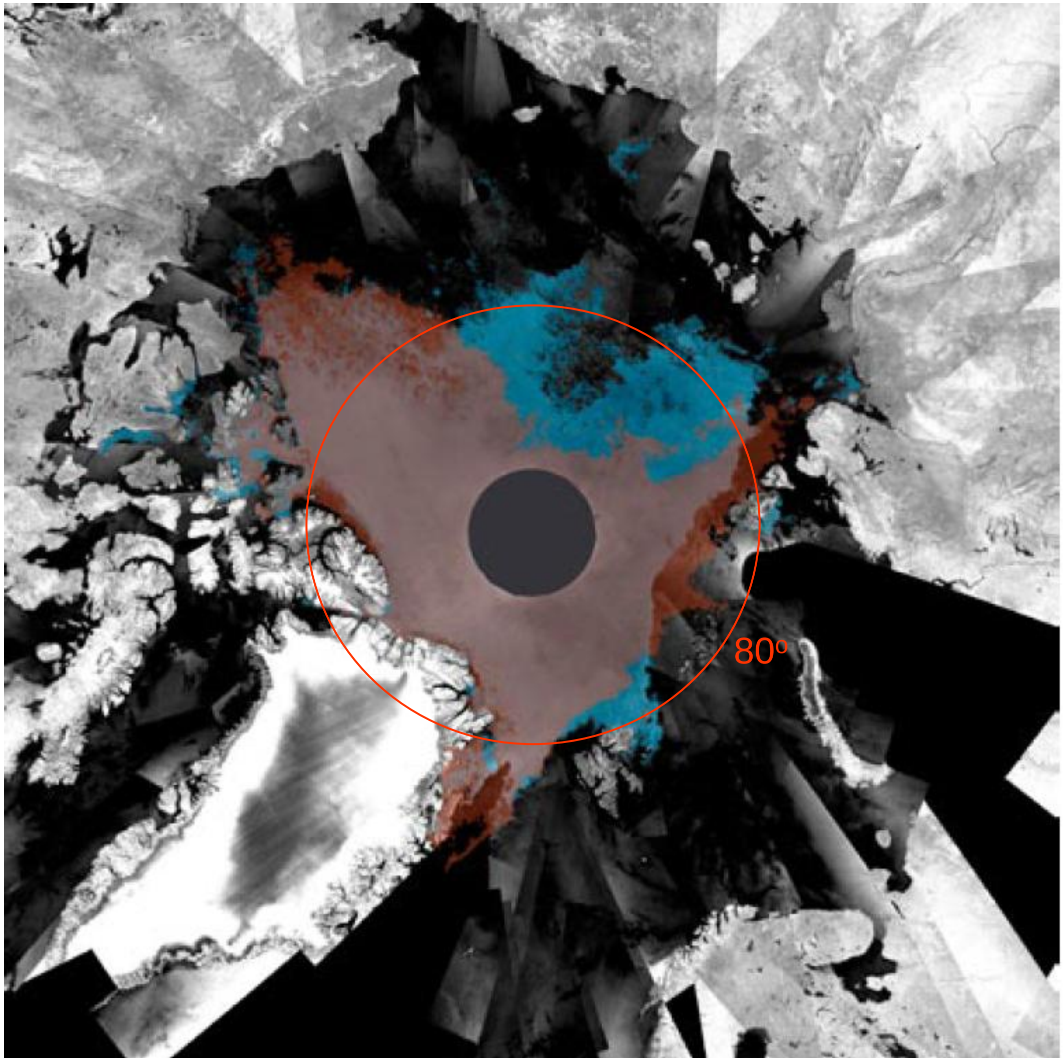
- Four processes (advection, diffusion, refraction and great-circle turning) related to ocean surface wave propagation are formulated on the SMC grid.

- Global 25km SMC grid has been coded into the WW3 model (Met Office version) and validated with Envisat RA2 SWH and buoy wave spectra. It matches the standard lat-lon WW3 model in accuracy and reduces overall CPU time by ~ 1/3.

Wave spectra (36) transport on 6-25 SMC grid

Basic time step 300 s for size-1 or 6km cells (1200 s 25km cells)
Single frequency 0.0625 Hz (T = 16 s) and 36 directions
Maximum Courant number = 0.929 (c_g = 12.5 m/s)
Max GCT Courant number = 0.133
Max Refraction angle per step to depth gradient direction.
Horizontal diffusivity = 3600.0 $m^2 s^{-1}$
Total time step no NTS = 6000 (500 hr)
Run time date 090225.473 20100604
End time date 105058.659 20100604 ~ 14833 s or 4 hr on
Dell Precision T3500 (desktop) for 36 elements.
For a single spectral element ~ 500 s.
Full spectra will have 900 elements and require over 100 hr.

Arctic ice by ESA Envisat ASAR in September 2007 and 2008



Arctic ice is retreating.

Global wave model needs extension to cover the Arctic.

Light brown: ice appear in both Septembers.

Dark brown: ice free in Sept 2008 but covered in Sept 2007.

Blue: ice free in Sept 2007 but covered in 2008.

Highest ice-free latitude 86º, present model 82º.

Worst scenario: Arctic ice free in summer 2013 (?)

Operational 35 km global model

3 grids lat ±80º dlat 0.333º and dlon 0.4/0.8º, lat coverage: [-80º, -65º], [-55º,72º], [60º, 80º] overlap 12º or 36 rows.