# *New vision of wave input terms*

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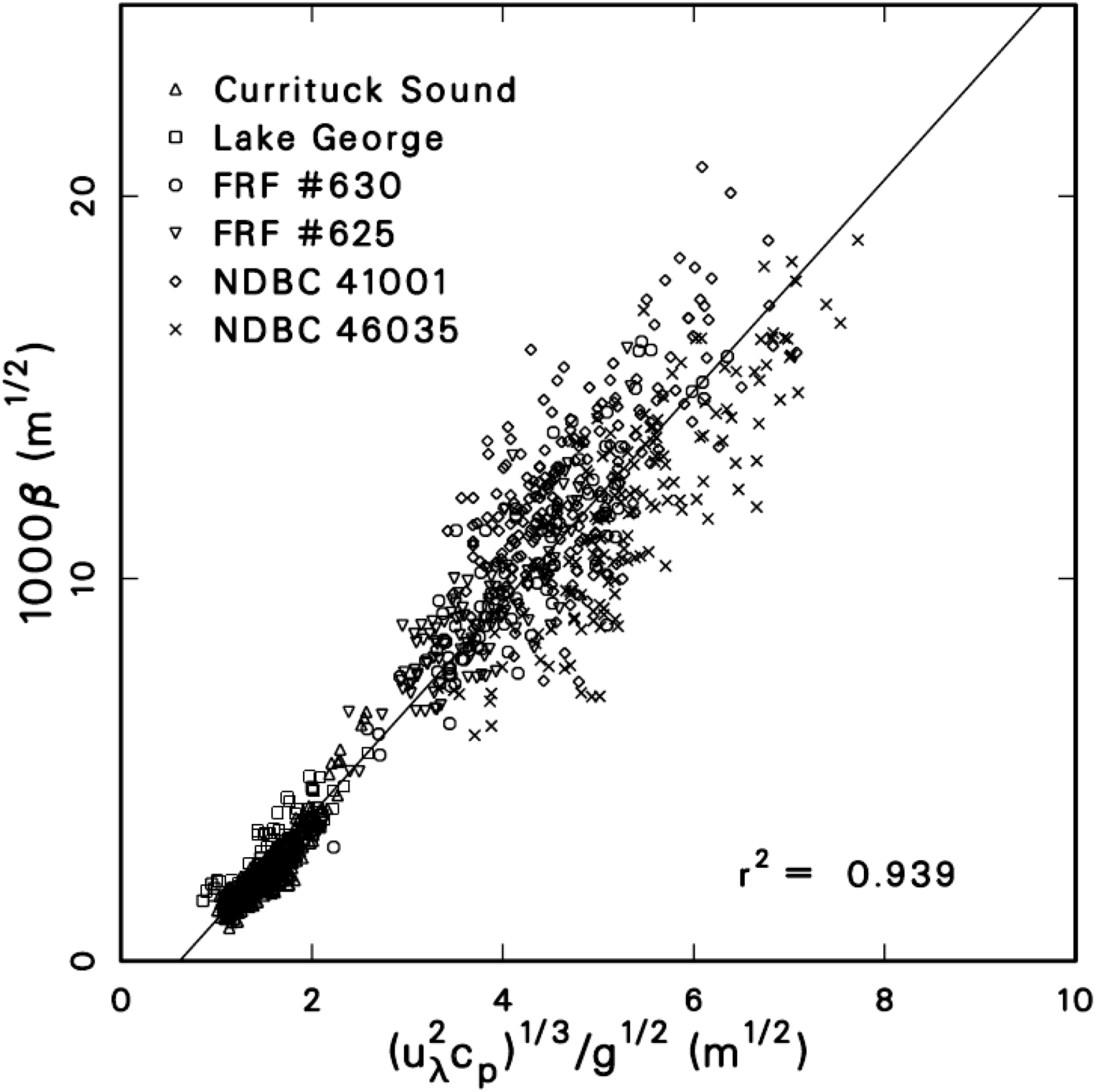


Currituck Sound
Lake George
FRF #630
FRF #625
NDBC 41001
NDBC 46035
$r^2 = 0.939$
$1000\beta$ ($m^{1/2}$)
$(u_\lambda^2 c_p)^{1/3}/g^{1/2}$ ($m^{1/2}$)
0
10
20
0
2
4
6
8
10

$$k^{5/2}F(k)$$

$$(u_\lambda^2 c_p)^{1/3} g^{-1/2}$$

$$\beta = \frac{1}{2}\alpha_4 \left[ (u_\lambda^2 c_p)^{1/3} - u_0 \right] g^{-1/2}$$

*Resio at al. 1987, 2004 and 2007*

$$\alpha_4 = 0.00553, u_0 = 1.93\, m/sec$$

$$u_\lambda = \frac{u_\star}{\kappa} \ln \frac{z}{z_0}$$

$$\kappa = 0.41$$

$$z = \lambda \cdot 2\pi/k_p$$

$$z_0 = \alpha_C u_\star^2/g$$

$$\alpha_C = 0.015$$

$$\frac{\partial n_{\vec{k}}}{\partial t} = S_{nl} + S_{wind} + S_{diss}$$

$$S_{wind}(\omega, \phi) = \gamma(\omega, \phi) \; n_{\vec{k}}$$

$$\gamma(\omega, \phi) = -\alpha \frac{\rho_{air}}{\rho_{water}} \omega \left(\omega/\omega_0 - 1\right)^{\frac{4}{3}} \cos\phi$$

$$\alpha = 0.2 \qquad \rho_{air}/\rho_{water} = 1.3 \cdot 10^{-3}$$

$$\omega_0 = g/u$$

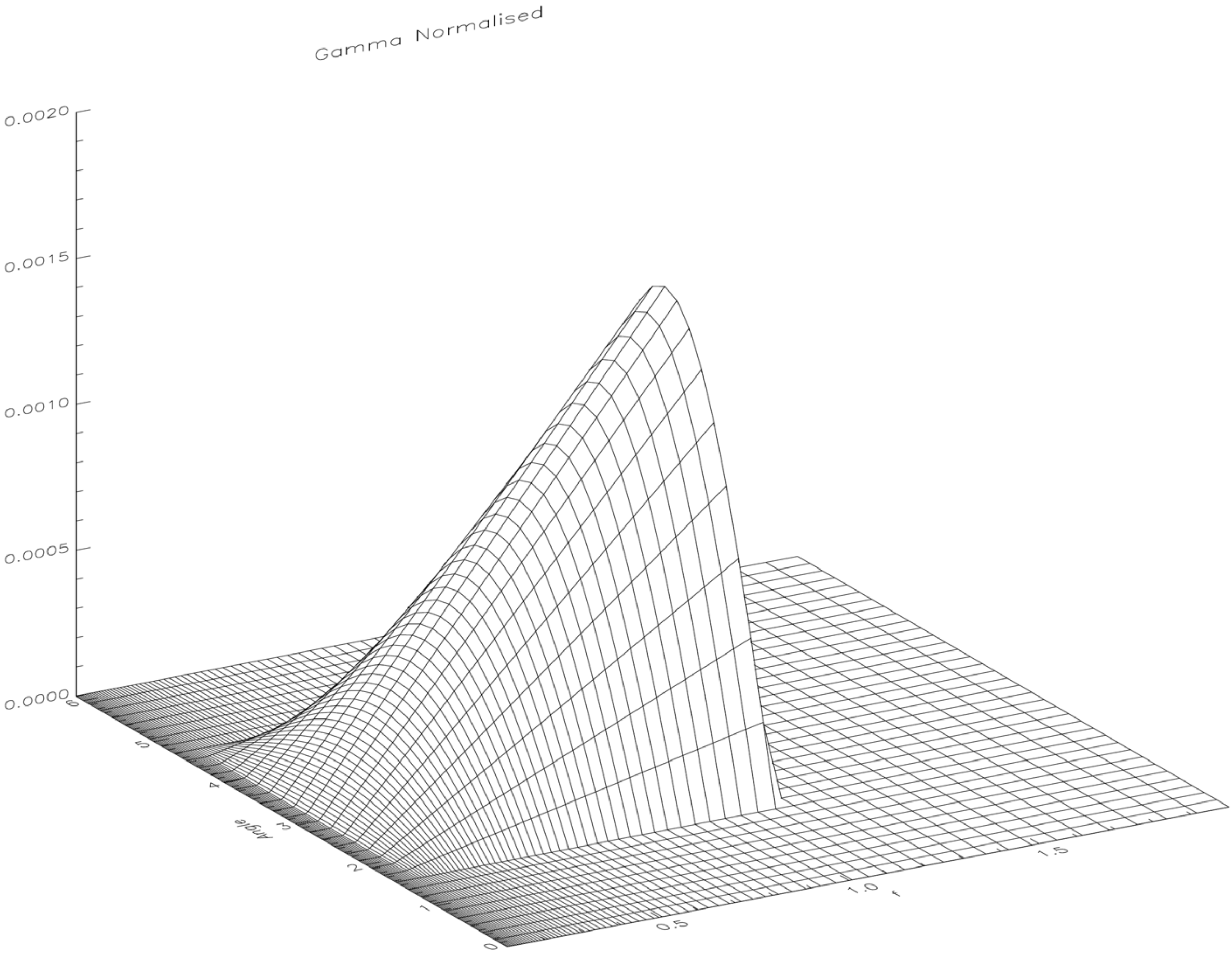
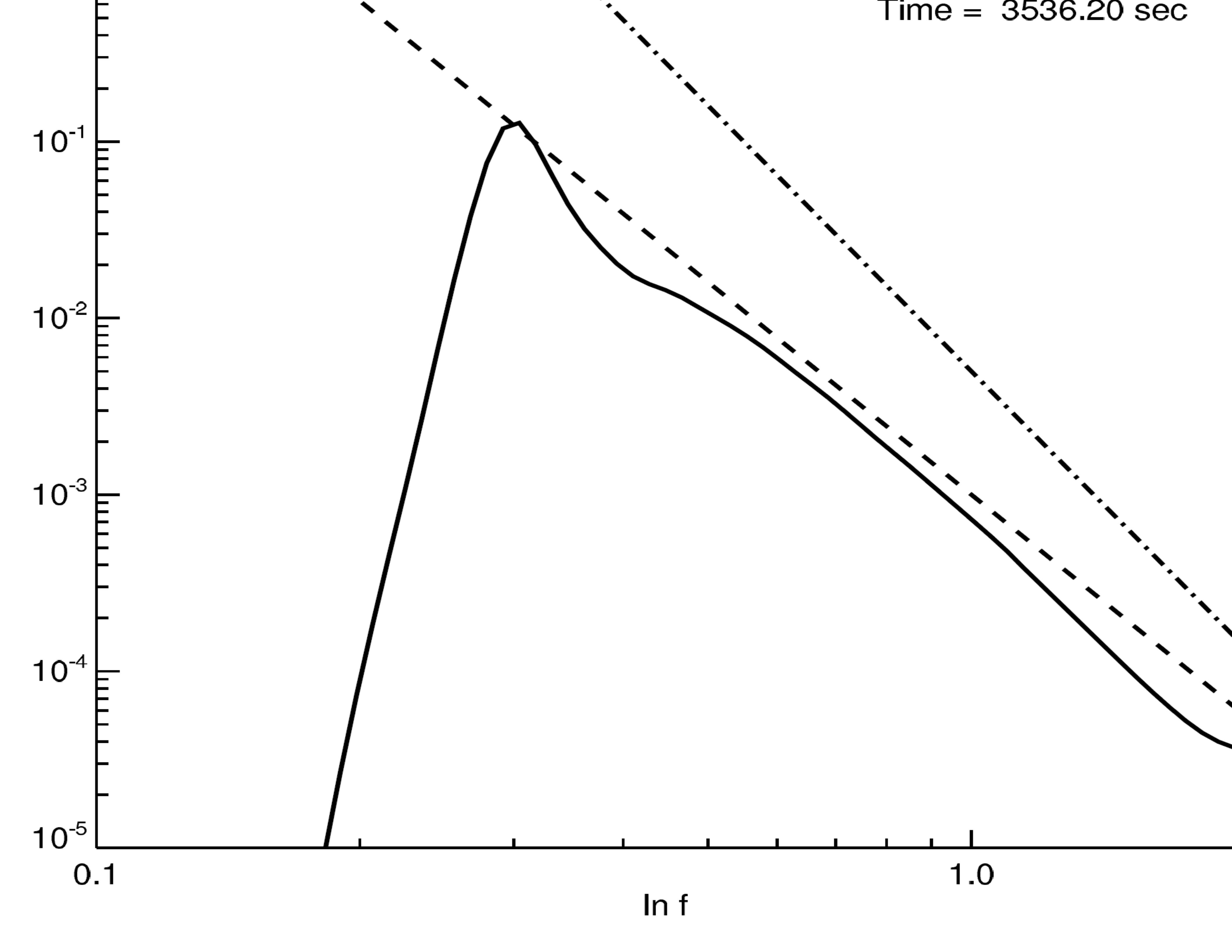
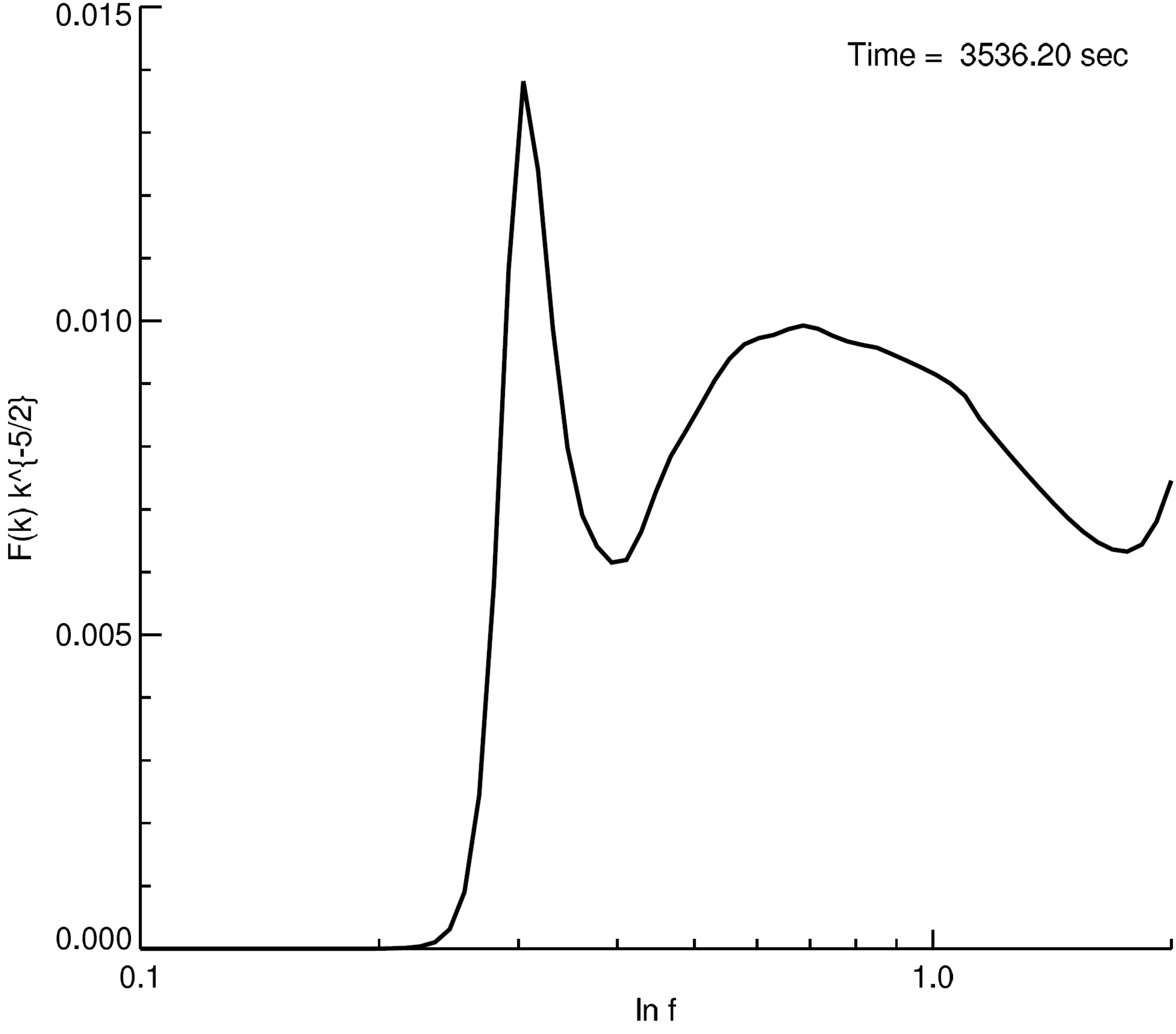


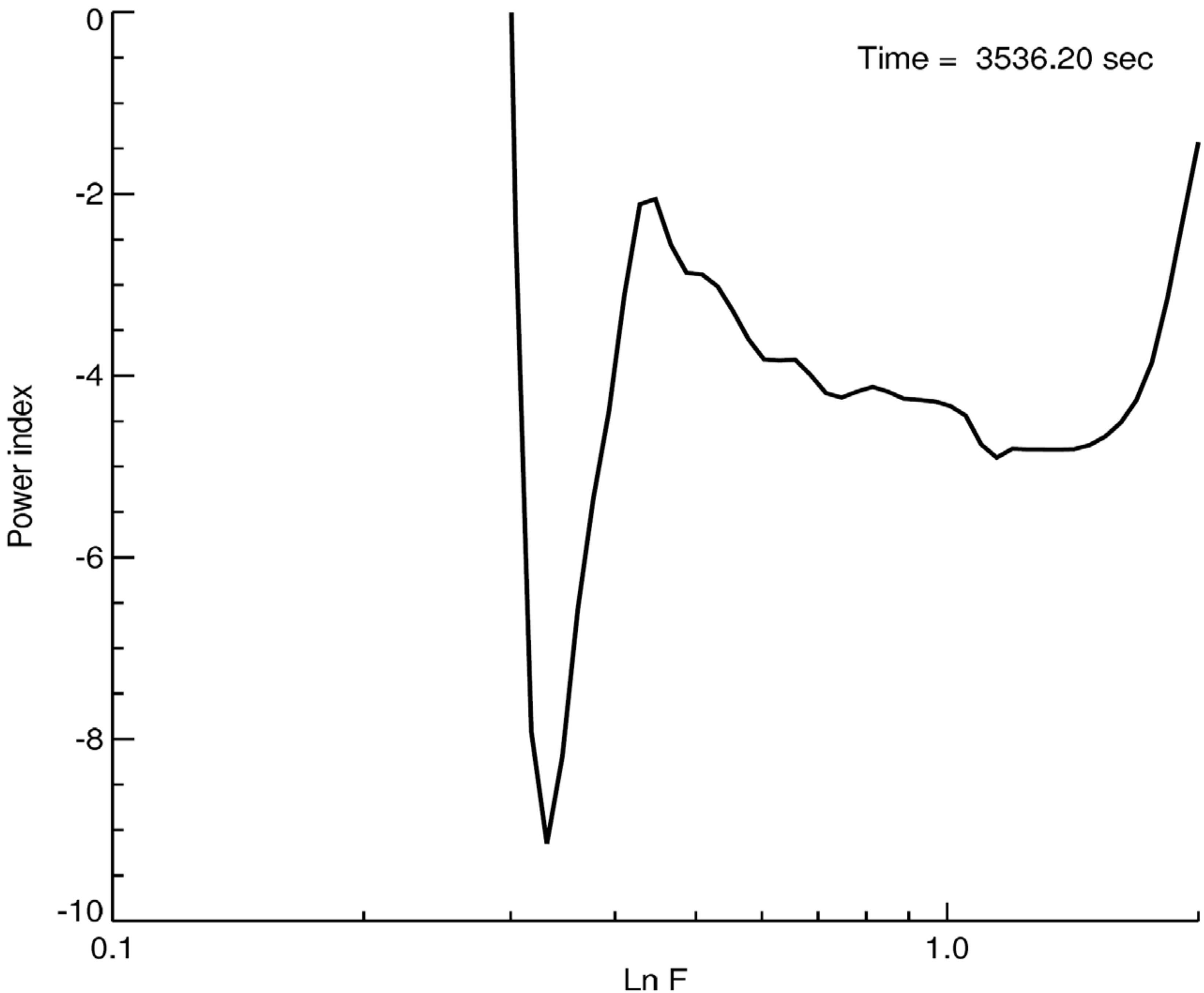

Figure 1: *Normalized wind input function* $\gamma(f, \phi)/\omega$



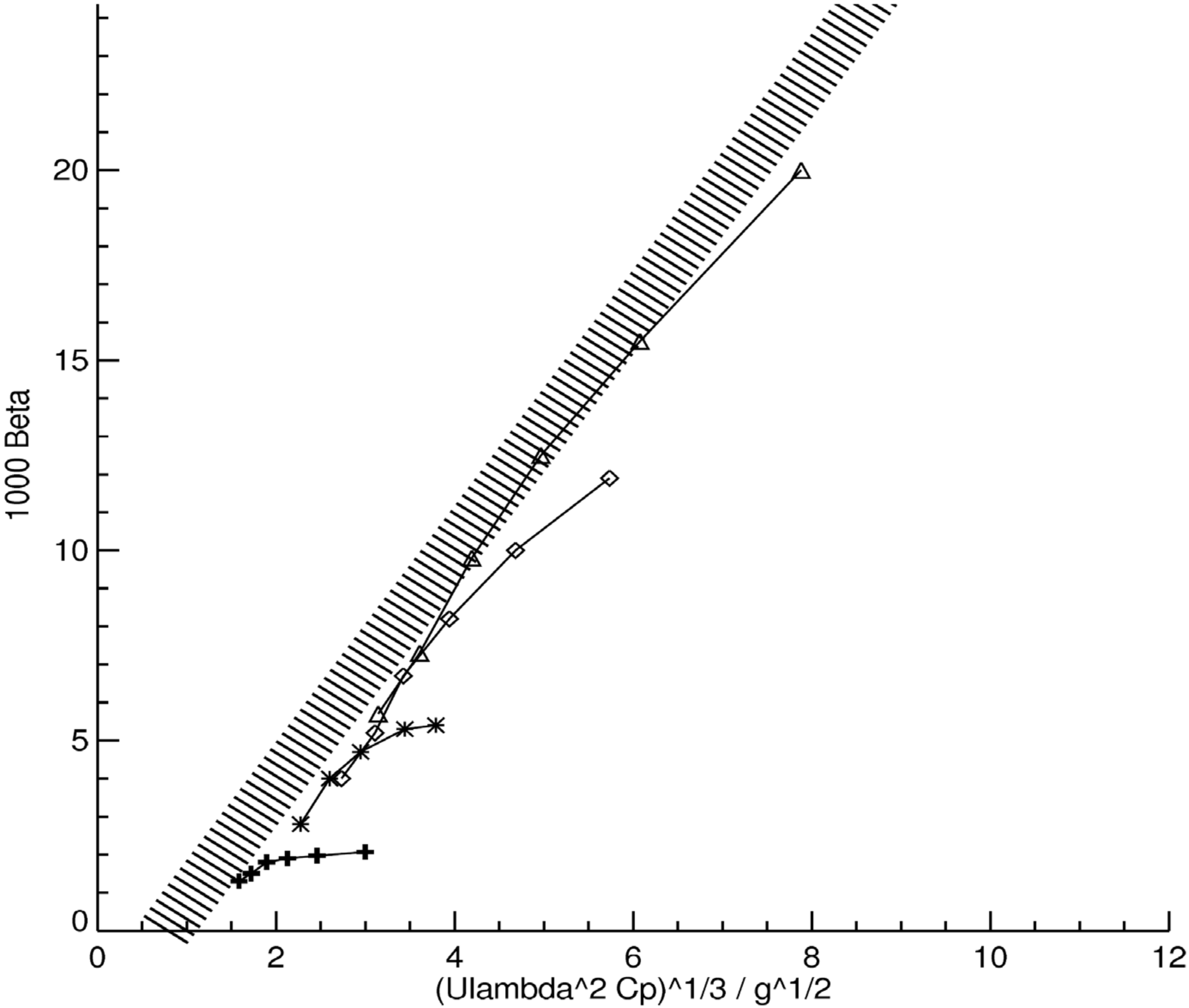
Time = 3536.20 sec
10-1
10-2
10-3
10-4
10-5
0.1
1.0
ln f



Time =  3536.20 sec
0.015
0.010
0.005
0.000
F(k) k^{-5/2}
0.1
1.0
ln f



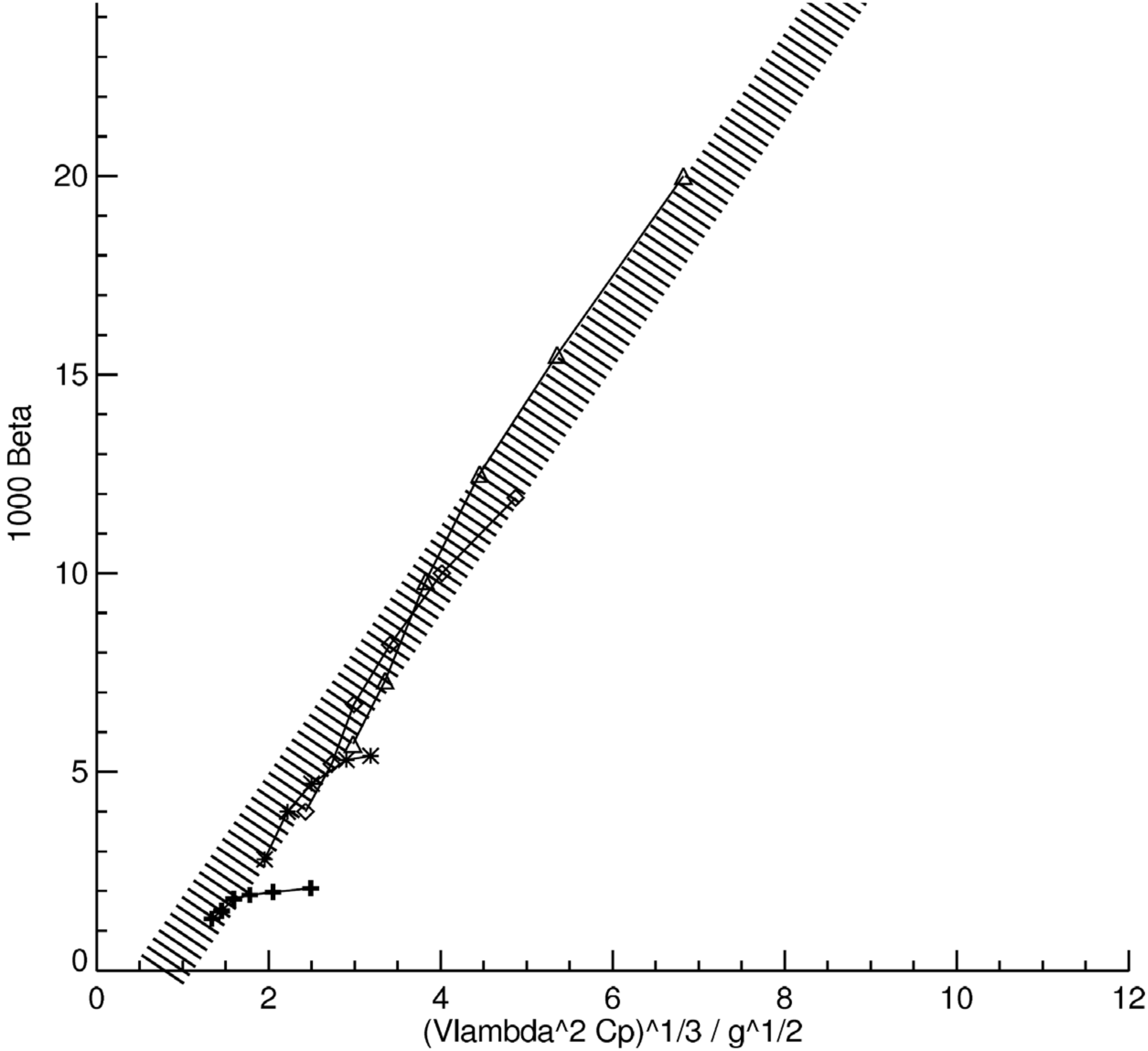

*Power index of energy spectrum* $\frac{d \ln \epsilon(f)}{d \ln f}$*, wind speed* $u = 10.0$ *m/sec*




*Crosses* – $u = 2.5$, *stars* – $u = 5.0$, *rectangles* – $u = 10.0$, *triangles* – $u = 20.0$

Let's rescale velocity: $v = \epsilon u$

$$S_{wind}(\omega,\phi) \quad = \quad -\epsilon^{4/3}\alpha\omega\left(\frac{\omega}{g}v - \frac{1}{\epsilon}\right)^{4/3}\cos\phi \;\; n(\omega,\phi)$$



1000 Beta
0
5
10
15
20
0
2
4
6
8
10
12
(Vlambda^2 Cp)^1/3 / g^1/2

$$v = \epsilon u$$

$$\epsilon = 0.7$$

| ***v*** | ***u*** |
|---|---|
| 2.5 | 3.6 |
| 5 | 7.1 |
| 10 | 14.3 |

# Self-similar interpretation of the regression line

$$\frac{\partial n_{\vec{k}}}{\partial t} = S_{nl} + \gamma(\omega, \phi)\epsilon$$

$$\gamma(\omega, \phi) = \alpha\omega^{1+s} f(\phi)$$

$$\int_0^{2\pi} f(\phi) d\phi = 1$$

$$S_{nl} = \omega \left( \frac{\omega^5 \epsilon}{g^2} \right)^2 \epsilon$$

# Self-similar solution:

$$\epsilon = t^{p+q} F(\omega t^q)$$

$$q = \frac{1}{s+1} \qquad\qquad p = \frac{9q-1}{2}$$

Zakharov-Filonenko asymptotics:

$$F(\xi) \simeq \xi^{-4} \;\; at \;\; \xi >> 1$$

Then for large frequencies

$$\epsilon \simeq \frac{t^{p-3q}}{\omega^4}$$

From another side

$$\epsilon(\omega, \phi) = \frac{\mu g u^{1-\xi} C_p^{\xi}}{\omega^4}$$

$$\omega_p \simeq t^{-q} \qquad t \simeq C_p^{1/q} \qquad C_p \simeq t^q$$

$$\epsilon \simeq \frac{C_p^{\frac{p-3q}{q}}}{\omega^4}$$

$$\xi = \frac{p-3q}{q} = \frac{3q-1}{2q}$$

Remembering $$q = \frac{1}{s+1}$$

we get $$\xi = \frac{2-s}{2}$$

$$\epsilon(\omega, \phi) \simeq \frac{\mu g U^{1-\xi} C_p^{\xi}}{\omega^4} g(\phi)$$

For $\gamma \simeq \omega^{7/3}$ or $s = 4/3$

we get $\xi = 1/3$

which is exactly experimental regression line prediction