

# A New Generation of Source Term Balance: Back to the Future or Forward to the Past?

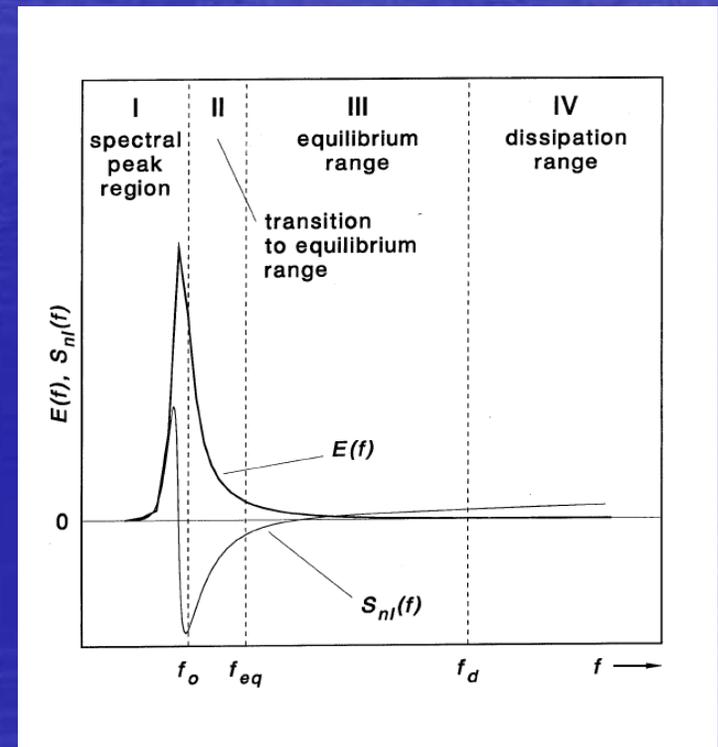
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11<sup>th</sup> International Workshop on  
Wave Hindcasting and Forecasting  
And  
2<sup>nd</sup> Coastal Hazards Symposium

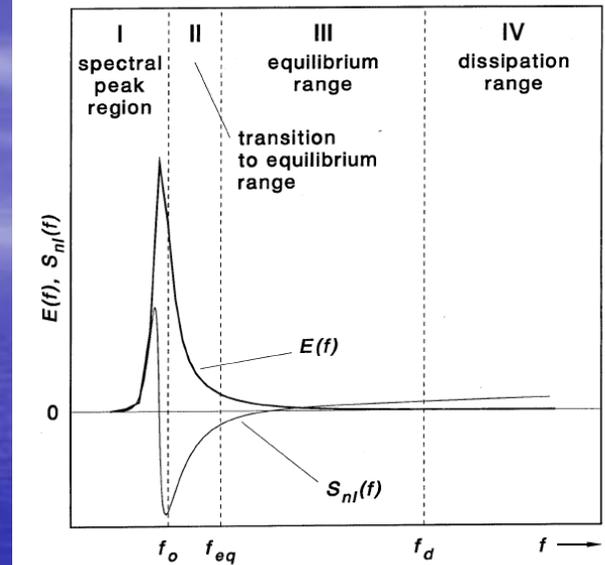
Halifax, N.S., Canada  
October 18-23, 2009



# MOTIVATION

The evolution of wave modeling has come a long way from its early stages:

- We have a lot more empirical terms with empirical coefficients
- And have journal papers to justify each term in these models and commonly state that more terms = better physics
- However, we still cannot replicate the observed spectral evolution along a fetch



Our progress in wave modeling is focused on wave forecasting which deals primarily with statistical skill scores rather than the physics of the processes

Whereas this may work for certain applications, for critical hindcasts of specific storms, particularly in complex situations such as coasts and hurricanes, these models rely heavily on local measurements for repeated re-calibration

This presentation addresses a way to the future via the past

**APPROACH:** “*entia non sunt multiplicanda praeter necessitatem*”  
Occam’s Razor

- Quick historical perspective on modeling
- Assume spectra should be self similar and that different regions in the spectrum have different shapes due to dominance of different processes
- Assume wind input is dominant near/in front of the spectral (thus, the very narrow directional spread at the spectral peak)
- Assume equilibrium range is controlled by  $S_{nl}$  in deep and shallow water, provided that slope is relatively mild
- Assume breaking primarily affects high-frequencies
- Simulate nondimensional energy and frequency growth with fetch and compare to observational data
- Simulate spectral evolution and compare to observational data

# SUMMARY/CONCLUSIONS

- The past two decades have been spent under the Phillips paradigm that all source terms are significant in all portions of the spectrum.
- This abandoned the older concept due to Kitaigorodskii that different regions of the spectrum had shapes that reflect the dominant source terms.
- No existing 3G model can produce a growth rate from very young waves to fully developed that is unconstrained and agrees with self-similar spectra and energy/fp growth rates.
- A simple hypothesized set of source terms under development here appear to come very close to accomplishing this task, including
  - Growth of energy with fetch
  - Change of peak frequency with fetch
  - Equilibrium range coefficient consistent with obs
  - Peakedness that can stay above 2 for young waves
  - Directional characteristics consistent with obs

# Historical Review of Wave Models

- 1940's 0-G models assumed that waves in nature could be understood in terms of dimensionless fetch and duration and swell decay
- Sverdrup, Munk, Bretschneider (1947) adapted data from observations to calibrate these models
- Life was good if we only had accurate winds!!

$$\hat{E} = q_1 \hat{X}^{q_2}$$

where

$$\hat{E} \text{ is dimensionless energy} = \frac{gE_0}{u^4}$$

with  $E_0$  taken to be total energy

$$\hat{X} \text{ is dimensionless fetch} = \frac{gX}{u^2}$$

with  $X$  taken to be fetch

and

$u$  is wind speed

In all studies  $q_2$  is  $\approx 1$  which implies that a constant fraction of momentum leaving the atmosphere enters and is retained in the wave field

# Historical Review of Wave Models

- 1960's 1-G models represent a balance of direct wind input and local breaking
- Local breaking is a “fast” process, so Phillips (1958) assumed it should be a universal constant
- Life was good if only we had accurate winds!!

$$E(f) \rightarrow \frac{\alpha g^2 f^{-5}}{(2\pi)^4}$$

$$\alpha = 0.0081$$

# Historical Review of Wave Models

- Except in the 1970's people began to try to use these models for fetch limited conditions
- They did not work.....
- AND alpha was not a universal constant
- AND Hasselmann's first principle theory for  $S_{nl}$  had been derived
- BUMMER

# Historical Review of Wave Models

- 1970's - Barnett, Ewing and Resio developed early versions of 2-G models – with variable alpha's but still  $f^{-5}$
- BUT simultaneously Toba and others were beginning to accumulate evidence that spectra were  $f^{-4}$
- This now implies a balance between nonlinear energy fluxes and net inputs near spectral peak

$$E(f) \rightarrow \frac{ugf^{-4}}{(2\pi)^3}$$

And life should have been good again if we only had good winds!!

# Historical Review of Wave Models

- 1980's Hasselmann argues that the SNL source terms cannot capture the behavior of the fluxes because they do not have a sufficient number of degrees of freedom in their stated form
- He cannot develop an appropriate approximation for the full 3-D imbedded integral, so he postulates that the Phillips 1-D integral might provide an OK surrogate
- 3-G models are born - and no they do not have better physics than 2-D models
- 1985-2009 Models are continually tuned and retuned – focus is on methods to improve skill scores and Quantile-Quantile plots
- BUMMER

Is there an alternative that can get us back to a focus on the physics of the processes, while at the same time moving us into a new generation of wave model?

Less empirical tuning factors

Better agreement with spectral characteristics

Directionally integrated shapes

Directional energy distributions

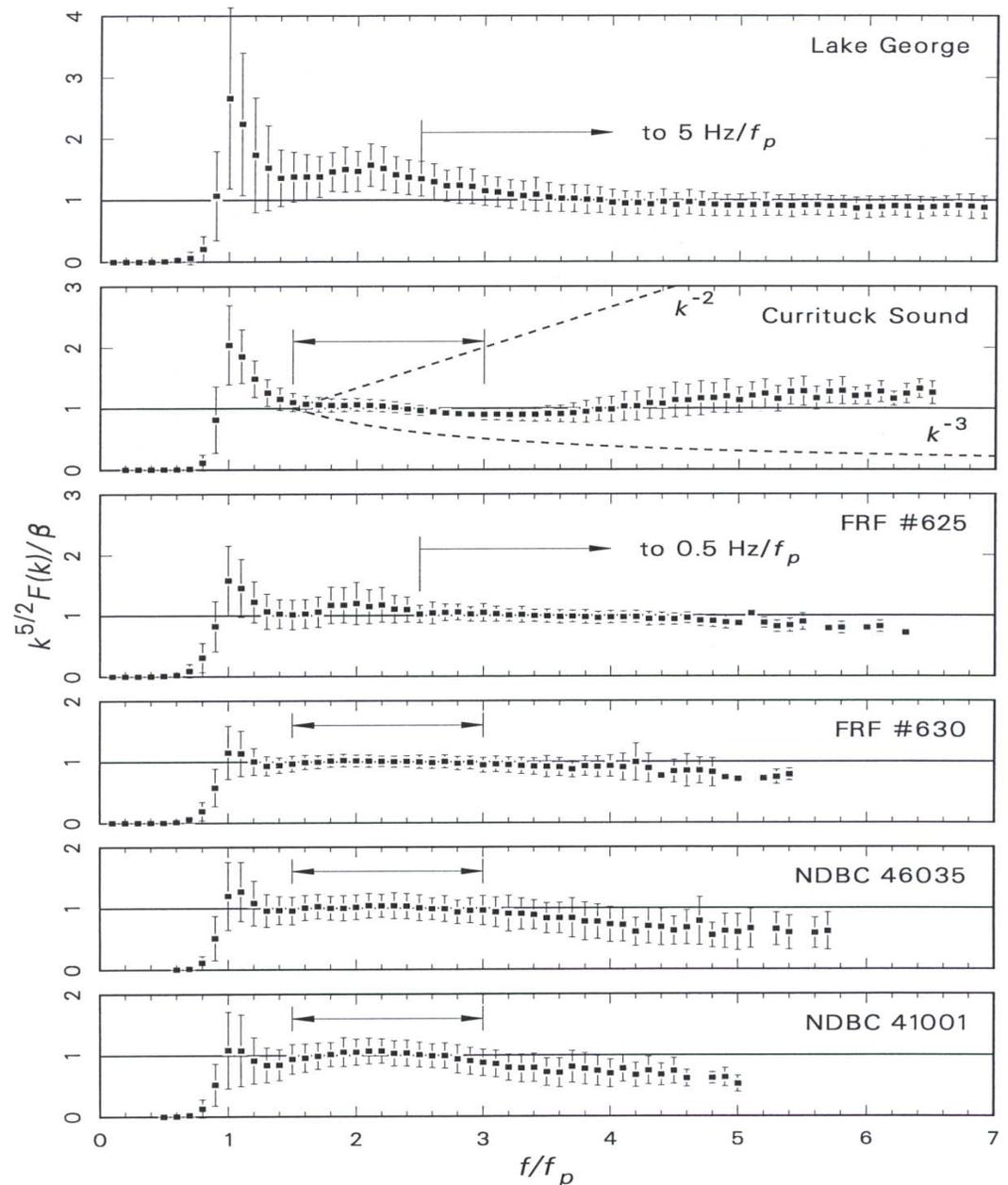
$$\hat{F}(k) = F(k)k^{5/2}$$

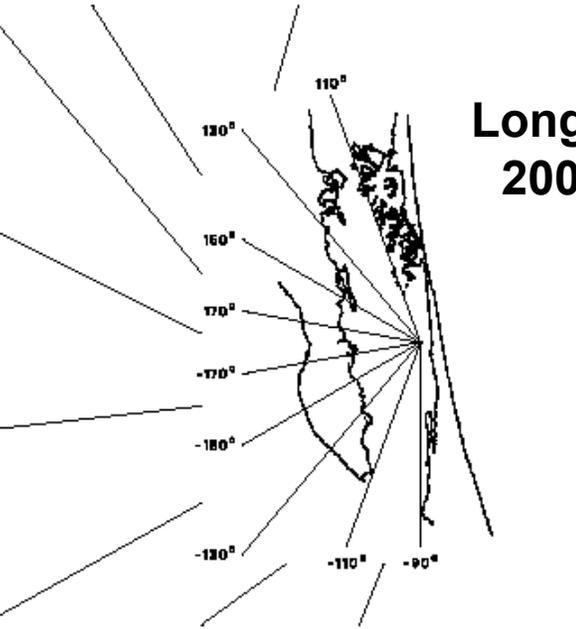
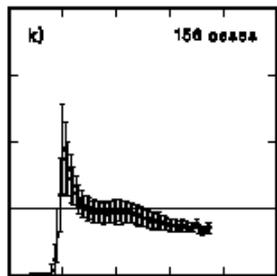
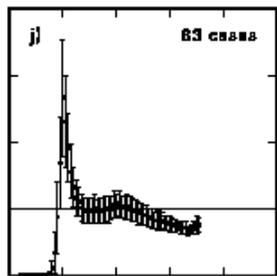
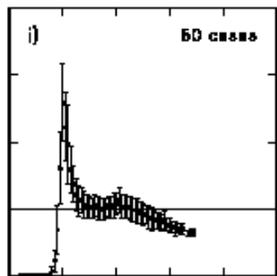
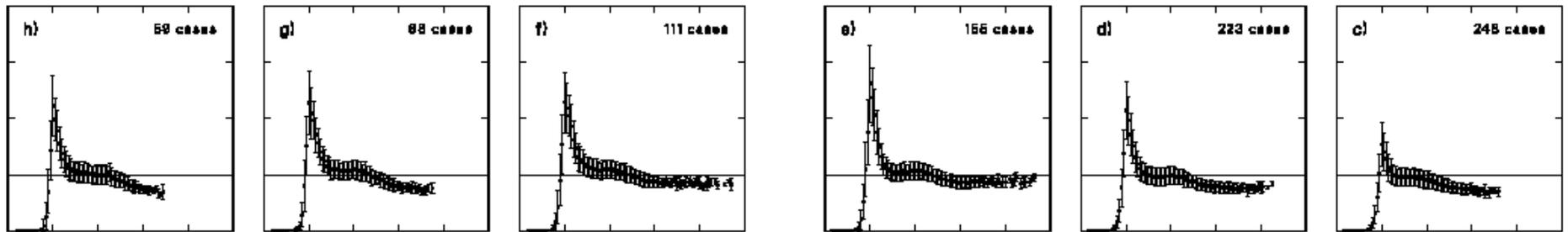
$$\hat{E}(f) = E(f)f^4$$

“Compensated” spectra:  
Straight line is equivalent to  
 $f^{-4}$  scaling in deep water.

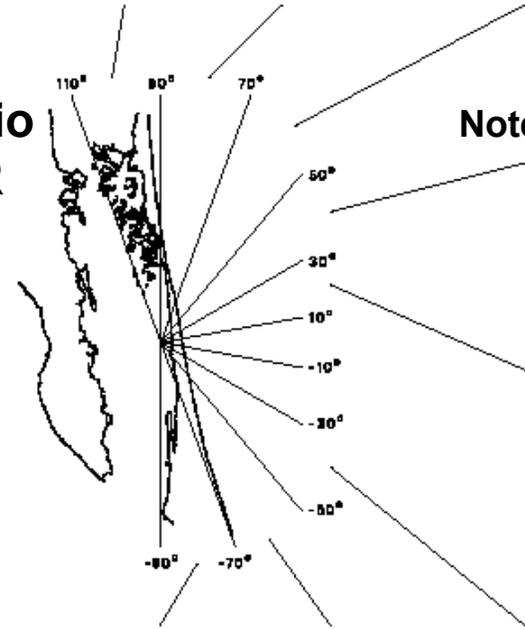
Shape suggests an  $f^{-4}$  basis  
for all these spectra; however  
what is  $\beta$  related to?

Equivalent to a constant energy  
flux toward high frequencies via  
nonlinear wave-wave interactions

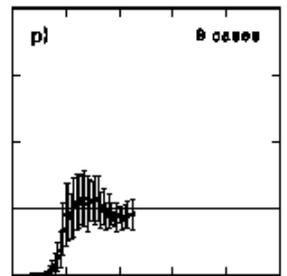
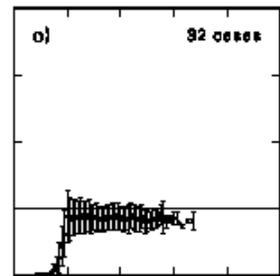
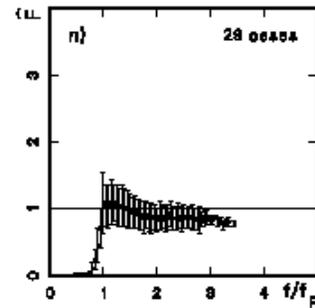
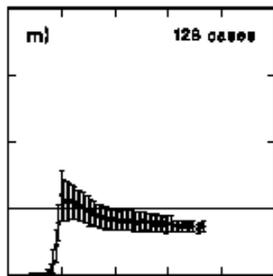
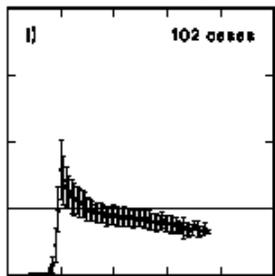
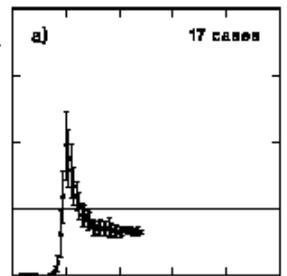
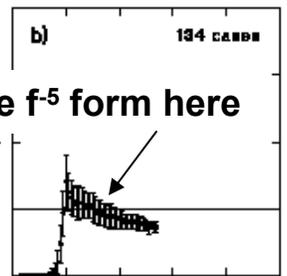


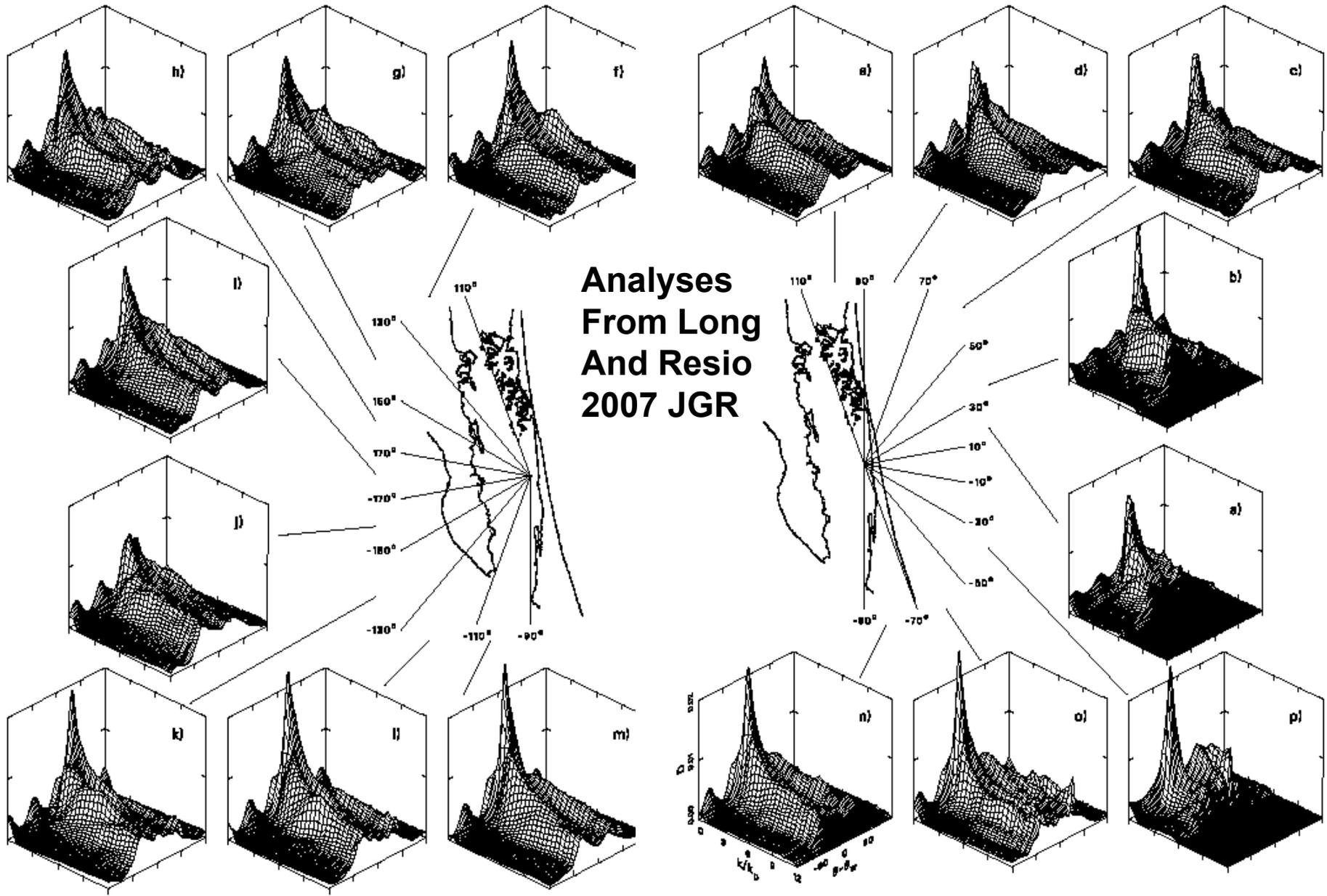


**Long & Resio  
2007 - JGR**

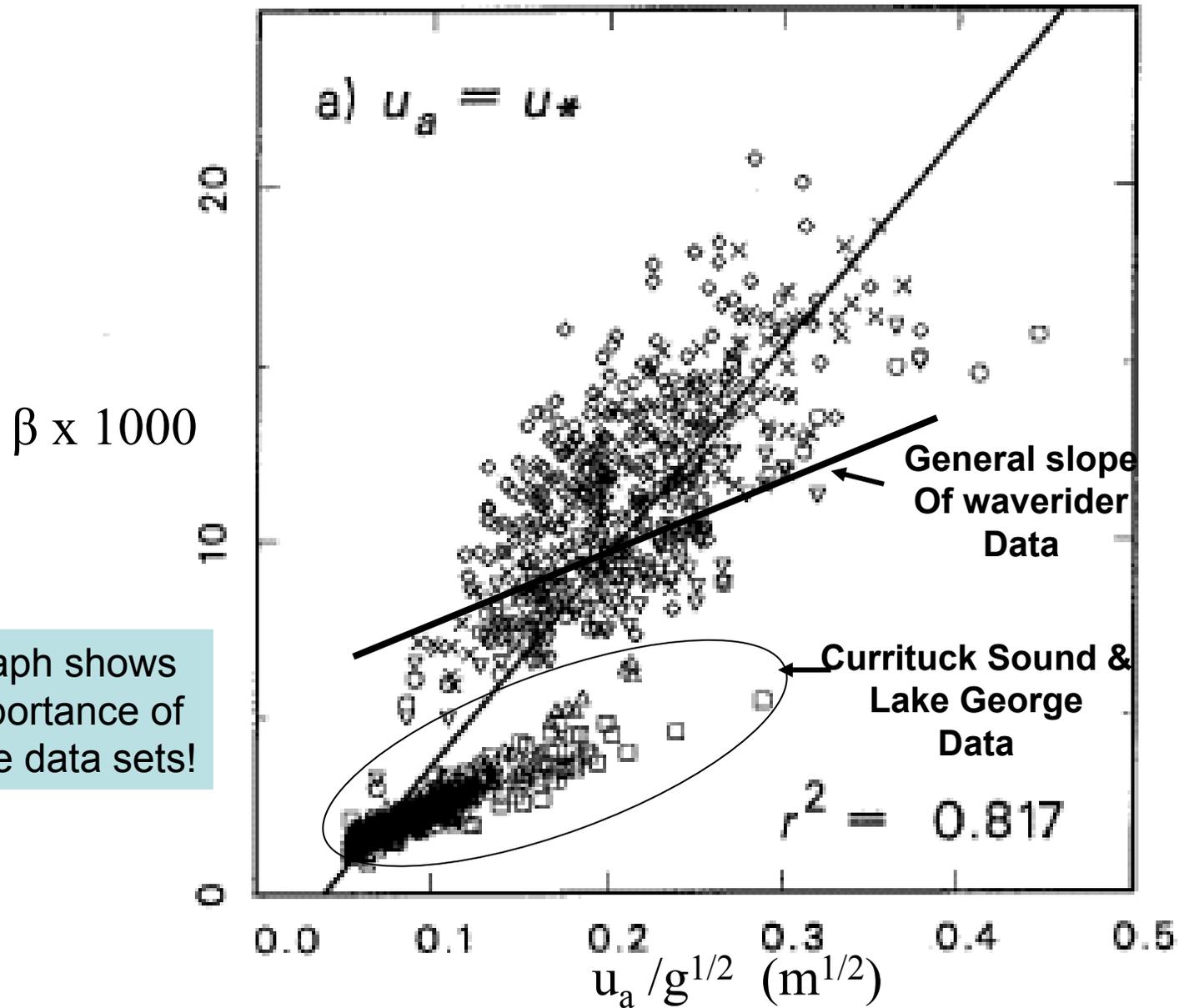


Note  $f^{-5}$  form here

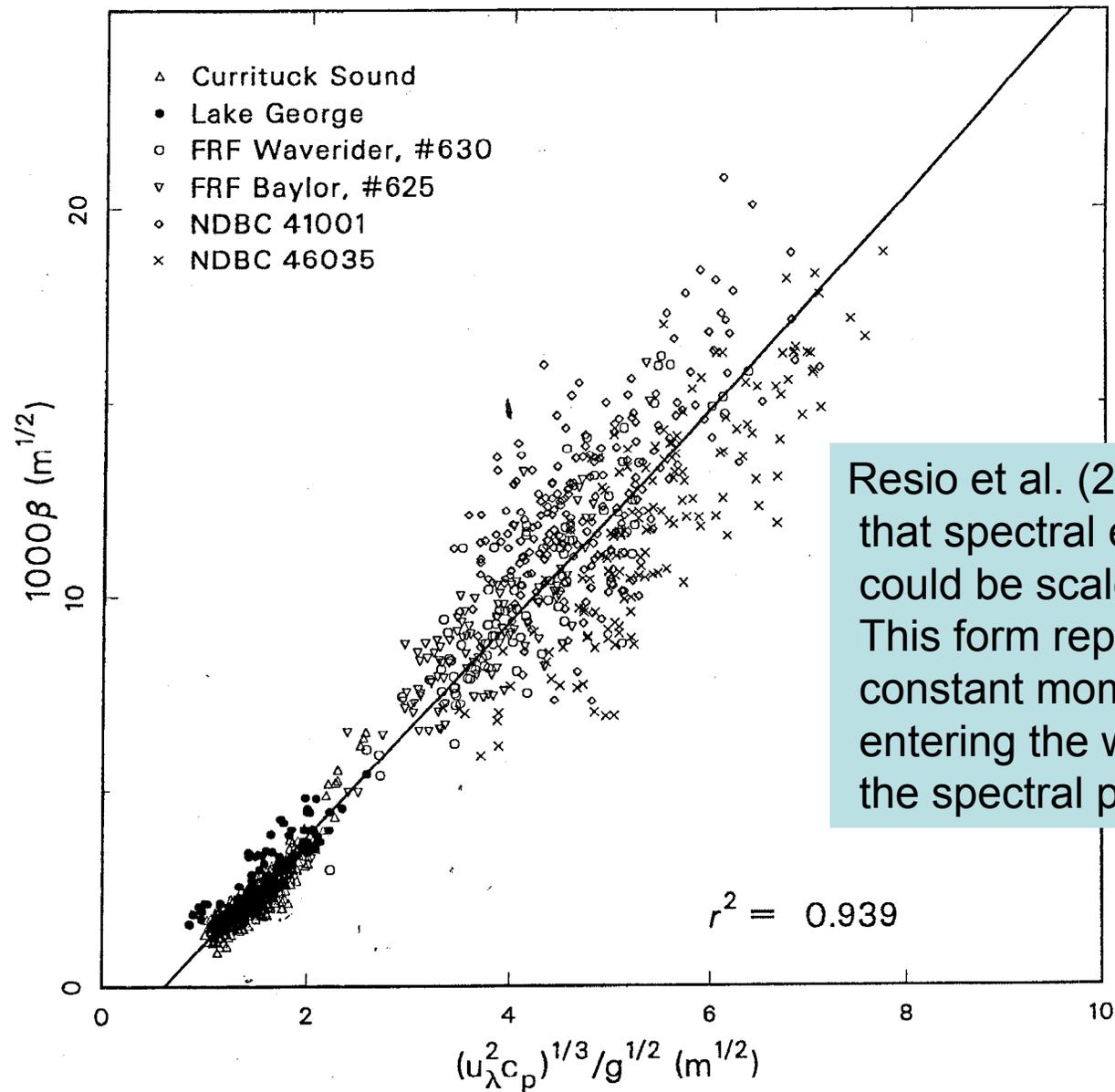




Toba, Belcher and others have postulated that  $\beta$  is linearly proportional to wind speed. This clearly does not work for multiple data sets.

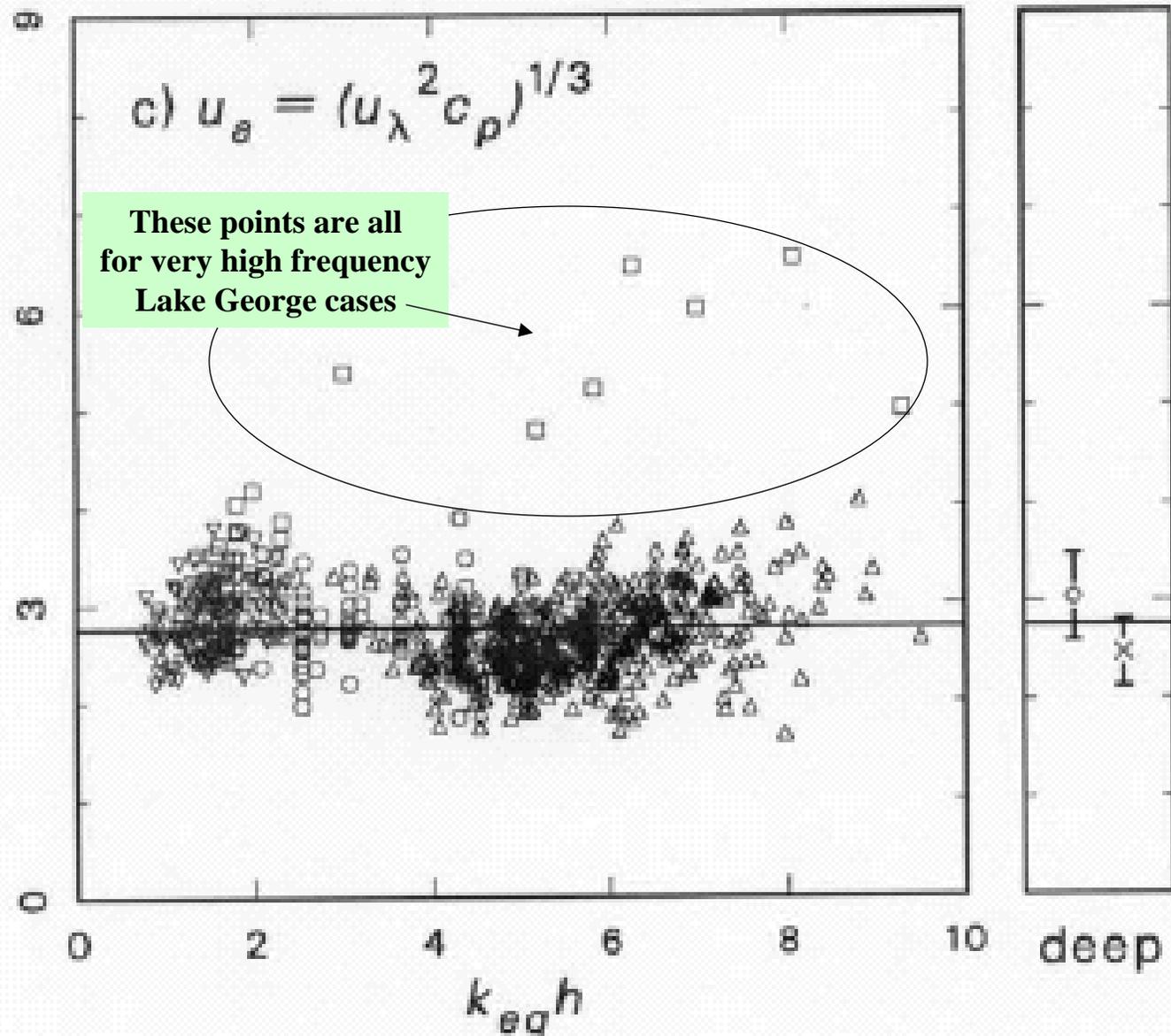


This graph shows the importance of multiple data sets!



Resio et al. (2004) showed that spectral energy levels could be scaled consistently. This form represents a constant momentum fraction entering the wave field near the spectral peak.

$$\frac{1000 \times \beta'}{u_a - u_0}$$



**Detailed analyses show no systematic deviation from  $k^{-5/2}$  form or variation of the equilibrium range coefficient with relative depth**

## Theoretical perspective for these rather robust results

Energy flux through equilibrium range is given by

$$\Gamma_E = \frac{\Lambda \beta^3}{g}$$

Momentum conserving wind input is given by

(Miles theory as interpreted physically by Lighthill, 1962)

$$\Gamma_{in} \sim u^2 c \quad (\text{for each component receiving energy})$$

Flux balance is, for a self-similar spectrum, simply

$$\beta^3 \sim u^2 c \quad \text{or} \quad \beta \sim (u^2 c)^{1/3}$$

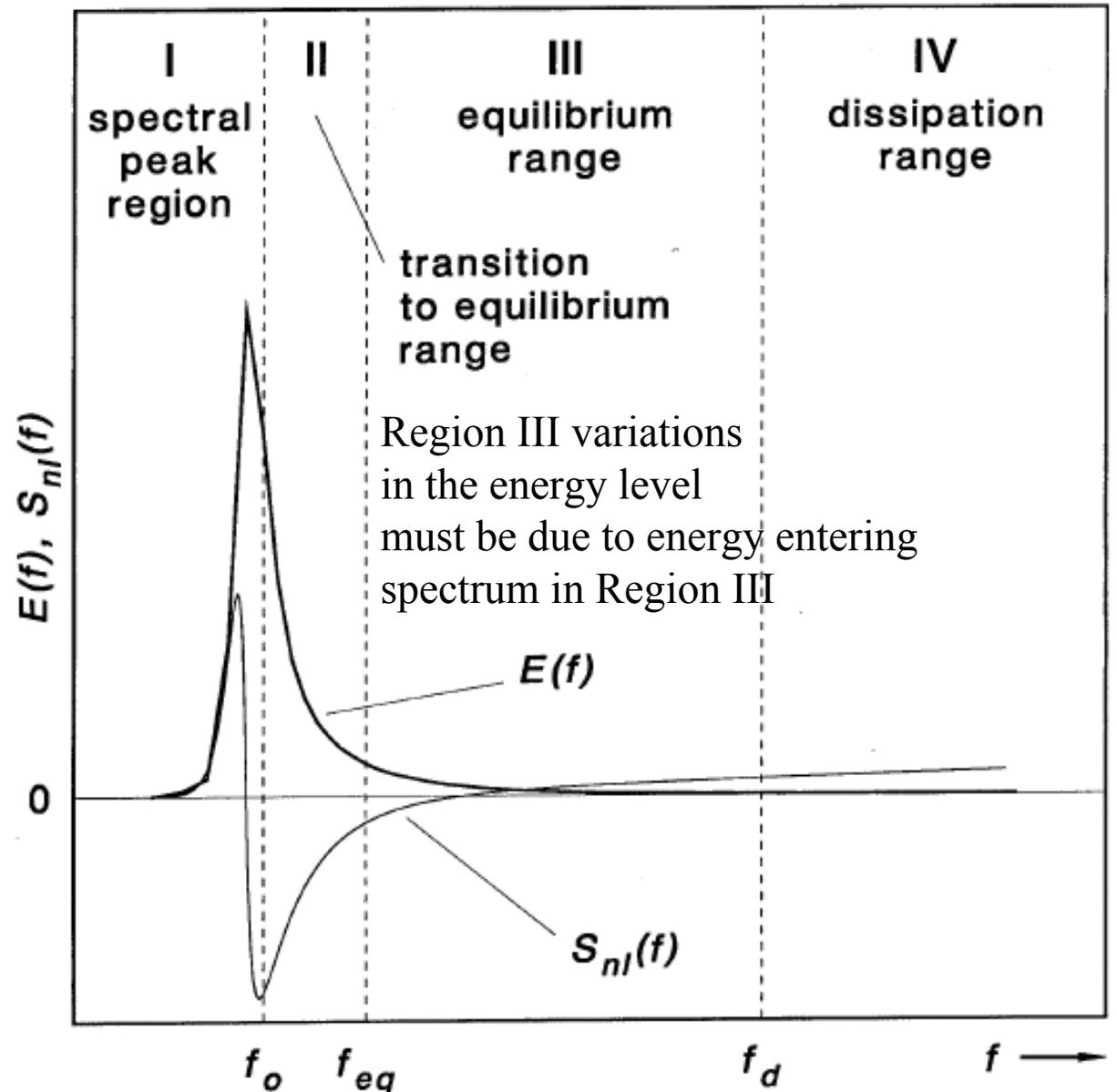
A simple way to understand the parameter  $f_0$  is that

$$\int_0^{f_0} S_{nl}(f) df = 0$$

It also is where the net Flux = 0

Region I – all net energy is retained – integral of wind input –dissipation from 0 to  $f_0$

Region II – must provide energy into equilibrium range



**Concept of Spectral Regions  
Based on Source Term Balances**

We want the compensated spectrum to look something like:

$$E(f) = \frac{2\beta g}{(2\pi)^3} f^{-4} \left[ z_4 \left( \frac{f}{f_p} \right)^4 \exp(-\Theta_4) + 1 \right]$$

where

$\beta$  is the equilibrium range constant as defined in Resio et al. (2004)

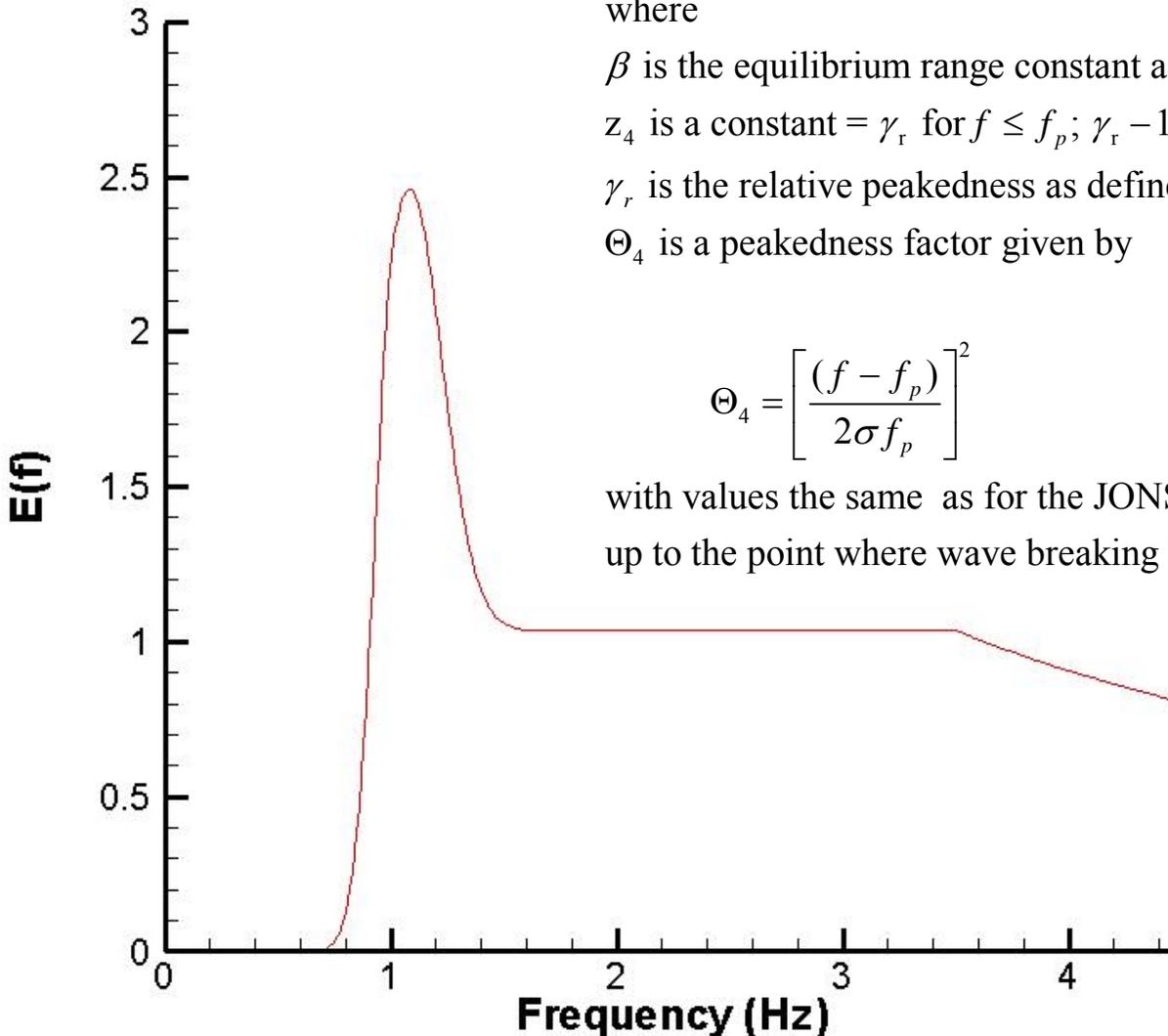
$z_4$  is a constant =  $\gamma_r$  for  $f \leq f_p$ ;  $\gamma_r - 1$  for  $f > f_p$

$\gamma_r$  is the relative peakedness as defined in Long and Resio (2007)

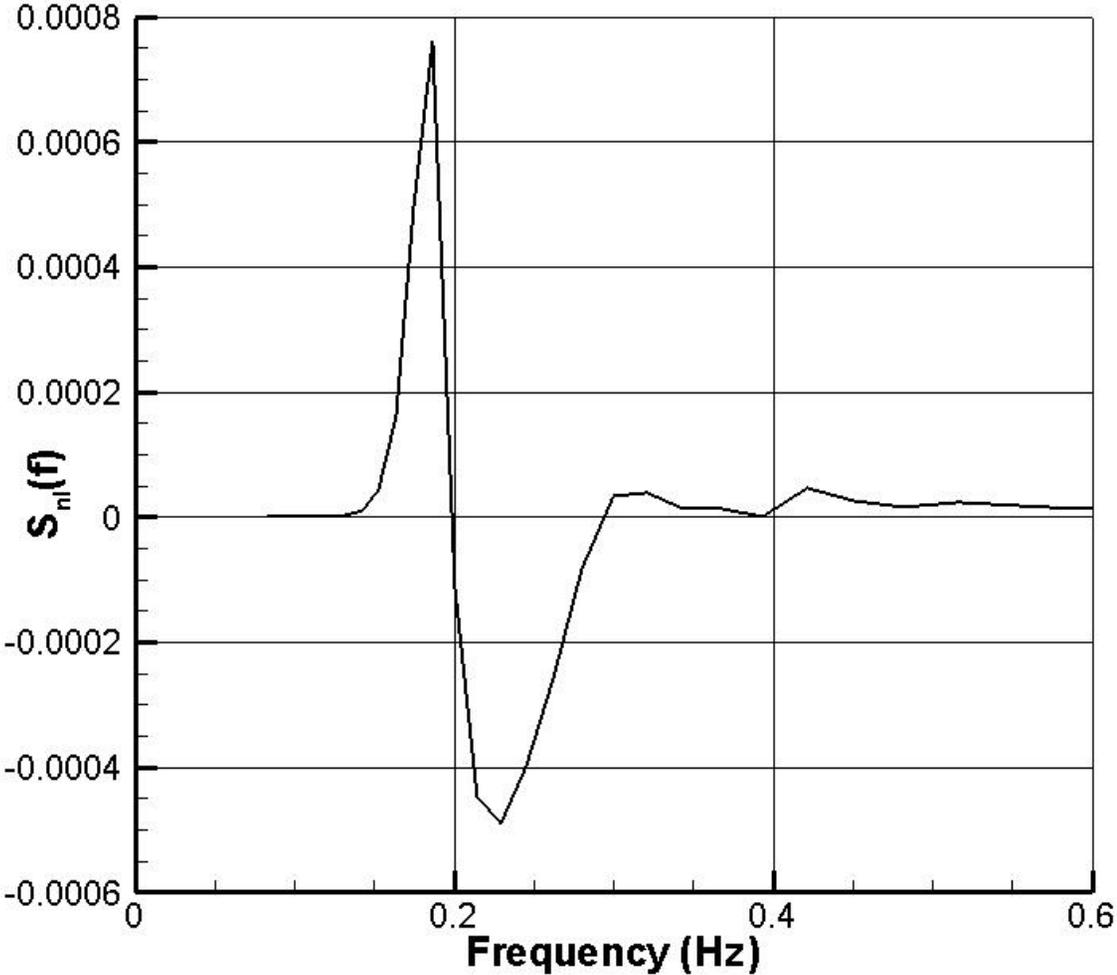
$\Theta_4$  is a peakedness factor given by

$$\Theta_4 = \left[ \frac{(f - f_p)}{2\sigma f_p} \right]^2$$

with values the same as for the JONSWAP spectrum, up to the point where wave breaking becomes dominant.

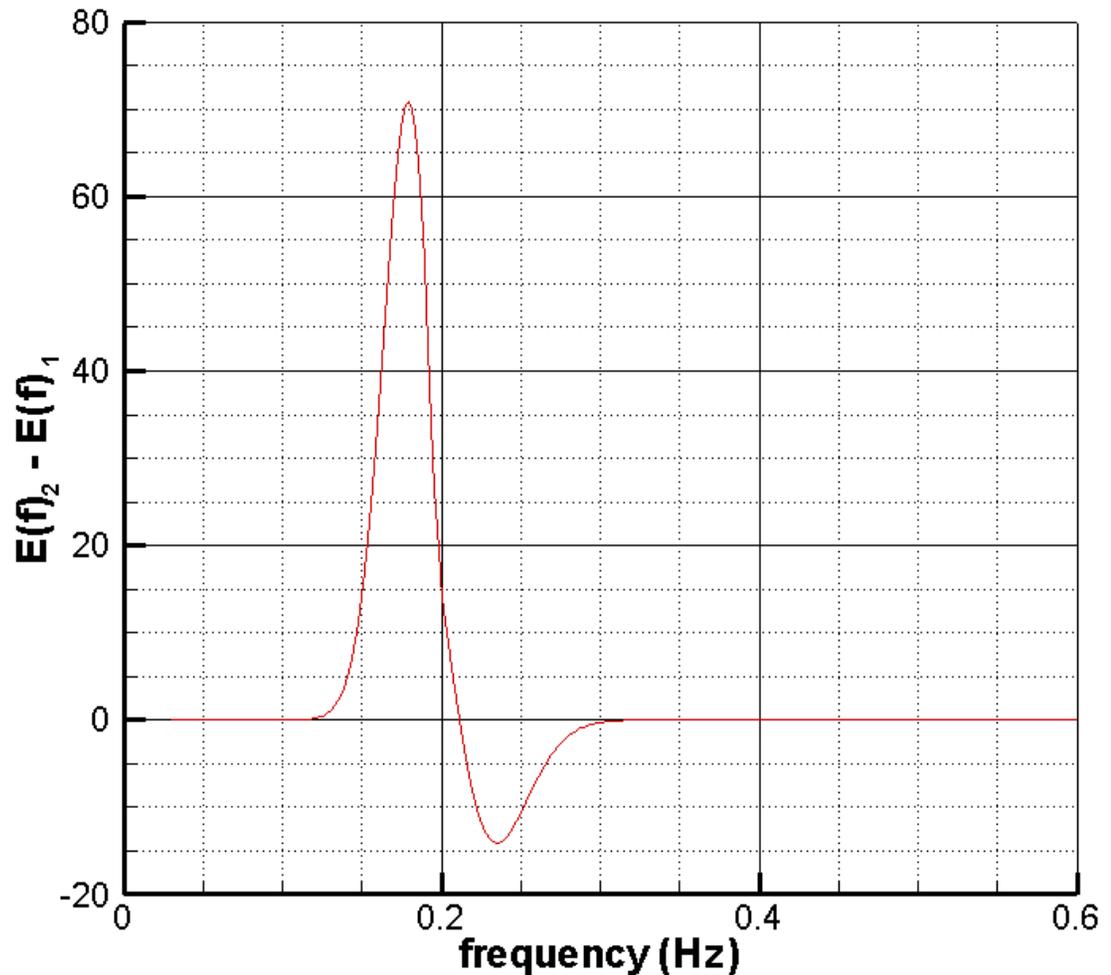


# **S<sub>n</sub>(f) for a spectrum of the self-similar type shown in the previous slide**

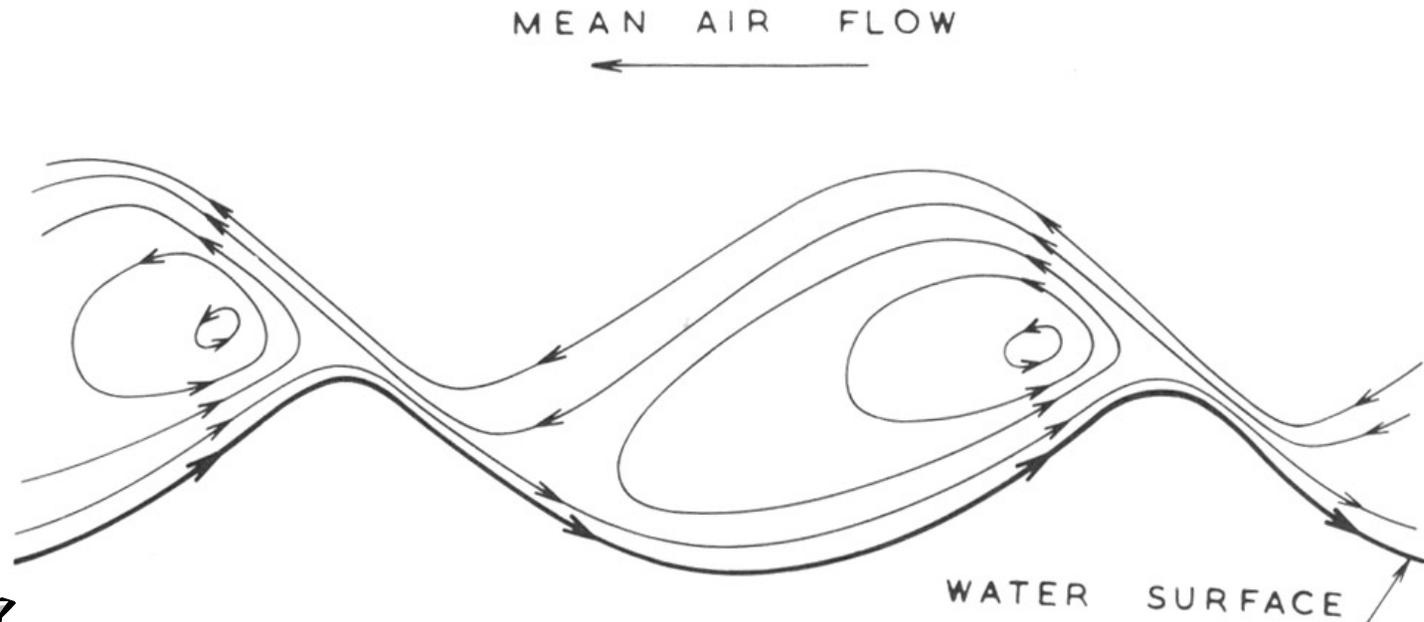


**If breaking is negligible near the spectral peak then wind input can be approximated as a closure term using the self-similar pattern of growth from observations minus  $S_{nl}$**

$$S_{in}(f, \theta) = \left[ \frac{\partial E(f, \theta)}{\partial t} - S(f, \theta) \right]$$



Theoretical perspective for these rather speculative results”



**Premise: The wind reacts to the water surface and not to individual spectral components**

## Examine the work rate on “zero-crossing” waves

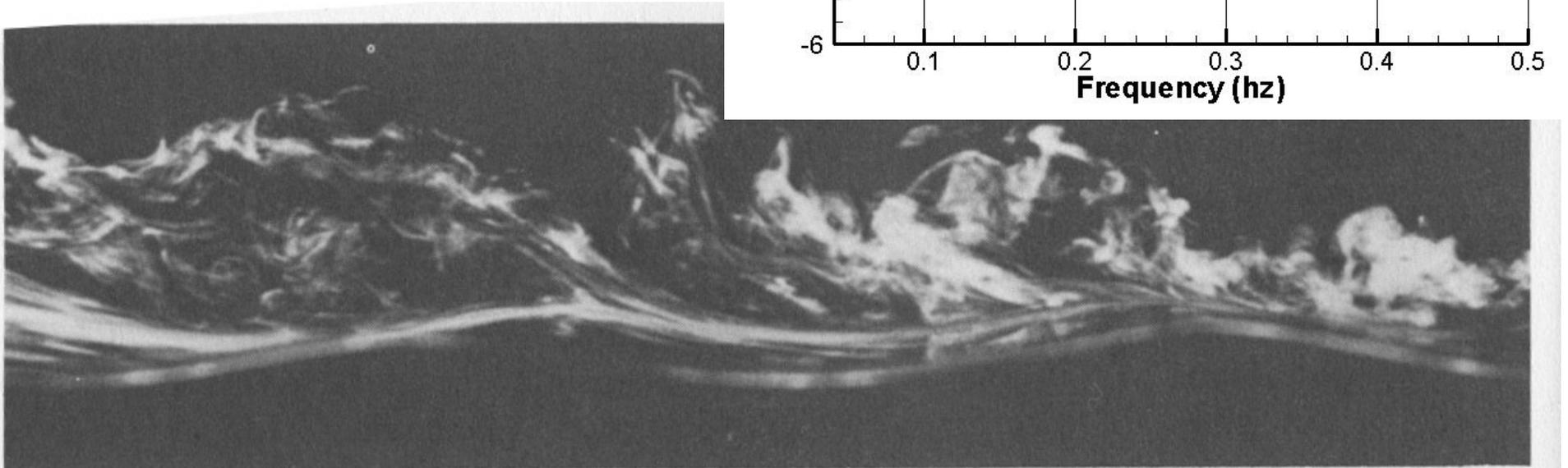
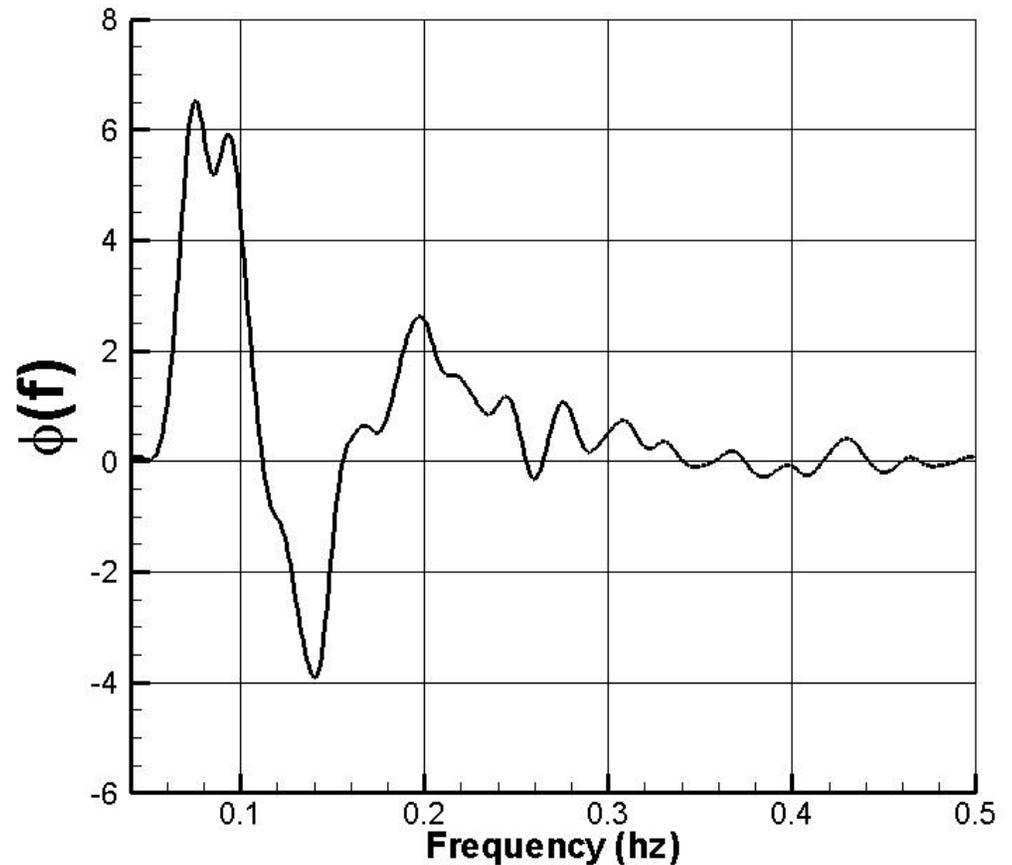
Energy Transfer Rate

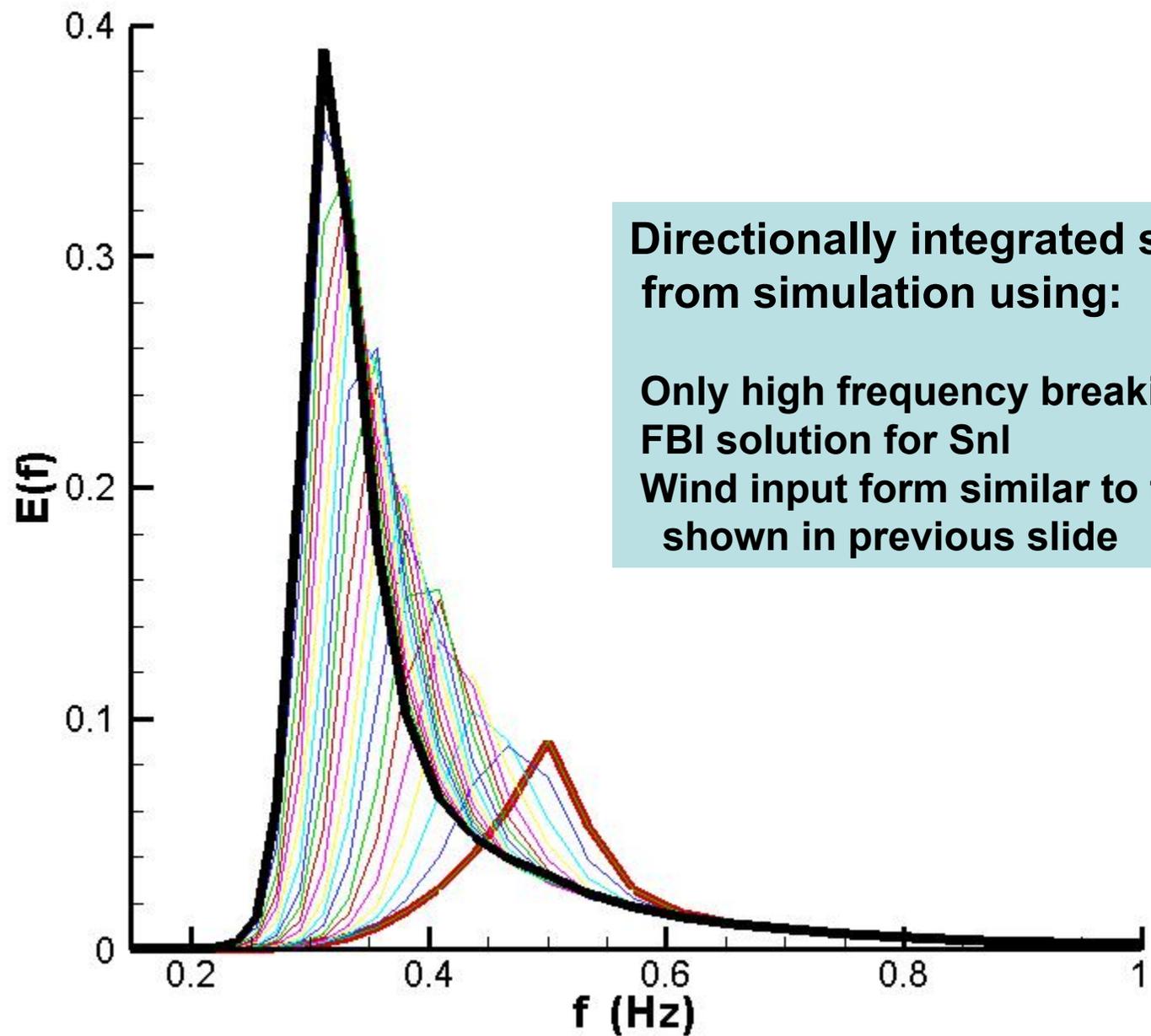
$$\phi_f \sim \langle p(t) \square v_n(t) \rangle$$

where

$p(t)$  is the pressure at the atmosphere  
water interface

$v_n(t)$  is the vertical velocity of the  
"zero-crossing" wave





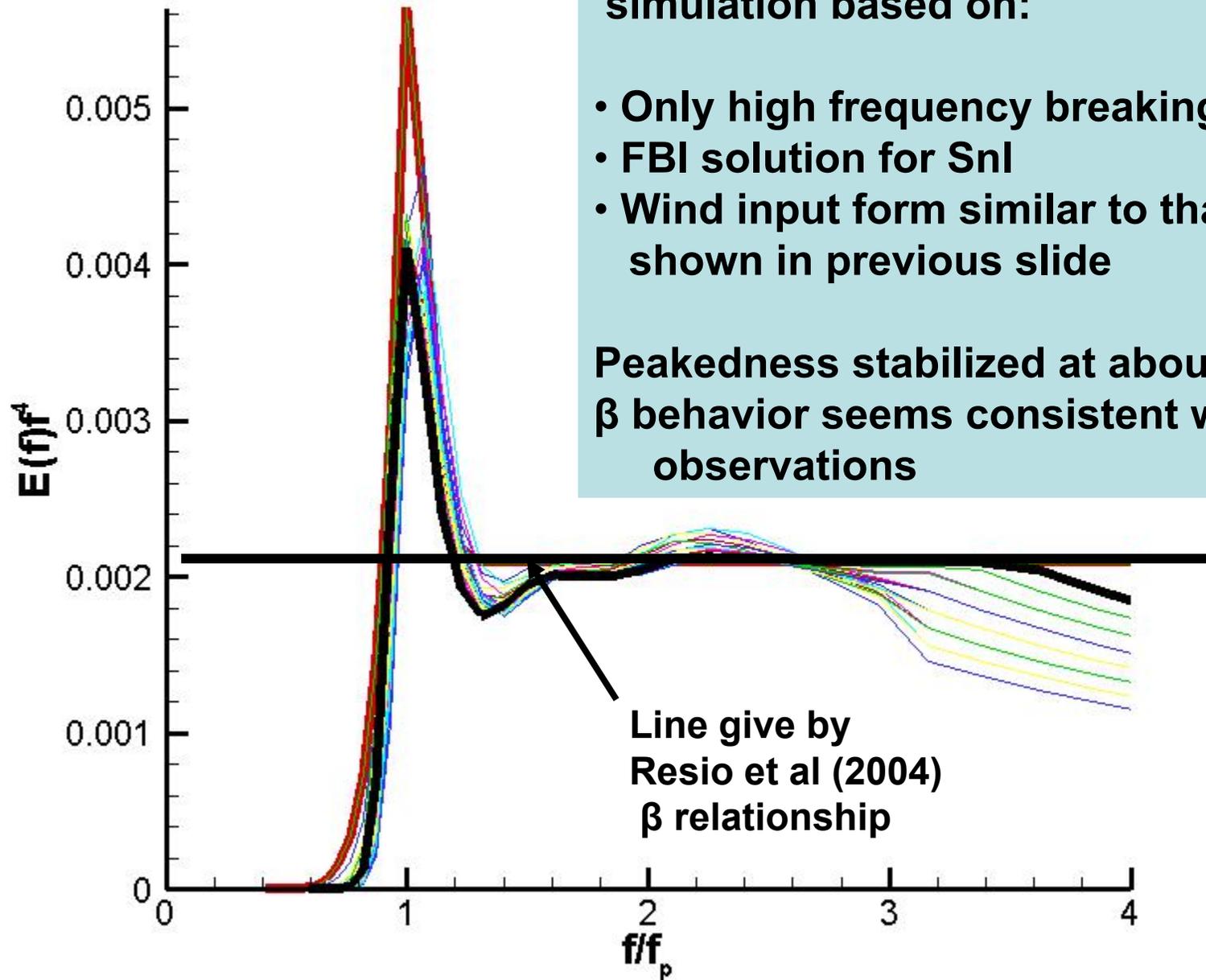
**Directionally integrated spectra  
from simulation using:**

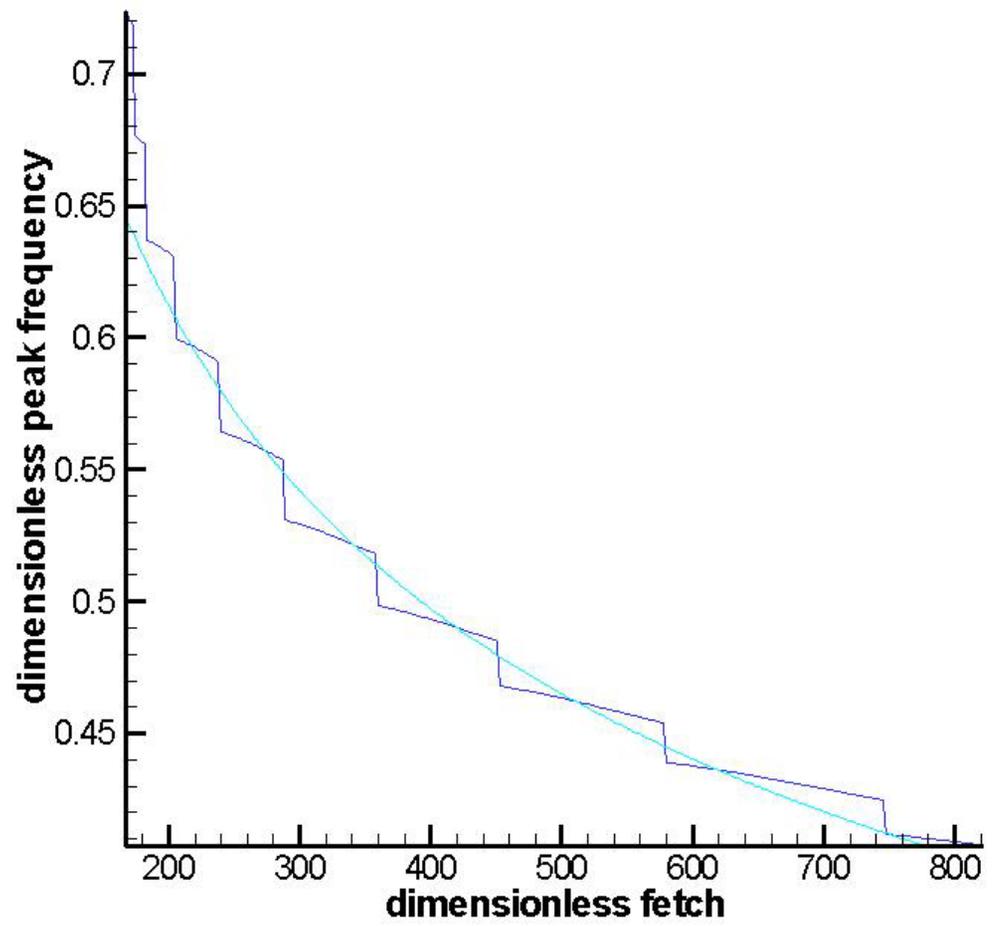
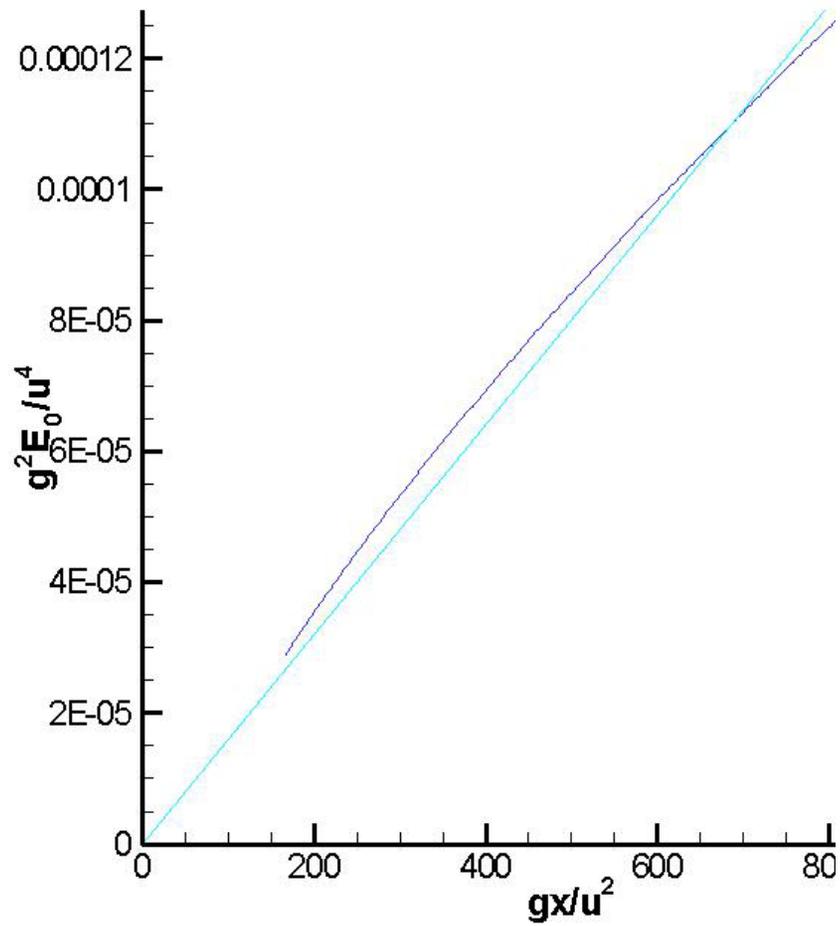
**Only high frequency breaking  
FBI solution for Snl  
Wind input form similar to that  
shown in previous slide**

Compensated spectra from simulation based on:

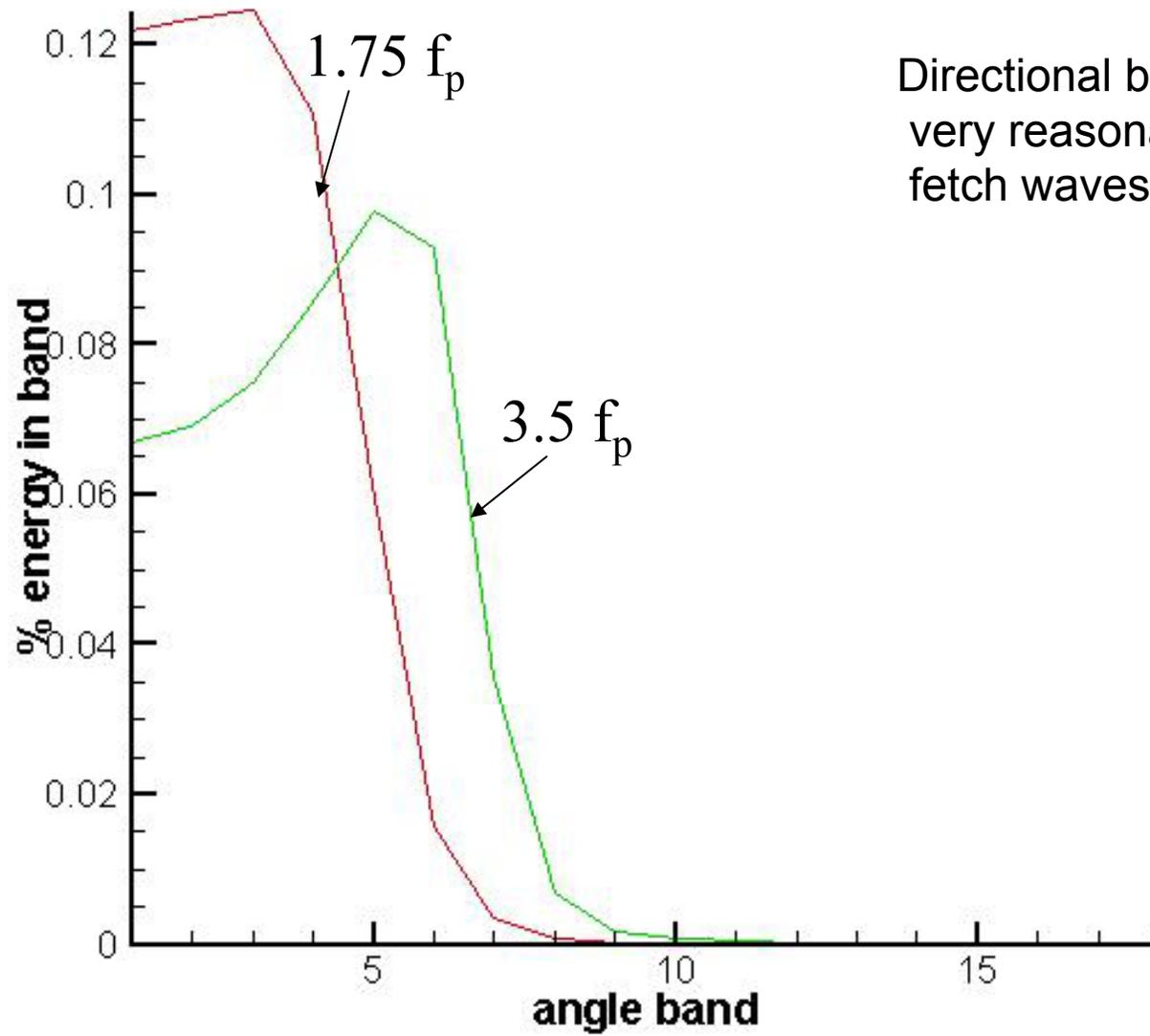
- Only high frequency breaking
- FBI solution for  $S_{nl}$
- Wind input form similar to that shown in previous slide

Peakedness stabilized at about  $2\beta$  behavior seems consistent with observations



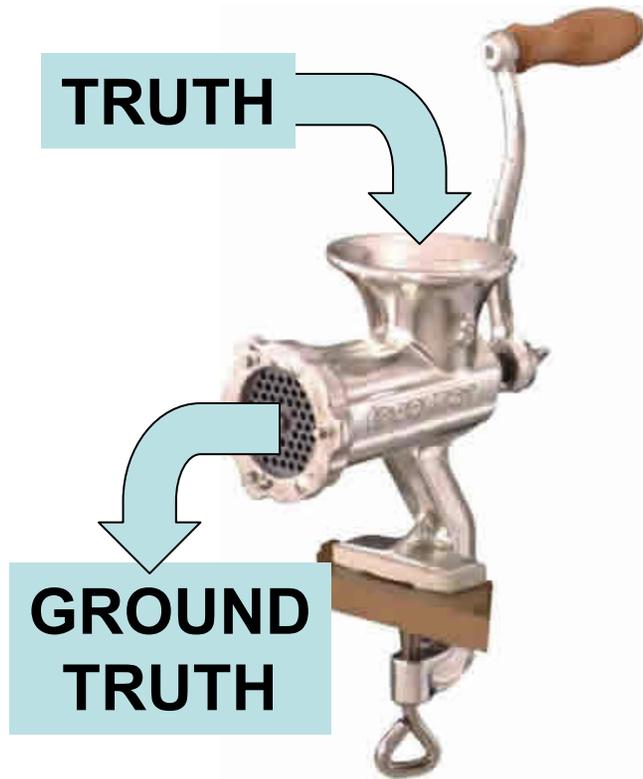


**Integrated energy and peak frequency characteristics**



Directional behavior seems very reasonable for short fetch waves

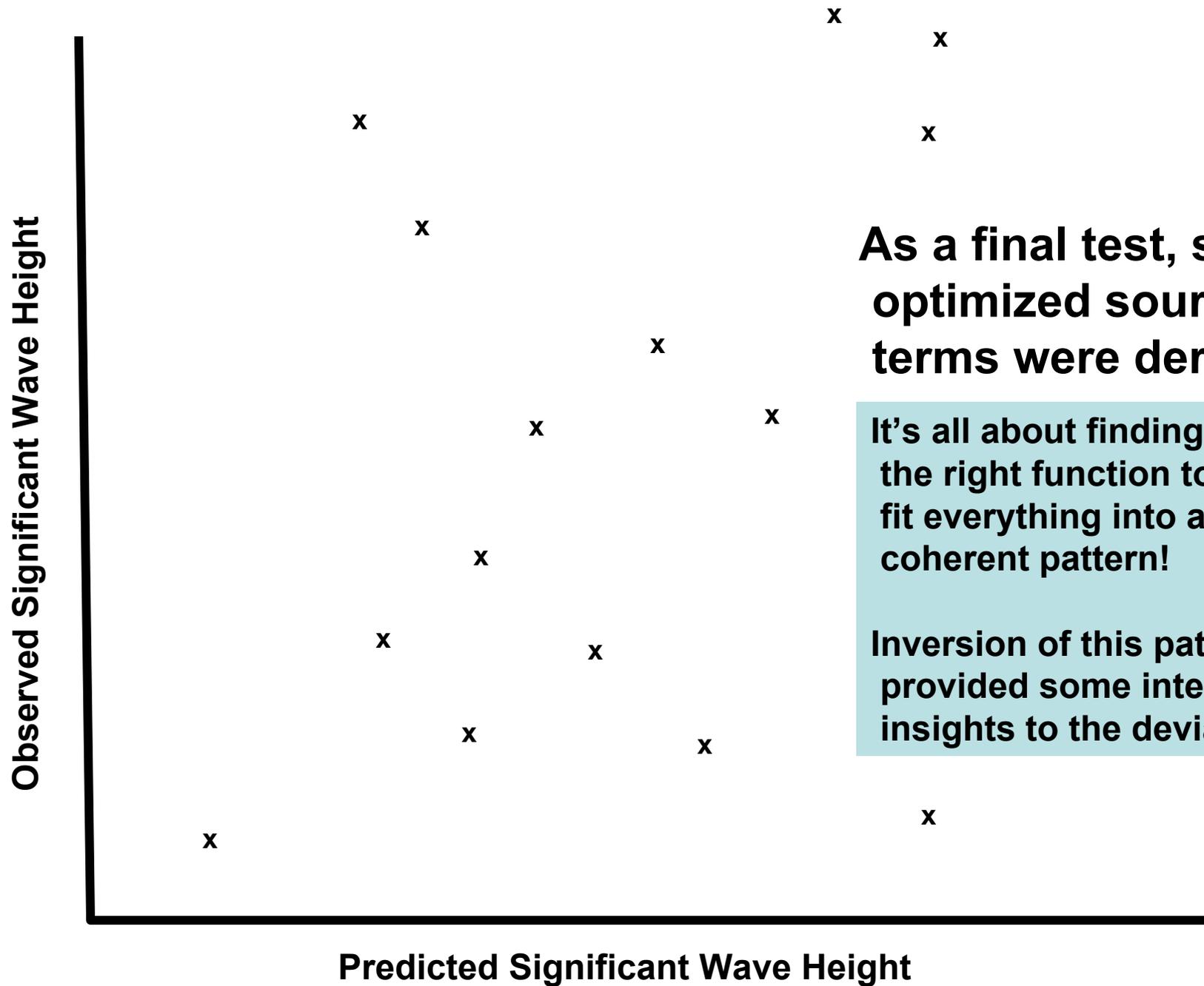
**But we need a lot more **ground truth** before this type of model can be considered to be validated.**



**New System for  
obtaining  
“ground truth”  
for wave models**

**Or**

**What about an  
independent group  
of assessors??**



**As a final test, some optimized source terms were derived.**

**It's all about finding the right function to fit everything into a coherent pattern!**

**Inversion of this pattern provided some interesting insights to the deviations.**

Observed Significant Wave Height

