EXTREME RESPONSE IN A HURRICANE GOVERNED OFFSHORE REGION: UNCERTAINTIES RELATED TO LIMITED AMOUNT OF DATA AND CHOICE OF METHOD OF PREDICTION

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1. INTRODUCTION

The aim of a design process is to ensure that a structure can withstand all foreseen forces with an adequate margin. This is in most cases done by selecting loads and responses corresponding to a given annual exceedance probability, q, where q is taken to be a rather low number. Here we will restrict the discussion to wave induced loads and responses. Throughout this paper a value corresponding to an annual probability of being exceeded equal to q will be referred as a q-probability value. For design against overload q is taken to be 10^{-2} for the basic design control (ULS). Additionally, some codes also require that accidental environmental loads and responses are considered, (ALS). For the ALS control q = 10^{-4}. The design recipe accounts for the fact that uncertainties are associated with the estimated q-probability loads/ -responses, r_q. The design responses are therefore obtained by multiplying the q-probability responses by a rule defined partial safety factor, γ_E. The capacity of a structure will also be associated with uncertainties. For the basic design control, the nominal capacity of a structural component is taken to be the 5%-value of the elastic component capacity, $c_{e,0.05}$. Uncertainties in this quantity is accounted for by dividing it by a rule defined material factor, γ_M.

Restricting the assessment to wave induced loads, the requirement that must be fulfilled by a proper design is given by Eq. (1). It must be kept in mind that this is merely an illustration. In practical design work one will also have to account for the effect of permanent loads and variable functional loads.

$$\gamma_E r_q \leq \frac{c_{e,0.05}}{\gamma_M} \qquad (1)$$

For q = 10^{-2}, typical values for steel structures are $\gamma_E = 1.3 - 1.35$ and $\gamma_M = 1.15$. For q=10^{-4}, both γ_E and γ_M are typically taken to be 1.0. Furthermore, $c_{e,0.05}$ is usually replaced by a higher capacity which accounts for the fact that the real capacity is larger than the elastic component capacity. Due to the reduced partial safety factors and the acceptance of larger utilization, ULS will in most cases govern structural design. An exception, however, is if the external load pattern changes abruptly in a worsening way for annual exceedance probabilities in the range 10^{-4} – 10^{-2}. An example where such a scenario is likely to be the case will be major wave-in-deck problems.

The aim of this paper is to discuss uncertainties associated with r_q and we will restrict the assessment to a case with q = 10^{-2}.

2. RESPONSE PREDICTION FOR DESIGN PURPOSES

In order to predict response values being in agreement with rule requirements all sources of inherent randomness should be included, i.e. long term variability in sea state characteristics and the short term variability of the response extreme value given the weather characteristics.

2.1 GENERIC RESPONSE CASES

Our aim is to predict an adequate estimate for a response quantity corresponding to a given annual exceedance probability, q. The basic random variable is the 30-min maximum response here denoted X_{30}. It is assumed that the distribution function of X_{30} is reasonably well modelled by the Gumbel model, see e.g. Bury (1975):

$$F_{X_{30}|H_sT_p}(x\,|\,h,t) = \exp\left\{-\exp\left\{-\left[\frac{x-\alpha(h,t)}{\beta(h,t)}\right]\right\}\right\} \quad (2)$$

Here we limit the problem to wave induced response. One could also include wind speed and current speed. The steps of the method will be the same

For a practical problem the challenge is to determine the short term distribution, i.e. the parameters $\alpha(\ )$ and $\beta(\ )$. For the illustrative purposes of this study, we will here consider a generic response problem. The location parameter is given by:

$$\alpha(h,t) = h^{\psi}\left[1+\cos^{r}\left(\frac{2\pi(t-t_0)}{p}\right)\right] \quad (3)$$

$$\beta(h,t) = \lambda\psi\,\alpha(h,t) \quad (4)$$

For a linear response problem, $\psi = 1$, while for a quadratic problem, $\psi = 2$. By selecting p >> t_0, we will have a response problem that is almost independent of the spectral peak period no matter of r. By selecting p and t_0 properly and use a relatively large value for r, a response problem sensitive to a period band around t_0 is achieved. λ is a measure of the variability around the mean 30-minute maximum. $\lambda = 0.1$ is adopted in this paper.

2.2 SHORT TERM DESCRIPTION OF RESPONSE

We will assume that we have time histories of hurricane characteristics, h_s and t_p, for all 30-minute hurricane steps (assumed to be stationary) of all (M) hurricanes exceeding 6m during a 50-year period. For each of the M hurricanes occurring within the selected area, we can estimate $\alpha(h,t)$ and $\beta(h,t)$ for all 30-minute stationary steps of the hurricanes using Eqs. (3 and 4).

The value of the location parameter represents the most probable largest 30-minute maximum response. If we could observe the actual response quantity during the hurricanes, we would observe that the actual 30-minute minute maxima where scattered around the calculated most probable largest values. This means that if we select the most probable maximum as the largest hurricane maximum response, we do <u>remove</u> a certain amount of inherent randomness, *the short term variability*, from the analysis. In order to include this variability in the response analysis, we generate a possible observation for each 30-minute stationary step of all hurricanes. This is done using Monte Carlo simulation, see e.g. Bury (1975). A random number, $u_{i,j}$, between 0 and 1 is generated. By replacing $F_{X_{30}|H_sT_p}(x\,|\,h,t)$ in Eq. (2) with $u_{i,j}$ and solve the resulting equation with respect to x, a possible realization for the 30-minute largest maximum of step no. j of storm no. i is achieved by:

$$x_{i,j} = \alpha_{i,j} - \beta_{i,j}\ln(-\ln(u_{i,j})) \quad (5)$$

An illustration of this process is shown for a particular hurricane in Fig. 1. The hurricane is one of the hurricanes of the base case 50-year generic hurricane data base (reference data base) discussed in a later chapter, see Table 1. It is seen that both the significant wave height and the spectral peak period follow an idealized time history. The difference between the most probable largest 30-minute maximum response and the observed (here simulated) 30-minute maximum response is clearly seen. Of course by repeating this process, the 30-minute maximum observations will change from simulation to simulation. This is illustrated by showing another simulation for the same storm in Fig. 2.

We will denote the largest most probable 30-minute response of hurricane no. i by $\tilde{y}_i$ and the largest observed 30-minute value during the hurricane by y_i. A priori, for an arbitrary hurricane, both these quantities will be random variables denoted by $\tilde{Y}$ and Y, respectively.

Following the ideas of Tromans and Vanderschuren (1995), we will consider the ratio $v_i = y_i / \tilde{y}_i$. We will assume that this ratio is a realization of a Gumbel distributed variable, V, which is assumed to be identically distributed for all hurricanes:

$$F_V(v) = \exp\left\{-\exp\left\{-\frac{v-\alpha_G}{\beta_G}\right\}\right\} \quad (6)$$

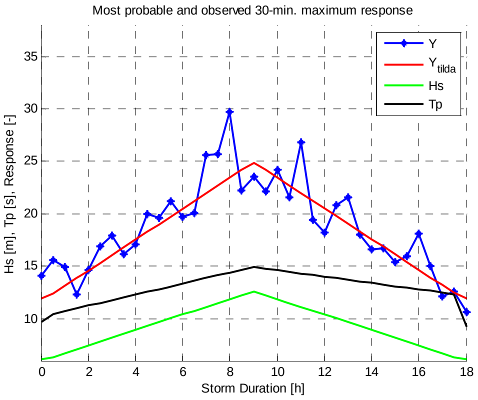

Fig. 1 Storm histories of h_s, t_p, $\alpha(h_s,t_p)$ and one set of simulation of x_{30} for a particular hurricane.

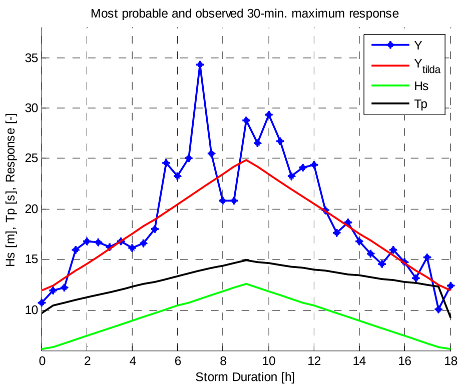

Fig. 2 Storm histories of h_s, t_p, $\alpha(h_s,t_p)$ and a new set of simulation of x_{30} for same hurricane as in Fig. 2.

By calculating this ratio for all available hurricanes, the mean, $\bar{v}$, and standard deviation, s_V, can be calculated from the generated sample. Applying the moment principle, the Gumbel parameters can be estimated by, Bury (1975):

$$\hat{\beta}_G = 0.7797 s_V \qquad (7)$$
$$\hat{\alpha}_G = \bar{v} - 0.57722\hat{\beta}_G$$

Using that

$$F_{Y|\tilde{Y}}(y|\tilde{y}) \equiv P[Y \le y | \tilde{Y} = \tilde{y}] = P[V\tilde{y} \le y] \qquad (8)$$
$$= P[V \le y/\tilde{y}] \equiv F_V(y/\tilde{y})$$

we have from Eq. (6):

$$F_{Y|\tilde{Y}}(y|\tilde{y}) = \exp\left\{-\exp\left\{-\frac{y-\alpha_G\tilde{y}}{\beta_G\tilde{y}}\right\}\right\} \qquad (9)$$

This is the *short term distribution* of hurricane maximum response given the most probable largest hurricane maximum.

2.3 LONG TERM ANALYSIS OF RESPONSE

In order to obtain a marginal distribution for Y, the *long term distribution*, we need to establish a long term distribution for $\tilde{Y}$. This can be done by fitting a probabilistic model to the observed most probable largest maxima for the M hurricanes, $F_{\tilde{Y}}(\tilde{y})$. We will here assume that $\tilde{Y}$ can be reasonably well modelled by a 3-parameter Weibull distribution:

$$F_{\tilde{Y}}(\tilde{y}) = 1 - \exp\left\{-\left(\frac{\tilde{y}-\tilde{y}_0}{\beta_W}\right)^{\gamma_W}\right\} \qquad (10)$$

A reasonable choice regarding the lower threshold for the most probable hurricane response, $\tilde{y}_0$, will be made in view of the sample values. Thereafter β_W and γ_W will be estimated using the maximum likelihood method, see e.g. Bury (1975). This is done by a Matlab function, *wblfit*.

As $F_{\tilde{Y}}(\tilde{y})$ is available, the long term distribution of Y is given by:

$$F_Y(y) = \int_{\tilde{y}} F_{Y|\tilde{Y}}(y|\tilde{y})\, f_{\tilde{Y}}(\tilde{y})\, d\tilde{y} \qquad (11)$$

The q- probability values are found by solving:

$$1 - F_Y(y_q) = \frac{q}{n_1}, \qquad (12)$$

where q is target annual exceedance probability and n_1 is the expected number of hurricanes per year (=M/50).

2.4 APPROXIMATE ESTIMATION OF LONG TERM EXTREMES

It is seen that the approach outlined above requires that the response analysis is done for a large number of sea states. This will be time consuming for a non-linear response problem where time domain simulations or model tests are required in order to determine the short term distributions.

An interesting question is therefore if we instead of considering all 30-minute steps of all hurricanes rather can consider the 30-minute maximum response of the worst q-probability 30-minute hurricane peak sea state. The sea state should be determined as the worst sea state (identified in view of the problem under consideration) along the q-probability contour of hurricane peak significant wave height, $h_{s,p}$, and the simultaneously occurring spectral peak period, $t_{p,p}$. An example of q-probability contour lines for $h_{s,p}$ and $t_{p,p}$ are shown in Fig. 3.

A reasonable probabilistic model for the 30-minute largest response given the significant wave height and spectral peak period of the hurricane peak, $h_{s,p}$ and $t_{p,p}$, is the Gumbel distribution shown in Eq. (2) with parameters given by Eqs. (3 and 4). When using this method for a real response case where the parameters, $\alpha(h_{s,p}, t_{p,p})$ and $\beta(h_{s,p}, t_{p,p})$, are not a priori known, they must be estimated from data of the 30-minute maximum response obtained by time domain simulations or model test experiments.

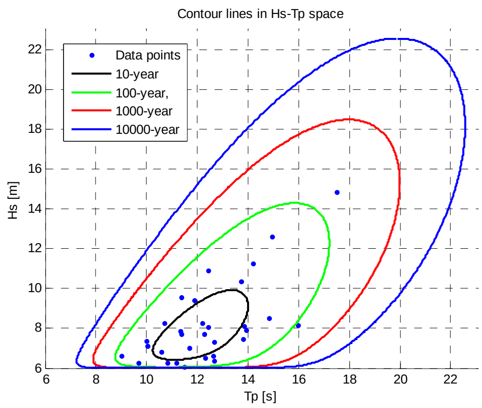

Fig. 3 q-probability contour lines for $h_{s,p}$ and $t_{p,p}$ based on 50 years of hurricane "observations".

The contour line method is presented into more details in Winterstein et al. (1993) and Kleiven and Haver (2004) and Haver and Kleiven (2004). Some examples of application are found in e.g. Haver and Winterstein (2008) and Baarholm and Haver (2009). We have long experience with the method in a North Sea type of wave climate. There the target quantity is typically taken to be the distribution of the 3-hour maximum response for the worst sea state along the q-probability contour line of h_s and t_p. h_s and t_p are the characteristics for an arbitrary 3-hour sea state. The sample space consists of all possible 3-hour sea states.

In Gulf of Mexico it is not convenient to consider an arbitrary 3-hour or 30-minute sea state due to the rather strong clustering of extreme sea states. Here it is more convenient to consider the largest hurricane response as the target quantity, Haring and Heideman (1978), Tromans and Vanderschuren (1995). The hurricane peak response will not necessarily occur during the 30-minute hurricane peak sea state. In spite of this, we will here assume that a properly selected hurricane peak sea state will be adequate for predicting design response.

We will establish q-probability contour lines for $h_{s,p}$ and $t_{p,p}$ from available hurricane data, see Fig. 3. This will be discussed into more detail later. For a given response quantity, we will determine the worst sea state along the 10^{-2}-probability contour. This sea state will be adopted as the ULS design sea state for this response quantity.

The challenge by this approach is to select a proper percentile of the Gumbel distribution for the 30-minute maximum response for the design sea state such that it becomes a good approximation to the "true" q-probability response obtained from Eq. (12). The assumption is that this percentile level should not be too dependent of the response quantity. This will be discussed later applying hurricane data qualitatively representing a 50-year period for a Gulf of Mexico site.

2.5 NEEDED METOCEAN INFORMATION FOR ESTIMATING DESIGN RESPONSE

The required metocean information for the two approaches outlined above are:

- The exact approach requires the time histories of significant wave height and spectral peak period for all 30-minute steps (at least all important steps) above the selected hurricane threshold.
- The approximate approach requires that the q-probability contour lines for $H_{s,p}$ and $T_{p,p}$ are available.

Both these subjects will be dealt with in the next chapter.

3. GENERIC 50-YEAR HURRICANE DATA BASE

For the Gulf of Mexico good quality hindcast data is available for the period 1950 – 2005, Cooper and Stear (2006). These data are not available for this study and a generic data base covering 50 years are therefore constructed using Monte Carlo simulation.

3.1 SIMULATION MODEL

The generic data base is developed as a part of a master thesis, Bergsvik (2009). The model is briefly described below:

Hurricane occurrences:
Hurricane occurrence within the hurricane season is assumed to be described by a Poisson process with a rate, γ. γ is varying over the hurricane season, July 1st - November 30th. The adopted rates (= expected number of hurricanes per day) are: 0.00024 for July 1st – August 15th, 0.008 for August 16th – September 30th , 0.003 for October 1st – October 31st. and, finally, 0.00024 for November 1st – November 30th.

A consequence of the Poisson process assumption is that the time until next occurring hurricane follows an exponential distribution with the rate, γ, as the only parameter:

$$F_T(t) = 1 - e^{-\gamma t} \tag{13}$$

Eq. (13) is used for simulating time until next hurricane. The simulation scheme which is repeated with new random seeds for every year is briefly described by:

Starting July 1st a realization of the waiting time [days] until next hurricane is obtained by Monte Carlo simulation using the rate valid for the first part of the season, July 1st – August 15th. If the waiting time corresponds to a day after August 15th, no hurricanes occur for this part of the hurricane season for this year. If the simulated number, t, corresponds to a day before August 15th , a hurricane is recorded as starting at day t after July 1st. Time until next hurricane is found by generating a new realization from Eq. (13) using the same rate. In this connection waiting time starts running from time of previous hurricane + 2 days. If the new realization brings us beyond August 15th, there are no more hurricanes for the first part of the season for this year. If it corresponds to a day earlier than August 15th, a second hurricane is identified for this part of season this year. This process goes on until one ends up after August 15th.

As we are finished with the first part of the hurricane season, rate is updated and the procedure is repeated until the second part of the hurricane season for this year is covered, i.e. until the simulated waiting time yields an event after September 30th. Then rate is again updated, the waiting time starts running from October 1st and this process goes on until the third part of the season is covered. Rate is adjusted from November 1st and the simulation process will be performed until we end up with an event after December 1st.

As we have reached December, we jump to July 1st and perform the same process for a new year. In this way, a generic 50-year data base of hurricane occurrences is obtained by repeating this for 50 years.

This data base can of course not be used for actual design work, but it is convenient for the purpose of indicating the effect of a limited amount of data. It is rather easy to generate data for many 50-year periods.

Peak hurricane significant wave height:
Given a hurricane occurrence, the peak significant wave height is simulated from a shifted exponential model, see Eq. (14). The parameter is selected such that a reasonable level for the underlying 10^{-2} – probability hurricane peak is obtained.

$$F_{H_{s,p}}(h) = 1 - \exp\left\{-\frac{h-6}{\rho}\right\}; \quad \rho = 2 \tag{14}$$

This model may be too simple for practical work, but it is selected for our illustrative study.

Spectral peak period at maximum hurricane significant wave height:
The simultaneous spectral peak period is thereafter simulated from a log-normal distribution with parameters being a function of the maximum significant wave height:

$$f_{T_{p,p}|H_{s,p}}(t\,|\,h) = \frac{1}{\sqrt{2\pi}\,\sigma(h)\,t}\exp\left\{-\frac{1}{2}\left(\frac{\ln t - \mu(h)}{\sigma(h)}\right)\right\} \tag{15}$$

$$\mu = a_1 + a_2\,h^{a_3} \tag{16}$$

$$\sigma^2 = b_1 + b_2\,\exp\{-b_3\,h\} \tag{17}$$

The parameters used in the simulations, $a_1 = 1.71, a_2\ 0.35, a_3 = 0.41$ & $b_1 = 0.005, b_2 = 0.12, b_3 = -0.31$, are taken from data for the North Sea and will not necessarily be very accurate for hurricane conditions in Gulf of Mexico. They are, however, considered of sufficient accuracy for the present investigation.

Duration of hurricane:
The duration of the event h_s > 6m is simulated by assuming that the duration follows a log-normal

distribution with parameters given as functions of peak significant wave height:

$$f_{D|H_{s,p}}(d\,|\,h) = \frac{1}{\sqrt{2\pi}\,\kappa(h)\,d}\exp\left\{-\frac{1}{2}\left(\frac{\ln d - \nu(h)}{\kappa(h)}\right)\right\} \qquad (18)$$

$$\nu = 1.74 + 0.16\,(h-6) \qquad (19)$$

$$\kappa^2 = \exp\{-\ 0.17\,(h-6)\} \qquad (20)$$

Time history of hurricane characteristics:
As the hurricane peak significant wave height and spectral peak period together with the duration above 6m is known for all hurricane occurrences, a full storm history of 30-minute sea states is obtained assuming a symmetric triangular history for the significant wave height, see Fig. 1.

For the spectral peak period a slightly skewed triangular history, see Fig. 1, is obtained by adopting a spectral peak period at hurricane start, i.e. as h_s crosses the 6m threshold, given by $t_p(0) = 0.75 t_{p,p} - 1$ and a spectral peak period at the end of the storm history equal to $t_p(d) = 0.75 t_{p,p} + 1$. Linear interpolation is used between the peak period at storm peak and at start and end, respectively. An example of hurricane histories for h_s and t_p is shown in Fig. 1.

3.2 CHARACTERISTICS FOR THE BASE CASE 50-YEAR HURRICANE DATA BASE

Some characteristics of the simulated hurricane occurrences during a 50-year period are given in Table 1. This hurricane data base will in the following be adopted as the reference data base. It is seen that the reference data base includes 32 hurricanes above 6m ranging from a peak significant wave height from 6.06m to 14.84m. Hs-max (= $h_{s,p}$) is the peak significant wave height of the hurricane. Tp_max (= $t_{p,p}$) is the peak period corresponding to Hs_max, and duration is the duration of significant wave height above a threshold of 6m.

4 EXTREME SEA STATE CHARACTERISTICS

The base case 50-year data base includes 32 hurricanes, i.e. we have 32 simultaneous observations of the peak significant wave height and spectral peak period at hurricane peak sea severity. Based on these observations, adequate extremes storm characteristics and, later, response extremes are to be predicted.

Table 1 Some hurricane characteristics of the generated 50-year hurricane data base.

Year	Occurence Day	Occurence Month	Hs_max	Tp_max	Duration
2	12	August	6,61	9,01	14
3	4	September	10,89	12,41	22
4	2	September	6,53	12,32	6
5	10	October	8,09	12,44	16
7	6	September	8,12	13,86	8
7	9	October	8,16	15,96	8
9	3	August	6,25	10,82	4
10	9	September	6,81	10,59	11
12	5	September	6,61	12,63	4
12	23	September	9,40	11,89	15
13	15	September	7,31	12,65	23
15	3	September	14,84	17,50	10
16	18	September	8,29	10,68	2
20	9	August	7,10	10,04	7
20	2	September	8,27	12,21	7
22	5	September	6,29	9,67	11
26	25	October	9,56	11,35	19
27	15	August	7,85	11,35	12
30	1	September	12,60	14,93	18
32	24	October	6,29	11,16	2
33	17	September	7,39	9,98	7
33	15	October	7,13	10,04	16
35	14	August	7,73	12,26	4
36	12	August	11,29	14,19	20
36	16	September	7,72	11,36	3
37	16	September	10,35	13,74	3
38	6	September	8,51	14,81	24
41	26	October	7,02	11,70	7
47	16	September	6,37	12,68	43
48	15	September	7,92	13,93	19
49	9	September	6,06	11,50	11
50	20	September	7,47	13,82	2

4.1 q-PROBABILITY HURRICANE PEAK SIGNIFICANT WAVE HEIGHT

The hurricane peak, $h_{s,p}$, minus the threshold (6m) raised to a certain power, k, is assumed to follow an exponential model, see Eq. (22). The moment estimate for the parameter, ρ , is determined from observations by:

$$\hat{\rho} = \overline{(h_{s,p} - 6)^k} = \frac{1}{M}\sum_{i=1}^{M}(h_i - 6)^k \qquad (21)$$

M is the number of hurricanes and h_i is the observed hurricane peak significant wave heights.

For illustrative purposes, $k \equiv 1$ is adopted in this subchapter. Under this assumption $\hat{\rho} = 2.15m$ is found. The sample distribution and the fitted exponential model are shown in Fig. 4. A reasonable fit is obtained.

The probability levels corresponding to annual exceedance probabilities of 10^{-1}, 10^{-2}, 10^{-3} and 10^{-4} are also shown in Fig. 4. The probability levels are found by using Eq. (12) with n_1 = 32/50 = 0.64. It is seen that the corresponding extremes are: 9.8m, 14.9m, 20.0m and 24.8m. One should not focus too much on the absolute values. Our concern is rather the uncertainties associated with the extremes when they are to be based from data of a 50-year period.

The observation of an acceptable fit using $k \equiv 1$ should be expected since the generic data base for the hurricane peak significant wave height is generated using an exponential distribution. For a real hurricane data base we will not know the underlying type of distribution. Due to this we will in the major part of this work skip the assumption of $k \equiv 1$.

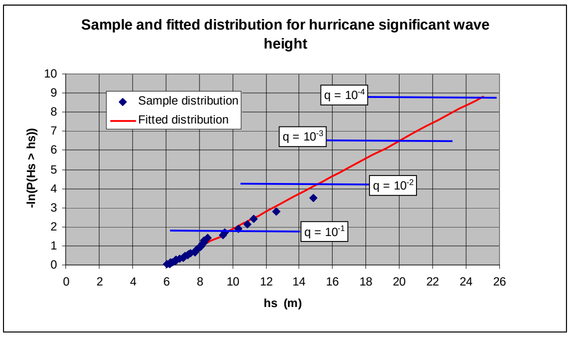

Fig. 4 Sample distribution and fitted exponential model for $H_{s,p}$.

4.2 q-PROBABILITY CONTOUR LINES FOR HURRICANE PEAK CHARACTERISTICS

In order to determine the contour lines, we need the joint probability distribution of $H_{s,p}$ and $T_{p,p}$, $f_{H_{s,p}T_{p,p}}(h,t)$. In order to estimate this distribution from data, it is convenient to write:

$$f_{H_{s,p}T_{p,p}}(h,t) = f_{H_{s,p}}(h)\, f_{T_{p,p}|H_{s,p}}(t\,|\,h) \qquad (22)$$

For the marginal distribution of $H_{s,p}$ we will assume that the data can be fitted to a 3-parameter Weibull model where the location parameter is taken to be 6m, i.e.:

$$F_{H_{s,p}}(h) = 1 - \exp\left\{-\frac{(h-6)^k}{\rho}\right\}; \qquad (23)$$

The parameters are fitted by determining k such that $E\left[(H_{s,p}-6)^k\right] = STD\left[(H_{s,p}-6)^k\right]$, which is the case if $(H_{s,p}-6)^k$ is exponential distributed. Thereafter ρ is estimated by Eq. (21). The coefficients estimated from the available hurricane samples for the marginal distribution of $H_{s,p}$, Eq. (23), and the conditional distribution of $T_{p,p}$, Eqs. (16 and 17) are given in Table 8 (at end of paper).

As the joint distribution is available, the contour lines are estimated as indicated in e.g. Kleiven and Haver (2004). The contour lines obtained for the reference data base are shown in Fig. 3. It should be kept in mind that the q-probability contour corresponds to a probability of exceedance per hurricane equal to q/n_1.

5 UNCERTAINTIES ASSOCIATED WITH THE PREDICTED EXTREMES

When estimating hurricane extremes of the significant wave height, the fitted model will be rather sensitive to the largest observed hurricanes. The estimated extremes will also be effected by the number of hurricanes observed during the 50-year period since this will define the target probability levels. The number of hurricanes occurring for the base case 50-year period is larger than we would expect during a 50-year period of the generic hurricane model. However, if this was our available observations, the best we can assume is that a typical 50-year period will include about 32 hurricanes.

The variability in number of hurricanes per unit time can easily be accounted for. By focusing on the distribution of the annual largest significant wave height instead of the distribution of hurricane peak significant wave height, the effects of randomness in annual number of hurricanes can be baked into the analysis as shown in Haring and Heideman (1978).

5.1 ALEATORY UNCERTAINTY

From our fitted model for the hurricane peak significant wave height in Ch. 4.1, we can estimate the value exceeded by an annual probability of 10^{-2}. The return period, i.e. average time between occurrences of exceeding this value, will equal 100 years and this quantity is therefore frequently referred to as the 100-year value. It should be noted that the return period is an average measure, i.e. the time period between adjacent occurrences could be very different from the return period. Accordingly, the largest value actually observed in a 100-year period will be a random variable itself.

Assuming that the fitted exponential model is the true model, we can estimate the variability of the largest hurricane peak significant wave height observed during a 100-year period. The 100-year hurricane peak significant wave height, $H_{s,p}^{(100)}$, is the largest out of 64 hurricanes, i.e. its distribution function reads:

$$F_{H_{s,p}^{(100)}}(h) = \left\{1 - \exp\left\{-\frac{h}{\hat{\rho}}\right\}\right\}^{64}; \qquad (24)$$

where $\hat{\rho} = 2.15$. This distribution function is shown in Fig. 5. It is seen that the 80% range for the variability of $H_{s,p}^{(100)}$ is given by 13m – 20m.

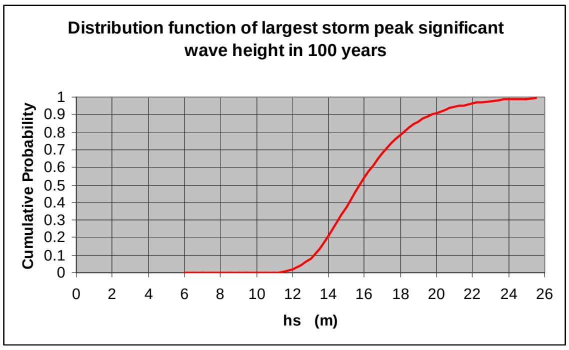

Fig. 5 Distribution function of the largest hurricane peak observed in a 100-year period.

There is not much that can be done with this variability. If one is concerned about robustness against this type of uncertainty one should check the structure against extremes corresponding to say 10^{-3} or 10^{-4} annual exceedance probabilities. This will be the same as selecting the 90-percentile or 99-percentile, respectively, in Fig. 5.

5.2 EPISTEMIC UNCERTAINTY

The variability discussed above is inherent to the problem. This type of variability is referred to as alatory uncertainties or Type I uncertainties. However, there are also epistemic (Type II) uncertainties associated with the predicted extremes. These uncertainties will be addressed later on by generating many 50-year hurricane data bases. Here we will consider the reference 50-year data base as our sole available information about the wave conditions. The Type II or epistemic variability related to a limited amount of data can be indicated either by classical bootstrapping (generating new samples of size 32 by drawing with replacement from the original sample), see e.g. Efron and Tibshirani (1994), or by a parametric bootstrapping. Here we will select the latter approach.

A probabilistic model is fitted to the original sample. Assuming that the fitted model for the hurricane peak significant wave height is the true model, we can generate a number of equally valid samples of size 32 using Monte Carlo simulation. In principle, each of these samples could just as well have been observed as the one we did observe.

The sample distributions are compared with the true distribution in Fig. 6. Exponential models are fitted to the various simulated samples and are shown in Fig. 7. It is seen that the sample distributions show a considerable variation relative to the underlying distribution. From the fitted model it is clear that the estimated 10^{-2}-annual probability hurricane peak significant wave height can deviate significantly from the underlying true figure. The 10^{-2}- annual probability of exceedance, 1/64, corresponds to 4.16 in the exponential scale used in Figs. 6 and 7.

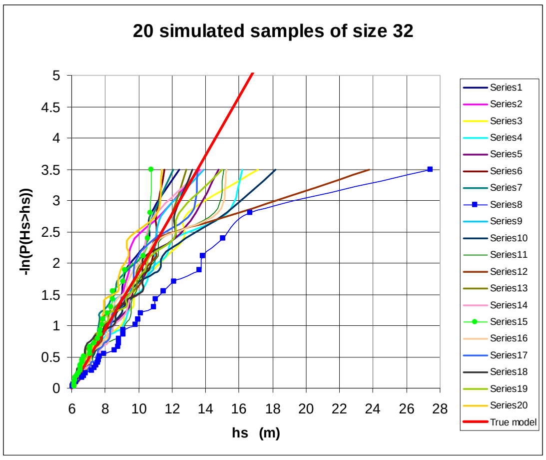

Fig. 6 Sample distribution of 20 simulated samples compared to underlying distribution

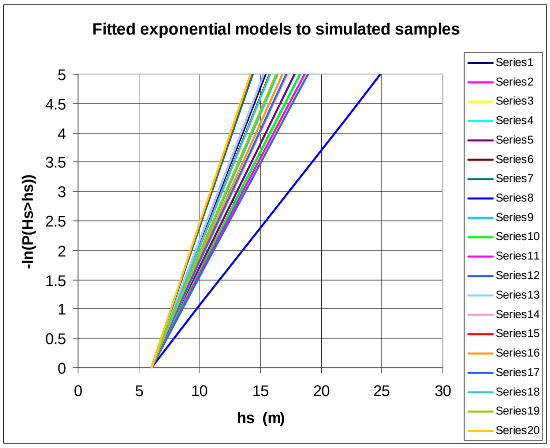

Fig. 7 Fitted exponential model to the 20 simulated samples (colour code is not the same as for Fig. 6)

Estimating the 10^{-2} – probability value from the 20 simulated samples give estimates from 12.8m to 21.7m. It is seen from Fig. 7 (and Fig. 6) that one of the simulated samples stands out as not a typical member of the 20 simulated samples. However, we will include the sample in this illustration of epistemic uncertainties. Assuming the variability in the predicted 10^{-2} – value to be close to Gaussian, a 80% band is estimated to be 12.6m – 17.7m. This range is slightly narrower than the aleatory variability discussed above.

A priori, we will not know which sample “mother nature” has given us relative to some underlying distribution. We must therefore conclude that based on 32 observations and an exponential model, we can not exclude that our fitted distribution function yields a 10^{-2} – value about 20% too low or about 10-15% too high.

In contrast to the aleatory variability, this type of uncertainty can be reduced by including more data e.g. obtained by covering a longer time period with hindcast data. But with a typical duration of about 50 years, the epistemic uncertainty related to the estimated 10^{-2} – probability hurricane peak significant wave height is considerable keeping in mind that a partial load factor of 1.35 (API) is meant to cover all uncertainties (except gross errors) associated with the characteristic design response.

In concluding this chapter, we will point out that if 50 years of hindcast data are available for the target area, a considerable epistemic uncertainty will be baked into our conclusions. Here we have illustrated this by assuming that the 50-year period includes always 32 storms. As will be shown below this will not be the case. There will be a considerable variability in the number of hurricanes exceeding 6m significant wave height during a 50-year period. The typical number is lower than 32. Meaning our indicated uncertainty is not conservative within the framework of the present hurricane simulation model.

5.3 VARIABILITY BETWEEN DIFFERENT 50-YEAR HURRICANE DATA BASES

In the previous section we estimated the uncertainty involved if we merely have one 50-year data period available and would like to make the best out of it. The advantage of a generic model for generating 50-year of hurricane occurrences is that we can generate as many 50 year periods as we would like. Here we have generated 20 data bases of duration 50 years in addition to the reference data base.

For each of these 50-year periods, we have estimated the 10^{-2} – annual probability hurricane peak significant wave height. The various fitted Weibull distributions are shown in Fig. 8. The range of the 10^{-2} – annual probability level is also indicated. The reason for a range of target probabilities per hurricane is the variation in number of hurricanes for the various 50-year samples.

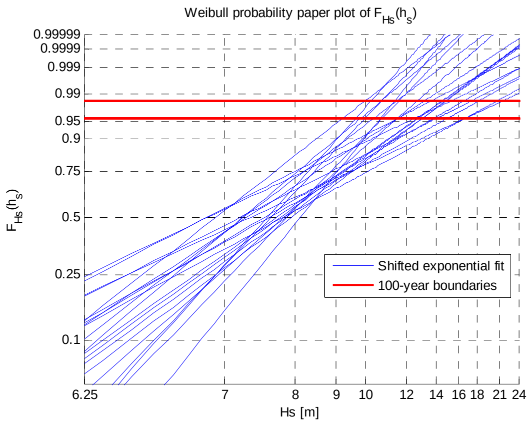

Fig. 8 Distribution functions for $H_{s,p}$ for the twenty 50-year data bases.

Estimated 10^{-2}- annual probability hurricane peak significant wave heights are shown in Table 2. An 80% band based on these estimates is given by 9.7m – 17.3m. The width of this range is slightly larger than the corresponding range obtained when we considered merely the reference 50-year period and kept the number of hurricanes fixed.

Table 2 Estimated 10^{-2} – annual probability values for the hurricane peak significant wave height for the 20 additional data bases.

50-year database [#]	$h_{s,p,0.01}$ (m)	$t_{p,p} \mid h_{s,p,0.01}$ (s)	No. of hurricanes
Reference	**14.3**	**15.8**	32
1	16.5	15.4	19
2	14.0	16.2	23
3	14.2	16.4	22
4	13.8	15.8	22
5	19.9	17.2	23
6	12.8	15.0	22
7	12.1	14.0	20
8	10.9	14.3	20
9	12.7	14.9	24
10	10.3	13.1	17
11	18.6	16.7	19
12	9.8	12.7	12
13	10.1	13.1	25
14	11.1	13.4	18
15	14.5	16.1	24
16	11.1	14.0	21
17	16.4	16.7	30
18	14.3	15.5	24
19	16.6	16.6	25
20	9.7	14.3	22
Mean (#1 - #20)	13.5	15.1	21.6
St. dev. (#1 - #20)	3.0	1.4	3.7

We have also generated 10^{-2} – probability contour lines for the hurricane peak characteristics, $h_{s,p}$ and $t_{p,p}$, for each 50-year period. The various contours are shown in Fig. 9. It is clear that for the generic model there will be a considerable variability from one 50-year period to another 50-year period.

In the next chapter we will indicate the response sensitivity to the observed variability both for the an exact response prediction method and the approximate response method.

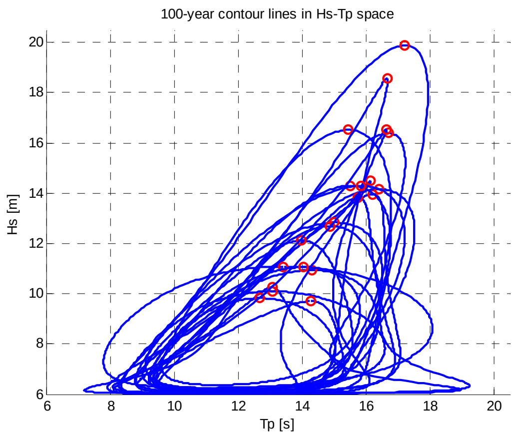

Fig. 9 10^{-2} – annual probability contour lines for $h_{s,p}$ and $t_{p,p}$ based on each of the twenty 50-year data bases.

6 FULL LONG TERM RESPONSE ANALYSES

6.1 CASE 1: LINEAR PROBLEM INSENSITIVE TO SPECTRAL PEAK PERIOD

With reference to Eqs. (3 and 4), this case is characterized by the following parameter values: $\psi = 1$, r=1, t_0 =12s, p = 400 and $\lambda = 0.1$. The analysis is carried out by establishing a distribution function for the hurricane maximum response given the most probable largest response, Eq. (9). Thereafter a 3-parameter Weibull distribution is fitted to the most probable response maxima for all hurricanes, Eq. (10). The latter distribution measures *the long term variability* as reflected by the hurricanes that has occurred during the actual 50-year period.

For the reference data period, the following extremes are estimated for annual exceedance probabilities of 10^{-1}, 10^{-2}, 10^{-3} and 10^{-4}, respectively: 24.0, 34.4, 44.4 and 54.5, see also Table 3. Our interest is here to see the variability in these extremes when estimated for 20 different realizations of 50 years of hurricanes. The results are shown in Table 3.

It is seen that a considerable variability is observed from 50-year period to 50-year period. It is seen from Table 3 that 1.35 (partial safety factor) times the 10^{-2} – annual probability value obtained for the reference period (34.4) is slightly exceeded by the largest estimated 10^{-2}-probability response of the 20 additional 50-year data samples.

It is, furthermore, seen that a couple of 50-year periods stands out. This can also be seen from Table 2 presenting the estimated 10^{-2}- probability hurricane peak significant wave height. These 50-year periods may in some sense be more severe than expected during a 1000-year period and may amplify the scatter. However, it is also a good illustration of the inherent variability of the observed largest within a given time period.

It may well be that the results of these 50-periods are too conservative since there are no physical limitations enforced by the simulation procedure used herein. In particular, this is worthwhile to keep in mind since an exponential distribution is used as probabilistic model for generating the hurricane peak significant wave height. This probability distribution has a rather fat upper tail.

Table 3 Response extremes for Case 1.

50-year database [#]	Response with return period [years]				Number of storms
	10.0	100.0	1000.0	10000.0	in 50 years
Reference case	**24.0**	**34.4**	**44.4**	**54.5**	**32**
1	22.5	39.5	58.6	79.3	19
2	23.3	34.6	45.8	57.1	23
3	22.9	40.4	60.8	83.6	22
4	22.3	34.4	46.8	59.6	22
5	24.1	46.1	72.9	102.1	23
6	21.1	33.7	49.1	66.7	22
7	23.0	32.6	41.4	50.3	20
8	20.2	28.5	36.7	45.3	20
9	22.1	34.0	47.0	61.1	24
10	20.0	28.5	36.5	44.7	17
11	23.9	47.3	75.4	106.2	19
12	17.9	28.3	40.0	53.1	12
13	21.6	29.2	36.3	43.5	25
14	22.5	32.7	42.5	52.6	18
15	22.2	35.4	51.0	68.6	24
16	23.4	33.3	42.6	52.3	21
17	25.9	41.6	58.7	77.3	30
18	24.4	36.6	48.4	60.2	24
19	23.0	38.2	55.4	74.0	25
20	19.7	26.3	32.5	39.0	22
Mean	22.3	35.1	48.9	63.8	21.6
Standard deviation	1.9	5.8	11.7	18.6	3.7
C.o.V	0.08	0.17	0.24	0.29	0.17
Max	25.9	47.3	75.4	106.2	30.0
Min	17.9	26.3	32.5	39.0	12.0

6.2 CASE 1: LINEAR PROBLEM SENSITIVE TO SPECTRAL PEAK PERIOD

The characteristics for Case 2 are with reference to Eqs. (3 and 4) given by: $\psi = 1$, r=16, t_0 =12, p = 64 and $\lambda = 0.1$. It is still a linear response case, $\psi = 1$, but as a consequence of the changes of r and p, this case becomes more sensitive to sea states with a spectral peak period around 12s. The results of this case are given in Table 4.

A certain scatter in extremes are observed, but the scatter for 10^{-2} -, 10^{-3} - and 10^{-4} – annual extremes are considerably reduced as compared to Case 1. This is most probably a result that there is less variability in the significant wave height level corresponding to a spectral peak period of about 12s.

For this case 1.35 x $y_{0.01}$(reference data base) is not exceeded by the predicted $y_{0.01}$ value for any of the additional 50-year data bases, but the margin is not large.

Table 4 Response extremes for Case 2.

50-year database [#]	Response with return period [years]				Number of storms
	10	100	1000	10000	
Reference period	**21.9**	**28.8**	**35.1**	**41.3**	**32**
1	20.6	32.2	43.1	54.0	19
2	20.5	28.0	34.4	40.6	23
3	20.2	27.7	34.4	41.2	22
4	21.0	31.9	42.6	53.7	22
5	21.7	32.1	41.2	50.2	23
6	19.2	26.7	33.3	40.0	22
7	20.0	29.5	38.8	48.6	20
8	18.1	24.3	29.8	35.4	20
9	20.6	29.5	37.4	45.5	24
10	19.3	26.7	32.9	38.7	17
11	21.7	32.3	42.4	52.9	19
12	16.4	23.7	30.6	37.4	12
13	20.9	28.1	34.7	41.2	25
14	20.7	29.9	37.9	45.7	18
15	19.1	26.2	32.4	38.6	24
16	21.0	28.2	34.6	40.9	21
17	23.5	34.2	44.5	55.2	30
18	21.8	29.6	36.2	42.5	24
19	21.9	32.4	42.2	52.2	25
20	18.6	24.8	30.7	36.7	22
Mean	20.3	28.9	36.7	44.6	21.6
Standard deviation	1.6	3.0	4.6	6.5	3.7
C. o. V.	0.08	0.10	0.13	0.15	0.17
Max	23.5	34.2	44.5	55.2	30.0
Min	16.4	23.7	29.8	35.4	12.0

6.3 CASE 1: NON-LINEAR PROBLEM INSENSITIVE TO SPECTRAL PEAK PERIOD

The characteristics of this case is similar to Case 1 except that $\psi = 2$, i.e. there is a quadratic relation between response and waves. The response analysis is done as for Case 1. The results are summarized in Table 5.

Results express the same qualitative tendencies. The major difference is that the non-linearity causes the epistemic variability in the predicted extremes to increase considerably.

The message is that if one deals with a quadratic response case, the uncertainty in the predicted 10^{-2} – annual probability is rather large. Our reference data period gave a 10^{-2} – probability response of 571.4, which after being multiplied by 1.35 gave a design response of 771.4. However, looking at the 20 other 50-year data periods obtained using the same model as the used to produce the reference period, the estimated 10^{-2} – probability value varies from 323.5 to 1050.8, see Table 5. This means that the design response is considerably exceeded by the 10^{-2} – probability predicted for the worst 50-year period.

Although one should keep in mind that the data bases are obtained using a simplified simulation model involving no physics that can limit spread, the results are of some concern keeping in mind that the partial safety factor of 1.35 is to account for all uncertainties in the load predictions.

Table 5 Response extremes for Case 3.

50-year database [#]	Response with return period [years]				Number of storms
	10	100	1000	10000	
Generic	296.4	571.4	876.6	1212.0	32
1	250.2	780.3	1594.8	2693.6	19
2	255.0	553.6	900.7	1290.3	23
3	247.3	622.3	1101.0	1670.2	22
4	251.6	625.0	1122.9	1735.9	22
5	275.3	1050.8	2394.4	4223.6	23
6	221.9	517.9	935.5	1468.8	22
7	230.5	460.9	717.5	1002.2	20
8	203.6	374.5	560.9	766.5	20
9	213.1	443.4	717.4	1031.6	24
10	190.7	365.0	556.4	769.6	17
11	262.9	855.1	1704.5	2771.1	19
12	158.7	365.7	638.8	976.8	12
13	211.7	358.4	511.6	676.6	25
14	219.8	426.4	646.4	883.8	18
15	230.1	558.3	1008.0	1559.7	24
16	241.1	452.2	675.5	916.5	21
17	316.1	784.6	1411.9	2185.2	30
18	287.5	623.4	1017.2	1470.0	24
19	271.3	722.6	1358.0	2158.4	25
20	186.1	323.5	484.7	672.4	22
Mean	236.2	563.2	1002.9	1546.1	21.6
Standard deviation	37.5	195.6	489.1	898.8	3.7
C.o.V	0.16	0.35	0.49	0.58	0.17
Max	316.1	1050.8	2394.4	4223.6	30
Min	158.7	323.5	484.7	672.4	12

6.4 CASE 1: NON-LINEAR PROBLEM SENSITIVE TO SPECTRAL PEAK PERIOD

Case 4 is the same as Case 2 except for that is a quadratic response problem. Results are given by Table 6. The variability is considerably increased as compared with the linear period sensitive case, Case 2, but the variability is reduced when compared to the non-linear case with no significant period sensitivity, Case 3.

The reference period suggest a 10^{-2} – annual probability value of 464.1, while the 20 50-year data bases suggest values from 238.7 to 669.2. The upper estimate exceeds 1.35 x 464.1 = 626.5. Again the results clearly illustrate the importance of epistemic uncertainties for response predictions, because here we merely include parts of all epistemic uncertainties.

Table 6 Response extremes for Case 4.

50-year database [#]	Response with return period [years]				Number of storms
	10	100	1000	10000	
Generic	**253.9**	**464.1**	**695.0**	**951.5**	**32**
1	239.4	625.5	1128.4	1742.3	19
2	223.3	412.6	601.5	802.3	23
3	214.9	413.7	620.0	839.5	22
4	213.7	418.9	639.2	881.6	22
5	268.6	669.2	1157.0	1731.9	23
6	183.5	354.8	534.7	727.6	22
7	212.0	421.4	656.3	918.4	20
8	186.0	335.1	492.3	660.8	20
9	205.6	373.7	542.1	721.8	24
10	183.7	357.0	540.6	743.5	17
11	228.4	582.5	1020.5	1529.4	19
12	130.3	293.5	511.9	775.0	12
13	202.0	342.0	483.6	634.4	25
14	175.9	359.2	561.1	782.1	18
15	202.1	389.2	593.1	817.2	24
16	213.9	402.6	610.0	842.2	21
17	271.6	535.2	824.6	1148.7	30
18	254.0	490.9	747.4	1030.9	24
19	239.3	532.2	866.0	1239.4	25
20	157.5	238.7	316.8	397.7	22
Mean	210.3	427.4	672.4	948.3	21.6
Standard deviation	35.4	112.0	221.9	360.2	3.7
C.o.V	0.17	0.26	0.33	0.38	0.17
Max	271.6	669.2	1157.0	1742.3	30.0
Min	130.3	238.7	316.8	397.7	12.0

7 APPROXIMATE ESTIMATION OF q-PROBABILITY RESPONSE

7.1 NEGLECTING THE INHERENT RANDOMNESS OF HURRICANE MAXIMUM RESPONSE

If the variability of the hurricane maximum response given the most probable maximum response is very small, the q-probability response can be estimated by merely considering the long term distribution of the most probable largest hurricane response, $\tilde{Y}$, see Eq. (10).

The 10^{-2} – annual probability values for the most probable largest hurricane response, $\tilde{y}_{0.01}$, are shown in Table 7 (last page) for all response cases and all available generic 50-year data bases. The 10^{-2} – annual probability response, $y_{0.01}$, is also shown for all cases in the same table. It is seen that the 10^{-2} annual probability response is considerably underestimated by neglecting the short term variability. For the linear cases, the effect of short term variability is an increase of about 25-30%, while for the quadratic response case the effect is 50-60%.

The effects indicated by these results are much larger than expected a priori, and the analyses will be further verified in the future. For a Northern North Sea climate our experience is that the short term variability increases the 10^{-2}-probability extremes by 10-15% for linear response cases. The reason for this may be that $\lambda = 0.1$ is too large for a linear response system. It can also be that the short term variability is more important in a storm extreme formulation instead of a 3-hour extreme value formulation.

7.2 ESTIMATING EXTREMES USING THE CONTOUR LINE METHOD

Background

The full long term analysis is time consuming. The most probable 30-minute maximum response must be calculated for all important 30-minute steps of the hurricane. This must also be done for all hurricanes of the 50-year period. From the maximum most probable largest response for each storm, we have to fit a probabilistic model reflecting the long term variability of the most probable largest hurricane response. This distribution ensures that we can account for non-observed events in our long term analysis. This is important if we are looking for extremes corresponding to a return period much longer than the time period covered by the available sample.

We must also determine the distribution function for the largest hurricane response given the most probable largest response of the hurricane. This also needs a number of response analyses.

It would be convenient if we could estimate a q-probability response value of reasonable accuracy directly from a limited set of environmental characteristics.

The environmental contour method

Here we will consider hurricane peak significant wave height and the associated spectral peak period as our primary weather characteristics. From these we have established q-probability contour lines, see Figs. 3 and 9. All combinations along the q-probability contour represent possible q- probability combinations of hurricane peak significant wave height and the associated spectral peak period. In the following we will merely consider the case $q=10^{-2}$.

Let us assume that we for a particular response case determine the most unfavourable combination of hurricane peak significant wave height and associated spectral peak period along the 10^{-2} – probability contour line. Let us furthermore assume that the 30-minute maximum response is deterministically given by the hurricane peak significant wave height and the associated spectral peak period, i.e. the extreme value distribution is more or less a Dirac delta function. We can then calculate the maximum 30-minute response for all sea states (30 minutes duration) along the 10^{-2}-probability contour. All of these will correspond to an annual exceedance probability of q. The largest value will occur for the most unfavourable sea state along the contour. The value obtained for the worst sea state along the contour will only occur once per 100 years, while the values obtained for the other sea states along the contour will occur more frequently. Accordingly, provided we can neglect the variability of the 30-minute maximum value, this will be the 10^{-2}-probability response.

In practise we can not neglect the short term variability. This is clearly indicated by the results of Ch. 7.1. However, if we in spite of this will use the worst sea state along the 10^{-2}-probability contour as our short term design sea state, what will we have to do in order to obtain a meaningful estimate of the underlying 10^{-2} – probability response?

When we neglect the short term variability we will remove variability from the analysis and this will result in an underestimation of our target extremes. The extent of underestimation will depend on the relative importance of short term variability versus the long term variability handled by the selection of the 10^{-2} – probability contour of the wave climate characteristics $h_{s,p}$ and $t_{p,p}$. A priori, we will therefore expect that we have to select a percentile of the upper tail (at least it must be larger than the median) in order to match the estimate obtained by the long term analysis.

Experiences from North Sea applications suggest a certain similarity among most response problems, see e.g. Baarholm and Haver (2009). It is often recommended to adopt 0.90-probability value of the 3-hour extreme value as a reasonable estimate for the long term extreme value, see e.g. recommendation in Norsok (2007).

Application of method to the generic data

We will investigate which percentile could be useful for Gulf of Mexico applications by investigating which percentile,100ε, we need to use in order to match the 10^{-2}- response obtained from the long term analysis with the 100ε-percentile value of the worst 30-minute sea state along the 10^{-2} – probability contours. Analysis will be done for the reference data base and the 20 additional simulated data bases for all 4 response cases.

The 10^{-2}-probability as obtained from the long term analyses, $y_{0.01}$, and the percentile, ε, needed to be used in order to match this estimate by the contour method are shown for all response cases in Table 7. It is seen that for all response cases, the variation in the proper percentile vary at most from about 0.8 to 0.99 from one 50-year period to another 50-year period. The mean value for the various response cases vary from 0.91 – 0.94. This variation is most probably negligible in view of all other uncertainties.

The scatter in the estimated probability level matching the 10^{-2} – probability value from the long term analysis is seen to be rather small. This investigation therefore suggests that the present implementation of the contour method may be a useful approach for estimating extremes for design – in particular for early phase considerations. A recommendation could be to recommend the 92.5-percentile as a proper estimate for the 10^{-2}- annual probability response. If one would like to be slightly more robust one could select the 95-percentile.

From the reference 50-year period the long term estimates for the 10^{-2}- probability response reads 34.4, 28.8, 571 and 464, respectively, for the 4 response cases, see Table 7. Adopting the 92.5 – percentile as the target short term characteristic, the corresponding values becomes: 35.9, 30.6, 617 and 479. These estimates are 3-8% conservative as compared to the long term results. Using the 95-percentile one finds: 37.1, 31.6, 651 and 505. The overestimation as compared to the long term results is now from 8 to 14%.

The best percentile level is shown versus data base severity in Fig. 10. Data base severity is in this connection measured by the predicted 10^{-2}- annual probability hurricane peak significant wave height. For the period insensitive cases, the percentile level decrease with increasing severity. For period insensitive case where the critical period is different from the mode of the contours, there is no decrease in the percentile level.

If merely one data sample is available, it is difficult to know the severity of the sample. The best one can do is to assume that it is close to the expected sample. Under such conditions, one should possible select the 95% value as the short term characteristic.

It is to be pointed out that the contour method is an approximate method. For final design a full long term response analysis is recommended.

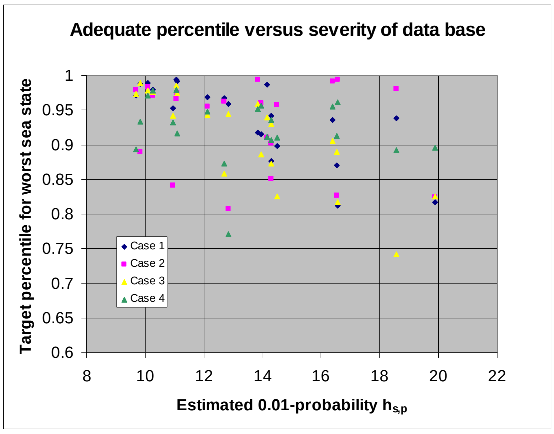

Fig. 10 Best percentile level versus severity of 50-year data base.

8. CONCLUSIONS

A generic model for simulation of 50-year hurricane data bases is used for investigating uncertainties related to extreme value predictions from a limited amount of data.

The severity of a typical 50-year period simulation is expected to be qualitatively representative for a possible 50-year hurricane sample for a Gulf of Mexico area. It should be pointed out that there is no physics baked into the simulation model. The results of the model should therefore merely be used for assessing uncertainties in wave and response characteristics when merely one 50-year period of data are available.

With the limitations of the present simulation model in mind, the main findings of the present study can be summarized as follows:

- As some minimum variability level, the aleatory variability suggest that the observed largest value in a 100-year period could well vary from -10% to +30% of the 10^{-2}- annual probability value.
- Fitting a probabilistic model to a 50-year period of data, introduces epistemic uncertainties into the analysis. If a fitted model suggests a most probable 10^{-2}-probability hurricane peak significant wave height of, $\tilde{h}_{s,p}$, we must accept that the underlying value can differ with +/- 20-25% from the result of the fitted model.
- The estimated 10^{-2} – annual probability contour lines for the hurricane peak characteristics, $h_{s,p}$ and $t_{p,p}$, show a considerable variation between the various 50-year periods.
- Regarding response predictions, the study clearly demonstrate that both the long term variability and the short term variability must accounted for. Neglecting the short term variability will significantly underestimate the target response quantity in view of rule defined safety factors.
- A considerable variation in predicted 10^{-2} – probability response is observed between the various 50-year data bases. For the period insensitive linear system the coefficient of variation is 17%, while as expected the quadratic system with preference regarding period is 35%. For the period sensitive cases variability is somewhat less.

Finally, the application of the contour line method to GoM problems has been investigated for the included response cases. The results suggest that this method can be a useful method for – at least - early phase considerations. Finding the 30-minute extreme value distribution for the target response for the worst sea state along the 10^{-2} – probability contour line, the long term extreme can be estimated by the 95-percentile of this distribution.

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Table 7 Long term extremes for the most probable maximum response, the actual maximum response and corresponding percentiles for the contour method.

50-year database [#]	Case 1			Case 2			Case 3			Case 4		
	$\tilde{y}_{0.01}$	$y_{0.01}$ (1)	ε (2)	$\tilde{y}_{0.01}$	$y_{0.01}$	ε	$\tilde{y}_{0.01}$	$y_{0.01}$	ε	$\tilde{y}_{0.01}$	$y_{0.01}$	ε
Reference	**27.8**	**34.4**	**0.88**	**21.9**	**28.8**	**0.85**	**375**	**571**	**0.87**	**296**	**464**	**0.91**
1	32.2	39.5	0.87	25.4	32.2	0.83	524	780	0.89	390	625	0.91
2	27.4	34.6	0.91	21.6	28.0	0.96	389	554	0.89	271	413	0.96
3	33.1	40.4	0.99	21.0	27.7	0.91	459	622	0.94	290	414	0.91
4	28.3	34.4	0.92	22.7	31.9	0.99	419	625	0.96	276	419	0.95
5	37.5	46.1	0.82	25.3	32.1	0.82	757	1051	0.83	438	669	0.90
6	25.9	33.7	0.96	20.1	26.7	0.81	307	518	0.94	245	355	0.77
7	24.4	32.6	0.97	22.1	29.5	0.96	310	461	0.94	275	421	0.95
8	22.2	28.5	0.95	18.7	24.3	0.84	237	374	0.94	209	335	0.93
9	26.4	34.0	0.97	21.2	29.5	0.96	318	443	0.86	241	374	0.87
10	22.2	28.5	0.98	20.7	26.7	0.97	240	365	0.98	222	357	0.98
11	38.7	47.3	0.94	23.4	32.3	0.98	650	855	0.74	424	582	0.89
12	22.0	28.3	0.99	19.4	23.7	0.89	224	366	0.99	214	294	0.93
13	22.4	29.2	0.99	20.7	28.1	0.98	233	358	0.98	223	342	0.97
14	23.5	32.7	0.99	22.7	29.9	0.97	279	426	0.97	273	359	0.92
15	26.8	35.4	0.90	20.4	26.2	0.96	359	558	0.83	253	389	0.91
16	23.9	33.3	0.99	21.2	28.2	0.97	295	452	0.99	258	403	0.98
17	32.0	41.6	0.94	22.3	34.2	0.99	515	785	0.91	333	535	0.95
18	29.1	36.6	0.94	23.0	29.6	0.90	410	623	0.93	309	491	0.94
19	30.9	38.2	0.81	23.1	32.4	0.99	475	723	0.82	354	532	0.96
20	19.0	26.3	0.97	16.8	24.8	0.98	190	324	0.97	158	239	0.89
Mean	27.4	35.1	0.94	21.6	28.9	0.93	379.5	563.2	0.91	282.9	427.4	0.92
St. Dev.	5.3	5.8	0.05	2.1	3.0	0.06	149.7	195.6	0.07	73.3	112.0	0.05
C.o.V.	0.19	0.17	0.06	0.10	0.10	0.07	0.39	0.35	0.08	0.26	0.26	0.05

(1): $y_{0.01}$ is the 10^{-2}-probability response obtained from a long term analysis

(2): ε is the percentile of the extreme value distribution matching the $y_{0.01}$.

Table 8 Coefficients of the fitted joint model of $H_{s,p}$ and $T_{p,p}$, Eq. (22)

50-year database #	Eq. (23)		Eq. (16)			Eq. (17)		
	k	ρ	a_1 [(1)]	a_2	a_3	b_1	b_2 [(2)]	b_3 [(2)]
Reference	1.078	2.211	0.000	1.741	0.174	0.0100	0.008	0.0711
1	0.866	2.368	0.000	1.985	0.115	0.0100	0.004	0.0000
2	1.120	2.400	0.000	1.879	0.149	0.0000	0.332	0.3618
3	1.134	2.525	0.000	1.868	0.152	0.0100	407.650	1.6513
4	1.098	2.333	0.000	2.001	0.122	0.0000	0.289	0.3695
5	0.722	2.163	0.000	1.877	0.139	0.0100	0.250	0.4133
6	0.809	1.318	0.000	1.721	0.178	0.0099	0.010	0.0000
7	1.364	2.350	0.000	1.927	0.126	0.0000	0.111	0.2465
8	1.185	1.643	0.000	1.849	0.152	0.0100	0.008	0.0000
9	1.025	1.788	0.000	2.109	0.097	0.0100	1105.800	1.5823
10	1.639	1.970	0.000	2.101	0.086	0.0000	11.813	0.9231
11	0.848	2.740	0.000	1.933	0.128	0.0000	0.130	0.3302
12	1.311	1.581	0.000	2.077	0.088	0.0100	0.008	0.0000
13	1.746	1.875	0.000	2.130	0.081	0.0100	0.022	0.0952
14	1.612	2.304	0.000	1.836	0.144	0.0100	0.035	0.0000
15	0.747	1.386	0.000	1.649	0.196	0.0001	10.393	1.0052
16	1.922	2.548	0.000	1.826	0.154	0.0080	0.047	0.2921
17	0.912	2.215	0.000	1.904	0.140	0.0050	3355400.000	2.8434
18	1.180	2.635	0.000	1.913	0.135	0.0100	2.107	0.8163
19	0.787	1.865	0.000	1.817	0.156	0.0000	1.795	0.6252
20	1.217	1.241	0.000	1.661	0.207	0.0000	0.465	0.4916

[(1)]: A reasonable fit to the data is found without using a constant term, therefore $a_1 \equiv 0.0$ is introduced.

[(2)]: Rather extreme values are seen for some few databases regarding b_2 and b_3. This is a result of an automatic default fitting of 3 parameters to very few data points. The outlier values will to a large extent compensate and the resulting curve give acceptable fit to the observations. However, the variance decays rapidly to a value equalling b_1 and for practical purpose one could just have used $\sigma^2 \equiv b_1$ for these cases.