The directional spreading of the Draupner wave and sea-state

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1. Introduction

On 1 January 1995 a large wave was recorded at the Draupner platform in the North Sea on a water depth of 70m. The time history including this wave is shown in Figure 1. The wave had a crest of 18.6m and height of 25.6m. The wave was measured with a downward pointing laser sampling at 2.1Hz. The platform had a sparse structure which would have had little effect on the free surface; it is therefore assumed to be an undisturbed field measurement. The wave caused minor damage to equipment below main deck level on the underside of the structure.

The storm properties were approximately stationary over a five hour period with $H_s = 12$m and $T_z = 12.5$s. The Benjamin-Feir index, as defined by Janssen (2003), was ~ 0.55. However, if finite depth is allowed for (Onorato et al. 2006) this reduces to ~ 0.07 and further if the effects of directional spreading are accounted for (Waseda et al. 2009; Janssen and Bidlot 2009). Unfortunately data were only recorded for 20 minutes in every hour. For more information on the structure, the instrumentation, the meteorological conditions and the wave record, see Haver (2004).

This wave has been much analysed as it is regarded as one of the few high quality measurements of a 'freak' or 'rogue' wave. See for example Walker et al. (2005) and Jensen (2005). In analysing the non-linear structure of this wave, knowledge of directional properties of the wave is vital. The degree of spreading will influence the size of an extreme crest (Johannessen and Swan 2001; Gramstad and Trulsen 2007; Onorato et al. 2009). Unfortunately no directional data was recorded at this location. In this paper we examine whether we can deduce anything about the directional spreading of this wave.

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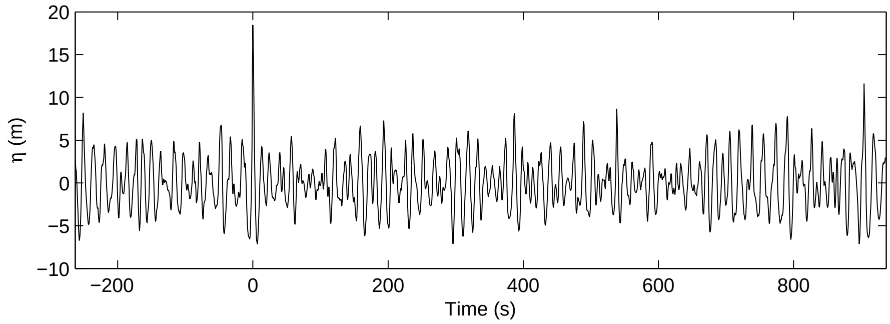


Figure 1: The surface elevation time history recorded at the Draupner platform which includes the New Year Wave. For convenience the time is set to zero under the big wave.

One unusual aspect of the Draupner wave was the low frequency set-up beneath the wave, first noted by Walker et al. (2005). Beneath a wave-group with a low level of directional spread there will be a set-down under the group (Forristall 2000). However, if waves are propagating in different directions then a set-up may occur (Toffoli et al. 2007). In this paper we explore this possibility.

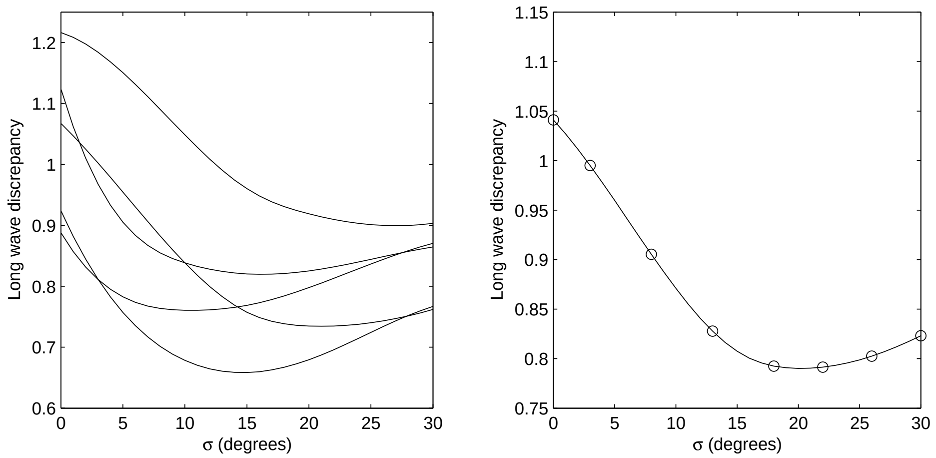
2. Known spreading information

The nearest location known to the authors where directional data for this storm was recorded is the Auk platform some 180km away (Ewans, private communication). The meteorological conditions suggest this was part of the same storm, and the other sea-state parameters are similar. At Auk, the wave directional spreading had a standard deviation about the mean direction of 20° when a weighted average is taken over all frequencies.

Adcock and Taylor (2009a) analysed the overall directional spreading of the Draupner storm using the low frequency bound waves to give an estimate of spreading. This was done by comparing the magnitude of the low frequency waves with that predicted for various spreading angles. The 'long wave discrepancy' was then found between these, where the discrepancy is given by

$$\text{long wave discrepancy} = \frac{\sqrt{\sum\left(\text{Estimate} - \text{Filtered data}\right)^2}}{\sqrt{\sum\left(\text{Filtered data}\right)^2}}. \tag{1}$$

The minimum discrepancy between these data was taken to be the spreading for the sea-state (see Adcock and Taylor (2009a) for more details). The discrepancy for the Draupner data is shown in Figure 2; (a) shows the discrepancy for each 20 minute record and (b) the average over the whole time series. Thus the average over the whole storm was 20° in agreement with the figure recorded at Auk. However, this analysis broke down in the vicinity of the large wave as discussed in section 6.



(a) The long wave discrepancy of each 20 minute section of the Draupner time histories.

(b) The average long wave discrepancy for Draupner time histories. The results are spline fitted.

Figure 2: Long wave discrepancies for the Draupner data

Thus, we have evidence that over most of the five hour storm the waves had a mean standard deviation of directional spreading of around 20° weighted across all frequencies, suggesting that there was little wave energy travelling at angles greater than about 40° to the mean wave direction.

3. The spectrum of the Draupner wave

If we consider the spectrum of the storm we can see that the local spectrum around the Draupner wave is significantly different to that of the rest of the storm. Figure 3 shows estimated spectra for all six of the hourly 20 minute records available during the storm, the 20 minute record containing the Draupner wave, and a five minute and two minute record centered on the Draupner wave.

It can be seen that for five minutes of data around the giant wave, the spectrum is similar to that for longer time periods, which in turn are similar in form to a JONSWAP spectrum with a peak at around 0.067Hz. In the 2 minute spectrum there is a clear second peak at 0.083Hz. We call the peak around 0.067Hz – peak 1, and the peak near 0.0833Hz – peak 2.

A broadening of the spectrum around a large wave event is well known. This result is found in experimental studies (Baldock et al. 1996; Johannessen and Swan 2001), numerical studies (Gibbs and Taylor 2005; Gibson and Swan 2007) and analytically (Adcock and Taylor 2009c,b). However, these show a smooth broadening of the spectrum, rather than the double peaked nature of the Draupner spectra. Furthermore, double peaked sea-states generally exist where two wave systems are propagating in different directions (Guedes Soares 1991). Thus the local double peaked nature of the spectrum is unusual.

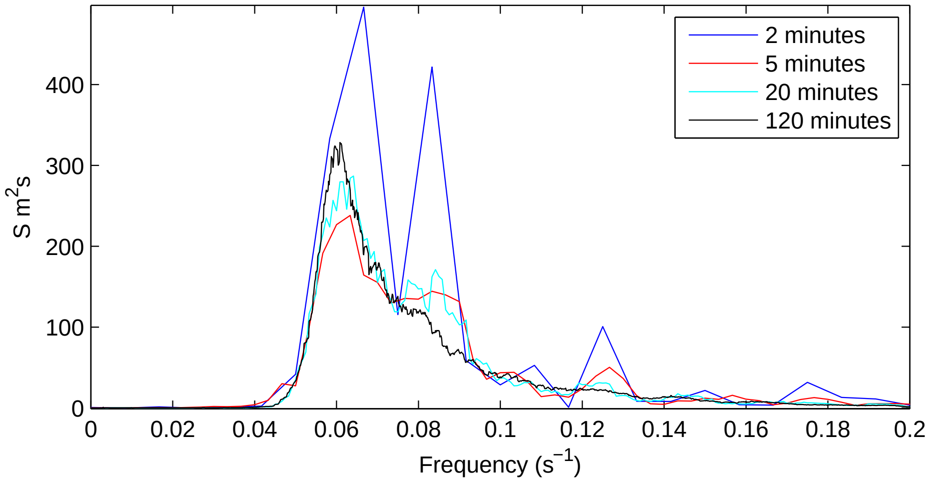


Figure 3: Spectra of time-records containing the Draupner wave. The 2 and 5 minute records are centered on the giant wave. The 20 minute is the spectrum of the data shown in 1. The 120 minute is all the available data for the storm. Data is windowed using a Hann window. The data is smoothed by taking an average over 0.001Hz to improve the clarity of the longer data sets.

4. Second order bound waves

Freely propagating, or linear, waves will interact to produce 'bound' waves which do not move independently. To second order, these waves will occur at the sum and difference of the frequencies of the linear waves. The freely propagating waves may be written as

$$\eta_{free} = \sum_{n=1}^{n=N} a_n \cos(\phi_n), \tag{2}$$

where a_n is the Fourier coefficient, N the number of Fourier components used, and

$$\phi_n = \omega_n t + \xi_n, \tag{3}$$

where ξ gives the relative phase of the component.

The linear waves in equation 2 will interact to give a second order sea state given by

$$\eta = \eta_{free} + \eta_{2+} + \eta_{2-}, \tag{4}$$

where

$$\eta_2^{\pm} = \sum_{n=1}^{n=N} \sum_{m=1}^{m=N} a_n a_m \kappa^{\pm} \cos(\phi_n \pm \phi_m), \tag{5}$$

where κ^+ and κ^- are the interaction kernels given in equations 6. The kernels for finite depth by Dean and Sharma (1981), see also Dalzell (1999) which gives corrections to the finite depth case.

$$\begin{aligned}\kappa^{\pm} \quad = \quad & \frac{\omega_n^2+\omega_m^2}{2g} + \frac{\omega_n\omega_m}{2g}\left(\frac{\cos\theta}{\tanh(|\mathbf{k}_n|\,d)\tanh(|\mathbf{k}_m|\,d)} \mp 1\right) \times \\ & \left(\frac{(\omega_n \pm \omega_m)^2 + g\,|\mathbf{k}_n \pm \mathbf{k}_m|\tanh(|\mathbf{k}_n \pm \mathbf{k}_m|\,d)}{(\omega_n \pm \omega_m)^2 - g\,|\mathbf{k}_n \pm \mathbf{k}_m|\tanh(|\mathbf{k}_n \pm \mathbf{k}_m|\,d)}\right) + \\ & \left(\frac{\omega_n \pm \omega_m}{2g\left((\omega_n \pm \omega_m)^2 - g\,|\mathbf{k}_n \pm \mathbf{k}_m|\tanh(|\mathbf{k}_n \pm \mathbf{k}_m|\,d)\right)}\right) \times \\ & \left(\frac{\omega_n^3}{\sinh^2(|\mathbf{k}_n|\,d)} \pm \frac{\omega_m^3}{\sinh^2(|\mathbf{k}_m|\,d)}\right). \end{aligned} \tag{6}$$

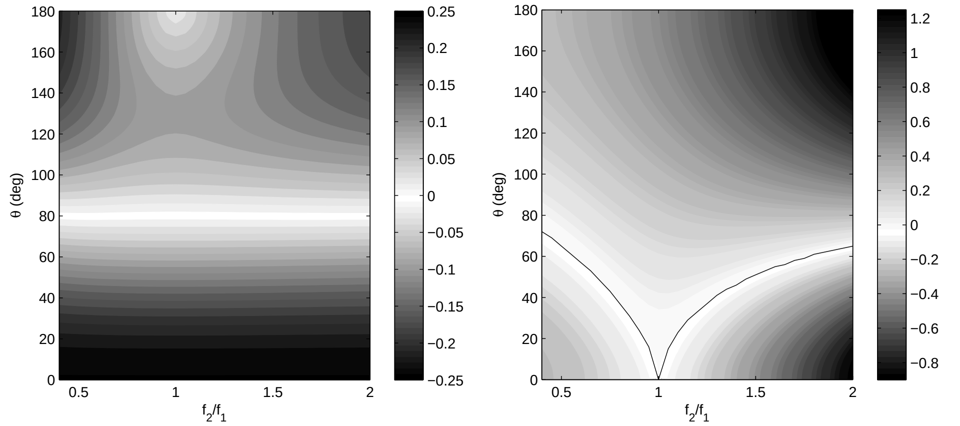
The magnitude of the wavenumber $|\mathbf{k}|$ and natural frequency ω are related by the linear dispersion relation. The angle between the interacting components is θ.

We now consider the form of these kernels for water depth at Draupner of 70m. The behaviour of of the sum term is comparatively straightforward. For any two interacting waves, this will be a maximum when they are travelling in the same direction (unless the ratio of the frequencies is large, in which case the assumption under which these equations are derived is invalidated). Figure 4 (a) shows the variation in κ^+ for the interactions with the first peak in the Draupner spectrum. It can be seen that the sum kernel goes to zero at around $80°$ for all frequency interactions and is negative for angles greater than this. For the difference term the behaviour is more complex. For waves travelling in the same direction, the kernel is always non-positive. However, if the waves are travelling at widely differing angles then the kernel is positive, the value at which the kernel is zero being strongly dependent on the frequencies of the waves interacting. This is shown in Figure 4 (b).

5. Second order sum waves during the Draupner event

We can see evidence of second order sum waves in Figure 3. Three peaks can be seen at approximately twice the frequency of peak 1 (0.125Hz), twice the frequency of the peak 2 (0.175Hz), and the sum of the frequencies of the two peaks (0.15Hz). Unfortunately, the frequency resolution limits the confidence in these assertions. It should also be noted that some linear waves will be in this part of the spectrum. The relative magnitude of these three second order peaks is interesting. The peak at 0.125Hz is much larger than one at 0.175Hz. This is to be expected as peak 1 is larger than peak 2. However, the peak at 0.15Hz is substantially smaller again. This is unexpected if the wave energy from both peaks is travelling in the same direction. However, consideration of Figure 4 (a) suggests this would be expected if some of the energy in the peak 2 was travelling at an angle between perhaps $60°$ and $150°$ to the mean wave direction of peak 1.

Thus, the second order sum term is consistent with a hypothesis that a substantial portion of the wave energy in peak 2 is travelling at a large angle to the waves in peak 1. We cannot go further than this when considering the sum term due to the amount of linear information in the spectrum at these frequencies.



(a) The bottom part of the contour map is positive - the top part is negative.

(b) The line where the kernel is zero is shown in black. Below the line the kernel is negative and it is positive above it.

Figure 4: The kernels of the second order interaction. $f_1 = 0.06$Hz and $d = 70$m.

6. Second order difference waves during the Draupner event

Adcock and Taylor (2009a) developed an approach to determining the spreading of a sea-state from the second order difference record. As noted in section 2, they found the whole storm to have a standard deviation of spreading of $20°$. However, their analysis did not work for a short section of time around the giant wave as described below.

The low frequency waves which are present in the time-history may be extracted by filtering in the frequency domain. These are shown in Figure 5, where the original time-series has been low-pass filtered at 0.03Hz. It can be seen that under the giant wave the low-pass filtered waves are positive. It is not expected that any linear waves will be propagating at frequencies this low. Therefore, the waves propagating in this part of the spectrum may be predicted from the linear waves using the interaction kernel given by equation 6. This predicts that energetic wave-groups, which are directionally narrowly spread, will tend to cause a set-down as pointed out by Forristall (2000). It is therefore surprising that there is a significant set-up under the giant wave in the Draupner record as noted in Walker et al. (2005). A set-down is observed under all other sizable wave-groups throughout the rest of the data set. Adcock (2009) found no evidence that third or higher order interactions could account for this set-up.

Walker et al. (2005) introduces an approach to linearising a free surface time history so as to estimate the freely propagating, or linear, waves. These may then be used to estimate the low frequency waves for given spreading as set out in Adcock and Taylor (2009a). Using this linearised data, we can use a model for spreading to predict the low frequency second order difference waves. If we use a frequency independent wrapped normal spreading

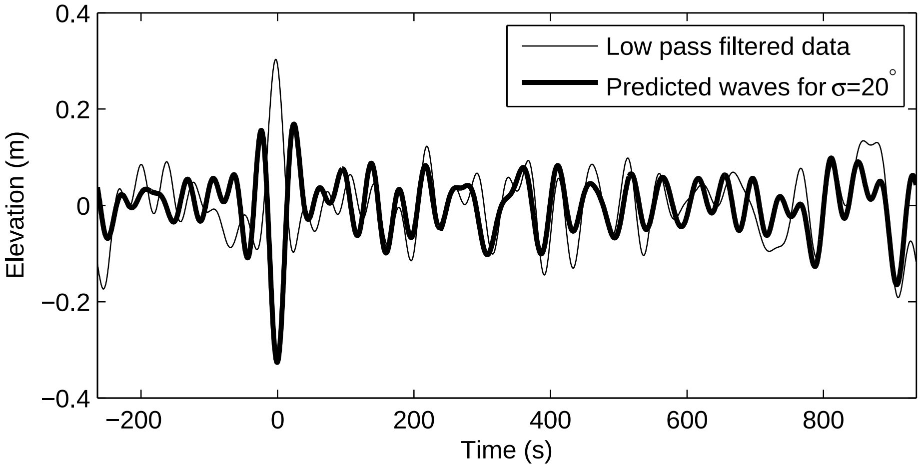


Figure 5: The low pass filtered data for the time-history containing the New Year Wave, together with the estimate of the long waves based on a wrapped normal spreading distribution with $\sigma = 20^{\circ}$.

function

$$D(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \text{Exp}\left(-\left(\frac{(\theta-\theta_0)^2}{2\sigma^2}\right)\right) \tag{7}$$

with σ, the standard deviation of spreading around the mean direction θ_0, of 20° then our estimate for the low-frequency wave is as shown in Figure 5. This is clearly completely out of phase for around 2 minutes near the giant wave.

This suggests that either there is unusual physics happening around the giant wave, or that our model of spreading is wrong. The interaction kernel (Figure 4 (b)) can be positive if the two interacting components are propagating in different directions. Thus, if the second peak of the spectrum was caused by a localised wave group moving in a direction different to the mean wave direction of the storm, a set-up would be possible. The remainder of this section examines this possibility.

The spectrum of the wave record for the section where the long waves are unusual is shown in Figure 6. This is 120 seconds before through to 75 seconds after the crest of the giant wave – note that this is a slightly different section of data to that used in Figure 3. We can fit a JONSWAP spectrum to the data excluding the region around the giant wave. This is also shown in Figure 6. We now assume that the linear waves are the sum of two systems of waves: one has a JONSWAP spectrum whilst the other has a spectrum given by the difference between the actual measured spectrum and JONSWAP spectrum (the area shaded in Figure 6). We call these the 'JONSWAP waves' (J) and 'transverse waves' (t) respectively. We have freedom to choose the phase of the individual Fourier components in the transverse wave. We can iterate these so as to try and recreate the difference wave observed in the original data. This iteration was carried out manually using the following assumptions

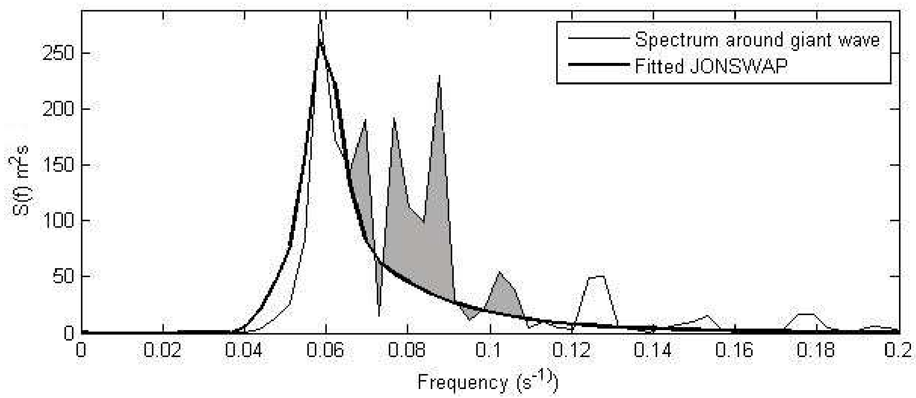


Figure 6: The spectrum around the giant wave for which the long waves are unusual. Also shown is a JONSWAP spectrum with parameters fitted to the time-series excluding this data.

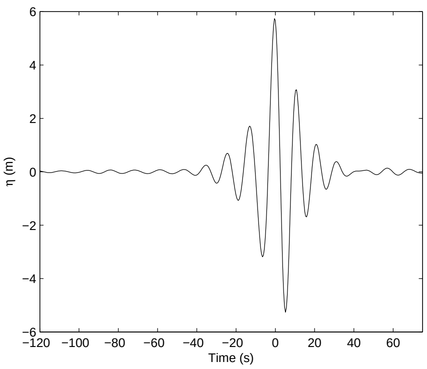
1. The sum of the JONSWAP waves and the transverse waves equals the original linearised time-series.
2. The JONSWAP waves have a JONSWAP spectrum.
3. For estimating the spreading it was assumed that both sets of waves had a spread of $20°$ about their mean value. The angle between the mean direction of the two systems of waves was ζ.

Figure 7 (a) shows the transverse wave which produces the difference waves shown in Figure when the angle between the mean direction of the wave-trains is $120°$. The JONSWAP waves are shown in Figure 7 (b). There is good agreement for around 60 seconds either side of the giant wave, regardless of exactly where the data is low-pass filtered.

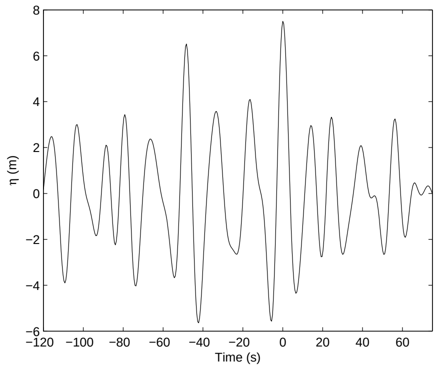
7. Other evidence

There are a number of other measurements and inferences which support the possibility that the Draupner wave may have been caused in a crossing sea.

- **Forces on the structure** The forces due to this wave on the northern platform were significantly lower than would have been expected for a wave of this size (Haver (2004) and Hansteen et al. (2003)). Given that the height of the Draupner wave is roughly the 50-100 year design condition for that part of the North Sea, the observed peak horizontal force on the structure, as recorded by strain gauges on the suction caisson foundations and lower structural members, was roughly half of that expected for a wave of this height. This is consistent with a simplistic Morison-drag estimate. If the wave components were at roughly $90°$ apart in direction, the total drag would

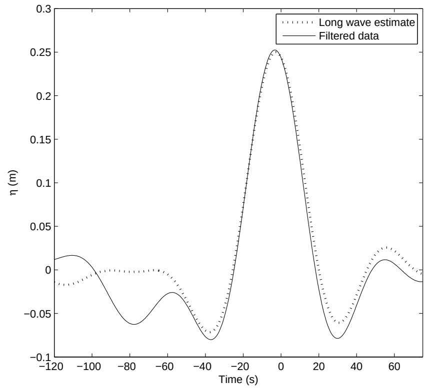


(a) The transverse wavegroup.

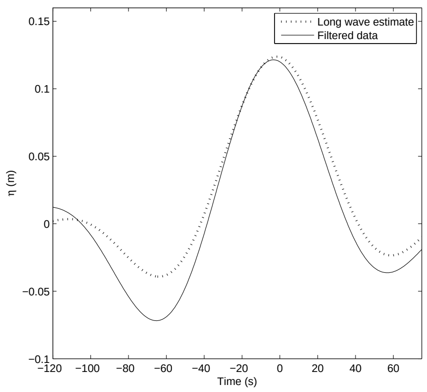


(b) The JONSWAP waves with high-frequency parts removed.

Figure 7: The combination of transverse waves and JONSWAP waves which produces a good fit to the observed difference terms.



(a) Data filtered at 0.0292Hz.



(b) Data filtered at 0.0146Hz.

Figure 8: The difference waves under the Draupner giant wave. Comparison of filtered data and predicted second order difference waves.

scale as the vector sum of the wave amplitudes for the wave trains roughly in phase in time, giving a force scaling as $(8^2+6^2) \approx 100$. In contrast with both wave components moving in the same direction the drag would scale as $(8+6)^2 \approx 200$.

- **Absence of wave breaking** The Draupner wave was an extremely steep event, with $ka \sim 0.45$, however there is no evidence in the wave-record that it was close to breaking. If the wave was the result of a crossing event then it could be far steeper without breaking.

- **Meterology** The weather conditions as described in Haver (2004) indicate that the sea-state was the result of two lows, one large-scale and centered over southern Sweden and a second smaller low which tracked down the centre of the North Sea to the west of the Draupner platform. Thus the platform was outside the area where the wind field seems to have been most severe, Haver (2004). The question arises as to whether this combined wind field could give rise to a 'crossing sea' at Draupner. This would depend on the local structure of the second low. If this was a locally intense polar low (Rasmussen and Turner 2003) with strong circulation around an 'eye' (such local storms have been called Arctic hurricanes) then the production of a strongly crossing sea-state would seem to be possible. The occurrence of polar lows in the North Sea in the winter is highlighted by Arkjær (1992), and the possible consequence for wave generation is discussed by Dysthe and Harkitz (1987), albeit only in the context of uni-directional wave propagation.

  The possibility of a locally intense polar low, which could produce a crossing sea-state, does not explain the apparently localised nature of the transverse wave packet. We have found no clear evidence of any other crossing components. However, weaker crossing components which did not coincide with a large wave group in the main roughly north-south wave field cannot be entirely discounted. Our analysis method based on the 2nd order difference bound waves relies on the occurrence of large linear components.

- **Other wave events** A number of accounts of unusual wave incidents suggest that isolated wave-packets may travel in directions different to those of the surrounding waves. An account in a BBC Horizon programme of an unusual wave hitting the cruise ship Caledonian Star in 2001 states that this wave was travelling at $30°$ to the waves around it. Video recorded during filming for *The Deadliest Catch* apparently shows a fishing boat being hit broadside by a large wave travelling at around $90°$ to the main swell waves. The wave which hit the Queen Mary in 1942 is described as hitting her 'broadside', so presumably this wave was travelling at roughly $90°$ to the majority of the waves. Similar accounts are given by the crews of the Gloucester dragger (Prybot 2007) and the RMS Etruria (Liu 2007). Thus, perhaps, the Draupner wave belongs to a population of freak wave events which are caused by short duration wave-packets propagating independently to the direction of the surrounding waves. Such waves would be potentially very dangerous to shipping, which might suggest why so few events have been observed. Where wave directionality is directly measured, this is usually averaged over an interval much greater than a few wave-periods, and even if an unusual directional reading was observed it might well be attributed to data error or averaged out.

8. Conclusions

In this paper we have suggested that the Draupner New Year Wave was the results of two waves propagating at around 90 to 120 degrees to each other. This is based on analysis of the low frequency difference waves, which exhibit an unusual set-up under the giant wave, which, to the second order, can only be explained if significant amounts of wave energy were travelling in different directions. Our explanation is consistent with other evidence. However, whilst the meteorology suggests that the Draupner sea-state may be a crossing sea, we offer no explanation as to why the energy from one wave-system should concentrate into one short wave-packet, as the evidence in this paper suggest.

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