



# Practical Nonlinear Interaction Algorithms

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The Generalized Multiple DIA (GMD) and the  
Neural Network Interaction Approximation (NNIA)

*Hendrik L. Tolman*  
*Chief, Marine Modeling and Analysis Branch*  
*NOAA / NWS / NCEP / EMC*

*Hendrik.Tolman@NOAA.gov*





## Motivation

- 25 years and counting: how to improve upon the Discrete Interaction Approximation (DIA).
  - Does not have to be *that* cheap.

## Methodology

- Generalized Multiple DIA (GMD)
  - Use proven improvements to DIA.
  - Address scaling for arbitrary depths.
  - Holistic optimization.
- Neural Network Interaction Approx. (NNIA)
  - Progress but feasibility of approach still in question.
  - Key is to show that wave growth can be modeled.
  - Hybrid-Compound system NN – QC – Filter.



## Generalized Multiple DIA

- GMD nearing maturity (algorithms, numerical optimization).
- Proper scaling behavior for arbitrary depths.
- Holistic parameter optimization approach well developed.
  - Initial results show massive improvement over DIA.
  - In depth parameter optimization under way.

## Neural Network Interaction Approximation

- Less mature, but model integrations is possible.
- Main issues:
  - Compound approach with internal QC needed.
  - High-frequency noise needs to be dealt with adequately (dual-NN, filter, ....).
- Clear path to further development as model accelerator.

## Generalized Multiple DIA

- Following original derivation / construction from Hasselmann *et al.* (1985)
  - Much previous work and authors.
    - ➔ Following Gerbrant van Vledder's earlier work.
  - Main goals:
    - ➔ More accurate than DIA yet affordable.
    - ➔ Proper shallow water scaling.
  - Need for holistic optimization.
    - ➔ Poor representation of  $S_{nl}$  for test spectra can still lead to good model integration (DIA).
    - ➔ Better representation can break model (DIA with relaxed quadruplet definition).
    - ➔ Only way to systematically improve model behavior is to optimizing parameters in full model integration.

## GMD main issues

- Need for 2 or three parameter quadruplet definition.
- Using multiple representative quadruplets increases accuracy.
- Shallow water version needs full shallow approach in both quadruplet layout and discrete interaction formulation.
- For shallow water scaling function needs to consists of two constituents:
  - Deep water asymptotic form (similar to previous)
  - Shallow water asymptotic form (new).
- Formulations for different spectral descriptions can be derived consistently, but give different (numerical) results.
  - Small but notable results for given spectra.
  - Unexpected large impact on model integration.
  - $F(f, \theta)$  and  $F(k, \theta)$  formulations far superior !

new results



## Holistic optimization

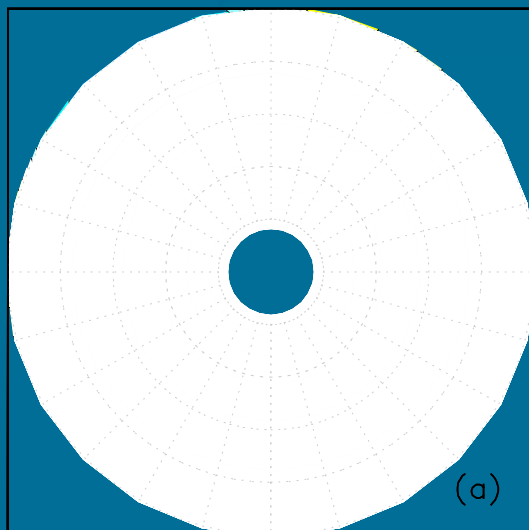
- Six deep water test cases.
- Three shallow water test cases.
- 15-19 error metrics per test case.
- Genetic optimization algorithms.
  - Augmented with steepest descent.
  - Augmented with error mapping where feasible.

Want equations and details?

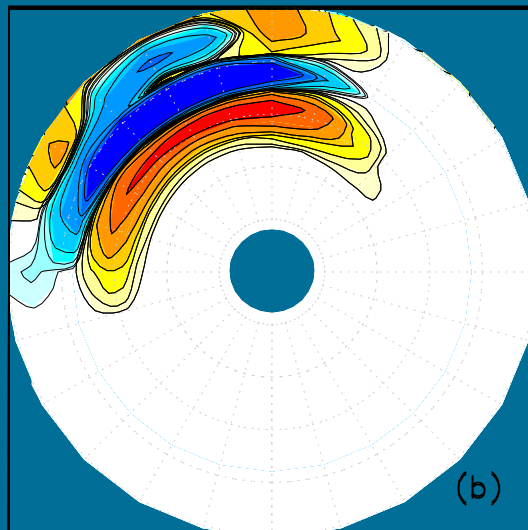
See conference paper.

Examples of impact follow here

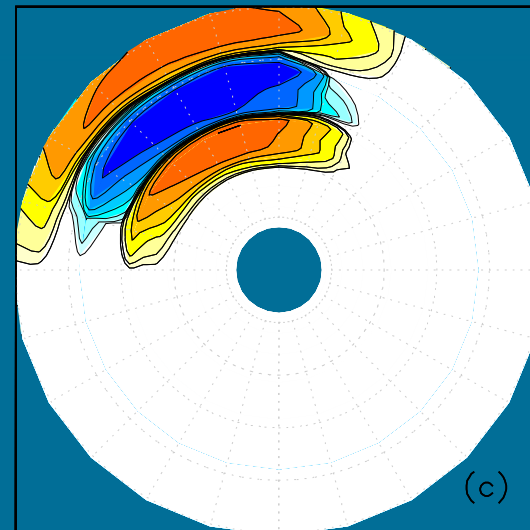
DIA



WRT

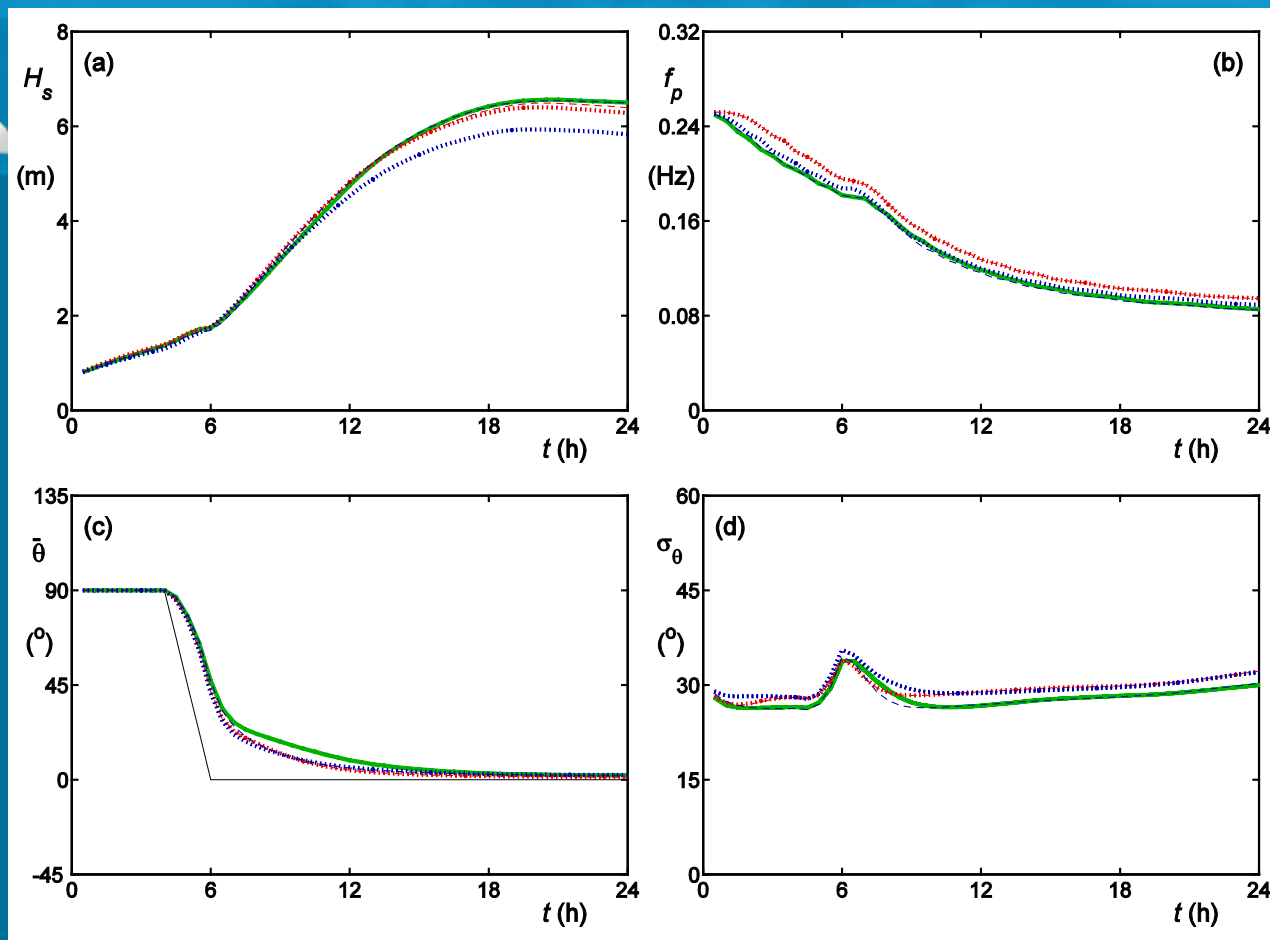


GMD



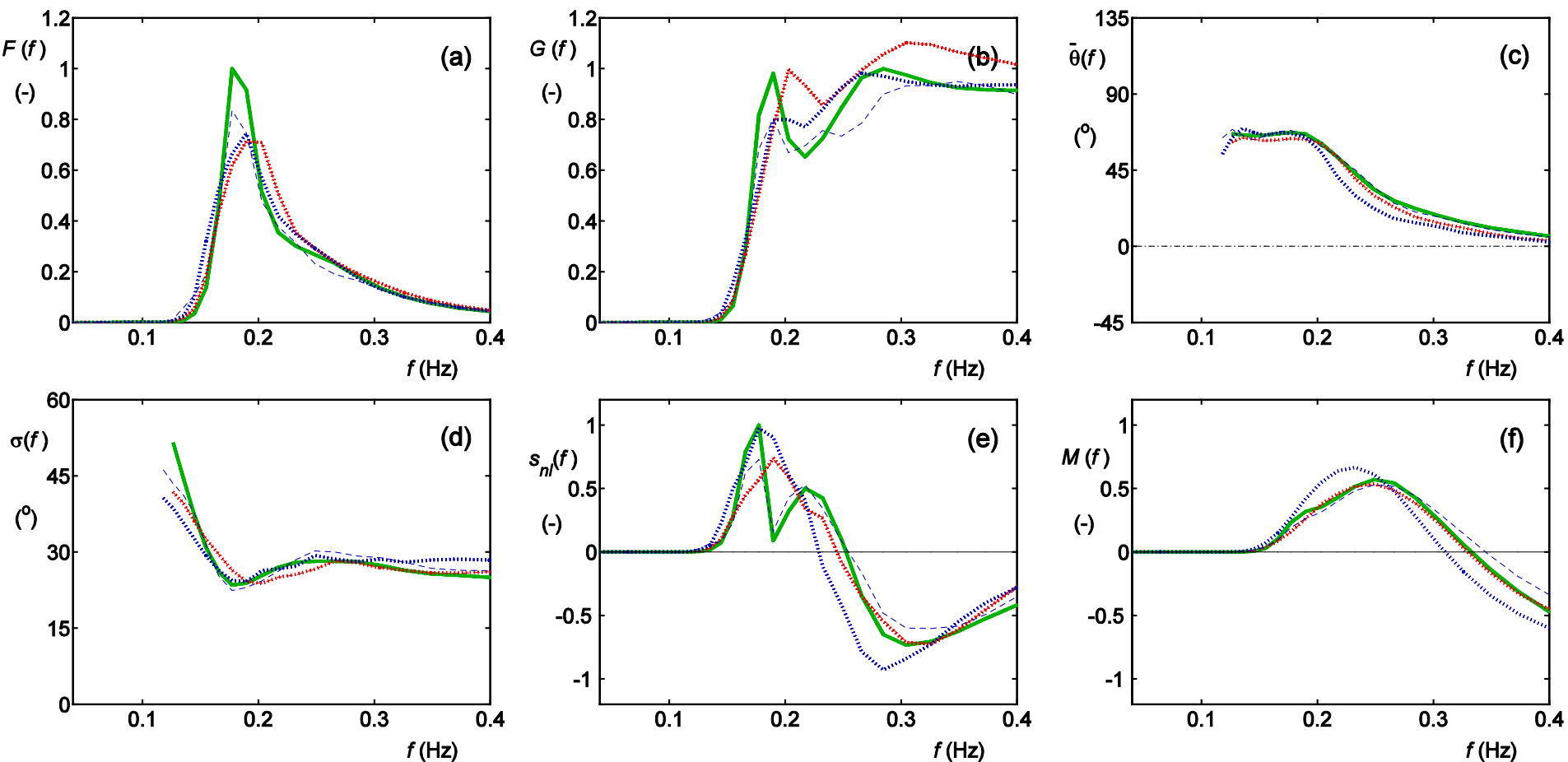
Interaction approximations for a given test spectrum from a wave model in deep water. DIA and GMD (1 trad. quad). are virtually identical.

Same spectrum but assuming 1m water depths. DIA is three orders of magnitude too weak, GMD shows proper scaling and reasonable shape.



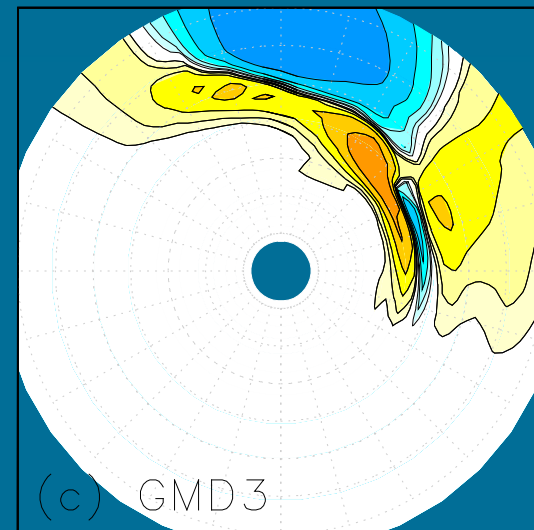
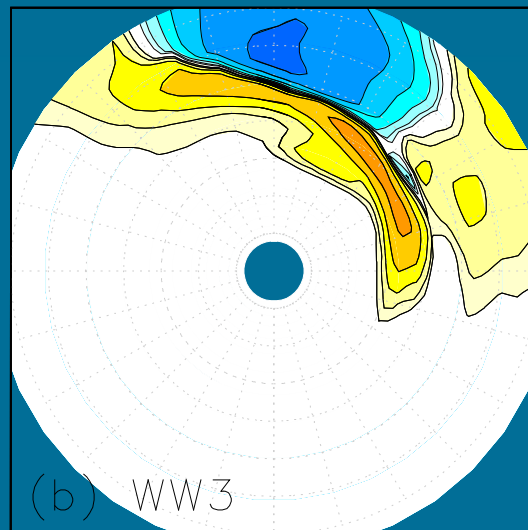
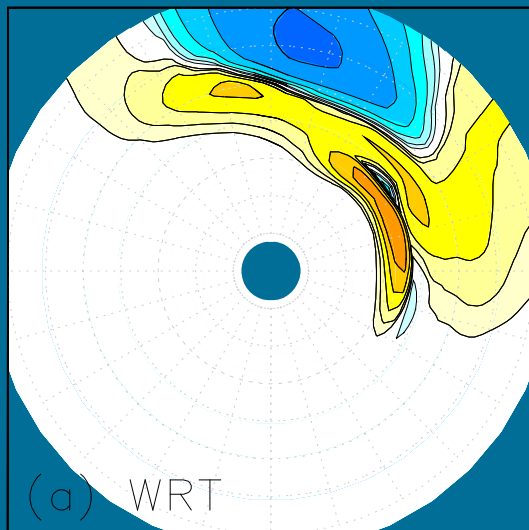
Mean parameters for deep water test with turning wind.  
 Green: WRT Red: DIA Blue GMD (1q dotted, 3q dashed).





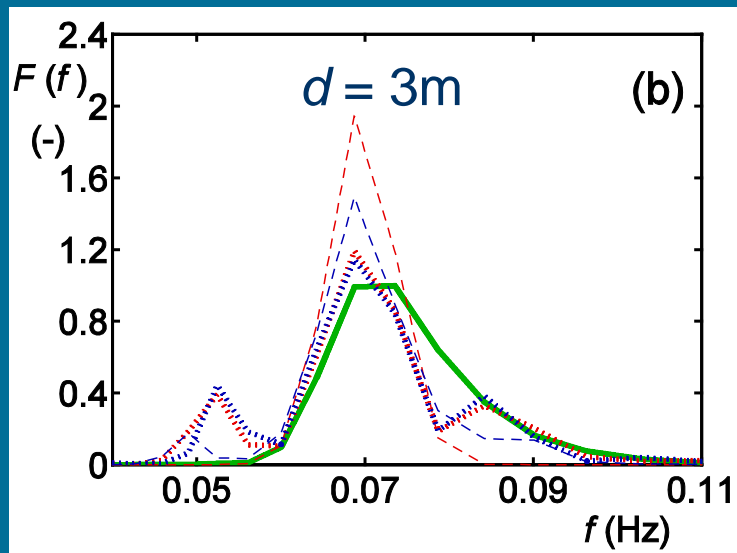
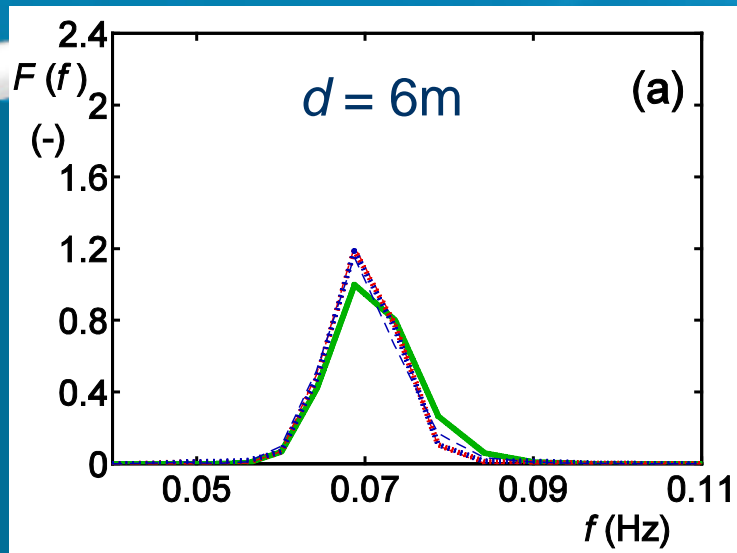
Corresponding spectral parameters at  $t = 6$  h

Green: WRT Red: DIA Blue GMD (1p,1q dotted, 2p,3q dashed).



Full two-dimensional interactions corresponding to previous slide for exact (WRT) traditional DIA (WW3) or GMD with three two-parameter quadruplets.

Note: interactions for identical spectrum are very different, but interactions for corresponding model integration are very similar, but with important differences in details (see 1-D interactions in previous slide).

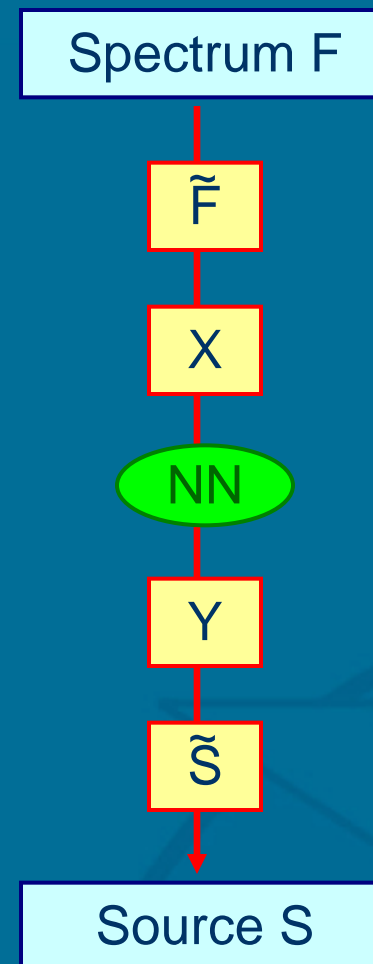


## Swell on beach

- Exact four-wave interactions do have a significant influence on spectral shape.
  - Can they be ignored?
- Initial optimization experiments suggest that more quadruplets are needed than in deep water.
- Note: this is a realistic beach with 1:250 slope. Both results are inside surf zone without triad interaction parameterization.

## Original NNIA.

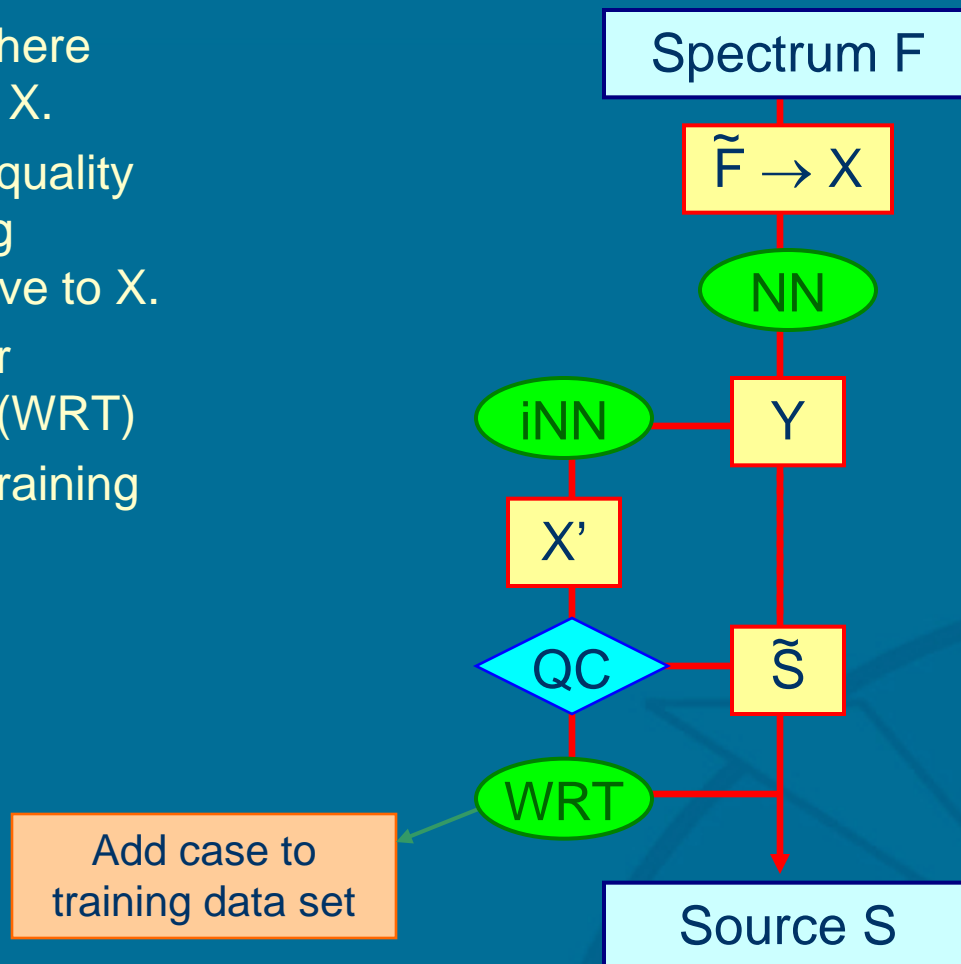
- Spectrum and source term are decomposed in EOFs to reduce dimensionality of NN.
- Both are normalized to enforce scaling behavior.
- NN is trained with spectra and exact interactions (WRT method).
- Focus on uni-modal wind seas because reproduction of wave growth is essential to success of NNIA.

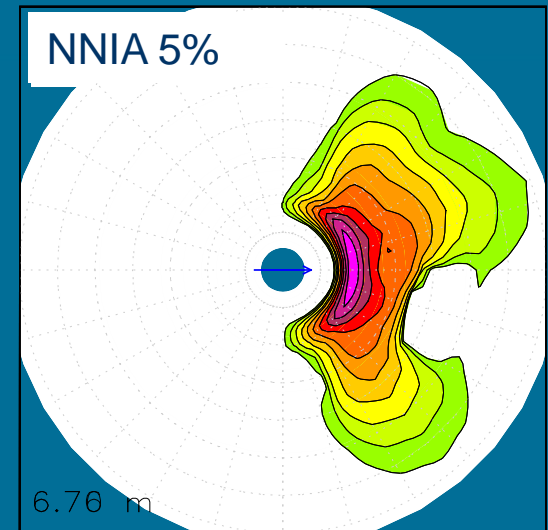
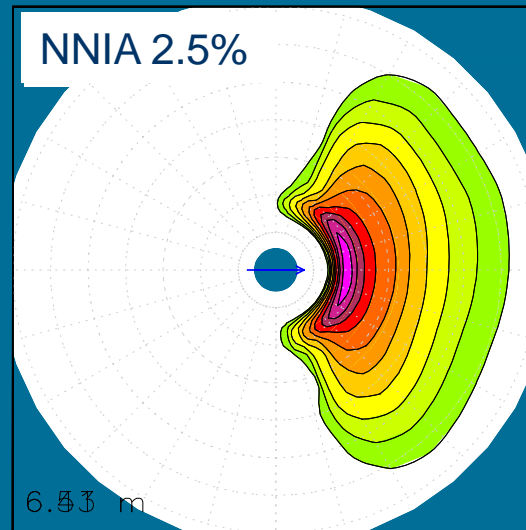
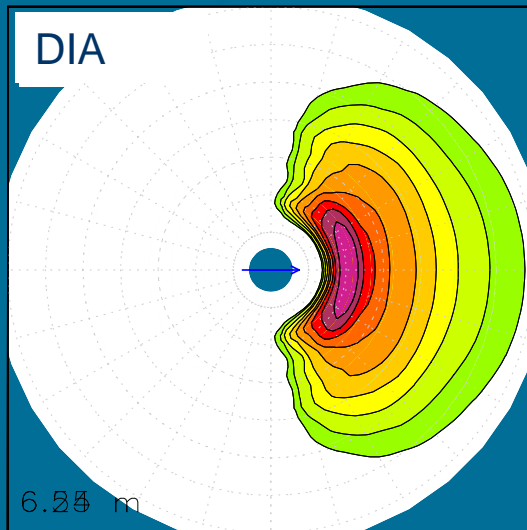


## Hybrid NNIA design with internal QC.

- Add “inverse” NN where  $X'(Y)$  is an estimate  $X$ .
- Introduce objective quality control by assessing accuracy of  $X'$  relative to  $X$ .
- Decide to use NN or alternative scheme (WRT)
- Dynamically make training data set broader.

Add filtering?



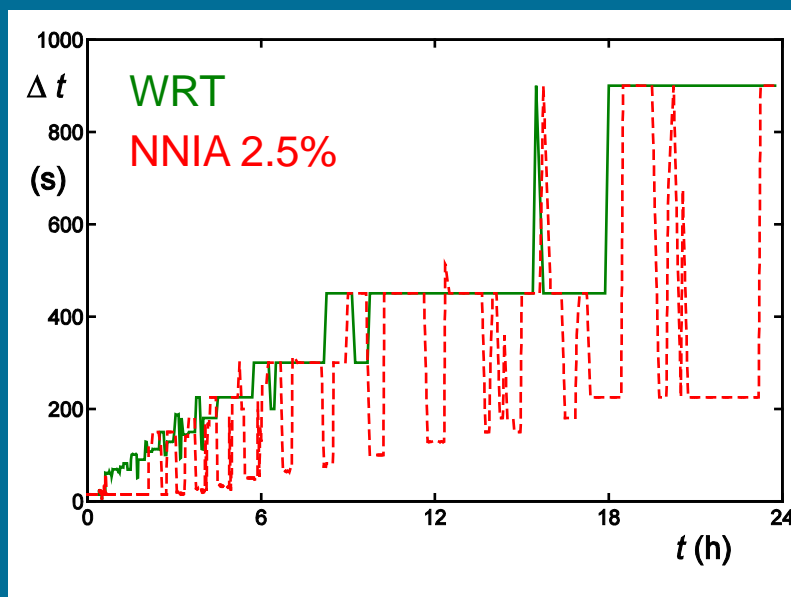


Spectra for wave growth with constant wind of 20m/s (not used in training) with various QC error levels. 2.5% gives good spectra throughout.

Adding filtering has modest impact positive on model performance. Note that differences with DIA are larger than filter impacts. Note that filter should be used integrally with NNIA development.

## Normalized model run time

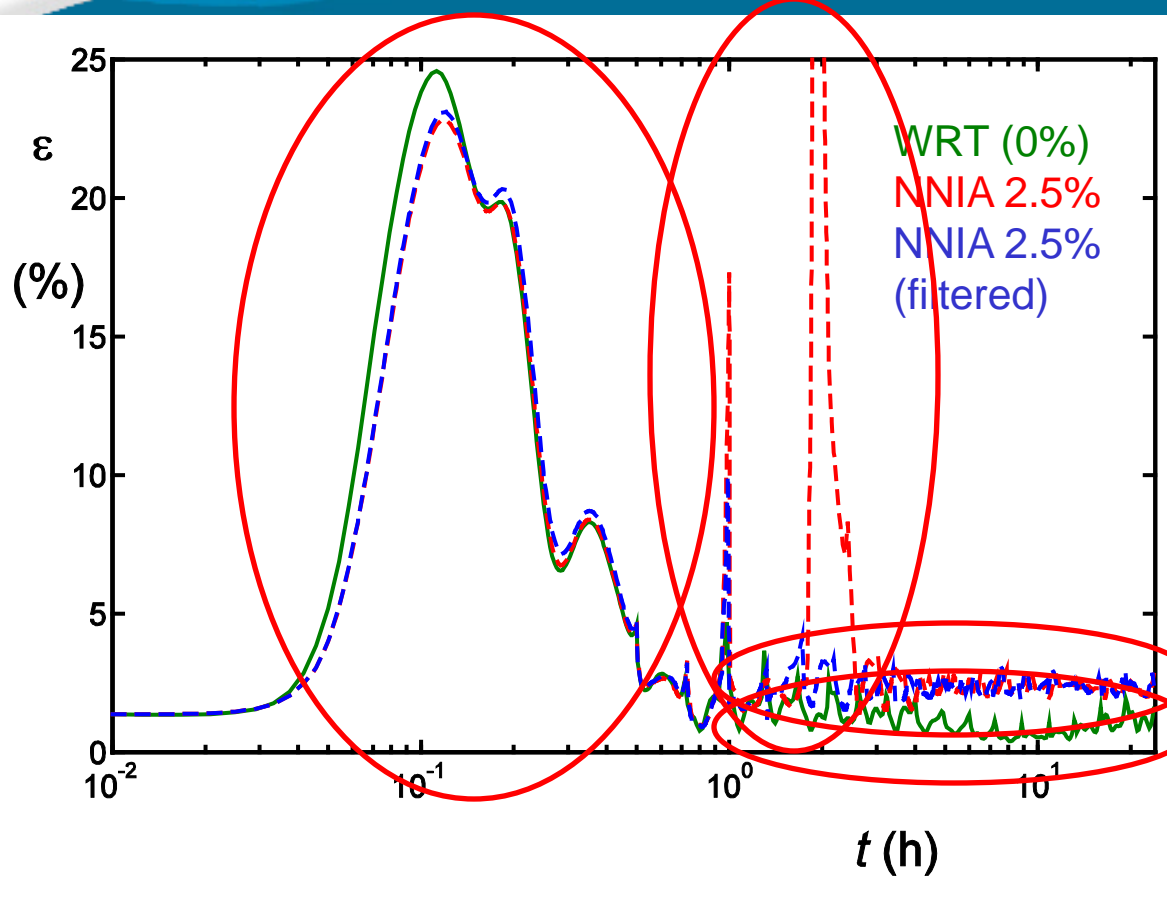
	WRT	NNIA			DIA
		1.25%	2.5%	5.0%	
Orig.	1	0.86	0.77	0.84	0.01
Filt.		0.83	0.56	0.68	



## NNIA economy.

- NNIA similar in cost to DIA. Model run time also depends on the dynamic time step.
- NNIA model integration cheaper than WRT, but nowhere near potential.
- Behavior can be attributed at least partially to model time steps.
- Cyclic behavior with NNIA noise growth and WRT noise suppression.
- Filter has positive impact, even though it was not integrated with NNIA.

## NNIA QC error evolution



## Observations :

- Insufficient training.
- Finite error levels
- Cyclic behavior with  $\varepsilon \cong \varepsilon_{\max}$
- Filtering works in principle, reducing error growth.





## How to go forward :

- Explore incremental training to reduce initial model error.
  - Labor intensive but trivial.
  - Better NNIA model integration behavior.
  - More economical NNIA mode integration.
- To make NNIA comparable in model integration economy to DIA, need to suppress cyclic error growth behavior.
  - Integrate filtering into NNIA in several possible ways.
    - ➔ Explicit filter.
    - ➔ NNIA / DIA hybrid approach.
    - ➔ NNIA with dual-NN.
  - Integration in NIA development essential.
- NNIA is not expected to be stand-alone approximation, but accelerator of more expensive approximations.



## Nonlinear interaction strategies for models and model development.

- Possible strategy for further model development:
  - Develop physics with exact interactions (WRT) and idealized cases and limited area models.
  - Optimize GMD to represent these physics.
  - Adjust original parameterizations as needed using original testing + full-blown application.
  - Develop NNIA version as accelerator.
- Parts of such a strategy can be used with other interaction approximations.



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