Second-order Crest Statistics of Realistic Sea States

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Motivation

- Freak waves hitting offshore platforms
  - Possible loss of air-gap $\longrightarrow$ wave-in-deck loads
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- Determine realistic environmental conditions that produce largest crest elevations
- Field measurements of freak waves indicate set-up in deep water (e.g. Draupner new year wave)
  - Investigate the role of sum- and difference-terms

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  - Bi-modal spectra
    $\longrightarrow$ Wind-sea: JONSWAP
    $\longrightarrow$ Swell: log-normal
  - Directional spreading
    $\longrightarrow$ Wind-sea: bi-modal and frequency dependent
    $\longrightarrow$ Swell: Ewans (2001)

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Spectral Surface Response Model (SRS)

- Very efficient method (Tromans & Vanderschuren, 2004)
- First Order Reliability Method (FORM)
    - Constant value of ocean surface elevation as the limit state
- Surface elevation, η, is a linear superposition of
    - Linear random wave theory
    - Second-order irregular wave theory (Sharma & Dean, 1981)
- Provides the probability of a maximum above the value η

Irregular Wave Theory

$$\eta^{(1)} = \sum_{i}^{N} a_i \cos(\psi_i)$$

$$\eta^{(2)} = \frac{1}{4}\sum_{i}^{N}\sum_{j}^{N} a_i a_j K_{ij}^{+} \cos(\psi_i + \psi_j) + \frac{1}{4}\sum_{i}^{N}\sum_{j}^{N} a_i a_j K_{ij}^{-} \cos(\psi_i - \psi_j)$$

$$\psi_i = \mathbf{k}_i \mathbf{x} - \omega_i t + \phi_i$$

K^{+} and K^{-} are kernels from Sharma & Dean (1981)

Simulations

- Investigate the effect of directionality
  - Unidirectional simulations
  - Directional simulations
- Investigate the effect of water depth
  - Deep, $d = 2000\mathrm{m}$
  - Shallow, $d = 30\mathrm{m}$
- 5 wind-sea spectra & 11 log-normal spectra
  $\rightarrow$ 220 cases in total
- Output: η/H_s for a probability of exceedence of 0.001

Bi-modal Sea States

- Wind-sea and swell components

$$G(f,\theta) = G_{sea}(f,\theta) + G_{swell}(f,\theta)$$

where

$$G_{sea}(f,\theta) = S_{sea}(f)D_{sea}(f,\theta)$$

$$G_{swell}(f,\theta) = S_{swell}(f)D_{swell}(f,\theta)$$

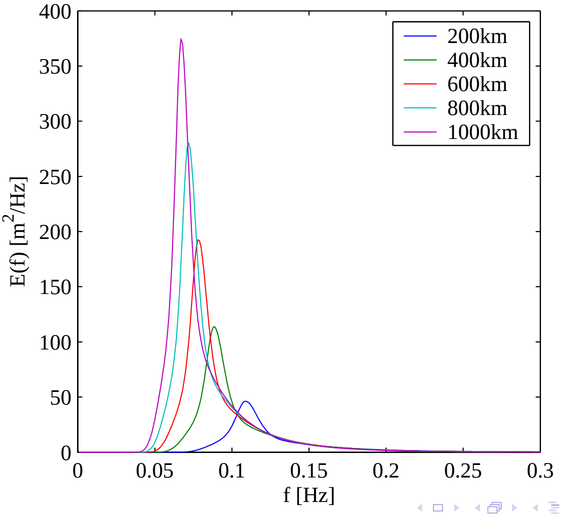
- $S_{sea}(f)$ is JONSWAP and $D_{sea}(f,\theta)$ from Ewans (1998)
- $S_{swell}(f)$ is log-normal and $D_{swell}(f,\theta)$ from Ewans (2001)

Wind-sea Spectra

- 5 JONSWAP spectra
- Fetch relationship of Carter (1982) with constant wind-speed of $20\mathrm{m/s}$

				H_s/λ_p [-]	
Spectrum	Fetch $[\mathrm{km}]$	H_s $[\mathrm{m}]$	T_p $[\mathrm{s}]$	Deep	Shallow
A	200	5.09	9.19	0.039	0.042
B	400	7.21	11.31	0.036	0.044
C	600	8.83	12.78	0.035	0.046
D	800	10.19	13.93	0.034	0.048
E	1000	11.40	14.89	0.033	0.049

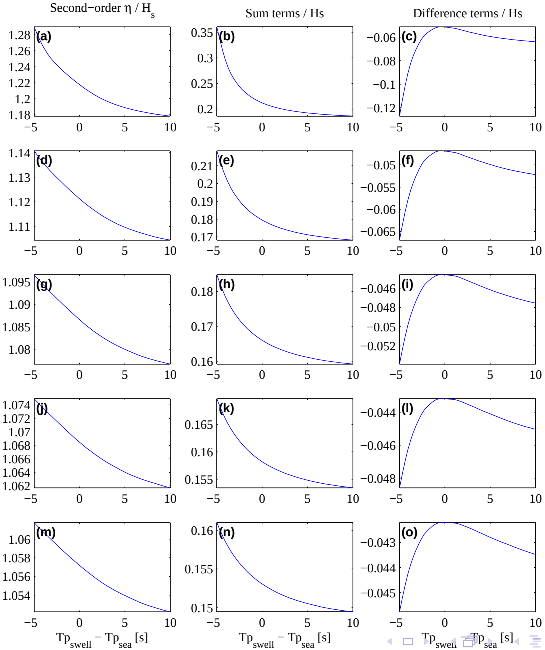
Wind-sea Spectra



Swell Spectra

- Log-normal distribution
- All 5 wind-sea spectra combined with swells with
  - Constant $H_s = 3\mathrm{m}$
  - 11 peak periods from within $T_{p,sea} - 5 \leqslant T_{p,swell} \leqslant T_{p,sea} + 10$
  - Standard deviation of $\sigma = 0.015\mathrm{Hz}$

Unidirectional, Deep Water ($d = 2000$m)



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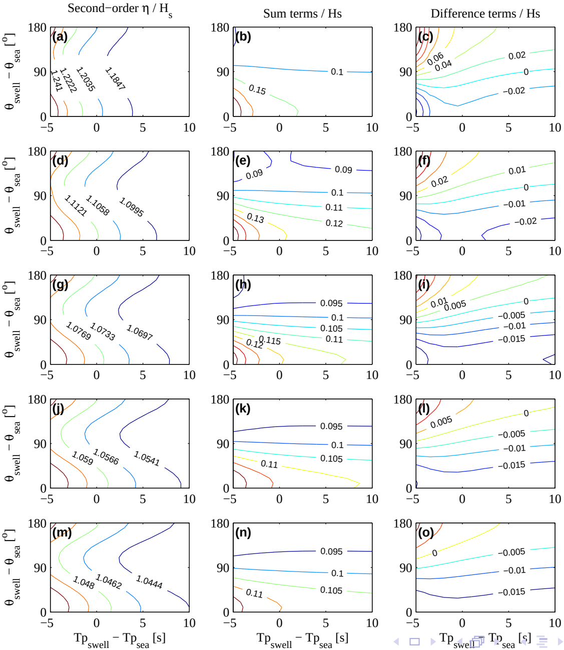

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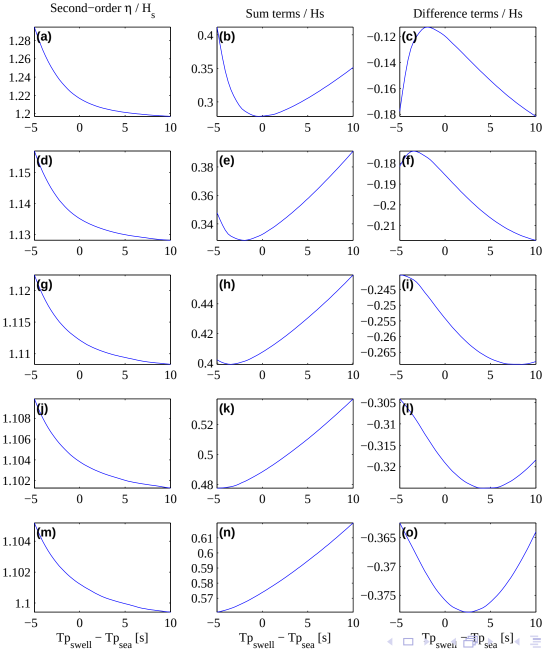

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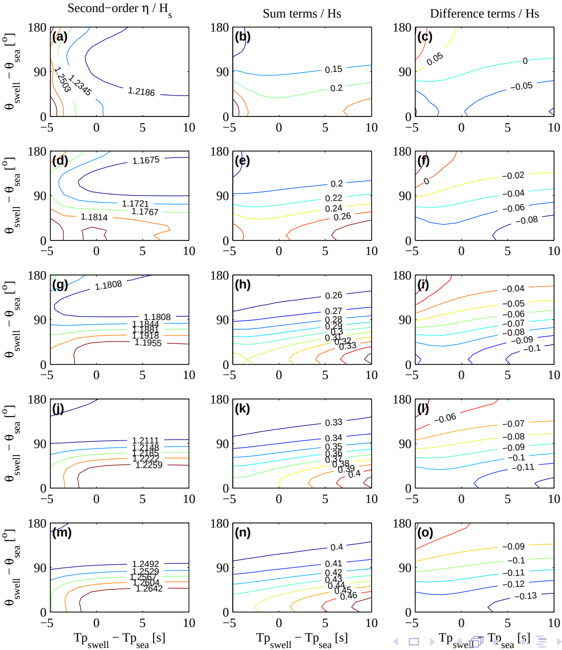

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The End

Thank you for listening to my presentation
Are there any (further) questions?

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