Progress in the operationalisation of the TSA for computing non-linear four-wave interactions in spectral models

Gerbrant van Vledder

Alkyon Hydraulic Consultancy & Research
Delft University of Technology

TU Delft
Delft University of Technology

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Alkyon

Conclusions

- The Two-Scale Approximation (TSA) is an elegant approximate method for computing quadruplets in a discrete spectral model
- The TSA uses correction terms (second scale) to exact transfer rates of pre-computed broad-band spectra (first scale)
- TSA is applicable to a wide range of spectra
- Applicability related to extent of pre-computed transfer rates
- Critical point is availability of a robust method to split arbitrary spectrum in a broad band spectrum and a residual spectrum
- The TSA is cast in experimental subroutine form

Motivation

Non-linear four-wave interactions play an important role in the evolution of wind-generated waves

Present models mostly use the DIA developed by Hasselmann et al. (1985)

DIA is fast but only a crude approximation (wrong)
Xnl accurate but very time consuming

DIA hampers further developments of source terms

Replace the DIA by a more accurate and computationally fast method.

Purpose

Two Scale Approximation (TSA) of (Resio & Perrie, 2007) attractive candidate

Make TSA operational for application in operational discrete spectral third-generation wave models (WaveWatch, SWAN, STWAVE, WAM, ...)

Challenge

Turn a research code into a flexible operational code in the form of a subroutine for general use

Determine limits of applicability of TS**A**

Principle of the TSA

- Split an arbitrary spectrum into two parts: a broad-band spectrum and a residual spectrum;

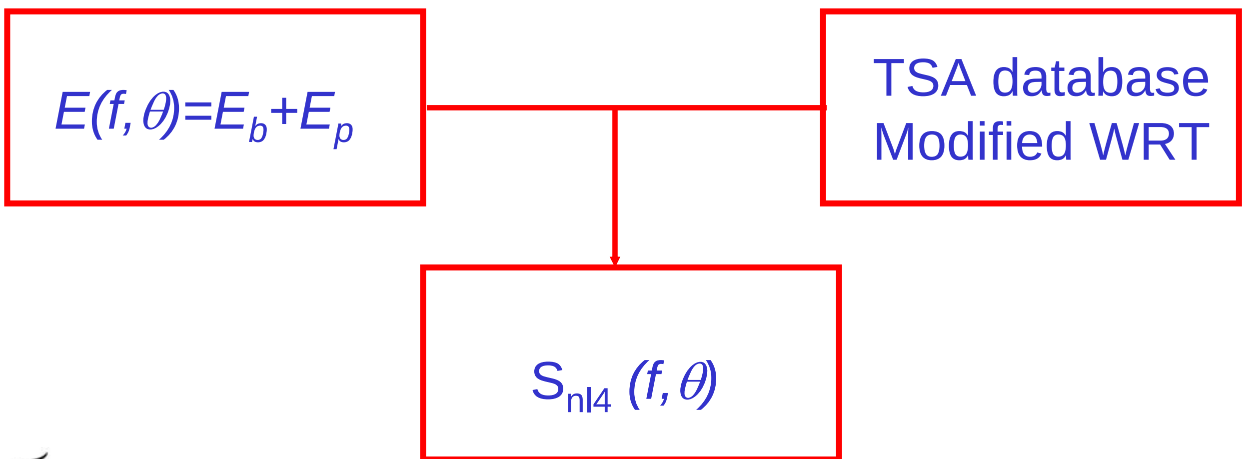
- Compute non-linear transfer rate using pre-computed exact transfer rates for broad band spectrum and apply corrections;

- The non-linear transfer rate of the broad-band spectrum **(first scale)** is pre-computed and stored in a database;

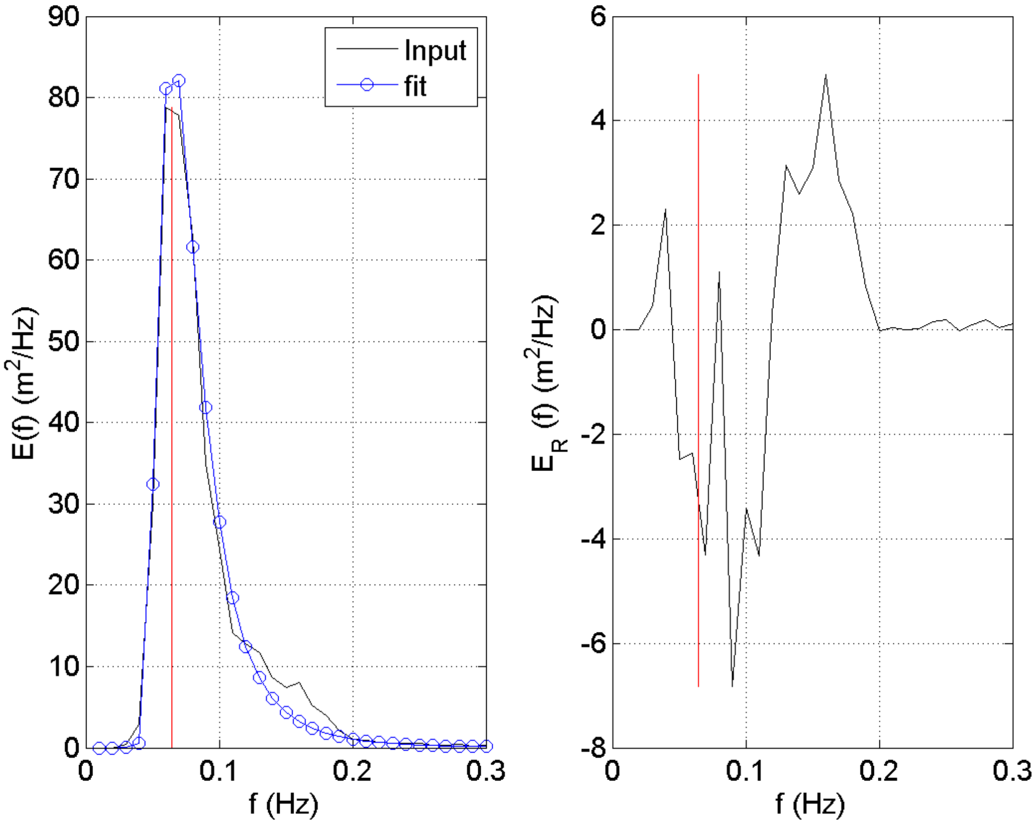
- Computation of correction terms **(second scale)** is based on product terms of spectral densities of the broad-band spectrum, residual spectrum and pre-computed correction terms.

Computational procedure

1: choose spectral grid (typically 30 f and 36 θ)
2: compute broadband transfer rates and TSA matrices
3: compute transfer rate for given spectrum



Decomposition of spectrum broad band and residue



WRT method for computation of four-wave interaction (Webb, 1978)

$$\frac{\partial n_1}{\partial t} = \iint k_3 dk_3 d\theta_3 T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right)$$

$$T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) = \iint d\boldsymbol{k}_2 d\boldsymbol{k}_4 \times G \times \delta\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4\right)$$
$$\times \delta\left(\omega_1 + \omega_2 - \omega_3 - \omega_4\right) \times \left[n_1 n_3 \left(n_4 - n_2\right) + n_2 n_4 \left(n_3 - n_1\right)\right]$$

$$T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) = \oint_s ds \times G \times J \times N_{1,2,3,4}$$

$$N_{1,2,3,4} = n_1 n_3 \left(n_4 - n_2\right) + n_2 n_4 \left(n_3 - n_1\right)$$

Description of the TSA

Split spectrum *n* into broad-band and perturbation

$$n_i = b_i + p_i \quad \text{for} \quad i = 1,4$$

Mathematical structure of TSA similar to WRT method

Save all terms that can be pre-computed

$$\frac{\partial n_1}{\partial t} = B(\boldsymbol{k}_1) + \iint (p_3 - p_1)\Lambda_d(\boldsymbol{k}_1,\boldsymbol{k}_3)k_3 dk_3 d\theta_3$$

$$+ \iint (p_1 p_3 + p_1 b_3 + b_1 p_3)\Lambda_p(\boldsymbol{k}_1,\boldsymbol{k}_3)k_3 dk_3 d\theta_3$$

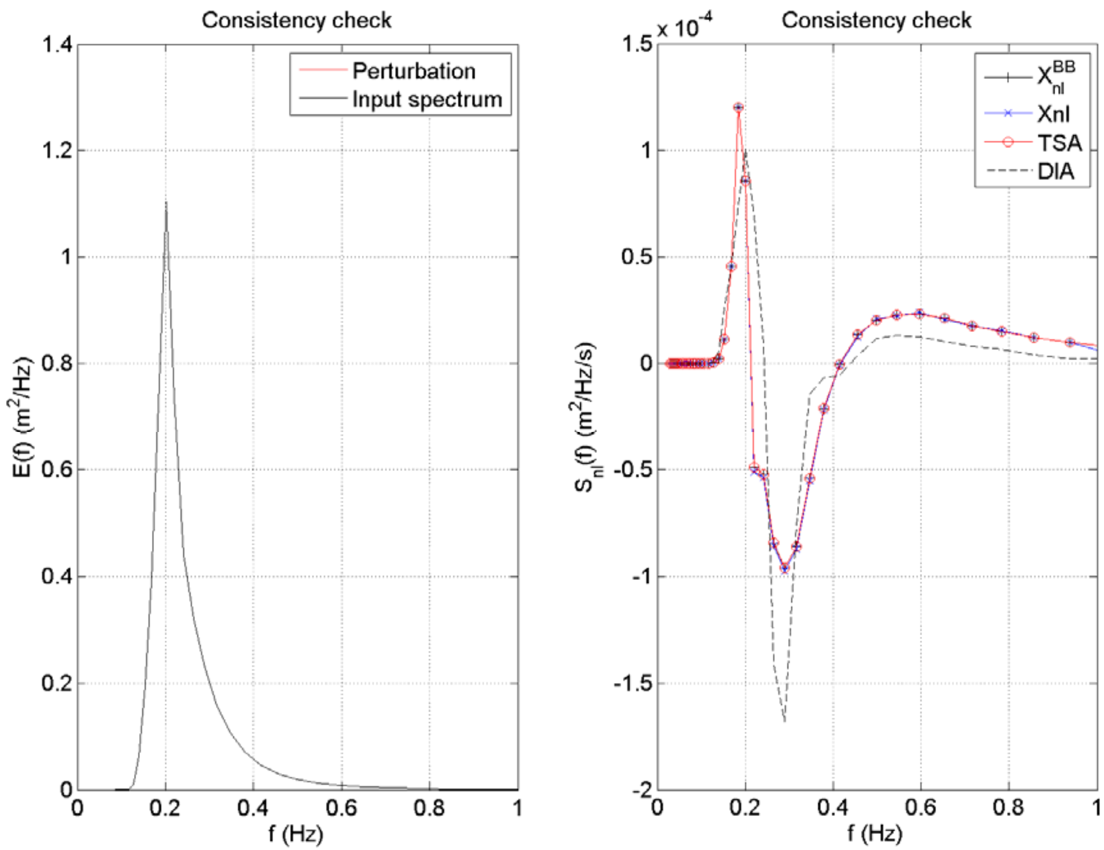
$$+ \quad \ldots\ldots$$

Primary sensitivities

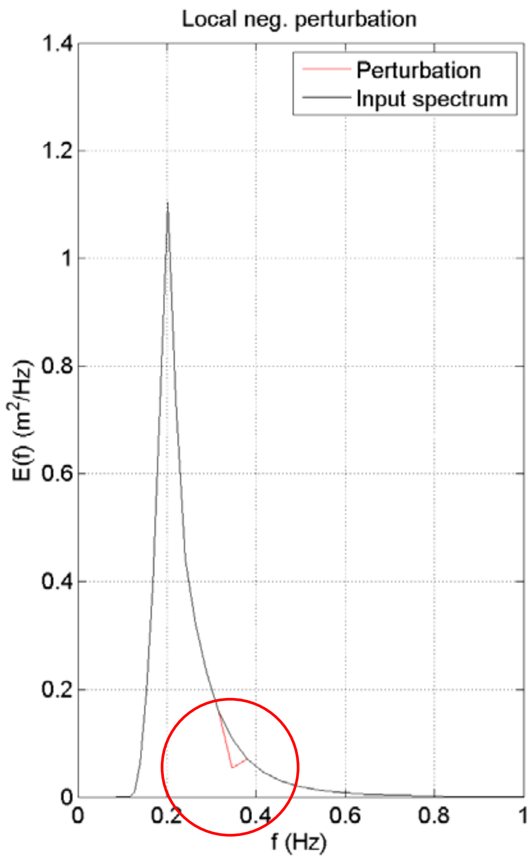
Investigate effect of various shape factors

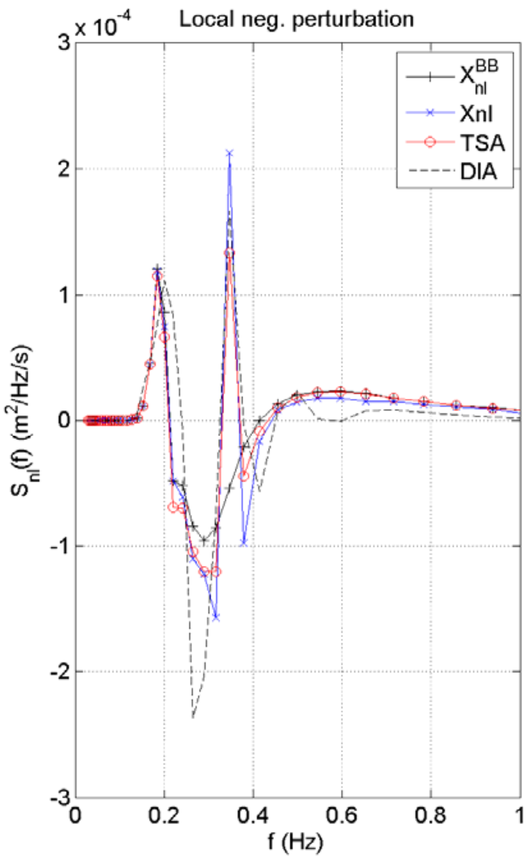
- Consistency test, no perturbation
- Local perturbation
- Peakedness (γ)
- Directional spreading (σ, $\sigma(f)$)
- Peak frequency (f_p)
- Scale (α)

Consistency test, γ=2, σ=30°
no perturbation

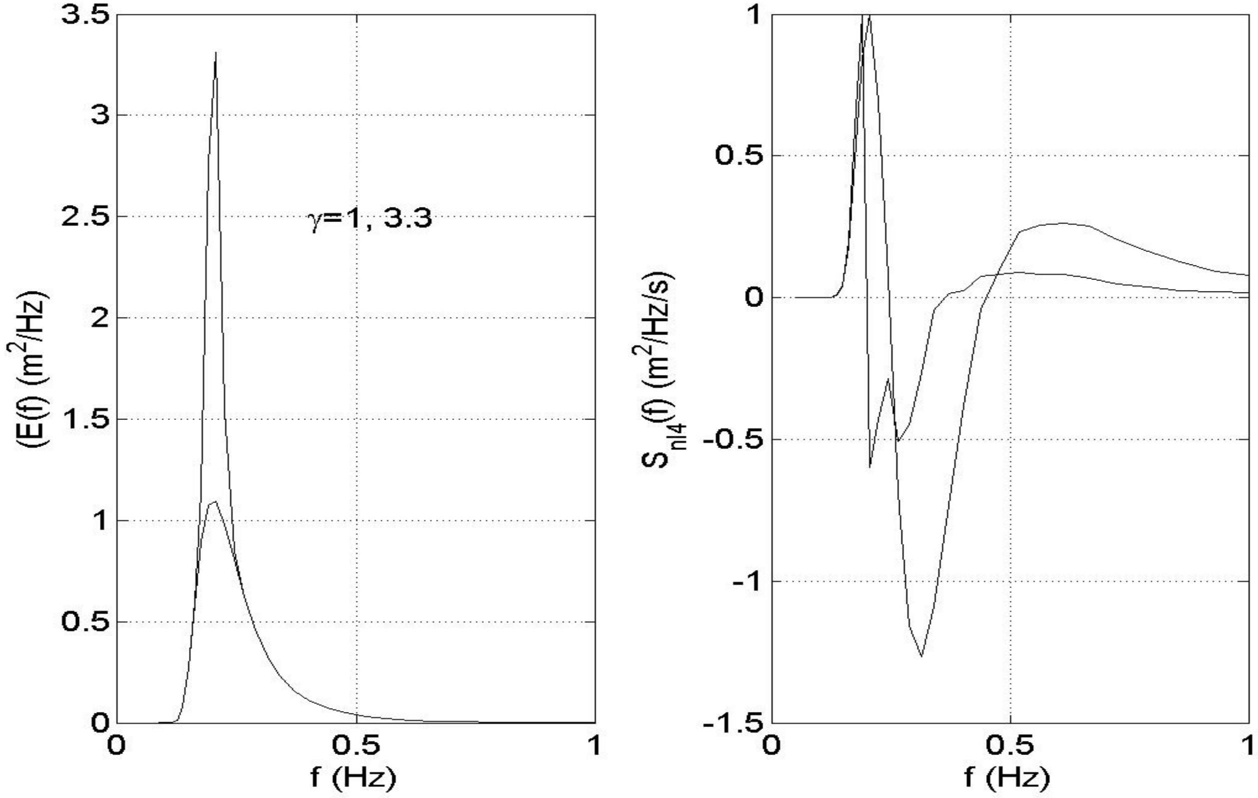


Local perturbation





Non-linear transfer rate depends on peakedness but also on directional spreading



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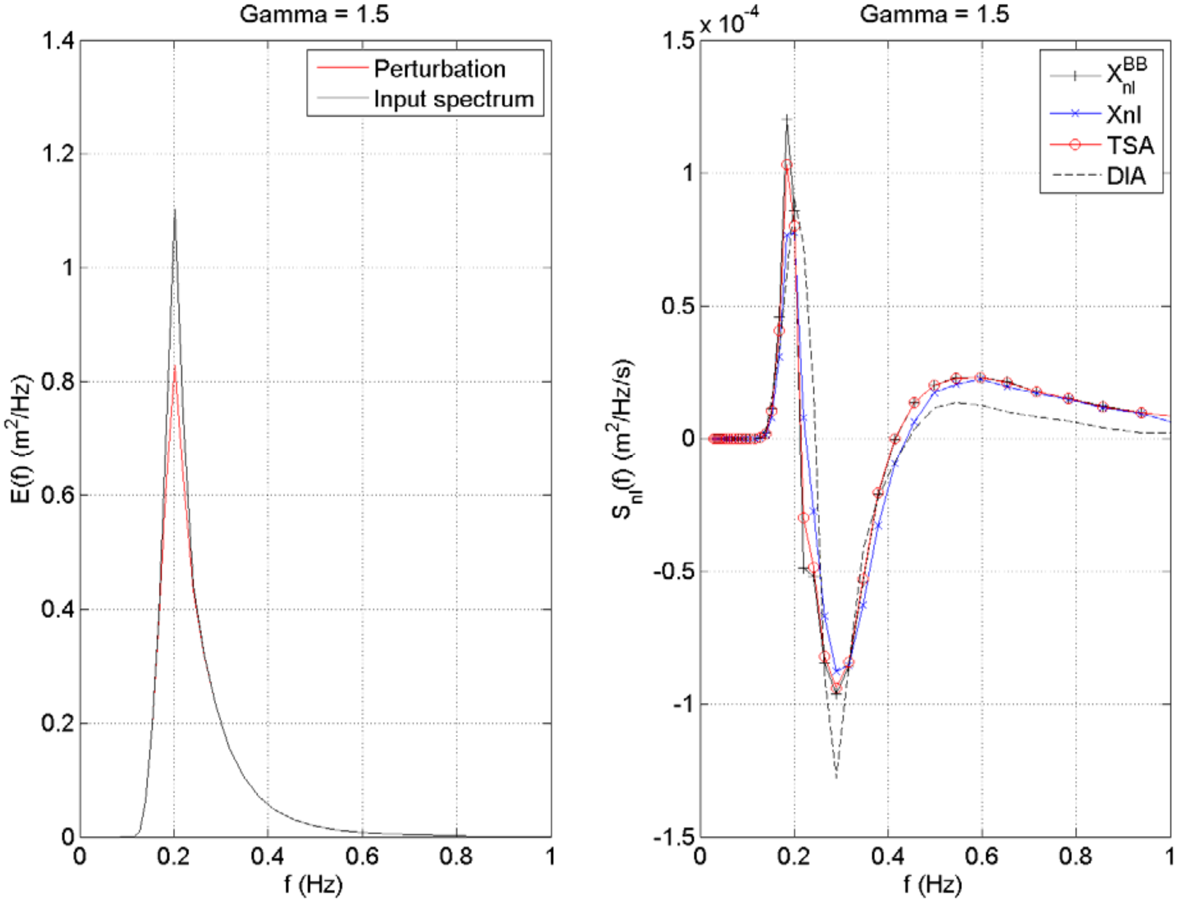
Peakedness, γ=1.5



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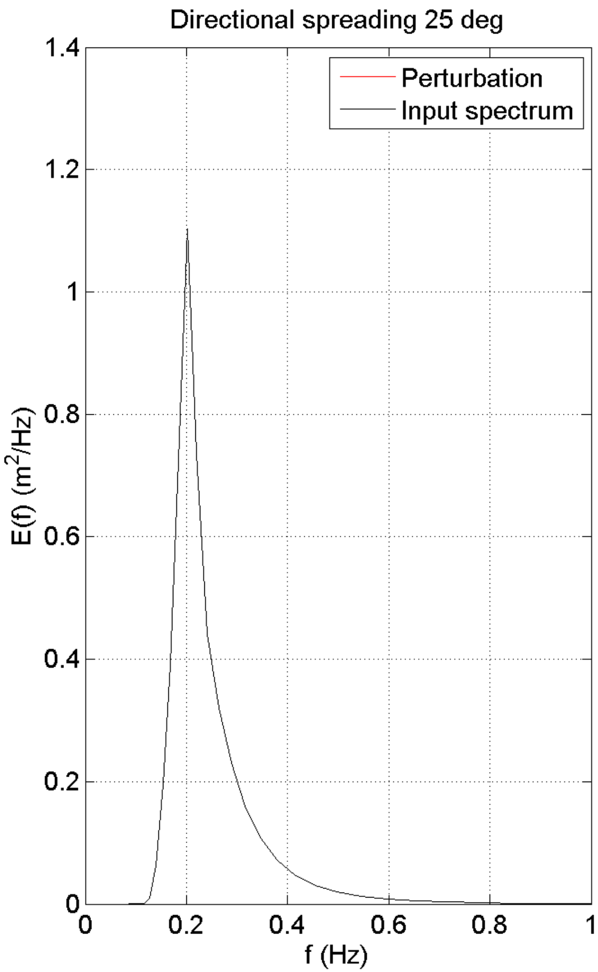
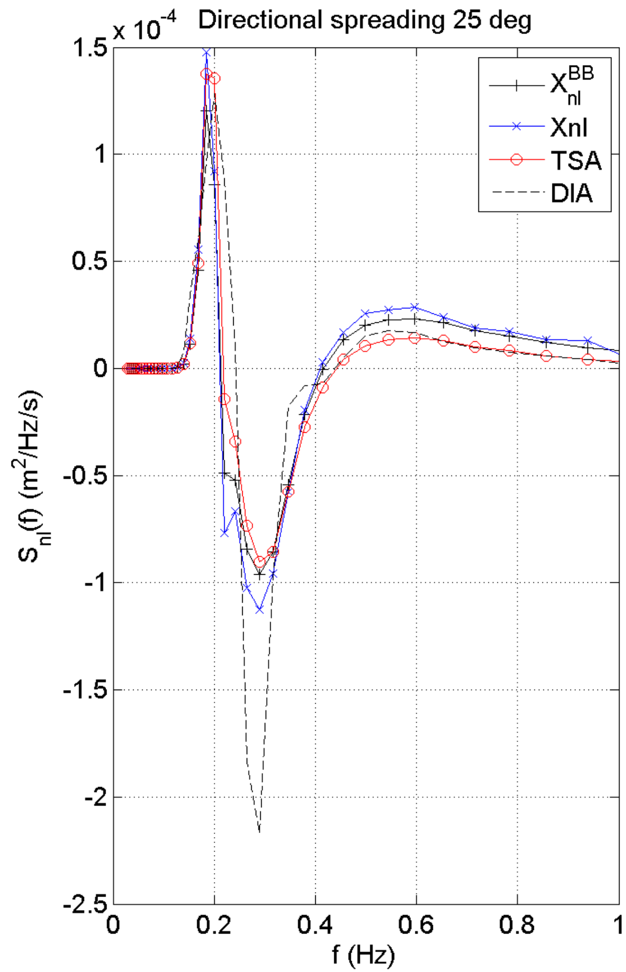
Directional spreading, σ=25°





Apply scaling laws to handle different f_p's and α's.

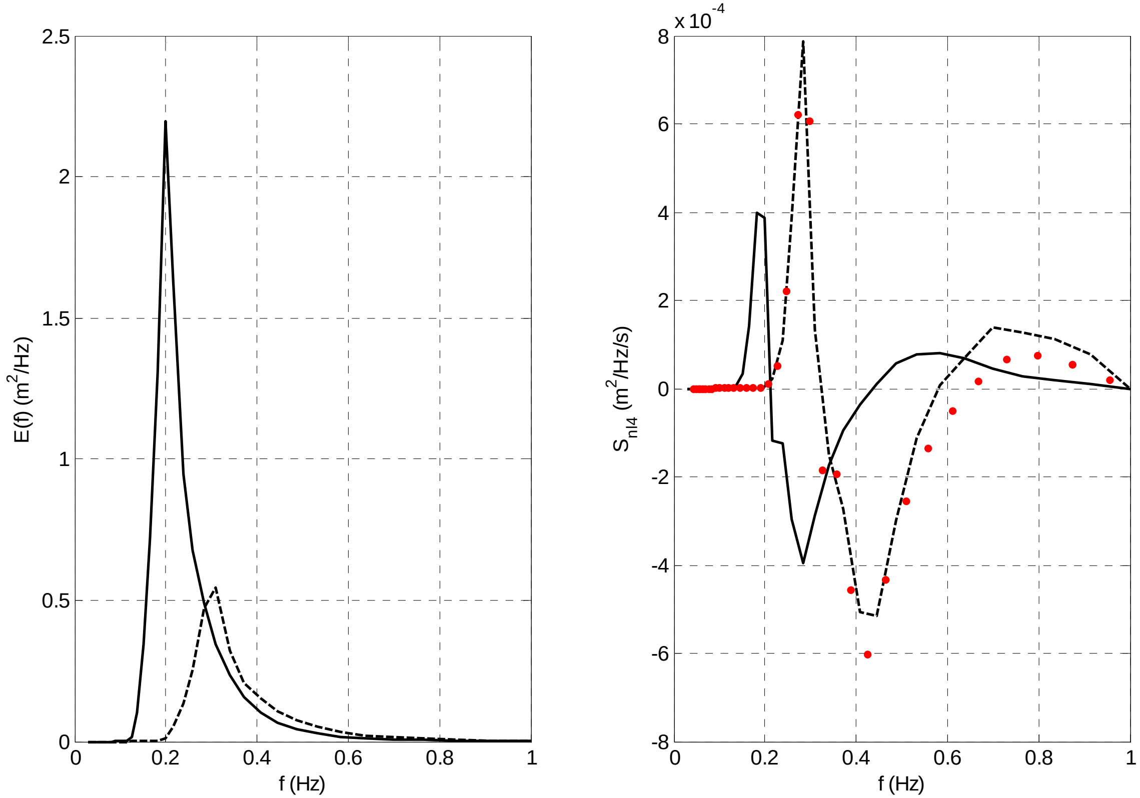
Non-linear transfer rate of similar spectra are related via scaling laws and rotations.

$$E(f,\theta)=\alpha f_p^{-n}\Psi(\upsilon,\theta)$$

$$S_{nl}(f,\theta)=\alpha^3 f_p^{11-3n}\Omega(\upsilon,\theta)$$

$$S_{nl}^{(2)}(f,\theta)=S_{nl}^{(1)}\left(f\frac{f_{p1}}{f_{p2}},\theta-\Delta\theta\right)\left(\frac{\alpha_2}{\alpha_1}\right)^3\left(\frac{f_{p2}}{f_{p1}}\right)^{-4}$$

Scaling of non-linear transfer rate for spectra with different α and f_p

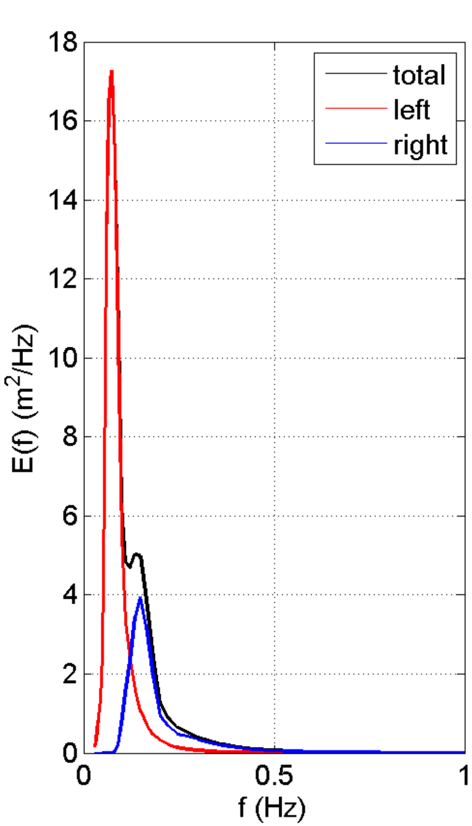


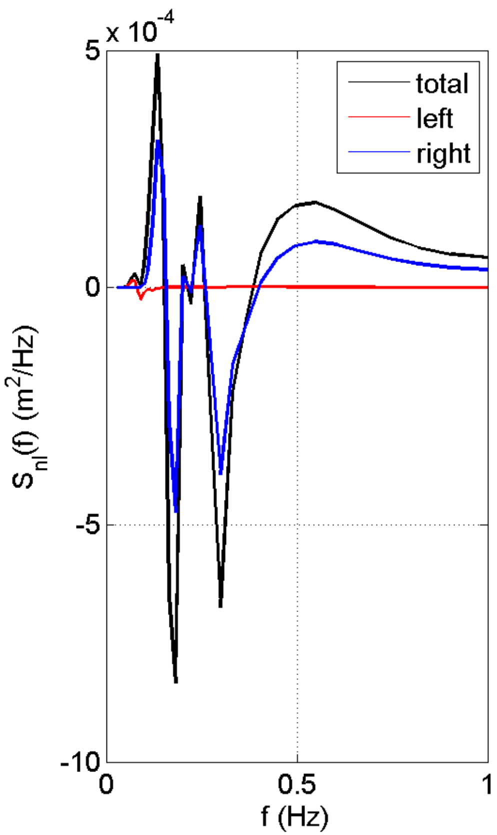
Red dots are transformed transfer rates

Small mismatch at higher frequencies

Double peaked spectra

decompose spectrum into components
need for robust method (e.g. method of Hanson), not a fundamental solution





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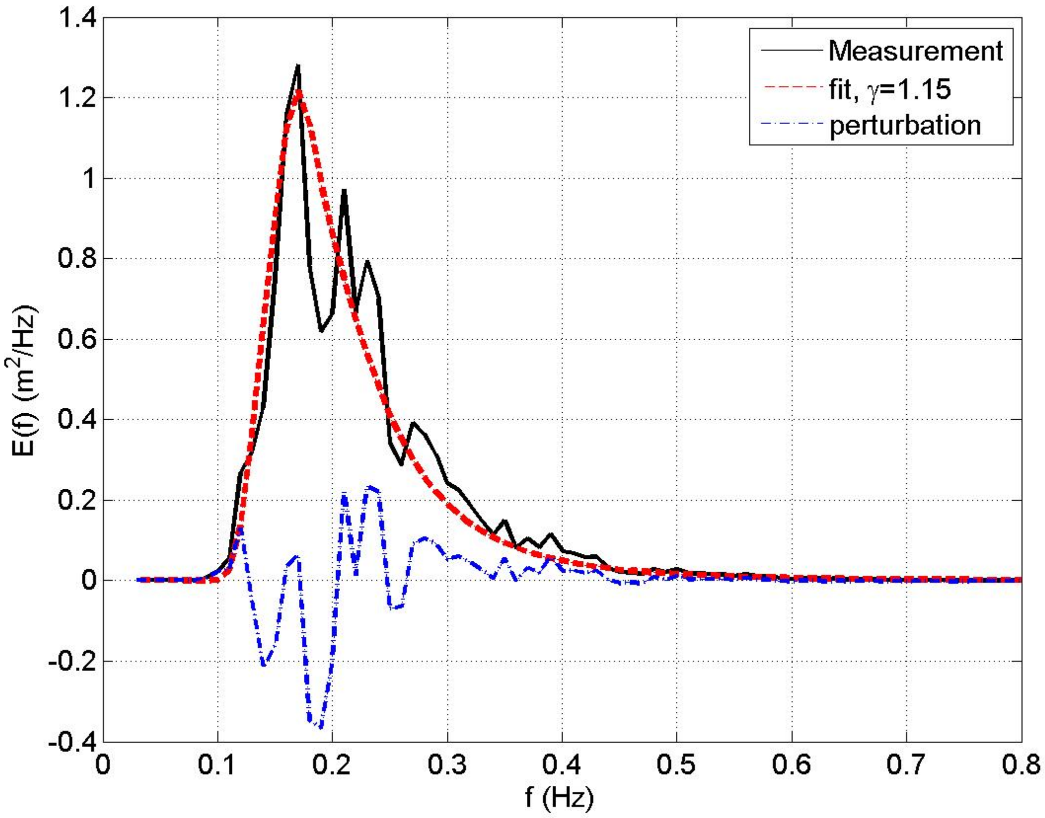
Elements of TSA subroutine

- Determine characteristics of arbitrary spectrum: f_p, α, peakedness γ, mean direction θ and directional spreading σ
- Search best fitting database in terms of peakedness, mean direction and directional spreading
- Split spectrum into broad-band (available in database) and residual part
- Retrieve non-linear transfer rate and related matrices
- Apply scaling laws and directional transformation where needed
- Compute correction terms to broad-band transfer rate

Fitting of broad-band spectrum to an arbitrary spectrum



Fitted γ (1.15) generally not in database, but only 'round' values, e.g. 1, 1.25, 1.5, 1.75, 2.0, etc.

Use γ=1.25 for broad band spectrum, rescale to conserve energy and re-computed residual spectrum

Similar arguments hold for directional spreading and mean direction ($\Delta\theta$)

Limitations of TSA

- Determination of broad band and decomposition of spectrum
  Requires robust fitting algorithm

- Extent of database, link with sensitivities
  Directional spreading has much effect
  TSA improves with improving number of databases with pre-computed data

- Treatment of non-standard spectra, bi-modal spectra
  Spectra of wave models generally smoother than measured spectra

Operational requirements of TSA subroutine

- Applicable in any discrete spectral wave model
- Input arrays: flexible range of frequencies, directions, depth, power of parametric tail
- Output: approximate non-linear transfer rate

- Settings: internal loops, fitting of spectra, location of database, logging options
- Error messaging
- Fortran 95, dynamic memory allocation, modules
- Documentation

Outlook

- Testing fitting procedure to obtain broad band spectrum
- Optimization of file i/o (keep data in memory when possible)
- Implementation in operational models (SWAN, WaveWatch, TSWAVE, WAM, …)

- Growth curve analysis, good performance for an individual spectrum is no guarantee that it works in a model run
- Assessment of model improvement (parameters, spectral shapes) based on field case applications

Acknowledgement

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