Incorporating Breaking Wave Predictions in Spectral Wave Forecast Models

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Motivational Aspects

- **wave breaking at sea** - widespread air-sea interfacial process with very significant geophysical and maritime importance
- present spectral wave forecast models **do not provide explicit forecasts** of breaking wave properties.
- recent advances in understanding wave breaking have made it possible to redress this deficiency
- to describe a novel methodology that adds to standard spectral wave model output - **accurate forecasts** of

(i) the **spectral density of breaking crest length per unit area** and

(ii) the associated **breaking strength**

We have done this initially for the **dominant wind waves**

Summary of Conclusions

- our framework provides predictions of wave breaking properties using **standard wave model outputs**

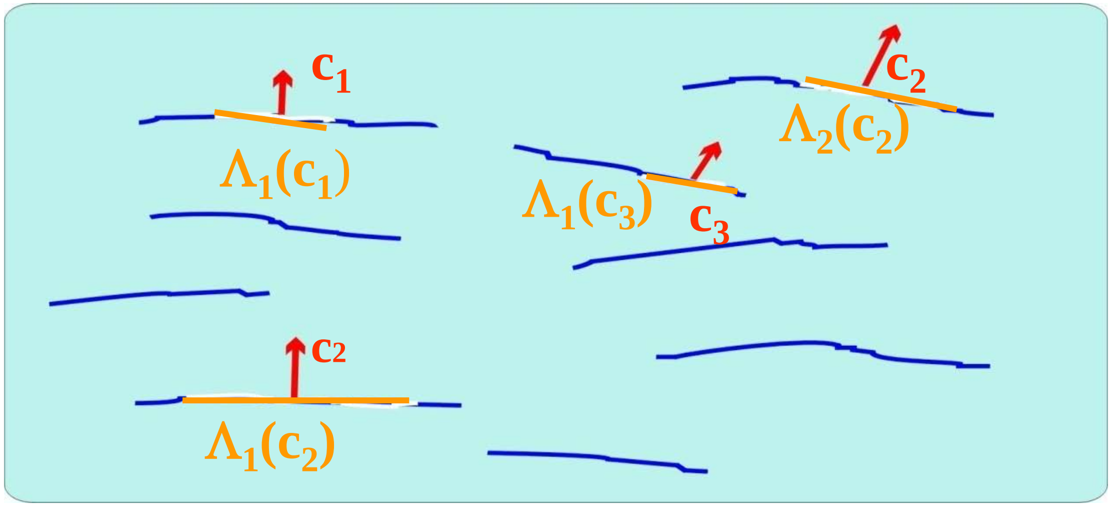
- it provides **accurate predictions** for the limited breaking data available for the dominant waves in developing and mature wind seas

- **further validation** against data will be made as suitable new data sets becomes available

- **easily added to existing spectral wave forecasting models**. Upgrading the DIA form of S_{nl} is desirable.

Brief description of the methodology - $\Lambda(c)$

Λ(c): spectral density of *breaking wave crest length* per unit area with velocities in the range (c, c+dc) (Phillips,1985)



$$b\frac{\rho}{g}c^5\Lambda(c)dc$$

wave energy dissipation rate at scale c

$$b\frac{\rho}{g}c^4\Lambda(c)dc$$

momentum flux from waves of scale c to currents

Brief description of the methodology (contd)

$\Lambda(\mathbf{c})$ is the spectral density of breaking wave crest length per unit area

$\Pi(\mathbf{c})$ is the spectral density of the total wave crest length per unit area

The breaking probability $P_{br}(\mathbf{c})$ for wave scales **c** is defined as:

$$\mathrm{Pr}_{br}(\mathbf{c}) = \frac{\int \mathbf{c}\,\Lambda\,d\mathbf{c}}{\int \mathbf{c}\,\Pi\,d\mathbf{c}} \sim \frac{\text{passage rate of breaking wave crests}}{\text{passage rate of wave crests}}$$

For the spectral peak region, it is easily shown from the definition that

$$\Pi(c_p) = \chi(c_p)\,g/(2\pi c_p^3)$$

where $\chi(c_p) \sim 0.65$ is the measured crest intermittency factor in the ±30% relative frequency bandwidth about the spectral peak

Hence

$$\Lambda(c_p) = (\chi(c_p)\,g/2\pi c_p^3) * \mathrm{Pr}(\tilde{\sigma}_p)$$

The sea state threshold variable used for breaking probability was the normalised spectral saturation

$$\tilde{\sigma}(k) = \sigma(k) / < \theta(k) >$$

where $\sigma(k)$ is the azimuth-integrated spectral saturation given by

$$\sigma(k) = k^4 \Phi(k)$$

$$= (2\pi)^4 f^5 G(f)/2g^2$$

and $<\theta(k)>$ is the mean spectral spreading width given by

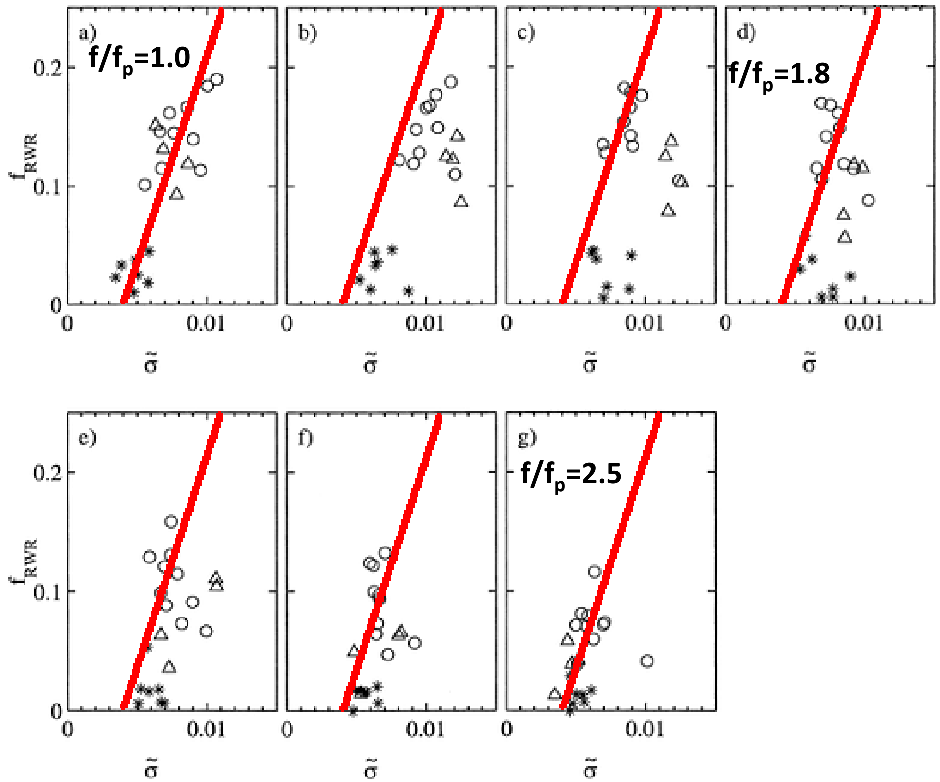
$$< \theta(k) > = \int_{-\pi}^{\pi} (\theta - \bar{\theta})\, F(k,\theta)\, k\, d\theta \Big/ \int_{-\pi}^{\pi} F(k,\theta)\, k\, d\theta$$

where $\bar{\theta}$ is the mean wave direction, and $\Phi(k)$, $G(f)$ and $F(k,\theta)$ are, respectively, the spectra of wave height as a function of scalar wavenumber, frequency and vector wavenumber.

Breaking Probability in the Wave Spectrum

- using spectral saturation normalized by the directional spreading, Banner, Gemmrich and Farmer, JPO, 2002 showed evidence for a *common threshold behavior* for the dissipation rate at different frequencies above the spectral peak

breaking probability



normalized saturation

- as a consistent approximation, $\mathrm{Pr_{br}}(\tilde{\sigma}_p) = \mathrm{H}(\tilde{\sigma} - \tilde{\sigma}_T) * \alpha_{br} * (\tilde{\sigma} - \tilde{\sigma}_T)$

where $\alpha_{br} \sim 33$. In our methodology, $\tilde{\sigma}(k)$ is calculated from our wave model.

Spectral Peak Breaking Strength Coefficient b_p

(Note that b may vary across the spectrum)

From Phillips (1985) definition: $$S_{ds}^{loc}(c)\,dc = b\,g^{-2}c^{5}\Lambda(c)\,dc$$

For the spectral peak, using the preceding result for $\Lambda(c_p)$ and transforming $S_{ds}(c_p)$ to $S_{ds}(k_p)$

$$b_p = 2\,\frac{g^3}{c_p^8}\,\frac{S_{ds}^{loc}(k_p)}{\Lambda(c_p)}$$

$$= \frac{4\pi}{\chi(c_p)}\,\frac{g^2}{c_p^5}\,\frac{S_{ds}^{loc}(k_p)}{Pr_{br}(\tilde{\sigma}_p)}$$

Recall that $$\Lambda(c_p) = (\chi(c_p)\,g/2\pi c_p^3) * Pr(\tilde{\sigma}_p)$$

From the wave model output at any time step, these equations provide the breaking crest length spectral density and breaking strength at the spectral peak

Conditions during the FAIRS Experiment

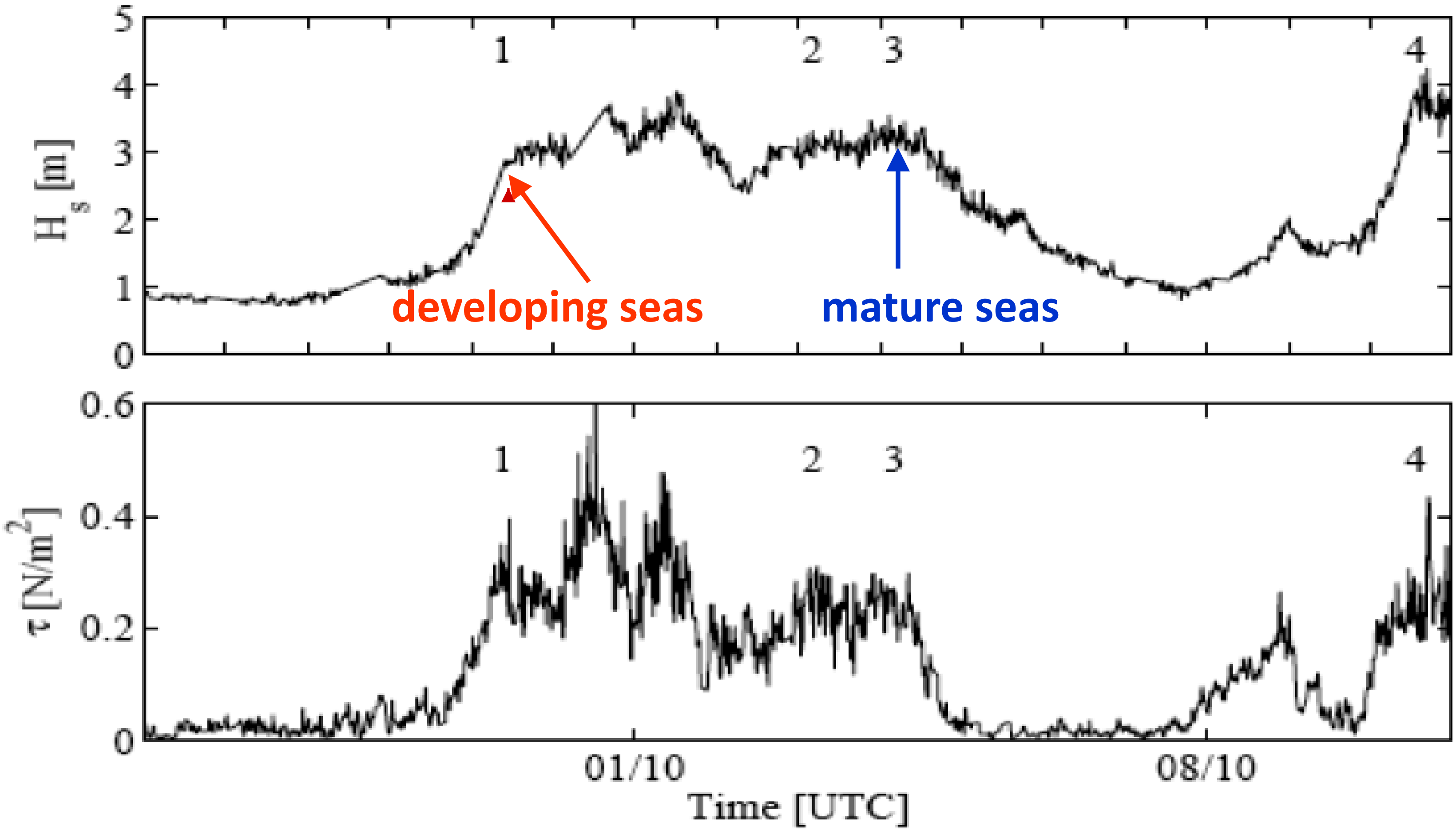


Figure 1. Significant wave height (H_s) and wind stress (τ) during the FAIRS experiment. The wind direction was around 300° for most of the observational period. Periods 1 (growing seas) and 3 (mature seas) are of particular interest in this study, during which the mean wind was measured to be 12 m/s.

Breaking Wave Histograms

period 1 – **growing** seas

period 3 – **mature** seas

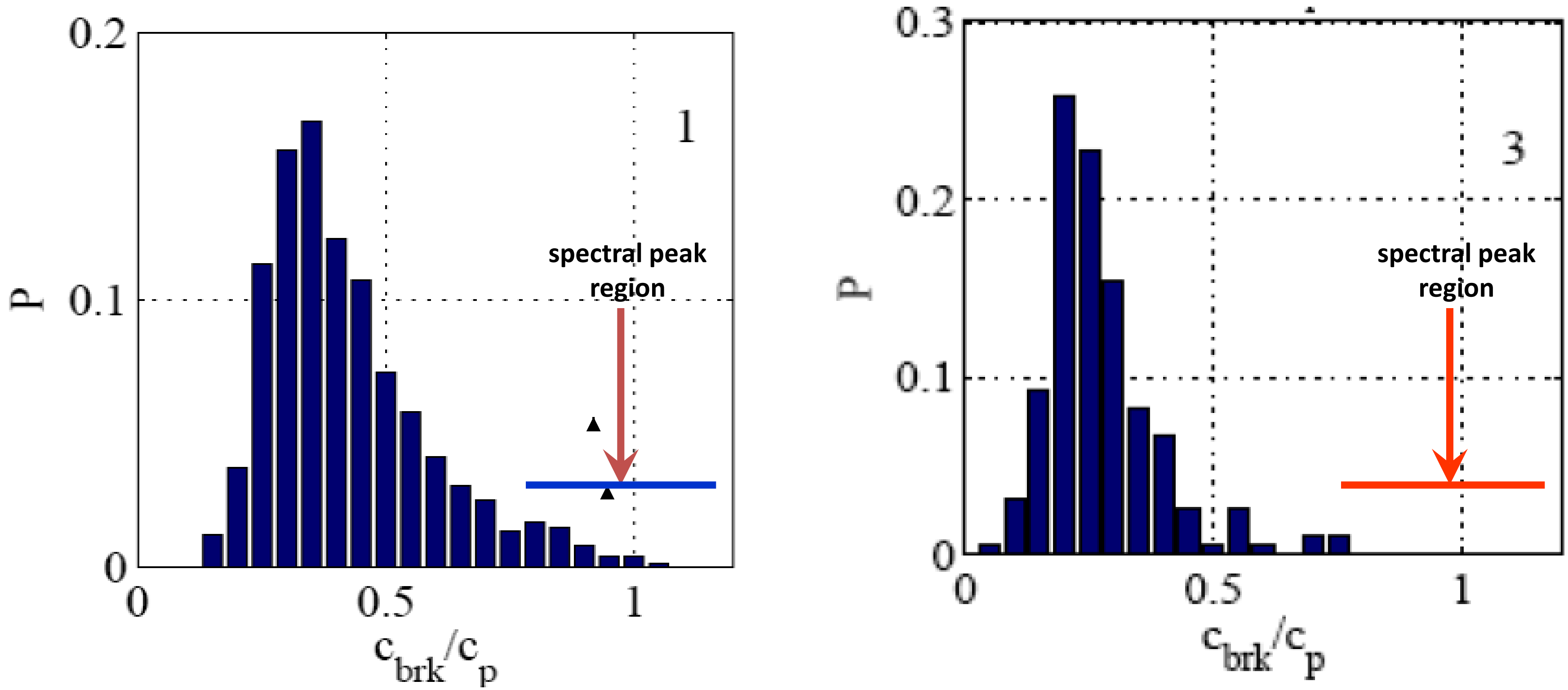


Figure 2. Probability distribution of breaking waves as a function of wave speed relative to the spectral peak, for period 1 (growing seas) and 3 (mature seas). Note that breaking events occur at the spectral peak ($0.8<c_{brk}/c_p<1.2$) for period 1, but not for period 3.

Predicted vs observed breaking probabilities

Observed spectral peak breaking probability:

$$\mathbf{P_{br}(peak) = 0.058}$$

Predicted spectral peak breaking probability based on normalized saturation and actual total crest count

$$\mathbf{P_{br}(peak) = 0.065}$$

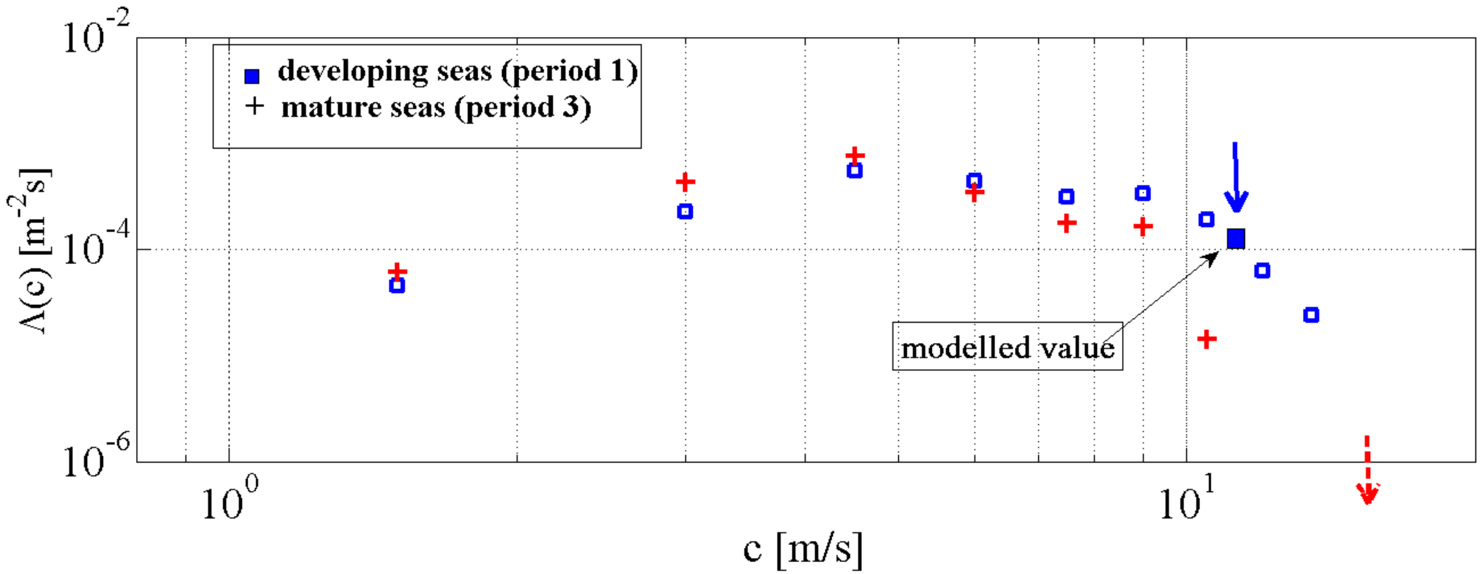
Predicted spectral peak breaking probability based on significant spectral peak slope and nominal crest count (based on f_p) (Banner, Babanin & Young (2000)

$$\mathbf{P_{br}(peak) = 0.038}$$

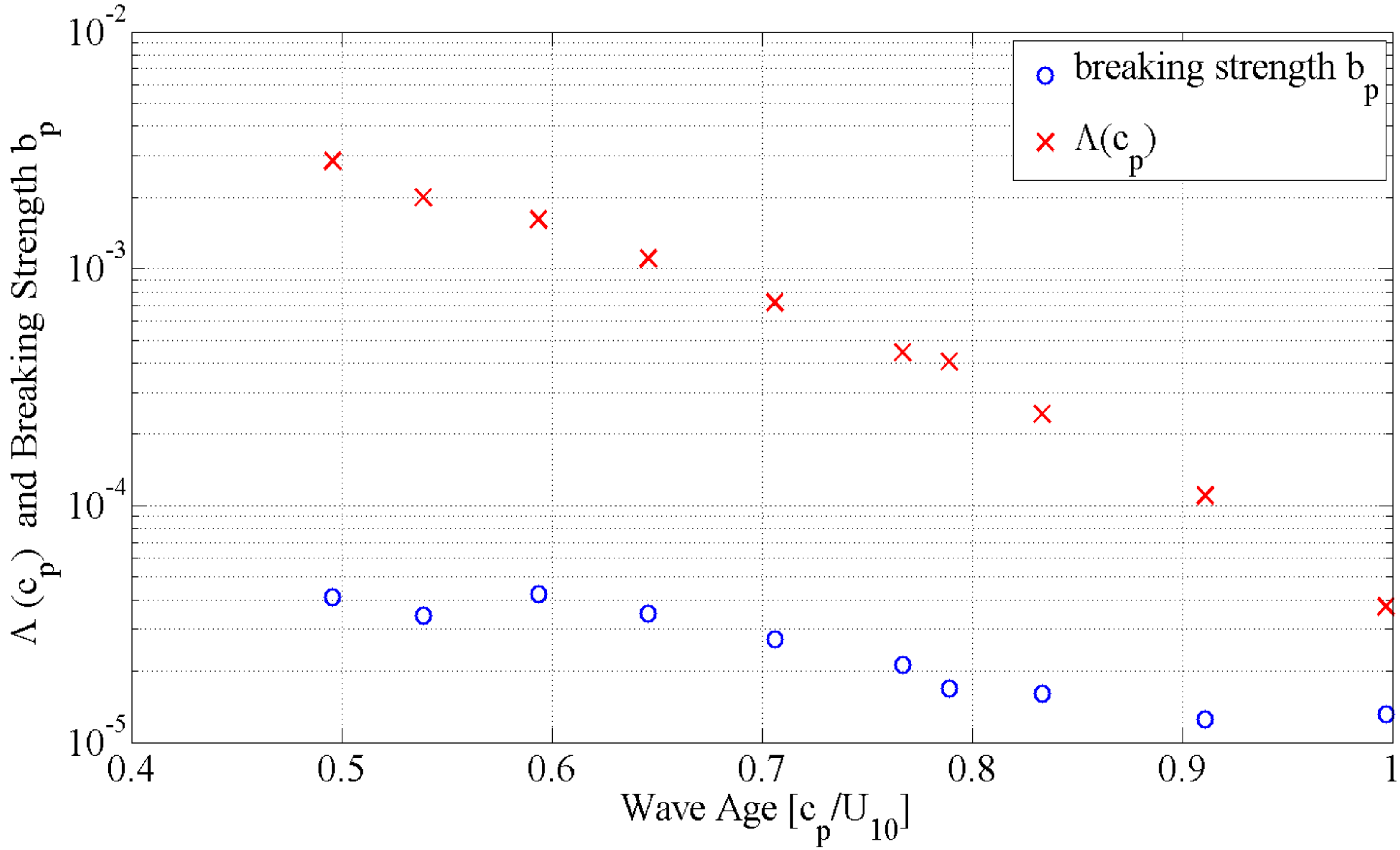
As above but corrected for actual crest count = 0.65*nominal based on f_p :

$$\mathbf{P_{br}(peak) = 0.059}$$

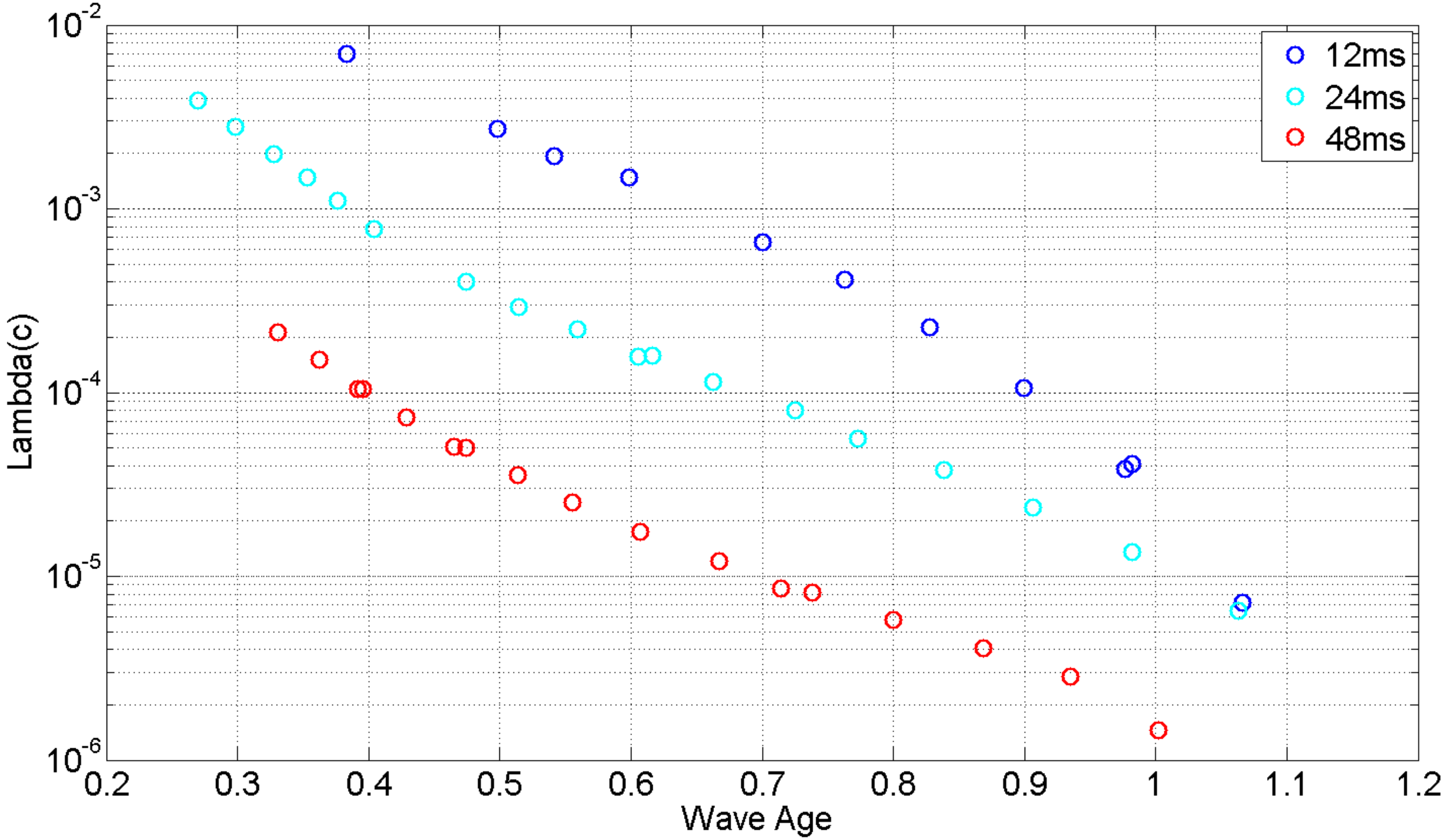
Predicted versus observed $\Lambda(c)$ at the spectral peak for U10 ~ 12 m/s during FAIRS experiment



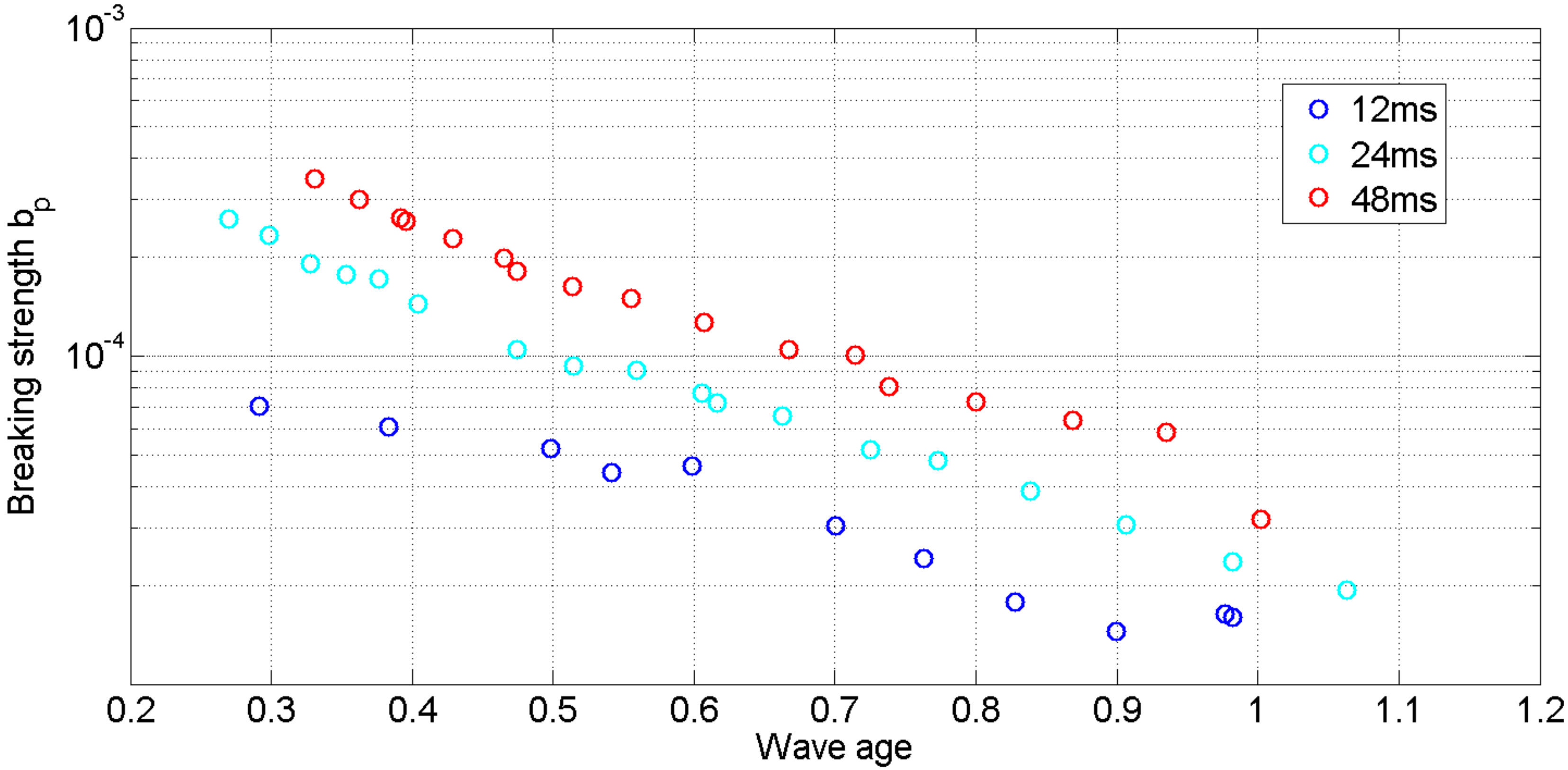
Predicted Variation of Λ and b with Wave Age for the Spectral Peak Waves for U_{10} = 12 m/s



Predicted Variation of Λ with Wave Age for the Spectral Peak Waves for various U_{10}



Predicted Variation of b with Wave Age for the Spectral Peak Waves for various U_{10}



Summary of Conclusions

• our framework provides predictions of wave breaking properties (crest length spectral density per unit area and breaking strength) using standard wave model outputs

• it provides accurate predictions for the limited breaking data available for the dominant waves in developing and mature wind seas

• further validation against data will be made as suitable new data sets becomes available (e.g. Santa Barbara Channel, Pacific Ocean off Hawaii)

• it is easily added to existing spectral wave forecasting models. Upgrading the DIA form of S_{nl} is desirable.